Kalman Filters to Reduce Noise Effects during External Kink Control

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Abstract

Magnetic feedback control of the resistive wall mode in tokamaks use derivative (and proportional) gain in order to optimize stabilization (e.g. M. Okabayshi, *et al.*, PoP2001; Y. Liu, *et al.*, NF2004.) and to adjust the phase response during control of rotating kinks (A.Klein, *et al.*, PoP2005.) **Derivative gain amplifies noise** and can lead to large and undesirable fluctuations in the feedback control current. In this poster, a recipe is presented for the implementation of a Kalman filter that tracks kink mode dynamics as recently described (M. E. Mauel, *et al.*, NF2005.) Numerical simulations demonstrate the use of the control algorithm for various configurations of magnetic field sensors and control coils used in the HBT-EP device. By properly tracking both the wall and plasma modes, feedback control is maintained up to the ideal wall limit in rotating discharges in the presence of measurement noise.

Outline

1. Modeling RWM/Kink Feedback
   - Chu/Fitzpatrick-Aydemir Dispersion Relation
   - HBT-EP & DIII-D (Example) Stability Diagrams
2. RWM and Kink Feedback Simulations
   - Simple digital filter
   - Kalman filter

Key Results

- Feedback control of HBT-EP and DIII-D requires derivative gain. In the presence of noise, the **control power** becomes large.
- Kalman filtering is **superior** to digital low-pass filters since Kalman filtering introduces little phase-shift.
- We illustrate the simplest Kalman filter: the growing, rotating rotor model. With ***both*** poloidal and radial more robust Kalman filters can be built.
- With direct coupling between control coils and plasma and with low-latency, kink modes near (and above) the ideal wall limit can be stabilized.

Non-Ideal Kinks (with Wall)

Chu, *et al.*, (1995)…

$$(\gamma + i\Omega)^2 K + (\gamma + i\Omega) D + \delta W_p + \frac{\delta W_v^b \gamma \tau_w^* + \delta W_v^\infty}{\gamma \tau_w^* + 1} = 0$$

Equivalent to Fitzpatrick-Aydemir, (1996)…

$$\left[\underbrace{(\gamma - i\Omega)^2 / \gamma_{MHD}^2}_{\text{Inertia}} - \overbrace{\bar{\alpha}(\gamma/\Omega - i)}^{\text{Viscosity}} + \underbrace{(1 - \bar{s})}_{\text{Kink}} \right] \overbrace{\left[\frac{\gamma}{\gamma_w}(1 - c) + 1 \right]}^{\text{WallMode}} = 1$$

Boozer/Coupling Parameters

$$s \;=\; -\frac{\delta W_p + \delta W_{v,\infty}}{\delta W_{v,\infty}} = 2\,\frac{m - nq_a - 1}{m - nq_a}$$

$$c \;=\; 1 - \frac{\delta W_{v,\infty}}{\delta W_{v,b}} = 1 - \frac{1}{\Lambda} = 2\,\frac{(a/b)^{2m}}{1 + (a/b)^{2m}}$$

$$\bar{s} \;\equiv\; \frac{s}{s_{crit}} = \left(\frac{m - nq_a - 1}{m - nq_a} \right) \times \left(\frac{2}{\Lambda - 1} \right),$$

RWM Stabilization by Rotation

$$-\bar{\alpha} \equiv \frac{(D/K)\,\Omega}{\gamma_{MHD}^2} = \frac{\nu\Omega}{\gamma_{MHD}^2}$$

For plasma rotation faster than the wall rate, then the RWM is stable for α > 0.5. In other words, the rate of energy dissipation (*i.e.* power) in the plasma must be greater than 1/2 the power available for dissipation in the wall.

HBT-EP and DIII-D Kink-Wall Parameters

Table 1: Examble RWM Control Parameters for HBT-EP and DIII-D.

