COMPARISON OF SENSORS FOR RWM CONTROL IN A SIMPLIFIED ANALYTIC MODEL

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A simplified version of analytic RWM feedback models

- Purpose: exploration of key qualitative features of feedback models
  - – Impact of the choice of RWM detection scheme
  - – Dependence on sensor location

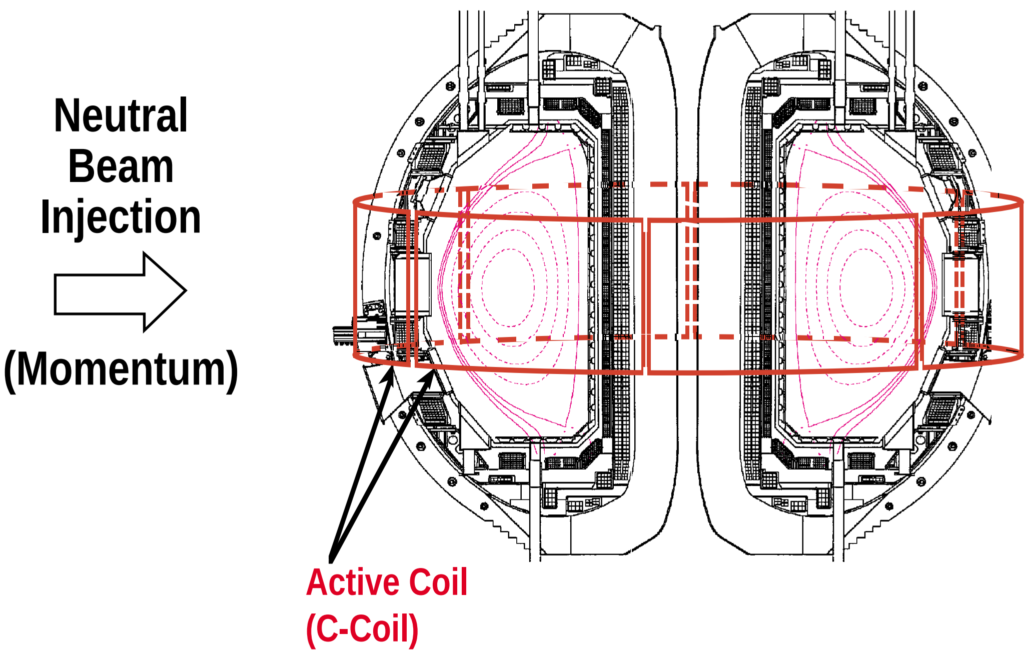
- Geometry-dependent quantities (self and mutual inductances) do not appear explicitly
  - – Variables are perturbed magnetic flux only
  - – All flux amplitudes are evaluated at the resistive wall
  - – Frequencies and growth rates are scaled by τ_{wall}

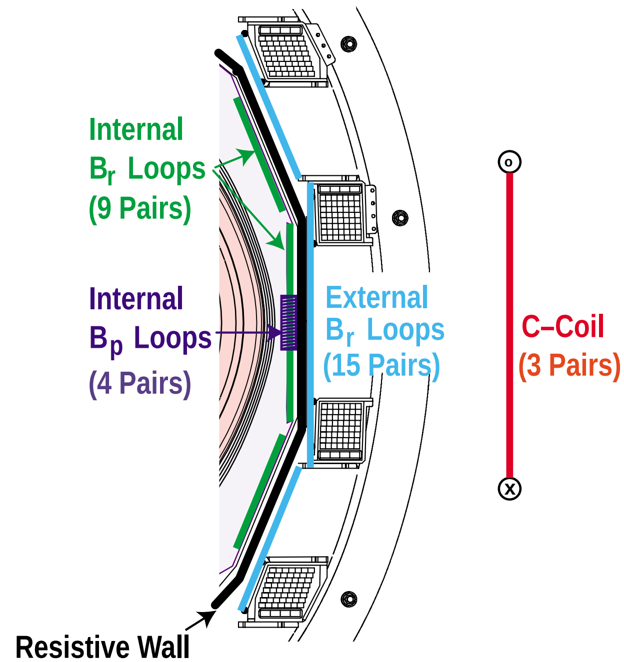
- Quantitative results would require explicit evaluation of the geometric quantities

This work is based on the model described in
A.M. Garofalo, T.H. Jensen, and E.J. Strait, Phys. Plasmas 9, 4573 (2002).

