Numerical Simulation of Phase-Space Flows in the Collisionless Terrella Experiment

Dmitry A. Maslovsky, Michael Mauel, *Member, IEEE*, and Benjamin Levitt

***Abstract*—Non-Maxwellian plasmas confined in a magnetic dipole can become unstable to hot electron interchange (HEI) instability, which saturates nonlinearly exciting drift-resonant fluctuations with complex time-varying frequency spectrum. A fully self-consistent nonlinear model, which includes μ, J preserving guiding center drift Hamiltonian dynamics of hot electrons, reproduces the frequency sweeping observed experimentally, and suggests spontaneous formation and consequent inward propagation of "phase-space holes"—coherent structures in phase-space, that can cause particle radial transport. We present the results of the numerical simulation showing the time evolution of the phase-space.**

***Index Terms*—Hot electron interchange instability, plasmas confined in a magnetic dipole, plasma waves, wave-particle resonance.**

THE COLLISIONLESS Terrella experiment (CTX) was designed to study dynamical processes of plasmas confined by a magnetic dipole field. In CTX, a population of energetic, deeply-trapped electrons, called an "artificial radiation belt," is created with electron cyclotron resonance heating (ECRH). As the electron pressure increases, the plasma becomes unstable to hot electron interchange (HEI) instability. Drift-resonant waves $(\omega \sim \omega_{dh})$ are observed with frequencies shifting upward in time [Fig. 1(a)]. Correlations between multiple probes show that the waves are flute-like, propagate azimuthally in the direction of electron ∇B drift, and have a broad, rigid radial structure. In addition, an outward radial expansion of energetic electrons is observed coincident with drift-resonant fluctuations [1], [2].

A self-consistent nonlinear numerical simulation was developed to further understanding of the dynamics of HEI instability [3]. Since μ and J are conserved, hot electron dynamics is described by a two-dimensional (2-D) canonical phase-space (φ, ψ), which is directly observable. Here, φ is an azimuthal angle and ψ is the magnetic flux (i.e., radial coordinate), which are the natural magnetic coordinates for a dipole magnetic field $\vec{B} = \vec{\nabla}\varphi \times \vec{\nabla}\psi$. The plasma is modeled with multiple, conservative fluids—several hot electron populations, having fixed μ, cold electrons, and cold ions. The cold fluids convect according to the local $\vec{E} \times \vec{B}$ and polarizations drifts. The equations of energetic electron phase-space motion are

$$\dot{\varphi} = \partial H/\partial\psi = \mu(c/e)\partial B/\partial\psi - c\partial\Phi/\partial\psi$$
$$\dot{\psi} = -\partial H/\partial\varphi = c\partial\Phi/\partial\varphi.$$

Here, $H = \mu c B/e - c\Phi$ is the guiding center drift Hamiltonian, e is the magnitude of the electron charge, c is the speed of light, B is the local strength of the magnetic field, $\mu \equiv m_e v_\perp^2/2B$ is the magnetic moment, m_e is the electron mass, and Φ is the electrostatic potential. Cold ions and cold electrons are described by field-line averaged particle continuity equations. In addition, bounce-averaged distribution function, $F(\mu, J, \psi, \varphi, t)$, with adiabatic constants, $(\mu, J = 0)$, is used to describe the complete dynamics of energetic electrons. For complete description of the details of numerical simulation refer to [3].

Computing the hot electron distribution function at fixed intervals in time on a Compaq Alphastation XP900 workstation generates the data for the images. The dynamics of six fluids interacting through the self-consistent electric field were obtained on a computational, 64×64, grid. The computation required 8 h to simulate the collisionless dynamics for 90 hot electron drift periods. The output was saved in hierarchical data format (HDF) type files and then transferred to an Apple PowerMac G4 computer, where NoeSys Visualization Pro package [4] is used to reconstruct phase-space flows.

Five distinct hot electron fluids corresponding to different values of $\mu\,(0.50\ \mu_0,\ 0.75\ \mu_0,\ 1.00\ \mu_0,\ 1.25\ \mu_0,\ 1.50\ \mu_0)$ are used in this simulation with only two energy levels—0.50 μ_0, 1.50 μ_0—shown in Fig. 1(b). All energies are normalized to $\mu_0 = 1$ keV. Color intensity plots are generated with Transform 3.4 after some smoothing and coordinate transformation are done with NoeSys 1.3. Red indicates the maximum in electron density, purple indicates the minimum. Data from all energy levels at a particular instant in time is then combined, interpolated and results are displayed with T3D 1.1.3 [Fig. 1(c)]. The z axis in three-dimensional (3-D) plots denotes the energy of particles with energy increasing in a positive direction. Selecting equal-density points across all energies generates the color surfaces. Note that the slight ripples seen on 3-D images are due to finite resolution effects when transferring from a rectangular (φ, ψ) grid to polar coordinates.

