Extended Lumped Parameter Model of Resistive Wall Mode
and
The Effective Self-Inductance

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OUTLINE

- Extended Lumped Parameter Model and RWM Effective Self-Inductance

- MOTIVATION
- APPROACH AND ASSUMPTIONS OF EXTENDED LUMPED PARAMETER MODEL
  - Relation to recent other approaches (models)
- "RWM EFFECTIVE SELF-INDUCTANCE"
  - Plasma response is conveniently characterized by one parameter
- APPLICATIONS
  - Dispersion relation / Initial value time dependent solution
  - Resonant magnetic braking,
  - Non-resonant braking,
  - Resonant Field Amplification

MOTIVATION

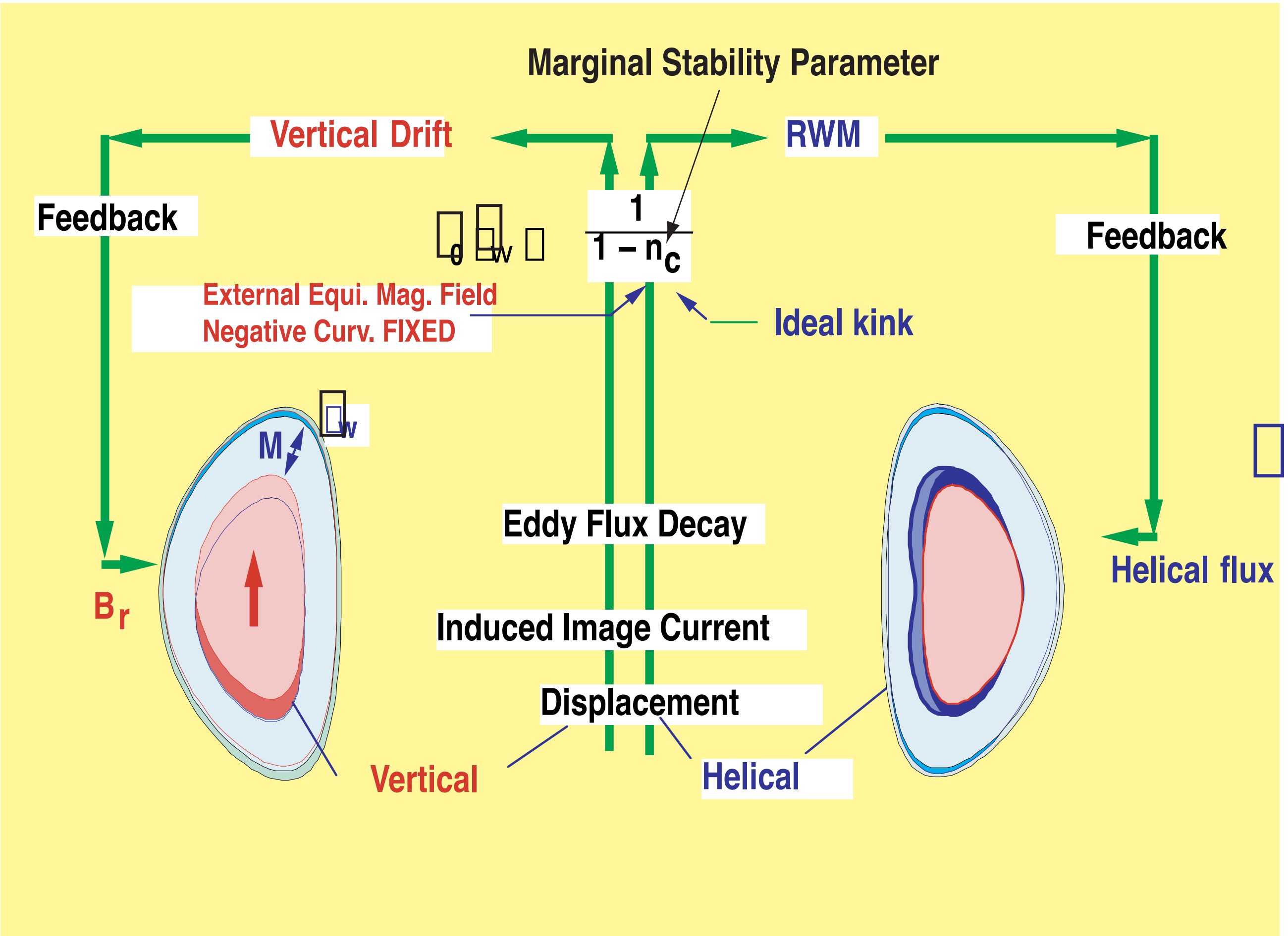
- Develop simple description of RWM stabilization process, useful for qualitative and semi-quantitative discussion.

  - M. Okabayashi, N. Pomphrey and R. Hatcher, N. Fusion. 38(1998) 1607
  - M. Okabayashi et al, EPS 2002 (Plasma Phy. and Controlled Fusion in press)

- The model to be consistent with MHD analysis models developed by various groups

  - A. Bondeson and D. Ward, Phys. Rev. Lett. 72 (1994)2709
  - J. Bialek, etal., Phys. of Plasmas, 8 (2001) 2170
  - M. Chu et al., (IAEA 2002)
  - R. Fitzpatrick, Phy. Plasmas 9 (2002) 3459
  - A. Garofalo, Sherwood meeting (2002)

RWM AND ITS FEEDBACK CONTROL CAN BE CONSIDERED AS THE n=1 HELICAL INSTABILITY ANALOG TO THE n = 0 VERTICAL INSTABILITY



• "Mode Rigidity" is the fundamental assumption

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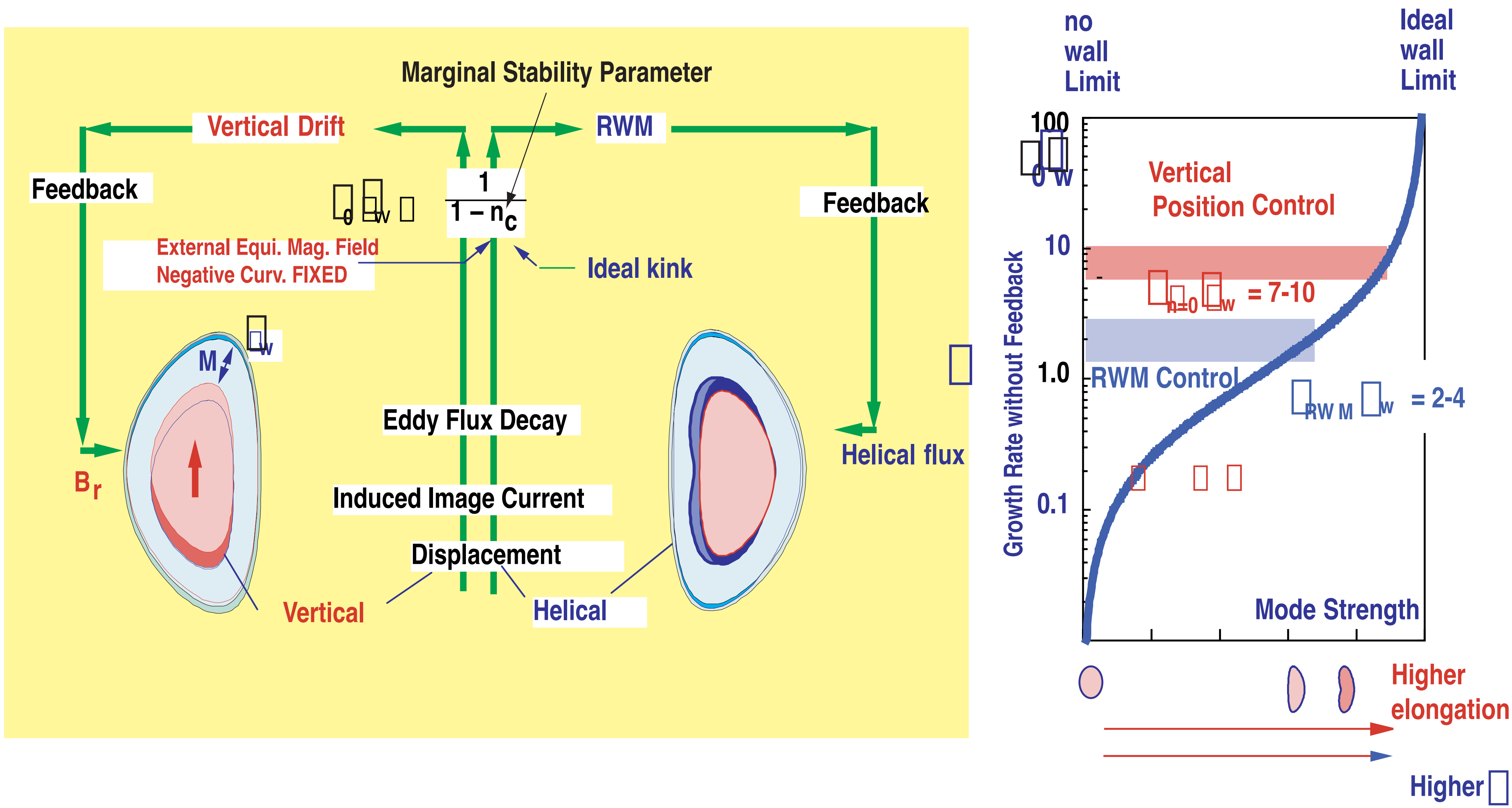