NON-AXISYMMETRIC "C-COIL" IS USED FOR ERROR FIELD CORRECTION AND RWM FEEDBACK CONTROL

- Six midplane coils (C–coil) connected in three pairs for n=1 control
- External and internal saddle loops measure δB_r
- Poloidal field probes measure δB_p with reduced coupling to the control coils





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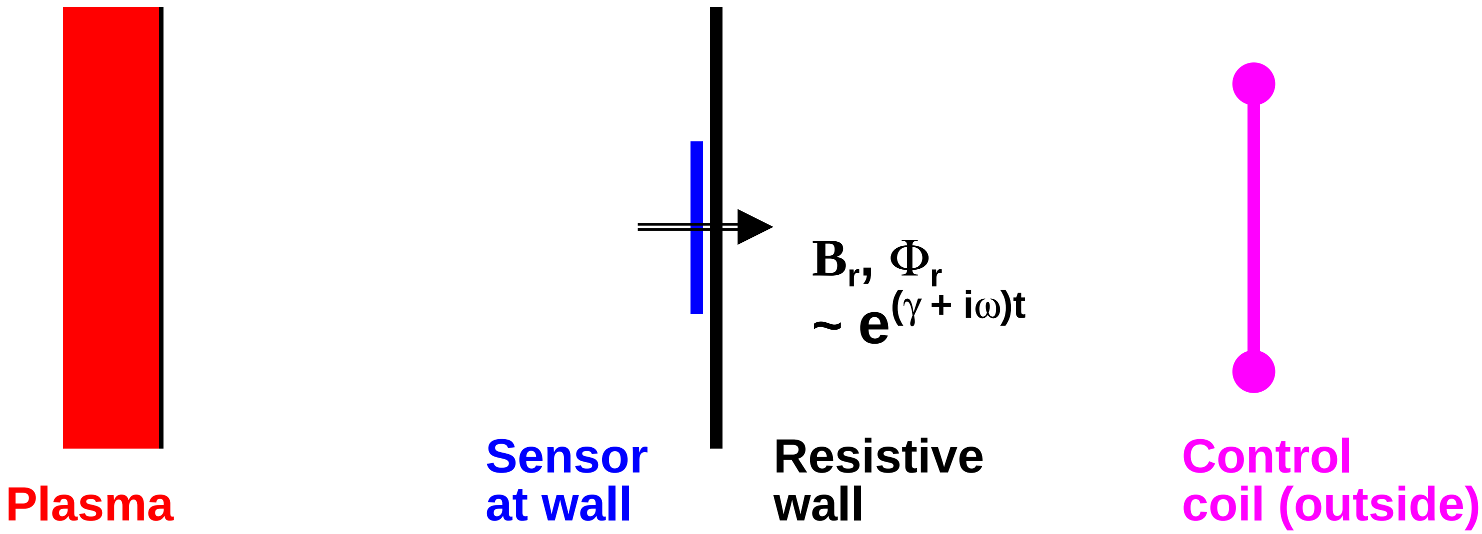
Feedback configuration (schematic)



- Br sensors: induced wall current opposes the driving field
  - coupled to control coils (normally)
  - decoupled from control coils (by analog or digital compensation)
- Bp sensors: induced wall current reinforces the plasma field for sensor inside wall
  - decoupled from control coils (midplane coils ⇒ purely radial field)
  - coupled to control coils (helical control coils)
  - outside the wall: flux from induced wall currents changes sign

Analytic feedback model can be reduced to simple form

$s - \gamma_0 + G(s)\,F(s) = 0$	Dispersion relation
$\Phi = \Phi_P + \Phi_{PW} + \Phi_C + \Phi_{CW}$	Contributions to perturbed flux at the wall
$\Phi_{PW,\,CW} = -\Phi_{P,C}\;\; s\,/\,(1+s)$	Flux from induced wall currents
$\Phi_P = (1+\gamma_0)\,\Phi$	Plasma response model
$\Phi_C = -G(s)\,\Phi_S$	Feedback model
$\Phi_S = \;\; F(s)\,\Phi$	Sensor model

