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Flowing Two-Fluid Equilibria of ST and CT

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This work was performed in collaboration with
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Outline

Purpose of Our Study:

To generalize Grad- Shafranov equation to describe well the following effects

1) the ion temperature and electric field,
2) the toroidal flow,
3) the poloidal flow,
4) the steep pressure gradient.

Outline:

Here I will present the topics related to the above items 1) and 2).

Why is two-fluid model required?

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Difference between Single- and Two-fluid Models:

$$m_i n\left(\partial \mathbf{u}_i / \partial t + \mathbf{u}_i \bullet \nabla \mathbf{u}_i\right) = -\nabla\left(p_i + p_e\right) + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}$$

- the same for both models

$$\mathbf{E} + \frac{1}{c}\mathbf{u}_i \times \mathbf{B} + \mathbf{F}_{2F} = 0$$

- $\mathbf{F}_{2F}$ **represents the difference.**

$$\mathbf{F}_{2F} \equiv \frac{1}{en}\left[\nabla p_e - \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}\right] = \frac{-1}{en}\nabla p_i - \frac{m_i}{e}\left[\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \bullet \nabla \mathbf{u}_i\right]$$

Single-fluid model is valid only for $\mathbf{E} \approx \frac{1}{c}\mathbf{u}_i \times \mathbf{B} >> \mathbf{F}_{2F}$ **.**

Therefore the single-fluid model requires no ∇p_i **for static equilibrium.**

Why is two-fluid model required? (Cont.)

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If the two-fluid model is used, the length scale of $\lambda_i \equiv c/\omega_{pi}$ appears naturally.

So the ratio L/λ_i , where L is the characteristic length of equilibria, may characterize the two-fluid effect.

For the single-fluid model, $L/\lambda_i \to \infty$.

For the H-mode region, $L \approx \rho_{i,p}$, $L/\lambda_i \approx 1$.

Other examples.

More quantitatively, we will measure the two-fluid effect using the term $\mathbf{F}_{2F}$.

Two-Fluid Model

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Assume that the density is constant. _ $\nabla \bullet \mathbf{u}_{\alpha} = 0$

The equations of motion: $$\boxed{\partial \mathbf{P}_{\alpha} \big/ \partial t = \mathbf{u}_{\alpha} \times \grave{\mathbf{U}}_{\alpha} - \nabla H_{\alpha}}$$

Here $\mathbf{P}_{\alpha} \equiv m_{\alpha}\mathbf{u}_{\alpha} + (q_{\alpha}/c)\mathbf{A}$: generalized momentum

$\grave{\mathbf{U}}_{\alpha} \equiv \nabla \times \mathbf{P}_{\alpha}$: generalized vorticity

$H_{\alpha} \equiv T_{\alpha} + (1/2)m_{\alpha}u_{\alpha}^{2} + q_{\alpha}\phi_{E}$: generalized enthalpy

$\mathbf{A}$, ϕ_{E} : vector and scalar potentials

Axisymmetric Equilibrium

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$\mathbf{u}_\alpha \times \grave{\mathrm{U}}_\alpha = \nabla H_\alpha$ **for** $\alpha = i$ **and** $\alpha = e$

$$\nabla \times \mathbf{B} = 4\pi enc^{-1}(\mathbf{u}_i - \mathbf{u}_e)$$

Introduce the five stream functions as

$\nabla \bullet \mathbf{B} = 0$ _ $\mathbf{B}(r,z) = \nabla \psi(r,z) \times \hat{\theta} / r + B_\theta \hat{\theta}$

$\nabla \bullet \mathbf{u}_\alpha = 0$ _ $\mathbf{u}_\alpha(r,z) = \nabla \psi_\alpha(r,z) \times \hat{\theta} / nr + u_{\alpha\theta} \hat{\theta}$

$\nabla \bullet \grave{\mathrm{U}}_\alpha = 0$ _ $\grave{\mathrm{U}}_\alpha(r,z) = (q_\alpha / c)\nabla \Psi_\alpha(r,z) \times \hat{\theta} / r + \boldsymbol{\Omega}_{\alpha\theta} \hat{\theta}$

Axisymmetric Equilibrium (Cont.)

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Since $\grave{\mathbf{U}}_\alpha = (q_\alpha / c)\mathbf{B} + m_\alpha \nabla \times \mathbf{u}_\alpha$ **,**

$$\Psi_\alpha(r,z) = \psi(r,z) + (m_\alpha c / q_\alpha) r u_{\alpha\theta}$$

$$\begin{array}{lll} \grave{\mathbf{U}}_\alpha \bullet \nabla H_\alpha = 0 & \_ & H_\alpha = H_\alpha(\Psi_\alpha) \\ \mathbf{u}_\alpha \bullet \nabla H_\alpha = 0 & & \psi_\alpha = \psi_\alpha(\Psi_\alpha) \end{array}$$

Equilibria are described by the coupled equations for the vorticity stream functions Ψ_i and Ψ_e .