• "Mode Rigidity" is the fundamental assumption

Approach and Assumptions of Extended Lumped Parameter Model

- Include essential RWM physics

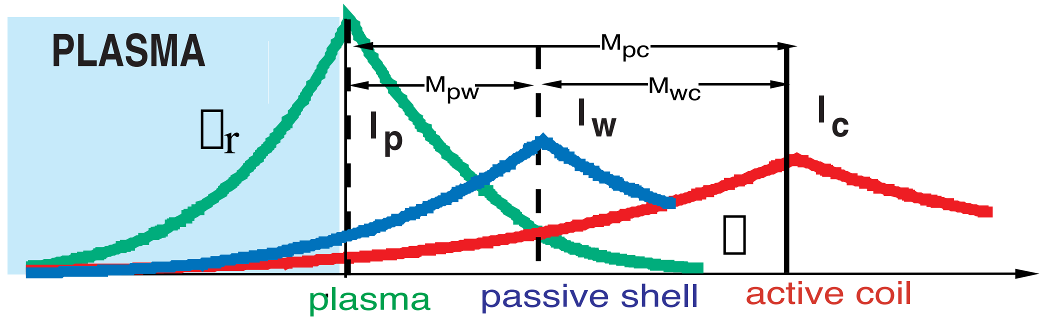
  - Pressure balance and B-normal continuity on plasma surface
  - Rotation dissipation in adhoc manner, however, consistent with existing theories
  -

- Assumptions

  - Rigid displacement:
    justifiable based on experiments [M. Okabayashi et al., Phys. Plasmas 8,2071(2001)]
  - Cylindrical geometry
  - One mode excitation
  - No coupling to other [stable] modes

EXTENDED LUMPED PARAMETER RWM MODEL

- **Explicit Usage of Boundary Conditions**
  - on the Plasma Surface, Wall and Coil (I_p , I_w , I_c)
  - Independent parameters: plasma skin current, wall current, feedback coil current



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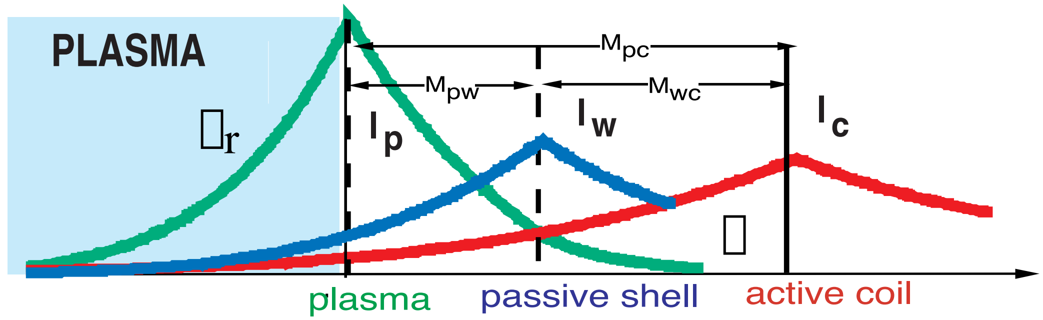

- **Pressure Balance and B-normal Continuity on Plasma Surface**

plasma displacement | rotational dissipation | plasma rotation | m - nq | external displacement | standard mutual inductance

$\square = \square\,\square_{i,j}\,\square_j$

$$(1/\square_r)\, d(r\square_r)/dr\,|_{r=a} + \square\,(\square + \square\square\square_{\square}) = (mf^2/\square^2\square_A^2 + f^2))(a\square'(a_+)/m\square(a_+) + 2/f)$$

toroidal number

fixed displacement approximation $\square_0$

(available from experiment)

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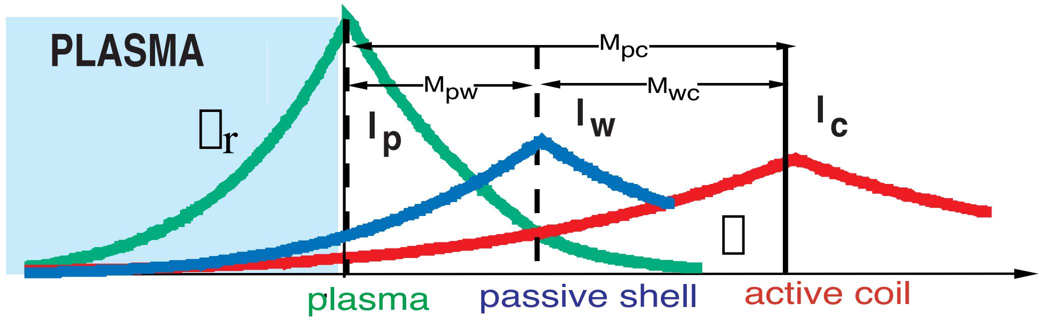

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$\|$

fixed displacement approximation $\square_0$

toroidal number

- **Conveniently formulated "RWM effective self-inductance"**

$$V = d/dt \begin{pmatrix} L_{eff} & M_{pw} & M_{cp} \\ M_{wp} & L_w & M_{wc} \\ M_{pc} & M_{cw} & L_c \end{pmatrix} \begin{pmatrix} I_p \\ I_w \\ I_c \end{pmatrix}$$

standard mutual inductance

$$L_{eff} I_p + M_{pw} I_w + M_{pc} I_c = 0$$

$$L_{eff} = (\square_0 \square \square_n + \square(\square + \square N\square\square))\, L \,/\, (\square_0 \square\square_n - 2 + \square(\square + \square N \square\square))$$