	HBT-EP	DIII-D
γ_w (msec^{-1})	5.0	0.26
r_w (m)	0.16	1.0
c	0.17	0.14
c_f	0.5	0.5
γ_{MHD} (msec^{-1})	100.0	100.0
ν_d/γ_{MHD} (msec^{-1})	4.5	1.6
M_c/L_c	0.3	0.3
R_c/L_c (msec^{-1})	10.0	2.0

RWM/Kink Stability

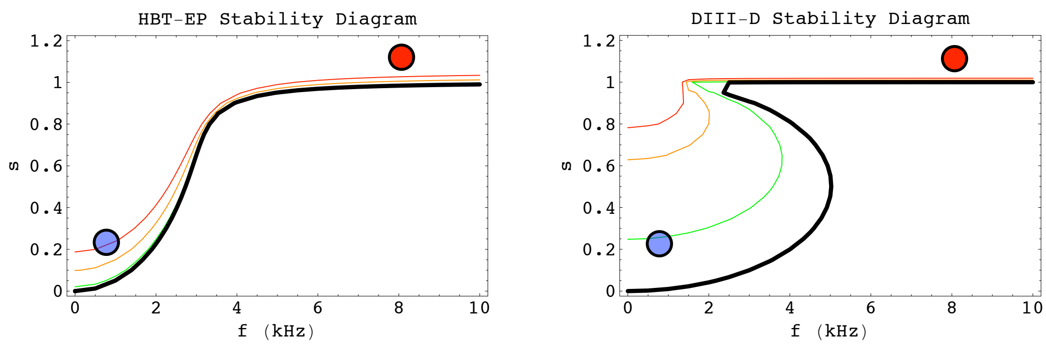

Figure 1: Open-loop stability diagrams characteristic of the HBT-EP and DIII-D tokamaks. The black curve represents marginal stability, and the orange and red curves represent exponential growth rates of 0.1, 0.5, and 1.0 $msec^{-1}$ respectively.

RWM Example:
Kink Example:

Simulating RWM/Kinks

- Fitzpatrick-Aydemir equations are (relatively) simple ODEs for time-analysis
- High plasma dissipation further simplifies kink/RWM dynamics (Shilov, Mauel, *et al.*)
- Assumption: plasma/wall parameters are time-invariant
- Feedback simulation models sensors and coils with fixed coupling parameters.

Simulating RWM/Kinks

See this article for notation and comparison between model and experiment…

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Dynamics and control of resistive wall modes with magnetic feedback control coils: experiment and theory

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Modeling Plasma-Wall

(The “Reduced” F-A Model)

$$\frac{d\vec{y}}{dt} = \mathsf{A} \cdot \vec{y} + \vec{R}\psi_c$$

$$\frac{d}{dt}\begin{pmatrix} \psi_a \\ \psi_w \end{pmatrix} = \begin{pmatrix} (1-\bar{s}+i\bar{\alpha})(\gamma^2_{MHD}/\nu_d) & -\gamma^2_{MHD}/(\nu_d\sqrt{c}) \\ \gamma_w\sqrt{c}/(1-c) & -\gamma_w/(1-c) \end{pmatrix}\cdot\begin{pmatrix} \psi_a \\ \psi_w \end{pmatrix} + \begin{pmatrix} -c_f\gamma^2_{MHD}/\nu_d \\ \gamma_w[1-c\,c_f/(1-c)] \end{pmatrix}\psi_c$$

where c_f is the direct coupling of the control coils to the plasma. (This is *required* for kink control.)

Control Coils

$$\frac{L_c}{M_c}\frac{d\psi_c}{dt} + \frac{R_c}{M_c}\psi_c = V_c$$

(Embarrassingly easy, but, well, it's easy!)
This simple model illustrates noise and filtering.

Sensors

$$\tilde{B}_p(r_w, \theta, \phi) = \Re\left\{b_p(r_w)\, e^{im(\theta - n\phi/m)}\right\}$$

$$b_p(r_w) = (3/r_w)(1-c)^{-1} \times \{2\sqrt{c},\ -(c+1)\} \cdot \vec{y}$$

$$b_r(r_w) = (3/r_w) \times \{0,\ 1\} \cdot \vec{y}$$

For these examples, only poloidal field sensors are used.
With both b_r and b_p sensors *and* with both sine and cosine detectors, then *both* unstable and stable modes can be used for a more robust Kalman filter.

“Smart-Shell” Controller

$$V_c = G_p \psi_w + G_d \frac{d\psi_w}{dt}$$

Only proportional and derivative gain needed for these examples.

Mode Control with Rotation

$$V_c = G_p e^{-in\delta\phi} \, r_w b_p(r_w) + G_d e^{-in\delta\phi} \, r_w \frac{db_p(r_w)}{dt}$$

This is the controller demonstrated by Klein, *et al.*...