$s = \gamma + i\omega$ = complex growth rate *(in units of the inverse wall time)*

γ_0 = growth rate without feedback *(in units of the inverse wall time)*

$G(s)$ = gain function for amplifier-coil system

$F(s)$ = transfer function for sensors

Φ = total perturbed flux *(all perturbed fluxes are defined at the wall)*

Φ_P = perturbed flux due to plasma

Φ_C = perturbed flux due to control coils

$\Phi_{PW,\,CW}$ = perturbed flux due to wall currents induced by plasma, coils

Sensors are defined in terms of coupling to the fluxes

Idealized Mode Detection	$\Phi_S = \Phi_P$
Br Sensor: Smart Shell	$\Phi_S = \Phi_P + \Phi_{PW} + \Phi_C + \Phi_{CW}$
Br Sensor: DC compensation	$\Phi_S = \Phi_P + \Phi_{PW} + \Phi_{CW}$
Br Sensor: AC compensation – or decoupled Bp sensor outside the wall	$\Phi_S = \Phi_P + \Phi_{PW}$
Bp Sensor: midplane coils (decoupled)	$\Phi_S = \Phi_P - \Phi_{PW}$
Bp Sensor: helical coils (coupled)	$\Phi_S = \Phi_P - \Phi_{PW} + \Phi_C + \Phi_{CW}$

- Bp sensor is defined in terms of the perturbed radial flux (with 90 degree phase shift from the actual measured poloidal perturbation)
- Sensor transfer function $F(s) = \Phi_S / \Phi$ is obtained by combining these sensor definitions with the rest of the model.

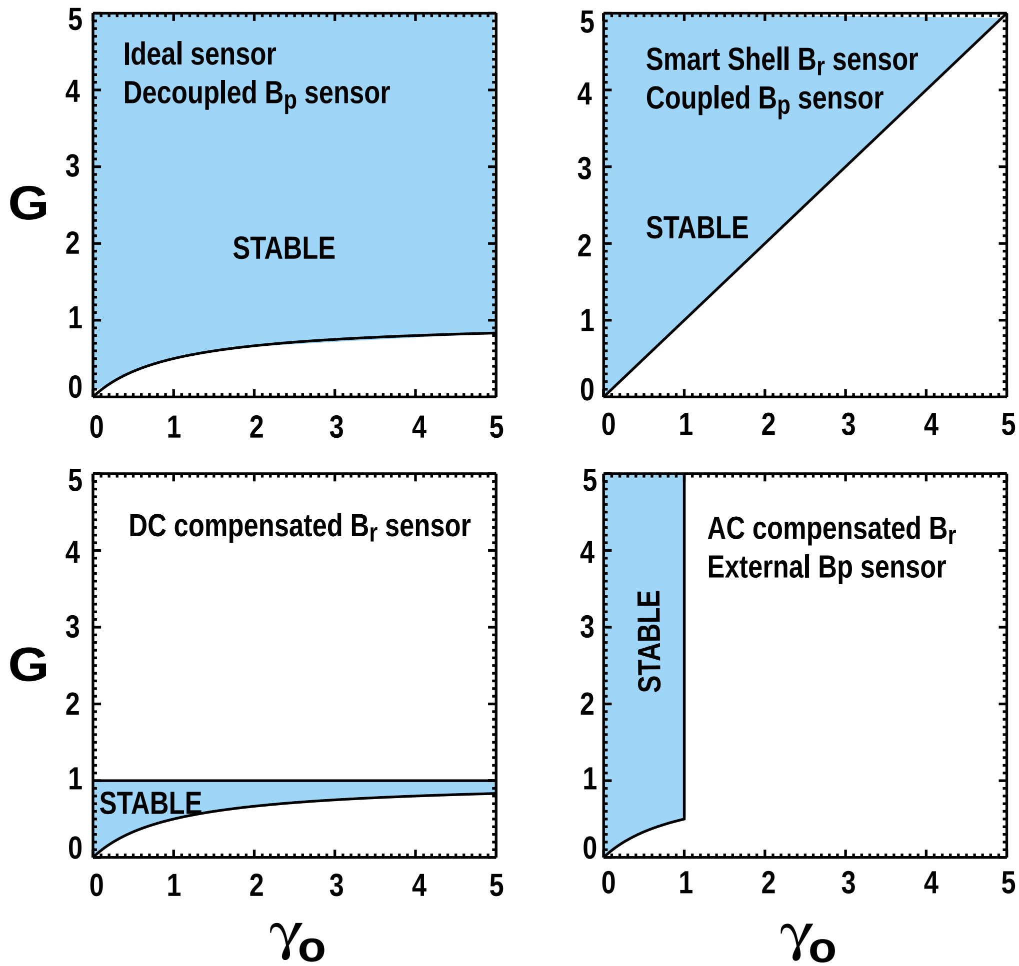
Critical gain for stability (γ<0) depends on type of sensor