$\square_n = 2/f - 1$ **(f=1 <--> no wall limit, $\square_n$ =1)**

- **L_{eff} represents all RWM characteristics currently considered**

EXTENDED LUMPED PARAMETER RWM MODEL

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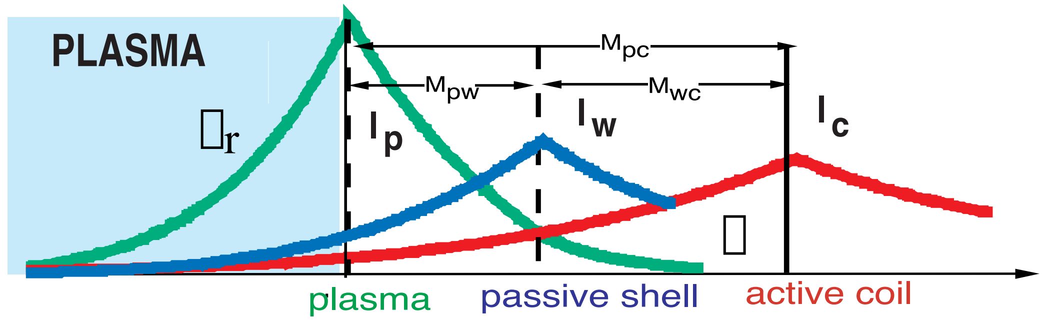

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□ = □ □ $_{i,j}$ □ $_j$

(1/□$_r$) d(r□$_r$)/dr |$_{r=a}$ + □ (□ + □□□$_□$) = (mf^2 / □^{2} □$_A^2$ + f^2))(a□'(a$_+$)/ m□(a$_+$) +2 /f)

||

fixed displacement approximation □$_0$

toroidal number

magnetic decay index

N''z + Br_ext =0 : vertical instability

- **Conveniently formulated "RWM effective self-inductance"**

$$V = d/dt \begin{pmatrix} L_{eff} & M_{pw} & M_{cp} \\ M_{wp} & L_w & M_{wc} \\ M_{pc} & M_{cw} & L_c \end{pmatrix} \begin{pmatrix} I_p \\ I_w \\ I_c \end{pmatrix}$$

standard mutual inductance

□$_n$ = 2/f -1

(f=1 <--> no wall limit, □$_n$ =1)

$L_{eff} I_p + M_{pw} I_w + M_{pc} I_c = 0$

L_{eff} = (□$_0$ □ □$_n$ + □(□+ □N□ □)) L / (□$_0$ □□$_n$ - 2 + □(□+ □N□ □))

- **L_{eff} represents all RWM characteristics currently considered**

RWM Dispersion Relation

- Dispersion Relation with Extended Lumped Parameter Model Expresses Explicitly Geomertical Elements

- Extended Lumped Parameter Model

kinetic

dissipation

plasma beta

wall time constant

geometrical term

$M_{ij} = (r_i / r_j)^m$

$$[\, c_k(\square + i\square)^2 + \square(\square + i\square) + \square_0 - \square_n\,](\square + \square_w^{-1}) - \square[c_k(\square + i\square)^2 + \square(\square + i\square) + \square_0 - \square_n - 2]\, (M_{pw}^2 / L_w L_p) = 0$$

RWM Dispersion Relation

• Dispersion Relation with Extended Lumped Parameter Model Expresses Explicitly Geomertical Elements

• Extended Lumped Parameter Model

kinetic | dissipation | plasma beta | wall time constant | **geometrical term** | $M_{ij} = (r_i/r_j)^m$

$$[\ c_k(\square + i\square)^2 + \square(\square + i\square) + \square_0 - \square_n\ ](\square + \square_w^{1}) - \square[c_k(\square + i\square)^2 + \square(\square + i\square) + \square_0 - \square_n - 2]\ (M_{pw}^2 / L_w L_p) = 0$$

• Fitzpatrick's " Simple Model of RWM... in Phys. Plasmas 9(2002)3459

E. RWM dispersion relation

Neglecting the error field, for the moment, Eqs. (5), (8), (9), and (10) can be combined to give the following simple cubic RWM dispersion relation:

$$[(\hat{\gamma} - i\hat{\Omega}_\phi)^2 + \nu_*(\hat{\gamma} - i\hat{\Omega}_\phi) + (1-\kappa)(1-md)] \times (\hat{\gamma}S_* + 1 + md) = 1 - (md)^2. \quad (12)$$

$$d = \frac{1}{m}\frac{(r_w/a)^{2m} - 1}{(r_w/a)^{2m} + 1},$$

$$d_c = \frac{1}{m}\frac{(r_c/a)^{2m} - 1}{(r_c/a)^{2m} + 1},$$

kinetic

dissipation*

plasma beta

wall time constant*

geometrical term

(M. CHU IAEA 2002)

GENERAL FORMULATION INCLUDING PLASMA ROTATION AND DISSIPATION

- The ìresistive wall modeî can be described in terms of circuits on the resisitive wall driven by the plasma or the external coils

$$\frac{\mu_0 \Delta z}{\eta \lambda_{l'_w}} \frac{\partial B_{l'_w}}{\partial t} + \sum_{l_w} M_{l'_w l_w} B^w_{l_w} = \sum_{l_p} C^{wp}_{l'_w l_p} B^p_{l_p} - \sum_{l_c} C^{wc}_{l'_w l_c} I_{l_c} \quad (7)$$

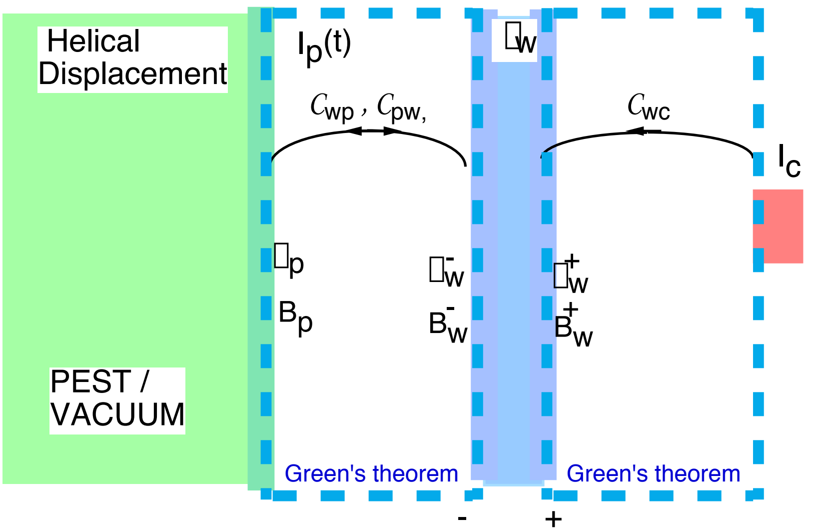
- The plasma responds to the magnetic field B_{lw} on the resistive wall through the relationship

$$B^p_{lp} = -\frac{1}{\delta W_{Iw} + i\Omega D} C^{pw} B^w_{lw} \quad (8)$$

- The response of the RWM to the coil currents is therefore

$$\frac{\mu_0 \Delta z}{\eta \lambda_{l'_w}} \frac{\partial B_{l'_w}}{\partial t} + \sum_{l_w} (M_{l'_w l_w} + \sum_{l_p} C^{wp}_{l'_w l_p} \sum_{l'_p} \frac{1}{(\delta W_{Iw} + iD\Omega)_{l_p l'_p}} C^{pw}_{l'_p l_w}) B^w_{l_w} = -\sum_{l_c} C^{wc}_{l'_w l_c} I_{l_c} \quad (9)$$

Schematics of RWM Feedback Analysis with Toroidal Geometry



Resistive Wall Thin Shell Approximation

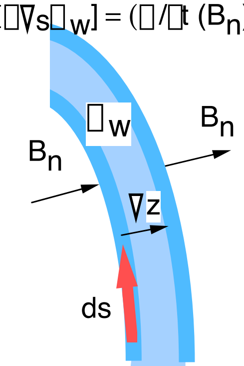
ï Introducing "skin current stream fluction" $\square_w$: j =∇Z x∇$\square_w$□(z-z_w)