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Suppression of rotating external kink instabilities using optimized mode control feedback

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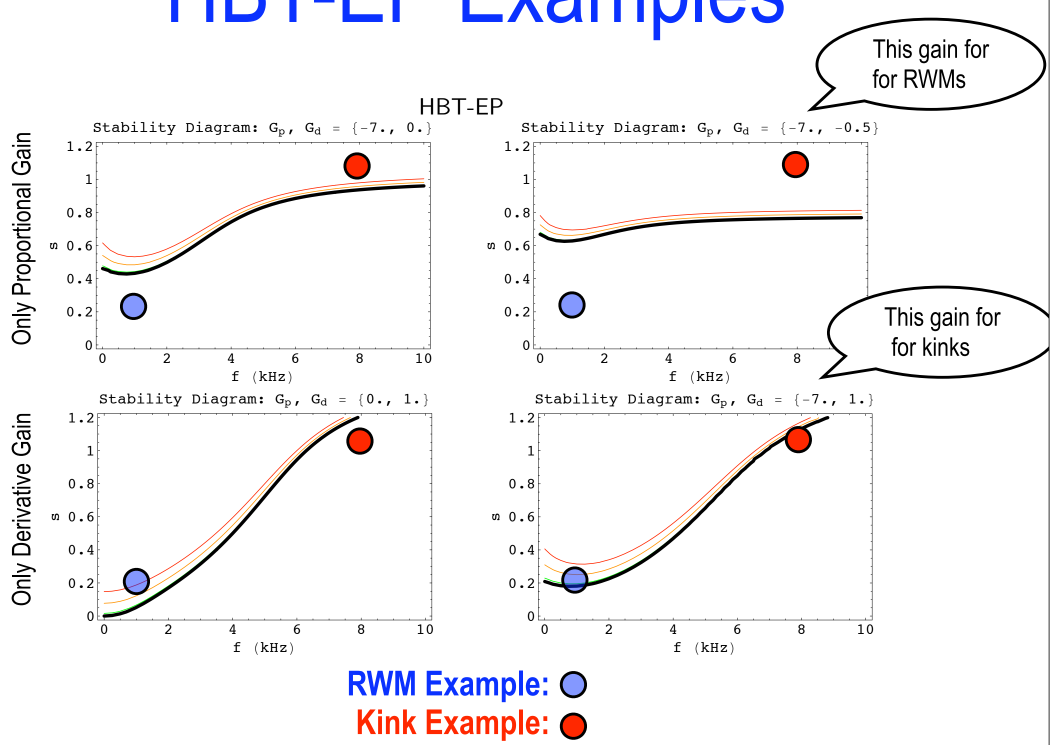
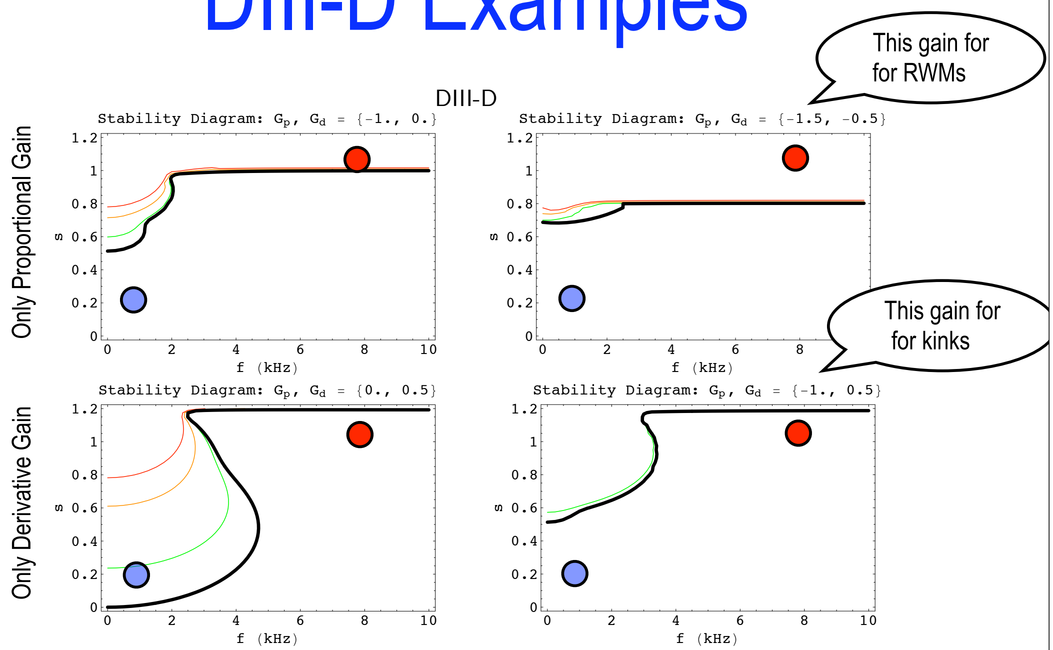
Rules of Thumb…