Coupled Equations for Ψ_i and Ψ_e

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$$\frac{d\psi_i}{d\Psi_i} r^2 \nabla \bullet \left(\frac{d\psi_i}{d\Psi_i} \frac{1}{r^2} \nabla \Psi_i \right) = S_*^2 (\psi_i - \psi_e) \frac{d\psi_i}{d\Psi_i} - S_*^2 (\Psi_i - \Psi_e) + r^2 \frac{dH_i}{d\Psi_i}$$

$$\Delta^* \Psi_e = S_*^2 (\psi_i - \psi_e) \frac{d\psi_e}{d\Psi_e} - S_*^2 (\Psi_i - \Psi_e) - r^2 \frac{dH_e}{d\Psi_e}$$

Here $S_* = r_s / l_i$. Every quantity is normalized by

r_s , $B_s = B_p(r = r_s, z = 0)$, $V_A = B_s / \sqrt{4\pi m_i n}$

$T_e - e\phi_E = H_e(\Psi_e)$; $T_i + u_i^2 / 2 + e\phi_E = H_i(\Psi_i)$

$rB_\theta = S_* \left(\psi_i(\Psi_i) - \psi_e(\Psi_e) \right)$; $ru_{i\theta} = S_*(\Psi_i - \Psi_e)$

Analytic CT Equilibrium

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By choice of arbitrary functions, the coupled equations can be reduced to

$$\Delta^{*}\Psi_e + (C_{He0} + C_{Hi0})r^2 = 0 \qquad (C_{He0}, C_{Hi0} \text{ are constant})$$

Sol. $\Psi_e(r,z) = \psi(r,z) = -\left(r^2/2\right)\left[1 - r^2 - (z/E)^2\right]$: Hill's vortex

$$(\; C_{He0} + C_{Hi0} = -4 - E^{-2} \;)$$

$$\mathbf{u}_i = \hat{\theta}\left(C_{Hi0}/S_*\right)r \; ; \quad \mathbf{u}_e = -\hat{\theta}\left(C_{He0}/S_*\right)r$$: Rigid rotations

$$\phi_E(r,z) = \left[-\frac{\gamma_T(C_{He0} + C_{Hi0})}{\gamma_T + 1} + C_{Hi0}\right]\Psi_e(r,z) + \frac{(C_{Hi0}/S_*)^2}{2(\gamma_T + 1)}r^2$$

$$T_i(r,z) = \gamma_T T_e(r,z) = \frac{\gamma_T(C_{He0} + C_{Hi0})}{\gamma_T + 1}\Psi_e(r,z) + \frac{\gamma_T(C_{Hi0}/S_*)^2}{2(\gamma_T + 1)}r^2$$

For no ion rotation, $T_i(r,z) = \phi_E(r,z)$

= Normalization & Geometry =

Normalization

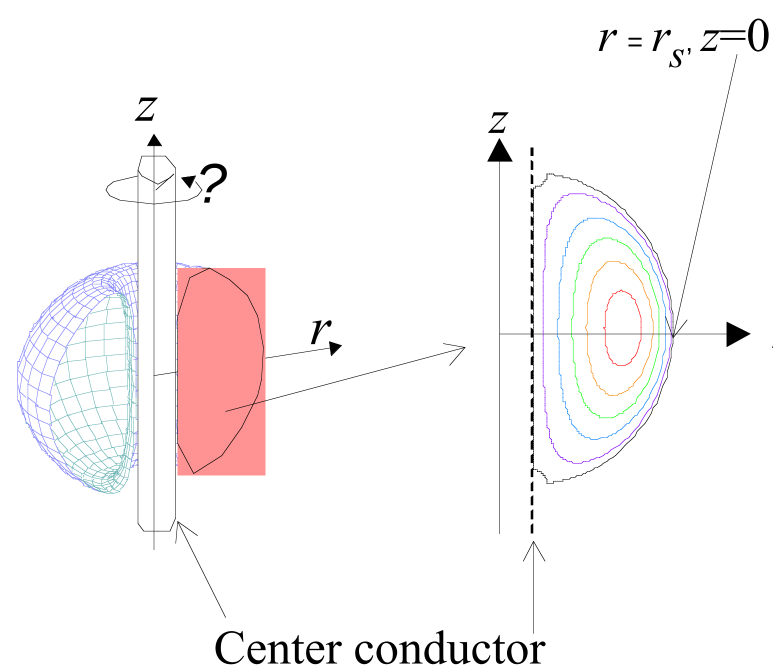
r_s **:** radius of the outer boundary on a symmetry plane (r=rs,z=0)