ï normal magnetic field continuity □□ /□$n^{(-)}$ = □□ /□$n^{(+)}$ = Bn

ï Ampere's Law: □(+) - □(-) = $\square_w$

ï Faraday's Law

∇_s [□$\nabla_s$$\square_w$] = (□/□t ($B_n$), assuming I∇z I = 1



$\square_w$ (□,□) can be expanded in eigenmodes

(M.CHU, IAEA2002)

THE LUMPED PARAMETER MODEL AND RFA

• For the least stable resistive wall mode and assuming the matrix elements of the energy and dissipation factors are constants, we arrive at a “lumped parameter equation”

the total RWM field at wall

$$\tau_w \frac{\partial B_w}{\partial t} + M \frac{1}{A} B_w = C^{wc} I_c \qquad (10)$$

external helical current

where A is the 'complex amplification factor'. Note that A = 1 in the absence of plasma

potential energy with ideal wall

$$A = \frac{\square W_{IW}(\square) + \square\square\square}{\square W_{nW}(\square) + \square\square\square} \qquad (11)$$

potential energy without wall

• In this equation we see that the dissipation on the wall and the dissipation is the plasma are separated

1. 'No External Coil': homogeneous equation, dissipation in plasma and the resistive wal

$$\gamma \tau_w = -M \frac{1}{A} \qquad (12)$$

2. '**Steady State with External Coil**': inhomogeneous equation. dissipation in plasma

$$B_w = \frac{A}{M} C^{wc} I_c \qquad (13)$$

□W FORMULATION AND LUMPED PARAMETER MODEL

M. Chu Formulation (IAEA 2002)

Extended Lumped Parameter Model

- **growth rate**

total B-field on wall

$$\tau_w \frac{\partial B_w}{\partial t} + M \frac{1}{A} B_w = C^{wc} I_c$$

$$1/A = \frac{\square W_{nW}(\square) + \square\square\square}{\square W_{iW}(\square) + \square\square\square}$$

$$\gamma \tau_w = -M \frac{1}{A}$$

- **resonant field amplification**

$$B_w = \frac{A}{M} C^{wc} I_c$$

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Extended Lumped Parameter Model

$$\square_w \frac{\partial I_p}{\partial t} + [-L_{eff} / (L_{eff} - M_{pw}^2 / L_w)]\, I_p = 0$$

$$L_{eff} = (\square_0 - \square_n + \square\square)\, I_p \,/\, (\square_0 - \square_n \square 2 + \square\square)$$

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• resonant field amplification

$$B_w = \frac{A}{M} C^{wc} I_c$$

$□□_w$

$$□L_w \frac{\partial I_p}{\partial t} + [-L_{eff} / (L_{eff} - M_{pw}^2 / L_w)]\, I_p = 0$$

$$L_{eff} = (□_0 - □_n + □□)\, I_p \,/\, (□_0 - □_n □2 + □□)$$

- marginal stability □ (no wall limit)

$□□_w = 0$ <----> $L_{eff} = 0$: $□_0 = □_{n.w}$

- Ideal wall limit

$□□_w$ = infinity --> $(L_{eff}(□_{I.w}) - M_{pw}^2 \, L_w) = 0$

$$□□_w = \frac{(□_{n.w} - □_n + □□□)}{(□_{I.w} - □_n + □□□)} \, (□_{n.w} - □_{I.w} □2) / (□2)$$

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$□□_w$

L_{eff} = ($□_0$ - $□_n$+ □□) I_p / ($□_0$ - $□_n$□2 + □□)

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normalization constant

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□$_w$

$$L_{eff} = (\square_0 - \square_n + \square\square)\, L_p / (\square_0 - \square_n \square 2 + \square\square)$$

- marginal stability □ (no wall limit)

$\square_w = 0$ <----> $L_{eff} = 0$: $\square_0 = \square_{n.w}$

- Ideal wall limit

$\square_w =$ infinity --> $(L_{eff}(\square_{I.w}) - M_{pw}^2 L_w) = 0$

$$\square_w = \frac{(\square_{n.w} - \square + \square\square\square)}{(\square_{I.w} - \square + \square\square\square)} (\square_{n.w} - \square_{I.w} \square 2) / (\square 2)$$

normalization constant

$$L_{eff} I_p + M_{pw} I_w + M_{pc} I_c = 0$$

(the $M_{pw} I_w$ term is struck through)

$$I_p = \frac{M_{pc}}{L_{eff}} I_c$$

RWM Time Evolution with Extended Lumped Parameter Model Plus Angular Momentum Balance

- Example(1): Magnetic Braking (Rotation slowing down with increasing mode amplitude)

Steady target

RWM amplitude

coil current or error field

$$\partial \square / \partial t = (\square_0 \square \square) / \square_m \square C_{vis} \mathrm{Im}[\square I_p(t) \square I_{c.}(t))]$$

momentum conf. time

~1/ □ (□ + i□) -> 1/(i□□)

known as rotation dissipation "induction motor model"

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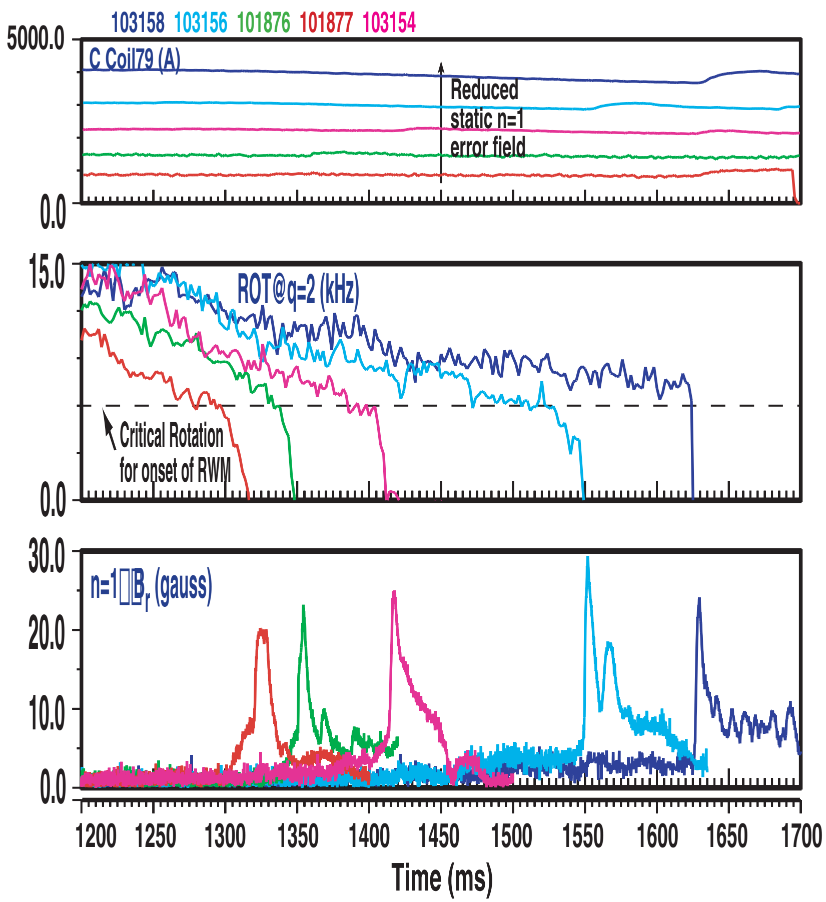
Experiment with Various External Fields



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Steady target — RWM amplitude — coil current or error field