- When the mode is slow, the control coils respond in the "resistive limit". The RWM is stabilized with proportional gain:
- $-G_p > \bar{s}(R_c/M_c)/(1-c)$
- HBT-EP: $G_p > 40\ s/s_{crit}\ msec^{-1}$
- When the mode is fast, the control coils respond in the "inductive limit". Rotating kinks are stabilized with derivative gain:
- $G_d > (\bar{s}-1)(1-c)(L_c/M_c)/6c_f\sqrt{c}$

HBT-EP Examples

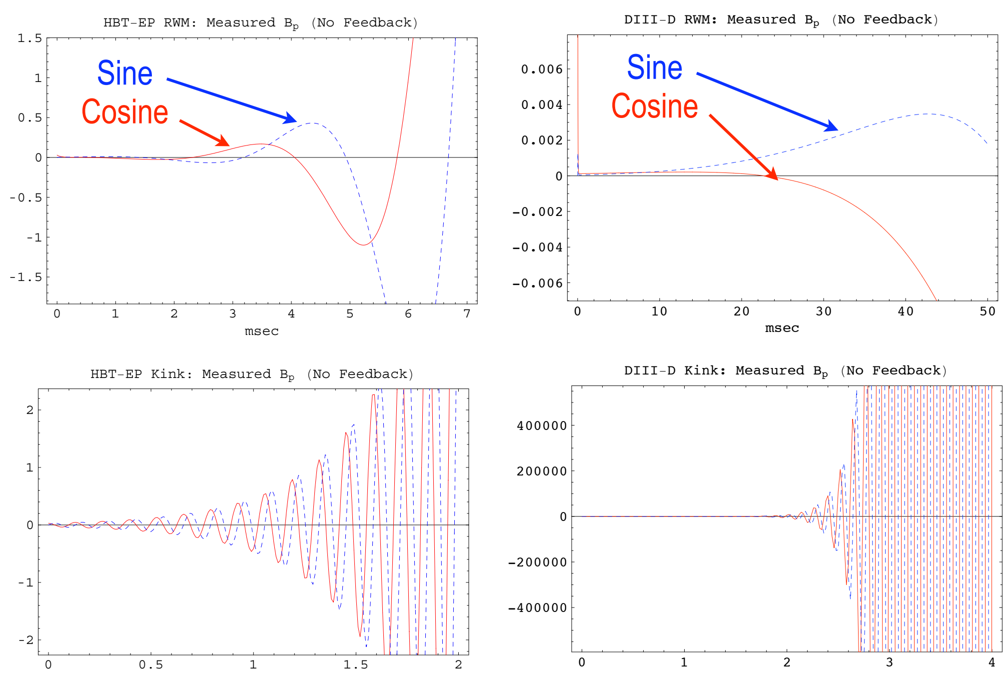


DIII-D Examples

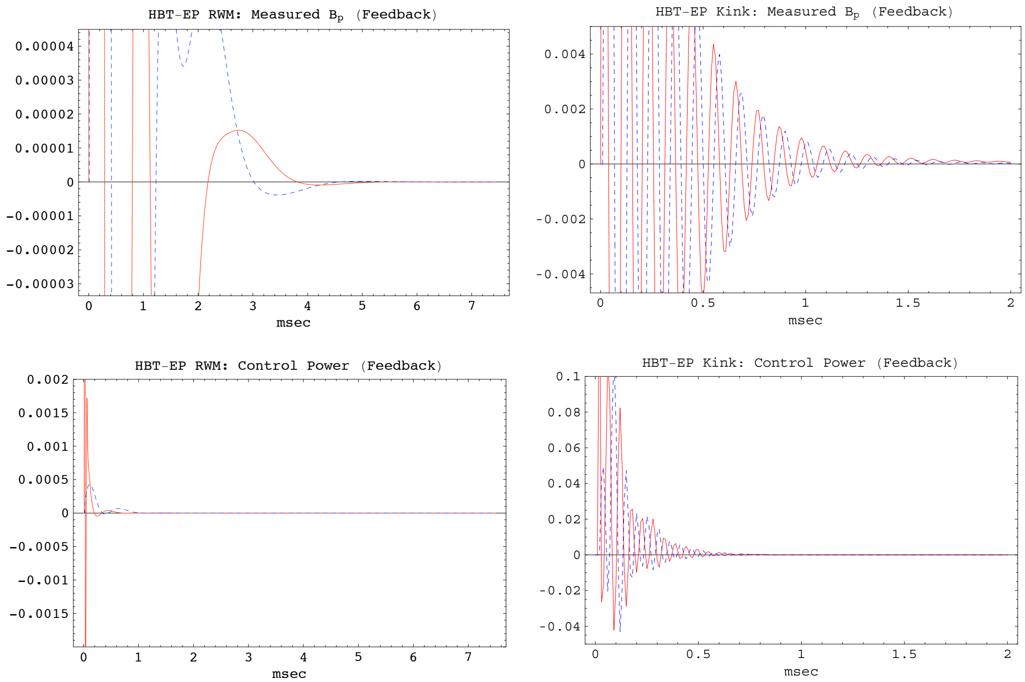


RWM Example:
Kink Example:

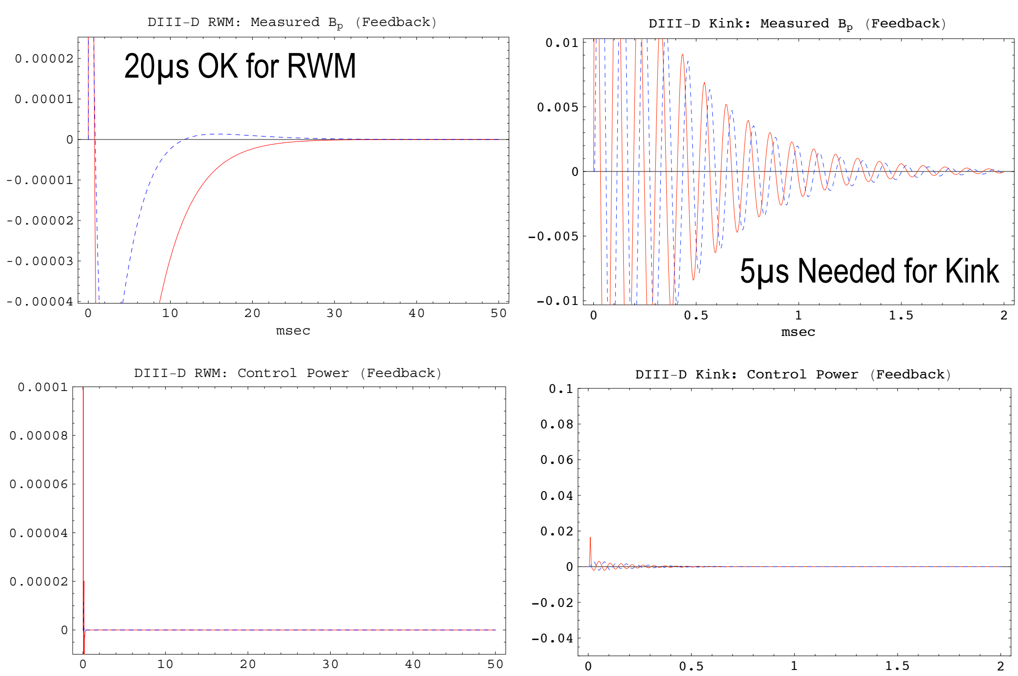
Without Feedback…



HBT-EP:Digital Feedback with 10μs Latency

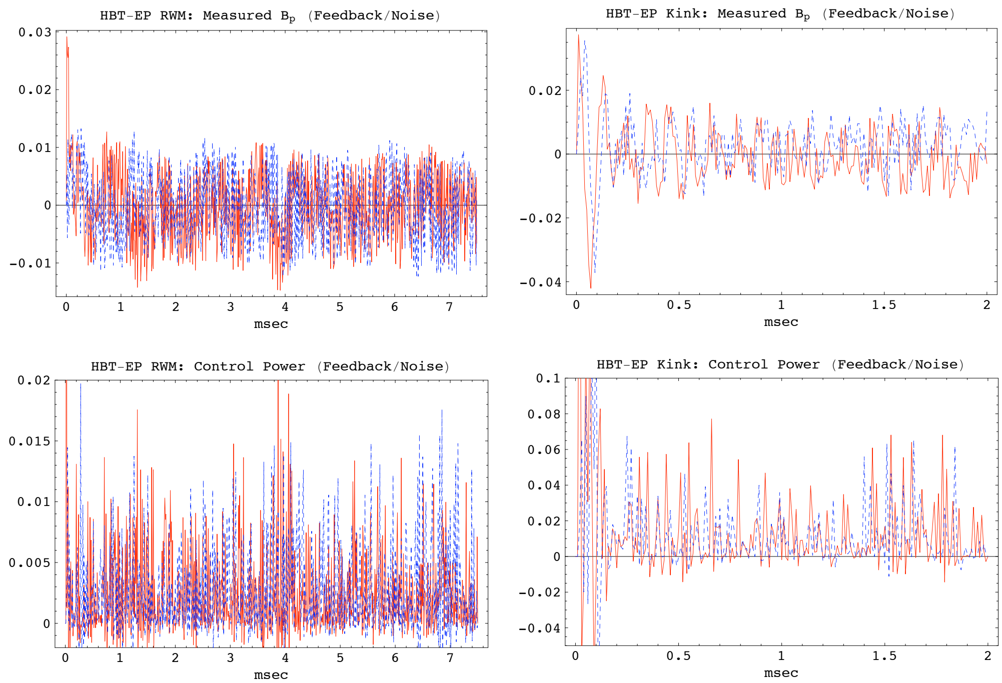


DIII-D:Digital Feedback with “20μs” Latency



Feedback With Random Noise

(Toroidal Phase & Amplitude)

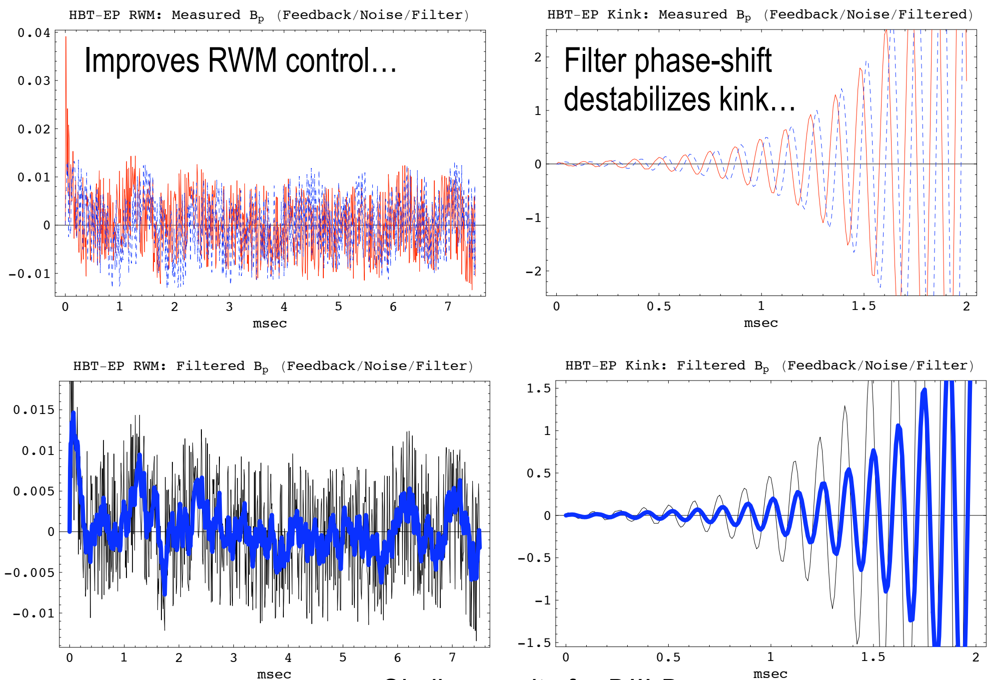


Adding Low-Pass Digital Filters

$$\begin{aligned}
\bar{b}_p[n] &= \bar{b}_p[n-1] + \frac{\delta t}{\tau_b}\left(b_p^m - \bar{b}_p[n-1]\right) \\
\bar{d}b_p[n] &= \bar{d}b_p[n-1] + \frac{\delta t}{\tau_{db}}\left(\frac{\bar{b}_p[n] - \bar{b}_p[n-1]}{\delta t} - \bar{d}b_p[n-1]\right)
\end{aligned}$$

For these examples $\tau_b = \tau_{bd} = 5\delta t$, and $V_c[n+1] \propto G_b\bar{b}_p[n] + G_d\bar{d}b_p[n]$.