From the presented images, one can clearly observe the formation of coherent structures—"phase-space holes"—which propagate azimuthally in the direction of electron $\vec{\nabla}B$ drift. As these holes move inward, plasma is being displaced outward. Low energy particles are the first ones to interact with the drift-resonant electrostatic fluctuations, which gives rise to the rising frequency chirping observed experimentally.

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The authors are with the Department of Applied Physics and Applied Mathematics, Columbia University, New York, NY 10027 USA (e-mail: dm341@columbia.edu).

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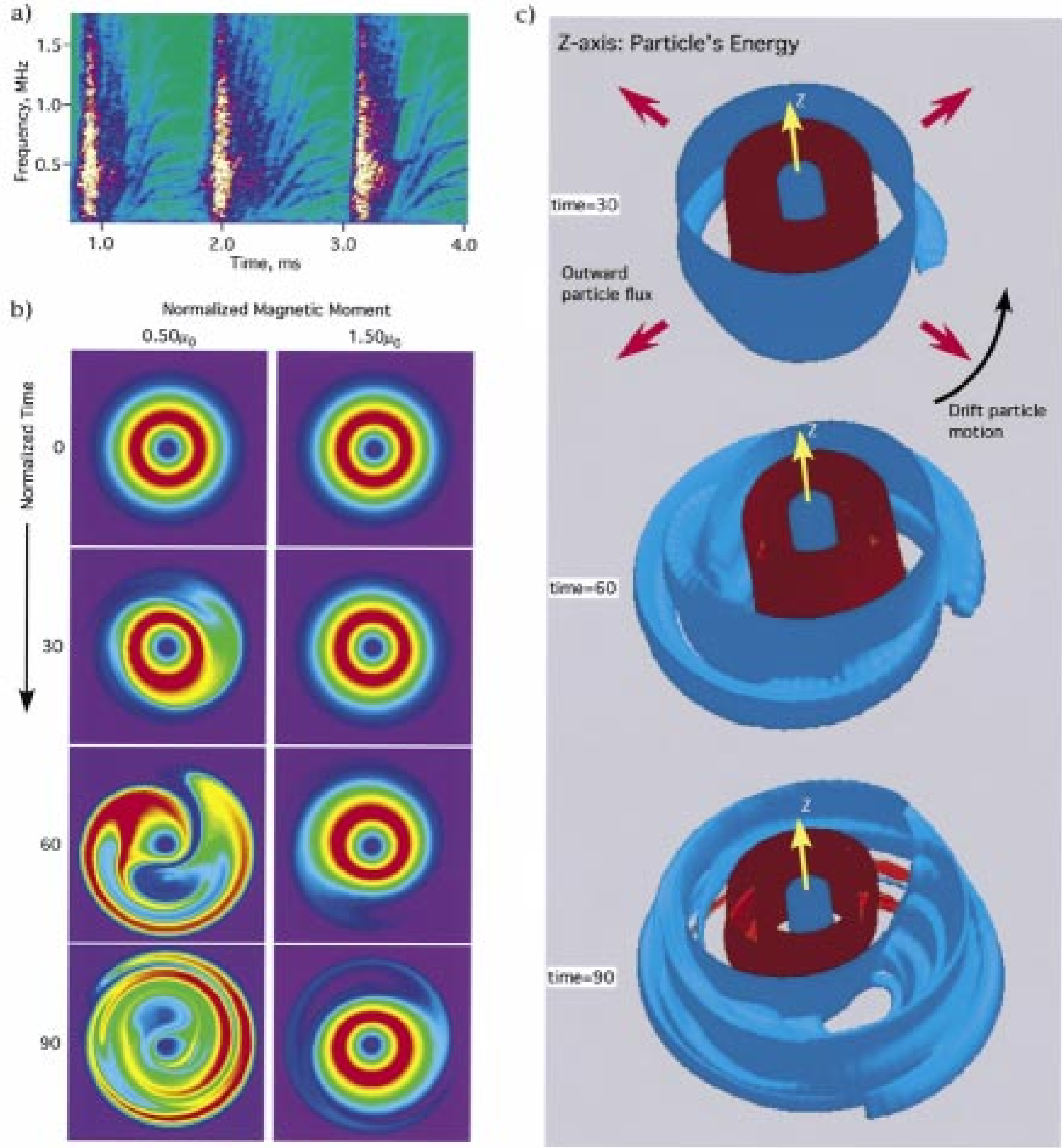

Fig. 1. Views of the evolution of the hot electron interchange instability: (a) time-frequency spectrogram of electrostatic interchange modes observed in the collisionless Terrella device; (b) phase-space evolution from self-consistent simulation; (c) 3-D reconstruction of phase-space flows. Z axis on all plots denotes the energy of particles, color labels equidensity surfaces across all energies.

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