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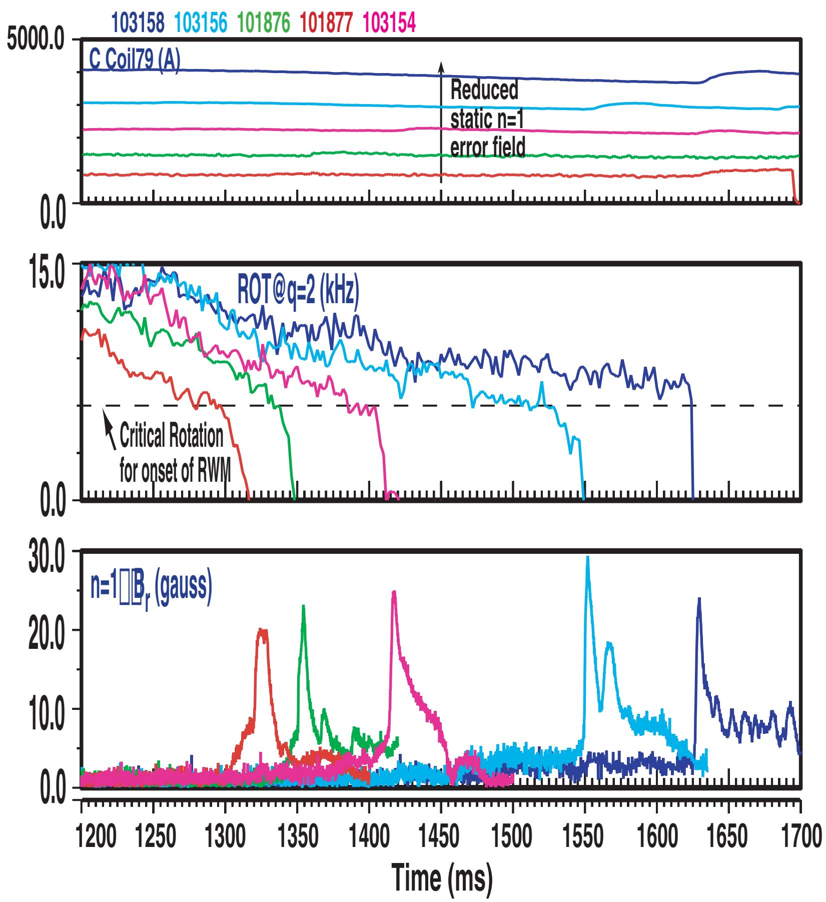
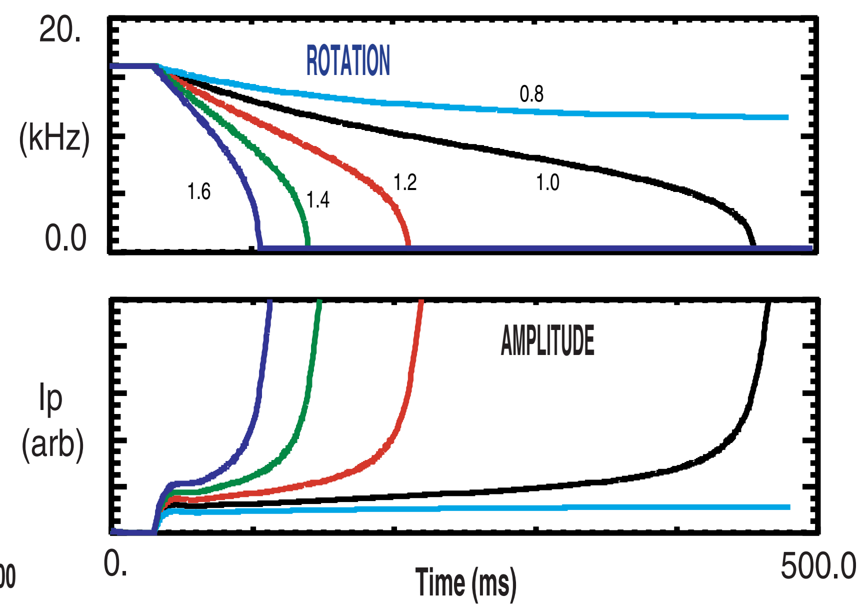


Extended Lumped Parameter Model Simulation

Magnitude of External Field: 0.8,1.0,1.2, 1.4,1.8 (arb.)



RWM Time Evolution with Extended Lumped Parameter Model Plus Angular Momentum Balance

- Example(2) : Active Magnetic Braking (Regulated rotation slowing down using mode amplitude controlled by feedback)

Steady target

RWM amplitude

coil current

$$\partial\Omega/\partial t = (\Omega_0 - \Omega)/\tau_m - C_{vis}\,\mathrm{Im}[\delta I(t)_p (\delta I(t)_{c.fb} + \delta I(t)_{c.pre.pro})]$$

momentum conf. time

$$\delta I(t)_p = G_{fb} / (1 - G_{fb})\,\delta I(t)_{c.pre.pro}$$

feedback gain

RWM Time Evolution with Extended Lumped Parameter Model Plus Angular Momentum Balance

- Example(2) : Active Magnetic Braking (Regulated rotation slowing down using mode amplitude controlled by feedback)

Steady target   RWM amplitude   coil current

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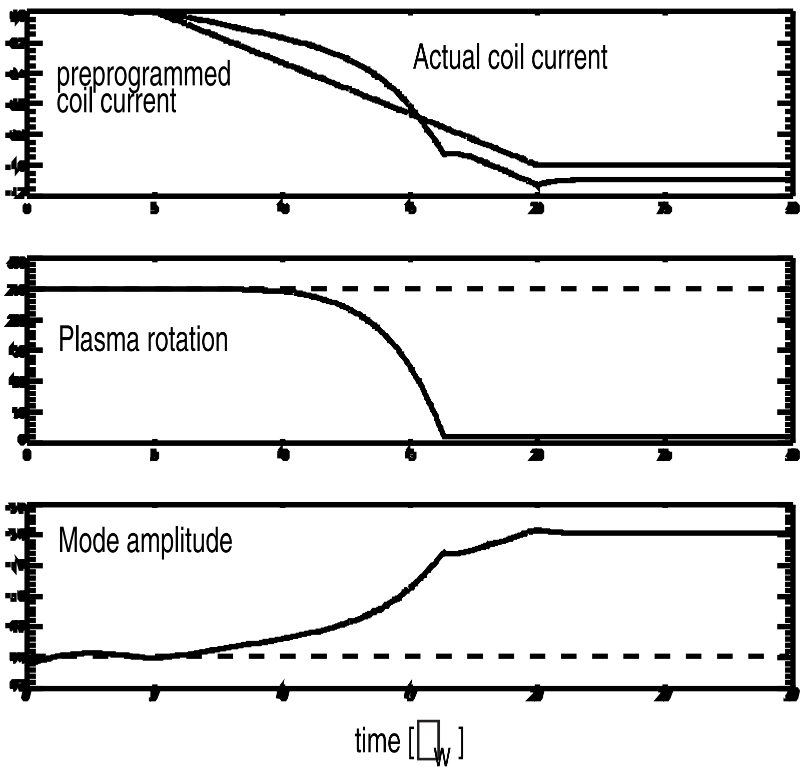
momentum conf. time

$$\square I(t)_p = G_{fb} / (1 - G_{fb}) \square I(t)_{c.pre.pro}$$

feedback gain

Simulation



RWM Time Evolution with Extended Lumped Parameter Model Plus Angular Momentum Balance

- Example(2) : Active Magnetic Braking (Regulated rotation slowing down using mode amplitude controlled by feedback)