HBT-EP: Low-Pass Digital Filters



Similar results for DIII-D.

Rudolf Emil Kalman (May 19, 1930 -) is most famous for his invention of the Kalman filter, a mathematical digital signal processing technique widely used in control systems and avionics to extract meaning (a signal) from chaos (noise).

Kalman's ideas on filtering were initially met with scepticism. He had more success in presenting his ideas, however, while visiting Stanley Schmidt at the NASA Ames Research Center in 1967. This led to the use of Kalman filters during the Apollo program.

He was born in Budapest, Hungary. He obtained his bachelor's (1953) and master's (1954) degrees from MIT in electrical engineering. His doctorate (1957) was from **Columbia University**. His worked as Research Mathematician at the Research Institute for Advanced Study, in Baltimore, from 1958-1964, Professor at Stanford University from 1964-1971, and Graduate Research Professor, and Director, at the Center for Mathematical System Theory, University of Florida, Gainesville from 1971 to 1992. Starting in 1973, he simultaneously filled the chair for Mathematical System Theory at the Swiss Federal Institute of Technology, (ETH) Zurich.

He received the IEEE Medal of Honor (1974), the IEEE Centennial Medal (1984), the Inamori foundation's Kyoto Prize in High Technology (1985), the Steele Prize of the American Mathematical Society (1987), and the Bellman Prize (1997).

He is a member of the National Academy of Sciences (USA), the National Academy of Engineering (USA), and the American Academy of Arts and Sciences (USA). He is a foreign member of the Hungarian, French, and Russian Academies of Science. He has many honorary doctorates. **This year's recipient of Columbia's Eggelston Prize!**

Simple Kalman Filter: Predictor & Corrector

The **"prediction"** step is

$$\begin{aligned}\mathbf{x}_n^* &= \mathbf{A}\cdot\mathbf{x}_{n-1} + \mathbf{u}_n \\ \mathbf{P}_n^* &= \mathbf{A}\cdot\mathbf{P}_{n-1}\cdot\mathbf{A}^T + \mathbf{Q}\end{aligned}$$

where $\mathbf{x}_n^*$ and $\mathbf{P}_n^*$ are predictions of the next step state vector and error covariance.

The **"correction"** step is

$$\begin{aligned}\mathbf{K}_n &= \mathbf{P}_n^*\cdot\mathbf{H}^T\cdot(\mathbf{H}\cdot\mathbf{P}_n^*\cdot\mathbf{H}^T + \mathbf{R})^{-1} \\ \mathbf{x}_n &= \mathbf{x}_n^* + \mathbf{K}_n\cdot(\mathbf{z}_n^m - \mathbf{H}\cdot\mathbf{x}_n^*) \\ \mathbf{P}_n &= (\mathbf{I} - \mathbf{K}_n\cdot\mathbf{H})\cdot\mathbf{P}_n^*\end{aligned}$$

$\mathbf{K}_n$ is the "Kalman Gain". $\mathbf{R}$ is the measurement noise covariance. With $\mathbf{R}$ large, the tracking is less sensitive to noise. With $\mathbf{H} = \mathbf{H}^T = \mathbf{I}$, these are especially simple.

Simplest Kalman Filter: Rotating, Growing Rotor

Let $\mathbf{x}_n = \{b_p[n]\cos\phi_n,\ b_p[n]\sin\phi_n,\ b_p[n-1]\cos\phi_{n-1},\ b_p[n-1]\sin\phi_{n-1}\}$, then

$$\mathbf{A} = \begin{pmatrix} \Re\{\gamma_k\}2\delta t & -\Im\{\gamma_k\}2\delta t & 1 & 0 \\ \Im\{\gamma_k\}2\delta t & \Re\{\gamma_k\}2\delta t & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

representing a rotating, growing "rotor". The complex growth rate, γ_k, is a filter parameter. Does not have to be too close to actual mode.

Note: this is a single-mode approximation, because **only two** independent measurements are available: $b_p\cos\phi$ and $b_p\sin\phi$. If we had **both** poloidal and radial sensors, then the Kalman model could be written in terms of $\bar{s}$ and $\bar{\alpha}$.