- Assuming constant proportional gain G:

Idealized Mode Detection	$G > \gamma_0 / (1+\gamma_0)$	
Br Sensor: Smart Shell	$G > \gamma_0$	
Br Sensor: DC compensation	$G > \gamma_0 / (1+\gamma_0)$	and $G < 1$
Br Sensor: AC compensation	$G > \gamma_0 / (1+\gamma_0)$	and $\gamma_0 < 1$
Bp Sensor: midplane coils	$G > \gamma_0 / (1+\gamma_0)$	
Bp Sensor: helical coils	$G > \gamma_0$	

- Decoupled Bp sensor (midplane coils) is equivalent to ideal sensor
  – can stabilize arbitrarily large γ_0 with $G \sim 1$.
- Smart shell Br sensor and coupled Bp sensor (helical coils) are equivalent
  – can stabilize arbitrarily large γ_0, but requires large gain as γ_0 increases.
- DC compensated Br sensor is not robust – narrow range of stable gain.
- AC compensated Br sensor (and external Bp) can only control weakly unstable modes.

Critical gain for stability ($\gamma<0$) depends on type of sensor



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Sensor's effectiveness is related to its coupling to plasma perturbation

Idealized Mode Detection	$\Phi_S = \Phi_P$	Ideal response
Smart Shell	$\Phi_S = \Phi_P\ / (1+\gamma_0)$	Reduced amplitude
Br Sensor: DC Compensated	$\Phi_S = \Phi_P\ [1 - s / (1+\gamma_0)]$	Destabilizing derivative term
Br Sensor: AC Compensated – or external Bp sensor	$\Phi_S = \Phi_P\ / (1+s)$	Low-pass filtered
Bp Sensor: midplane coil	$\Phi_S = \Phi_P\ [1 + s / (1+s)]$	Enhanced high-freq response
Bp Sensor: helical coil	$\Phi_S = \Phi_P\ [1/(1+\gamma_0) + s/(1+s)]$	Reduced low-freq response

- Sensor response in terms of the plasma perturbation, assuming that the control coil current is determined by the feedback model.