Steady target · RWM amplitude · coil current

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momentum conf. time

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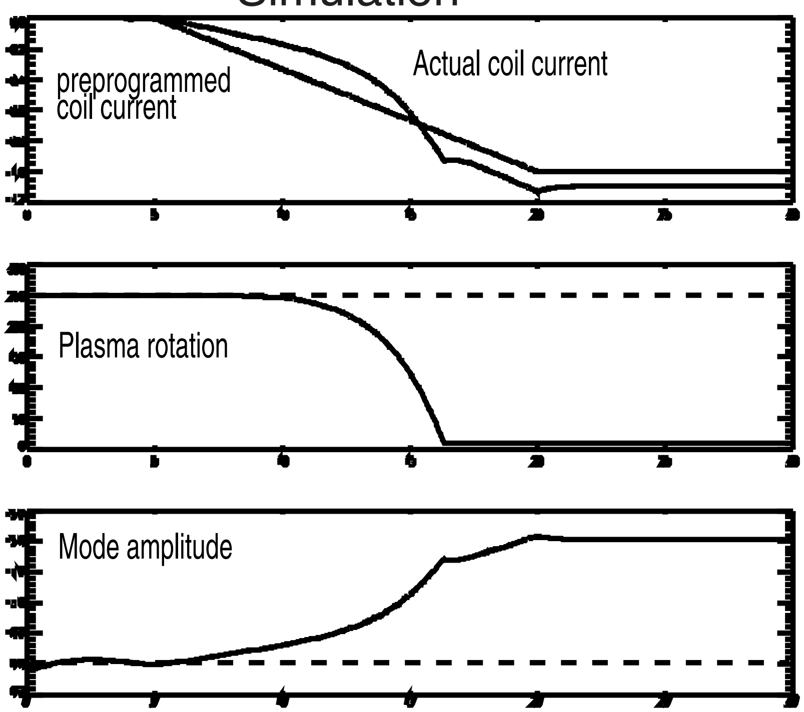
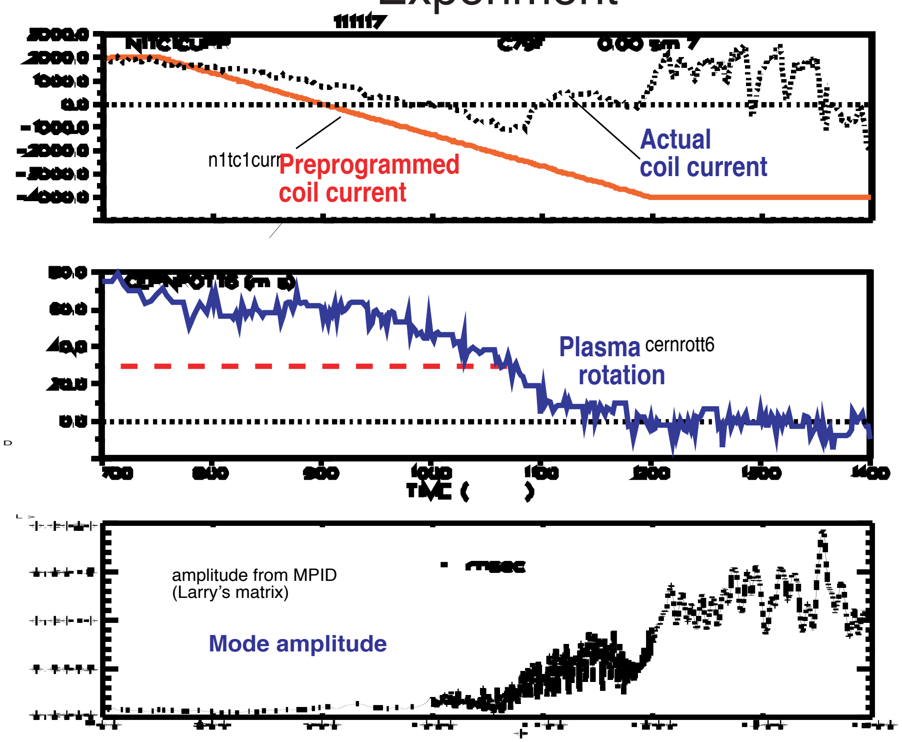
feedback gain

Simulation



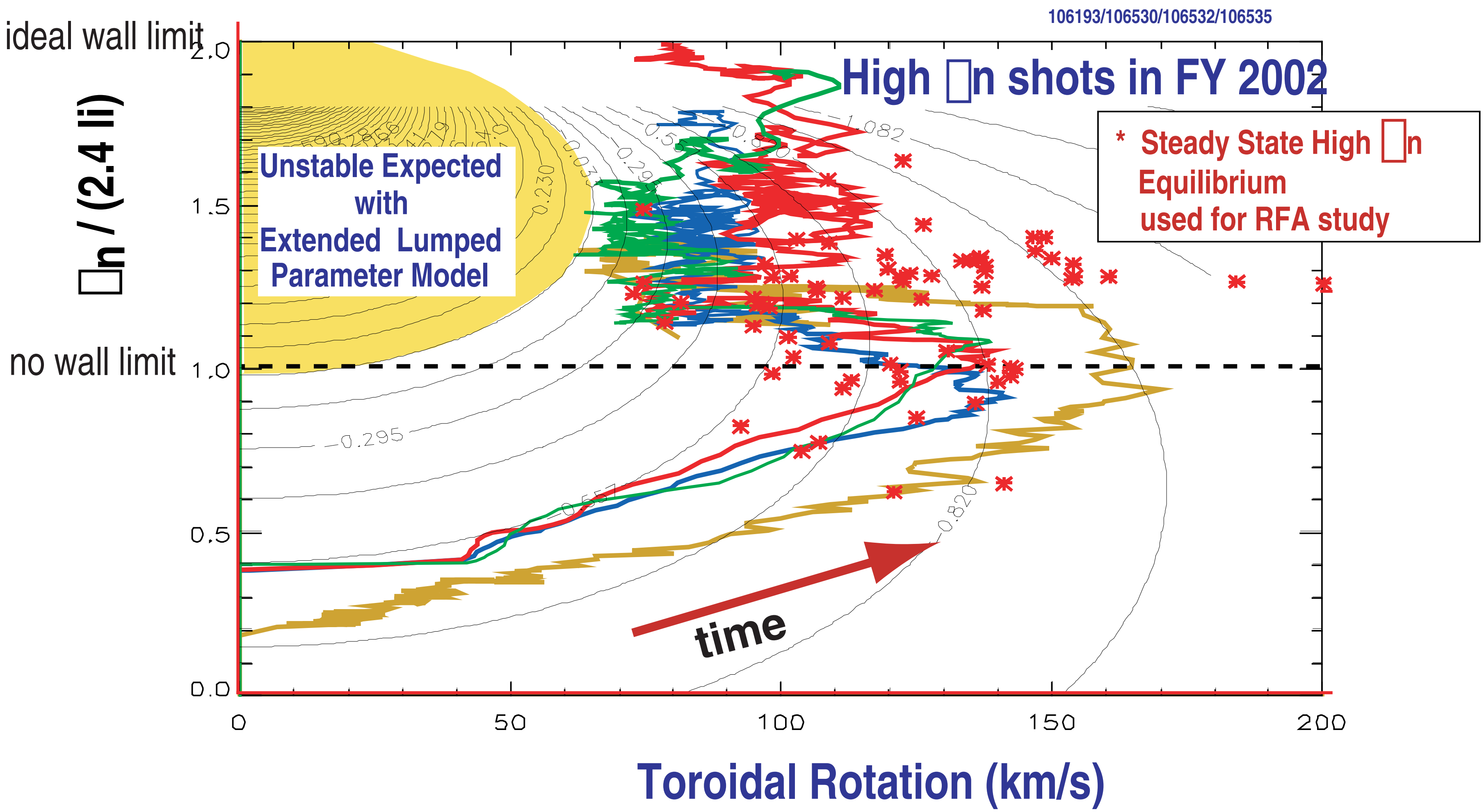
time [$\Box_W$]