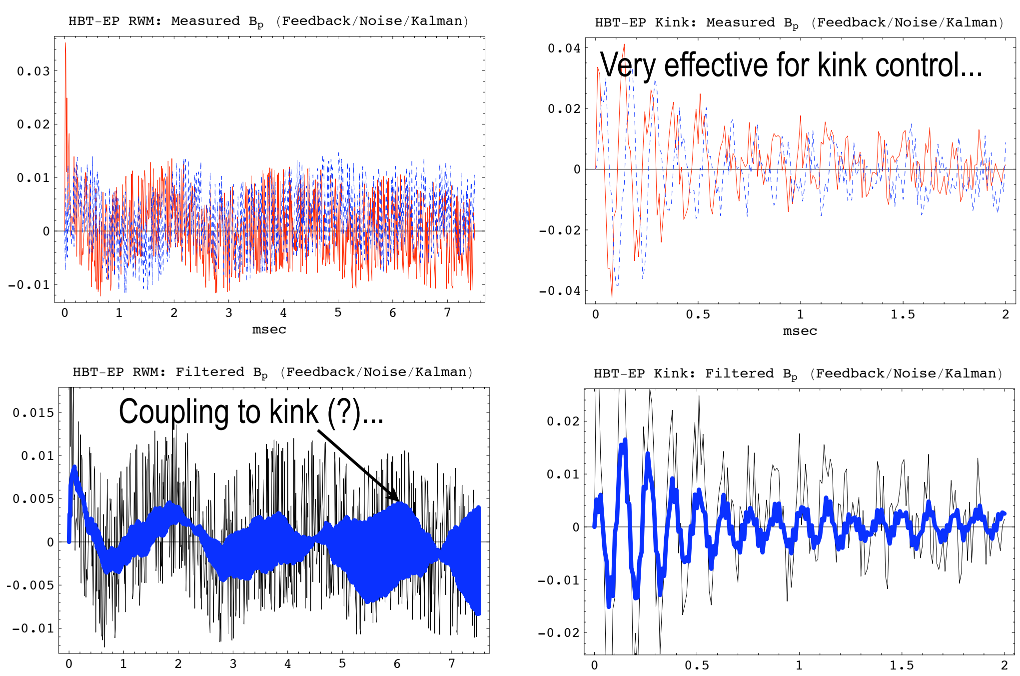
Control Vector is Constructed from F-A Eigensystem

Let $\mathbf{V}$ be the eigenvectors of the "reduced" Fitzpatrick-Aydemir Equations. $\mathbf{V}_i = \{\psi_a,\ \psi_w\}$ for the ith eigenvector (*i.e.* either the RWM or the kink). Then, with $\mathbf{C} = \{-c_f \gamma_{MHF}^2/\nu_d,\ [1 - cc_f/(1-c)]\gamma_W\}$, the control vector (for the single-mode, or "rotor") is

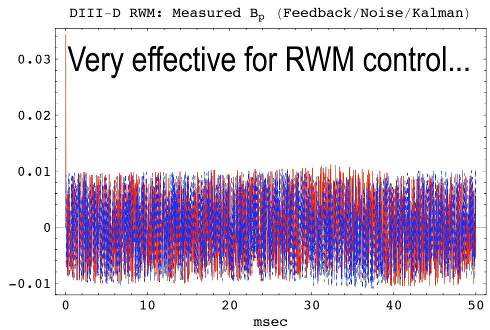
$$\mathbf{U}_n = 2\delta t\ [\mathbf{m}\cdot\mathbf{V}_i]\ \left[(\mathbf{V}^T)^{-1}\cdot\mathbf{C}\right] \times \psi_c[n]$$

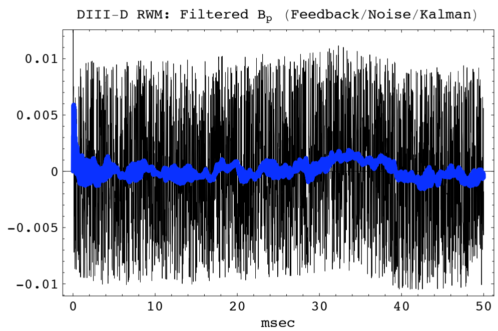
where (in this simple model) ψ_c is proportional to the "measured" control coil current.

HBT-EP: Kalman Filter



DIII-D: Kalman Filter





> 2-fold control coil power reduction as compared to digital low-pass filter!

Summary

- Kink mode/Wall system is characterized by parameters and coupling coefficients. (See Maurer.)
- When these change slowly in time, then a simple set of coupled ODEs can be used to simulate feedback control and allow rapid prototyping of control algorithms.
- When noise is reduced (low-latency and Kalman filtering), then derivative gain can be used for kink/RWM control.