B_R **:** poloidal field at (r=rs,z=0)

$$V_A = B_R \Big/ \sqrt{4\pi m_i n}$$

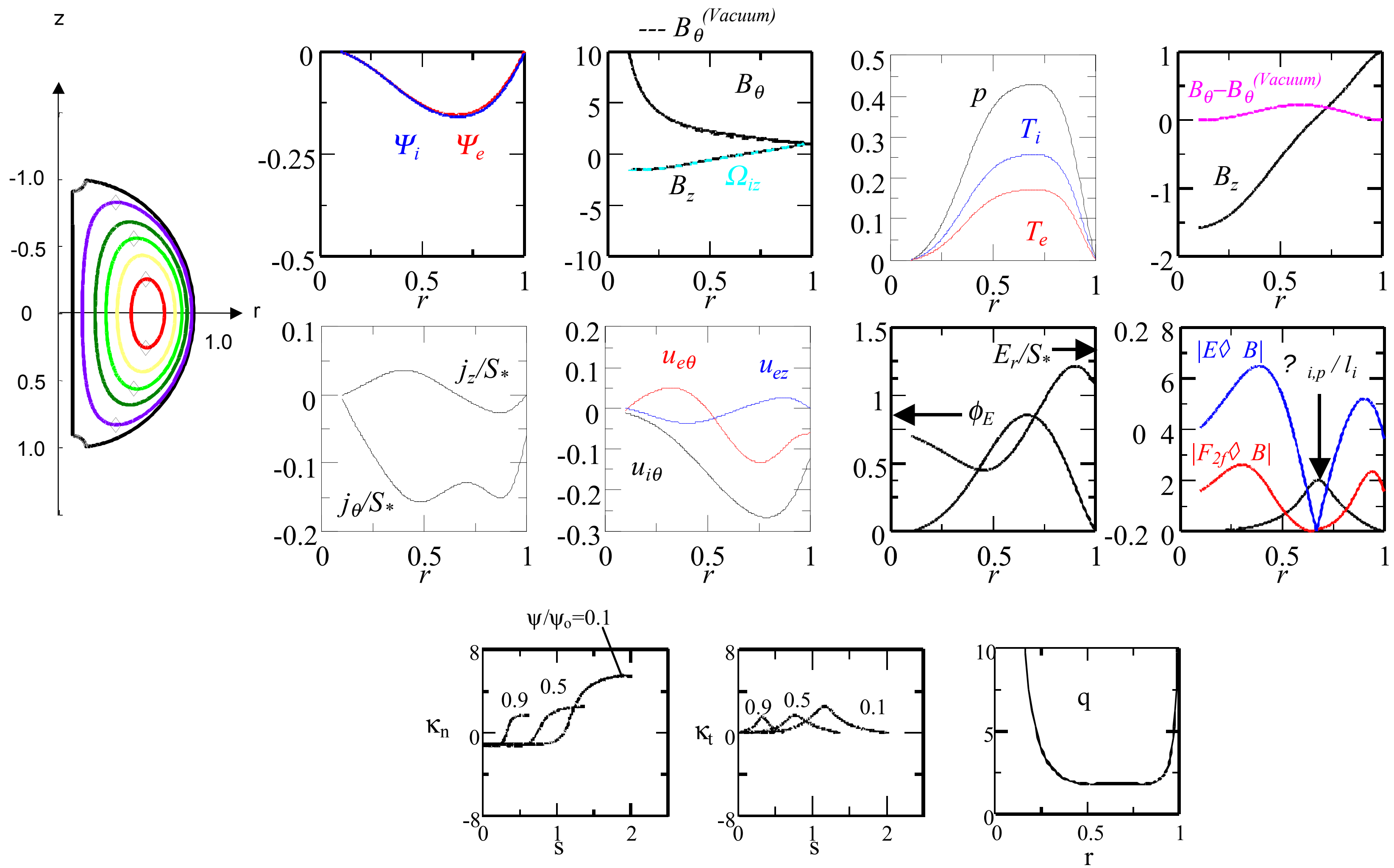
$S_* = \dfrac{r_s}{l_i}$ is adopted.

Geometry for the numerical computation



2D ST Equilibrium_(S*=30)

Profiles at the mid-plane



2D ST Equilibrium (Cont.)

When the following NSTX values are allocated

$r_s = 1.52[m]$ **: the outer radius at mid-plane**

$n = 0.4 \times 10^{20}[m^{-3}]$**: the average density** _ $S_* = 30$

$B_\theta^{(vacuum)} = 0.4[T]$ **at the magnetic axis,**

then $I_\theta = 1.1[MA]$, $u_{i\theta\,\max} = 142[km/s]$,

$T_{i\,\max} = 1.5[KeV]$, $\langle \beta_T \rangle_M = 0.17$

These are in agreement with the 1MA, NB driven NSTX.

2D ST Equilibrium (Cont.)