Experiment

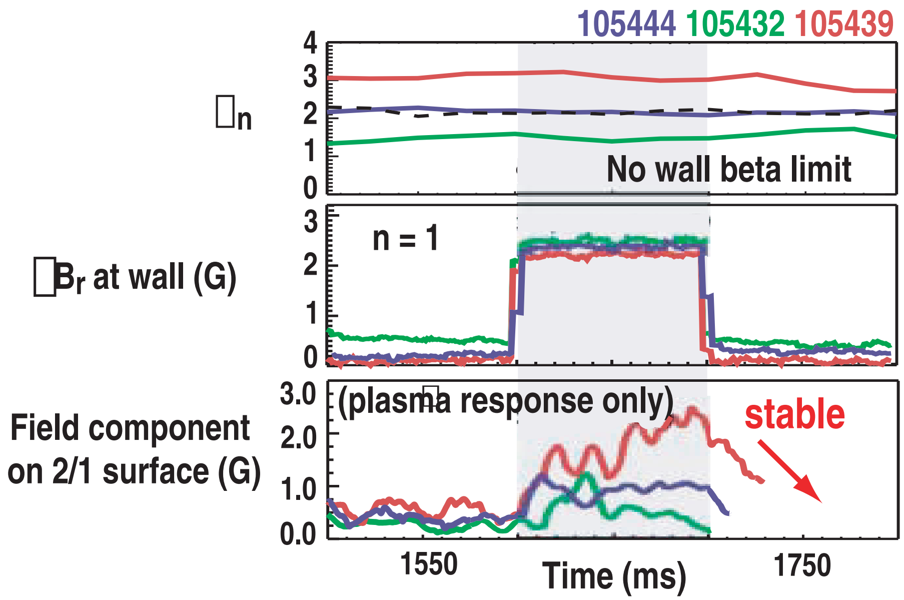


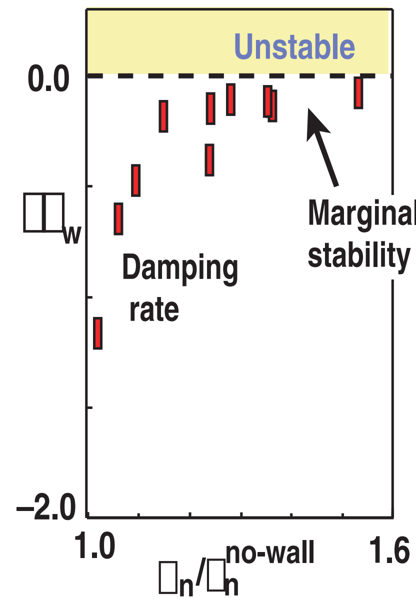
time (ms)

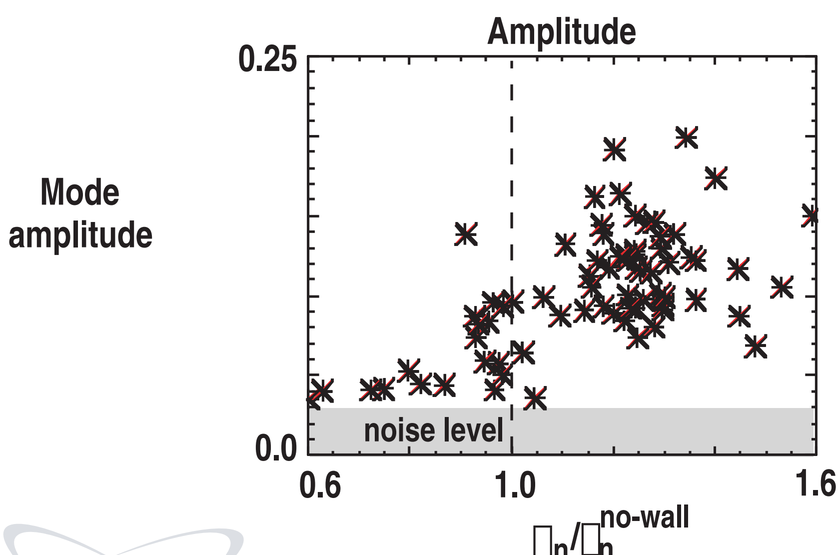
STABLE REGIME PREDICTED BY EXTENDED LUMPED PARAMETER MODEL IS CONSISTENT WITH EXPERIMENTALLY ACHIEVED PARAMETERS

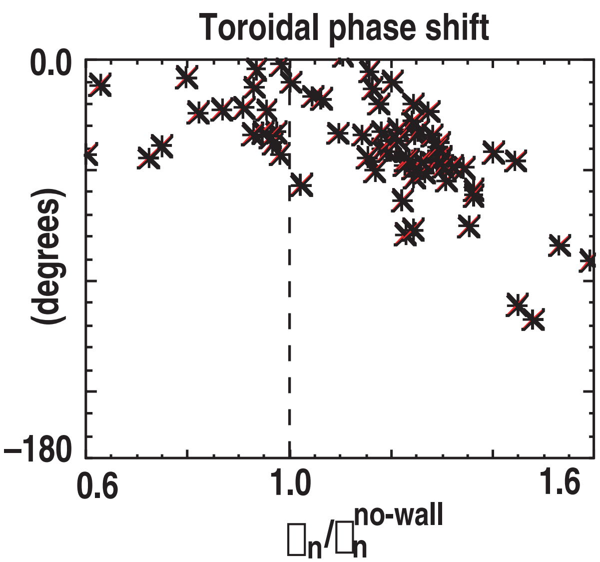


RFA AMPLITUDE AND PHASE SHIFT INCREASE WITH INCREASE OF β_n TO MARGINAL STABILITY









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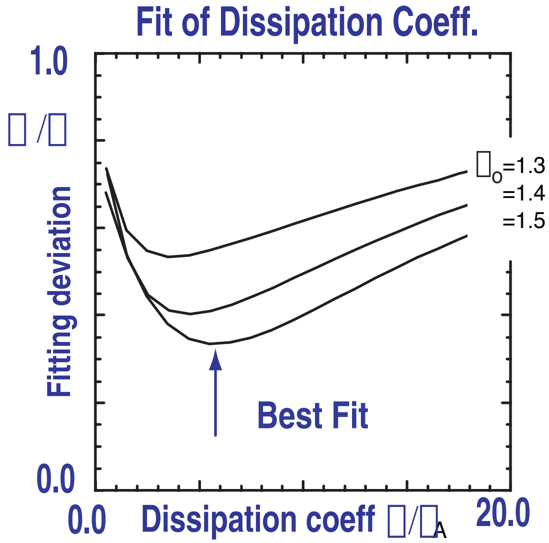
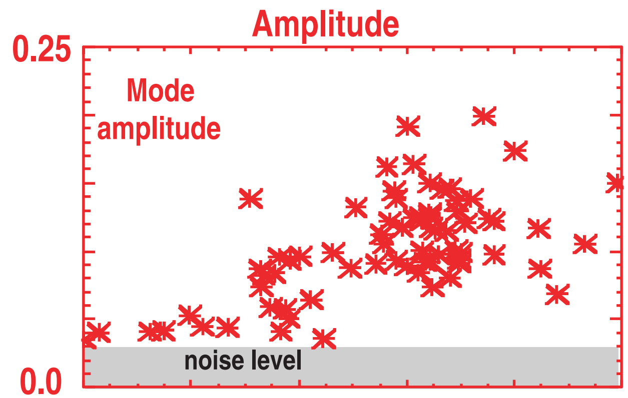
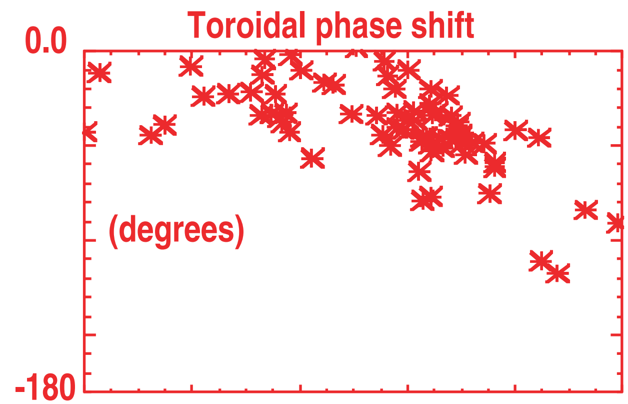
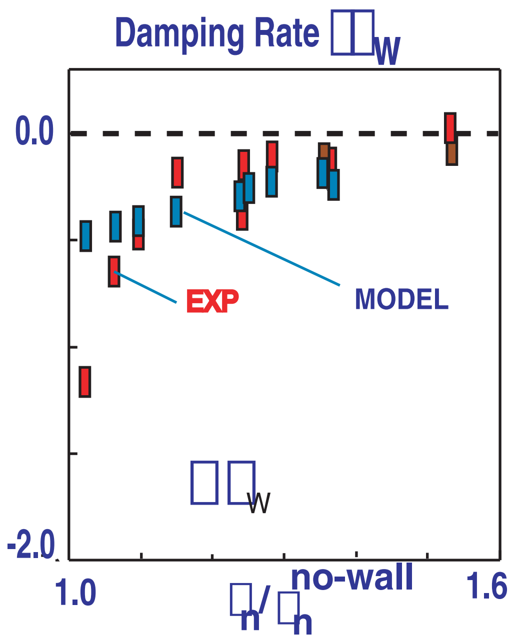
COMPARISON OF MODEL AND OBSERVATIONS SHOWS THAT DAMPING OF RFA ARISES FROM ROTATIONAL DISSIPATION