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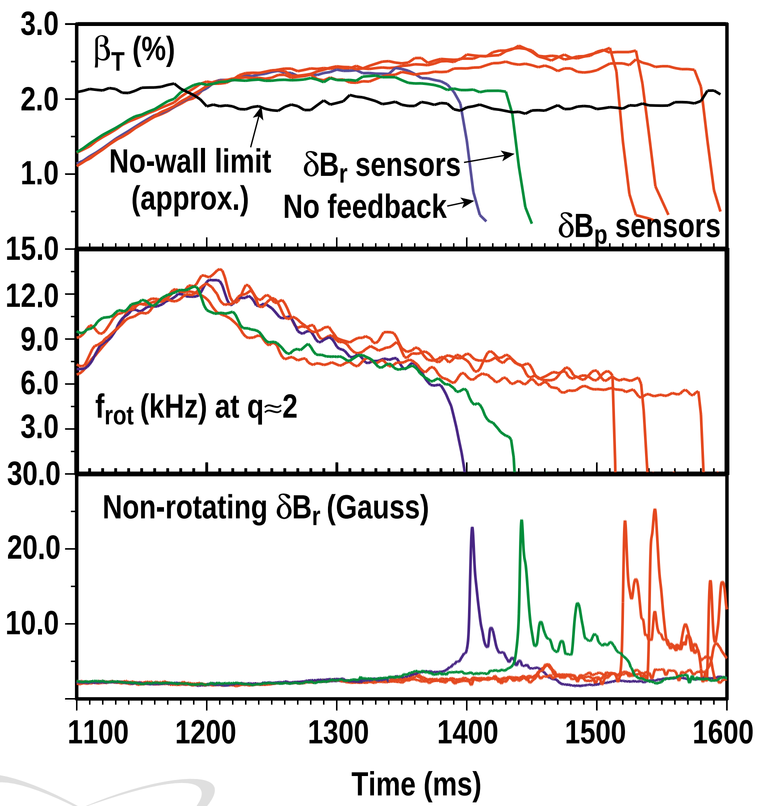
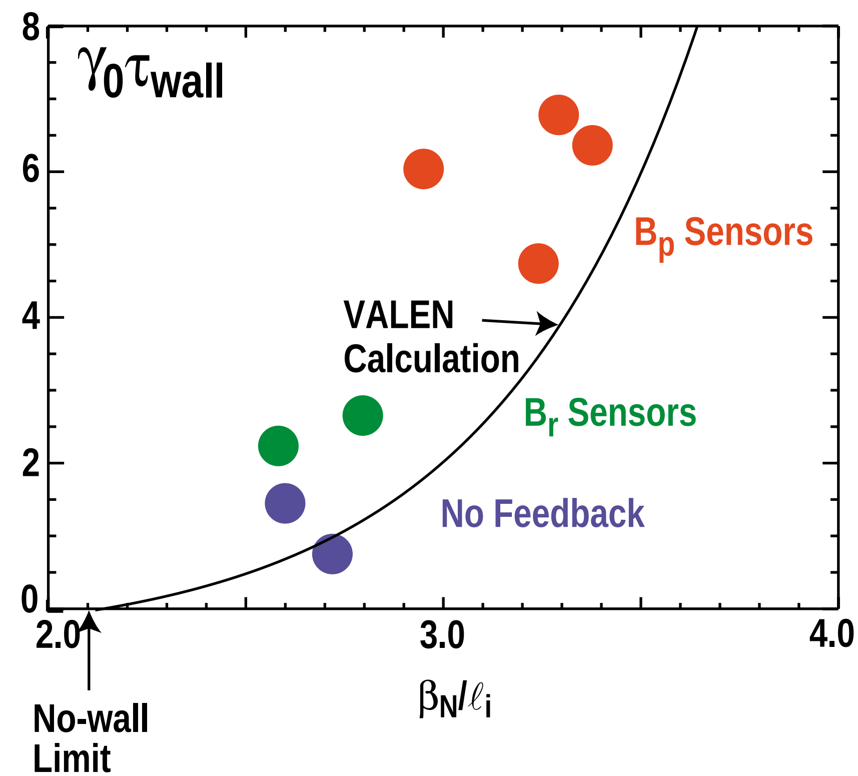
INTERNAL B_p SENSORS IMPROVE ACTIVE CONTROL OF THE RWM

- Stable duration and $\beta/\beta^{no\text{-}wall}$ increase with internal B_p sensors
- Internal B_p sensors stabilize RWM with larger open-loop growth rate γ_0
- Measured open-loop growth rate is consistent with VALEN prediction





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Finite amplifier bandwidth

- Add a single-pole high frequency cutoff to the gain function:

$$G(s) = G \frac{\Omega_0}{s + \Omega_0}$$

- Previous results are essentially unchanged, except for an additional constraint on the maximum growth rate that can be stabilized:

$$\gamma_0 < \Omega_0$$

Ideal sensor,

Br Sensor: Smart Shell

$$\gamma_0 < \Omega_0 + \frac{1}{2}$$

Bp Sensor: midplane coils

- With proportional gain only, feedback cannot stabilize a mode with a growth rate that is significantly faster than the cutoff frequency.
  - Bp sensor's high frequency enhancement gives a modest extension of γ_0.