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Other Characteristics of Computed Equilibrium:

$\langle \beta_T \rangle_M = 0.17$ **;** $\langle \beta \rangle_M = 0.09$ **where** $\langle \beta \rangle_M \equiv \dfrac{\langle n(T_i + T_e) \rangle_M}{\left\langle n(T_i + T_e) + B^2 / 8\pi \right\rangle_M}$

$$\langle p_i \rangle_M \approx 0.04 \left\langle B_\theta^{(vacuum)^2} \right\rangle_M$$

$$\left\langle u_{i\theta}^2 \right\rangle_M \approx 0.01 \left\langle B_\theta^{(vacuum)^2} \right\rangle_M \text{ :}$$

– Rotation energy is 1/4 of the ion thermal energy and 1/100 of energy of the external magnetic field.

$$\left\langle B_{self}^2 \right\rangle_M \approx 0.1 \left\langle B_\theta^{(vacuum)^2} \right\rangle_M \text{ :}$$

– Energy of the self magnetic field is 1/10 of energy of the external magnetic field.

2D ST Equilibrium (Cont.)

Two-Fluid Effect:

$f_{2F} = 0.31 \cong T_{i\max} / \phi_{E\max}$, **where** $f_{2F} \equiv \dfrac{\langle [\mathbf{F}_{2F} \times \mathbf{B}] \rangle_M}{\langle [\mathbf{E} \times \mathbf{B}] \rangle_M}$,

- **Two-fluid effect is fairly large.**
- **the second relation results form a fact that the ion flow velocity is smaller than the ion thermal velocity.**

Current Ratio:

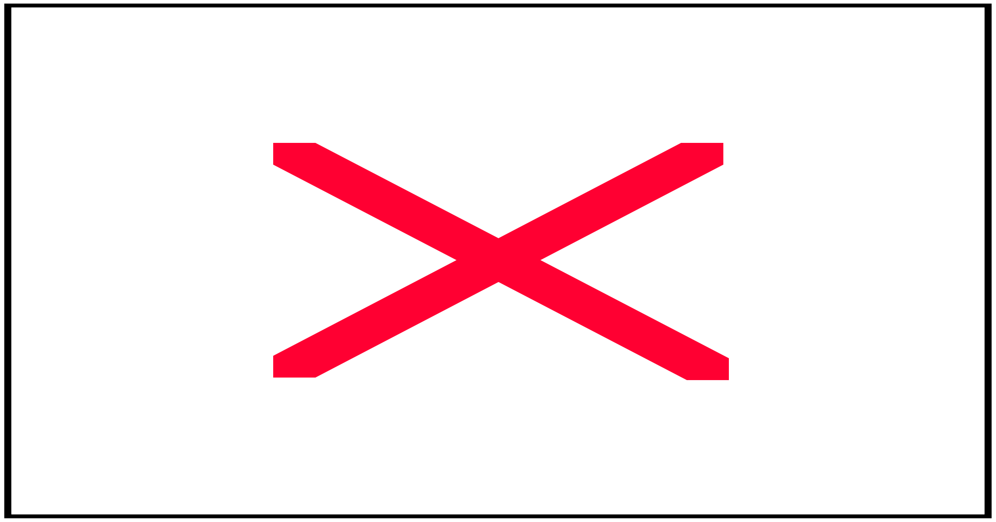
$I_{i\theta} / I_\theta = 1.2$ **and** $I_{e\theta} / I_\theta = -0.2$

- **The almost entire current carried by the ion fluid.**
- **The electron current flows in the opposite direction to the total current.**

Measurement of the two-fluid effect

1-D STs with S∗=30 **1-D CTs with S∗=5**

Each symbol represents
an equilibrium with
corresponding beta value
<_>M and maximum
value of ion flow (_) _



The two-fluid effect is significant for plasmas with…

Smaller S*

Higher beta value

Ion rotation closer to the ion diamagnetic drift

Summary

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@ To generalize Grad-Shafranov equation to describe well the effects of

1) the ion temperature and electric field,
2) the toroidal flow,
3) the poloidal flow,
4) the steep pressure gradient,

the reason was explained why the two-fluid model is necessary.

@ Assuming constant density, the coupled equations are derived for the stream functions of the ion and electron generalized vorticities.

Summary (Cont.)

@ Analytic CT solution and numerical ST solution are shown. Some properties are discussed.

@ Criteria were shown for when the single-fluid model is adequate and when the more general two-fluid model is necessary.

For more detail, see our recent paper:

"Equilibrium analysis of a flowing two-fluid plasma"
H.Yamada, T.Katano, K.Kanai, A.Ishida, L.C.Steinhauer
Phys. Plasmas, November issue of 2002

Summary (Cont.)

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For Future Plan:

- To study the effect of poloidal flow and
  the steep pressure gradient.
- To release the constant density assumption.