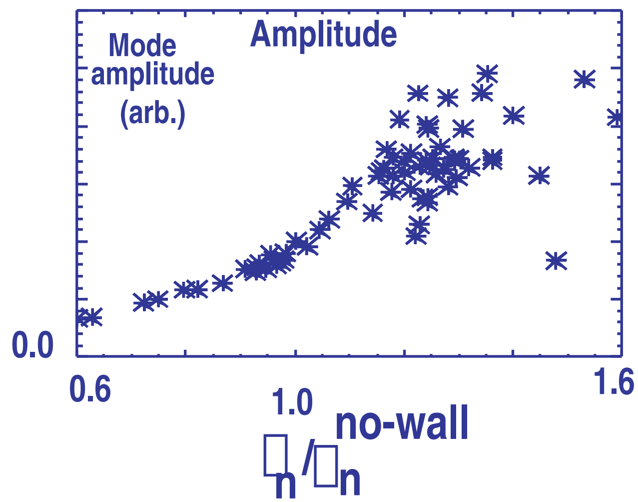
Observed Plasma Response to

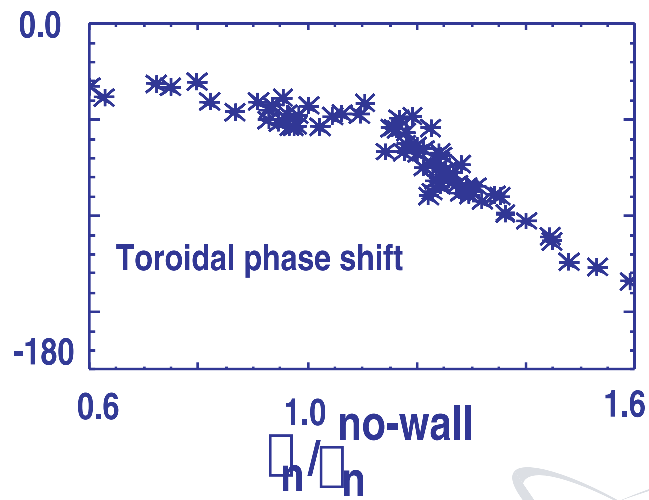






Best fit to the model with the dissipation parameter □ /□A =5.





- **dissipation / damping rate : □ □□/ | □exp□w | □ 0.5 - 2**

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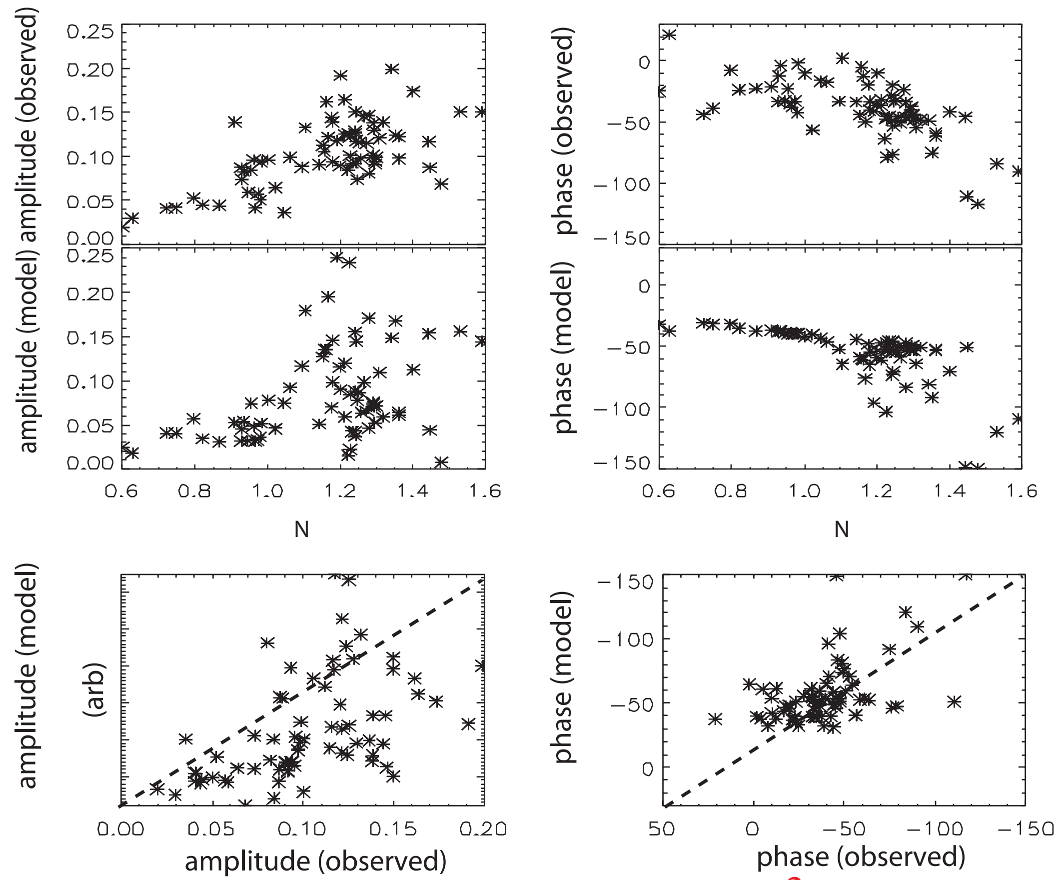
COMPARISON OF LUMPED PARAMETER MODEL WITH EXPERIMENT



$$A = \frac{[\square W_{lW}(\square=0) + \square\square^2] + \square\square\square}{[\square W_{nW}(\square=0) + \square\square^2] + \square\square\square} \quad (14)$$

Qualitative agreement is also obtained by using the present lumped parameter model with different forms for A. This indicates more details should be included in future comparisons

SUMMARY

- Extended Lumped Parameter Model includes the essence of RWM physics
- The present approach is consistent with other approaches
- Comparison with experimental results provides semi-quantitative discussion of the present RWM understandings

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