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Sensor displaced away from the wall

- Assume small displacement of the sensor from the wall:

  $\Phi_i \propto (1 \pm \delta)$, $\delta << 1$ Slab model: $\Phi(x) \propto \exp(\pm kx)$, $\delta = kd$

  – sign for each Φ_i depends on whether sensor moves closer or farther from the source

- Conditions for stability become

  Br Sensor: Smart Shell ($\delta < 0$) $G > \gamma_0 / [1 + |\delta| (1 + 2\gamma_0)]$

  Br Sensor: Smart Shell ($\delta > 0$) $G > \gamma_0 / [1 - |\delta| (1 + 2\gamma_0)]$ and $\gamma_0 < (1 - \delta) / 2\delta$

  Bp Sensor: midplane coils ($\delta < 0$) $G > \gamma_0 / [(1 + \gamma_0)(1 + |\delta|)]$

- Br sensor inside the wall can stabilize an arbitrarily large growth rate with finite (but large) gain, $G = 1 / 2|\delta|$

- Br sensor outside the wall acquires an upper limit to the growth rate that can be stabilized.

- Bp sensor inside the wall has only weak dependence on sensor position

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Error field amplification

- Add a static perturbation Φ_0 to the model (equation is no longer homogenous).

$$s\Phi - \gamma_0\Phi + G(s)\,F(s)\,\Phi - \Phi_0 = 0$$

- Consider the limits of static response ($s = 0$) and large gain ($G \rightarrow \infty$).
Assume the RWM is stable ($-1 < \gamma_0 < 0$). The model becomes

$\Phi_S = 0$ — Ideal control in limit of high gain

$\Phi = \Phi_P + \Phi_C + \Phi_0$ — Perturbed flux, without induced currents

$\Phi_P = (1+\gamma_0)\,\Phi$ — Plasma response model ($\gamma_0 \rightarrow -1$ if no plasma)

- Assume the static error field is <u>not</u> detected directly
(reference level for detection is taken after vacuum fields are established).

$\Phi_S = \Phi_P$ — Bp Sensor: midplane coils

$\Phi_S = \Phi_P + \Phi_C$ — Br Sensor: Smart Shell

Suppression of resonant field amplification depends on type of sensor

- Assuming RWM is stable ($-1 < \gamma_0 < 0$), solving for the plasma perturbation yields

$\Phi_P = \Phi_0 (1+\gamma_0) / \lvert\gamma_0\rvert$	No feedback ($\Phi_S \neq 0$, $\Phi_C = 0$)
$\Phi_P = 0$	Bp Sensor: midplane coils
$\Phi_P = \Phi_0 (1+\gamma_0)$	Br Sensor: Smart Shell

- No-feedback case shows resonant behavior as $\gamma_0 \rightarrow 0$.
- Bp sensor reduces the plasma perturbation to zero.
- Br sensor can only reduce the plasma perturbation to a level comparable to the external error field as the plasma approaches marginal stability ($\gamma_0 \rightarrow 0$).
  – could still allow significant drag on the rotation

Summary

- Simplified analytic model allows qualitative study of RWM feedback
- Radial field sensors with "smart shell" control can stabilize arbitrary growth rates.
  – Br sensors are sensitive to radial position (for better or worse performance)
  – Br sensors with compensation for coupling to control coils perform poorly.
- Internal poloidal field sensors can stabilize arbitrary growth rates with finite gain. Two features are essential to this superior performance:
  – Fast time response
    (compare external Bp: decoupled from coils but slow $\Rightarrow$ poor performance)
  – No coupling to control coils
    (compare coupled Bp: fast response to plasma but performance similar to Br)
- Amplifier bandwidth sets an upper limit to the growth rate that can be stabilized.
- Poloidal field sensors that are decoupled from the control coils may be superior also for feedback-controlled error correction.