Analysis of control schemes for resistive wall modes in tokamaks

Y.Q. Liu, A. Bondeson, D. Gregoratto

Department of Electromagnetics, EURATOM/VR Fusion Association

Chalmers University of Technology, S-412 96 Göteborg, Sweden

C.M. Fransson, B. Lennartson, C. Breitholtz

Department of Signals and Systems, Control and Automation Laboratory

Chalmers University of Technology, S-412 96 Göteborg, Sweden

Y. Gribov

Plasma and Field Control Division, ITER Naka Joint Work Site, Naka, Ibaraki, JAPAN

Workshop on Active Control of MHD Stability: Extension of Performance

November 18, 2002, Columbia University

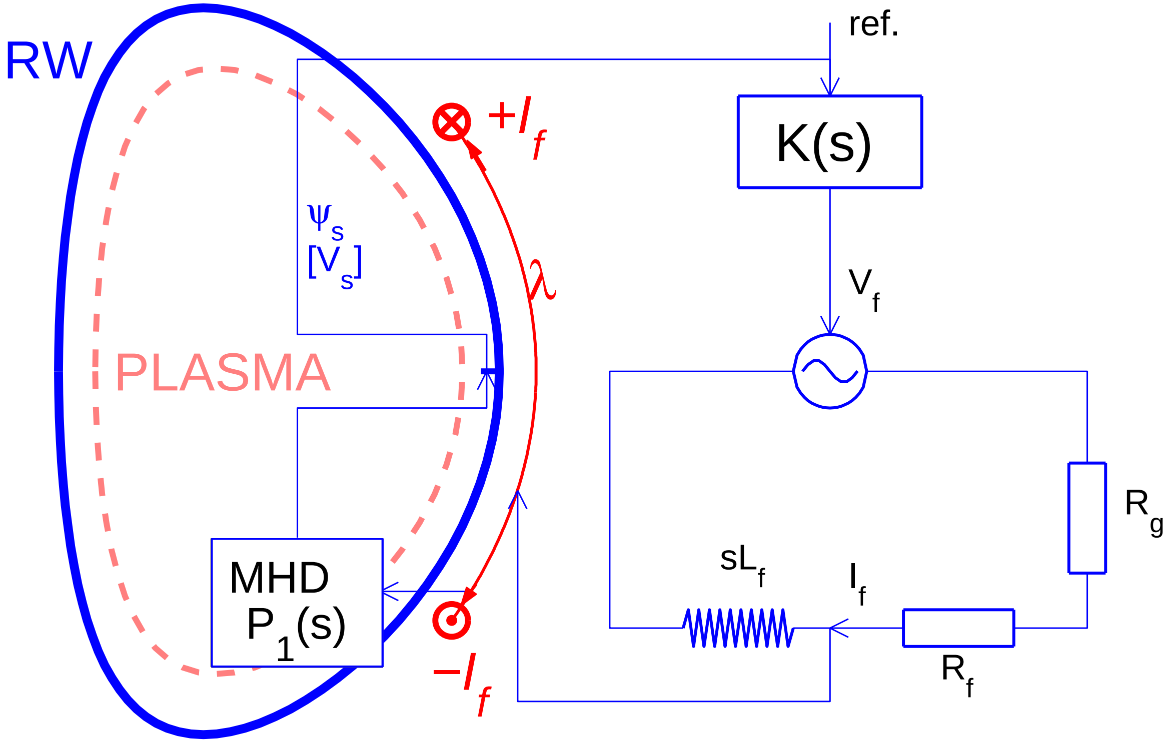
Outline

1. Plasma Response Model
   - Cylindrical Model
   - Toroidal Model
2. SISO Control and Robust Control
3. MIMO Control
4. MISO Control
5. RWM Control in ITER
6. Conclusions

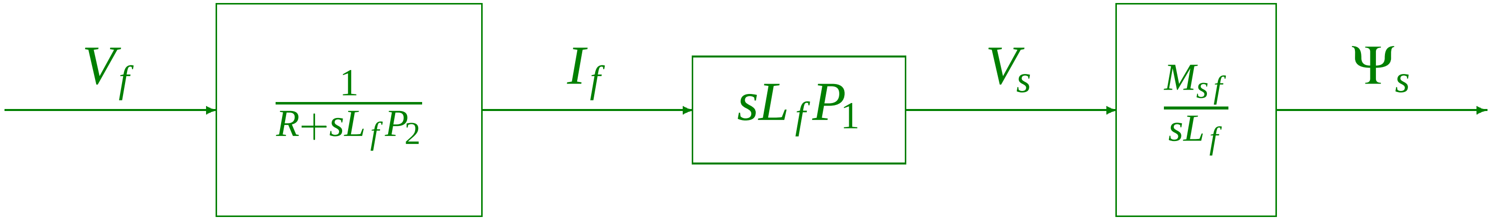
Outline

1. Plasma Response Model
   - Cylindrical Model
   - Toroidal Model
2. SISO Control and Robust Control
3. MIMO Control
4. MISO Control
5. RWM Control in ITER
6. Conclusions

RWM Feedback Control Diagram



- Input signal: current I_f or voltage V_f
- Output signal: flux Ψ_s or voltage V_s
- Plasma dynamics: $P_1(s)$ – frequency dependent transfer function
- $\lambda \equiv$ fraction of poloidal width subtended by active coil



- Current control: $I_f = -K\Psi_s/M_{sf}$

  Frequency response of the plasma-wall system to feedback currents is determined by a non-dimensional transfer function $P_1(s)$.

  Characteristic equation of closed loop $1+K(s)P_1(s)=0$.

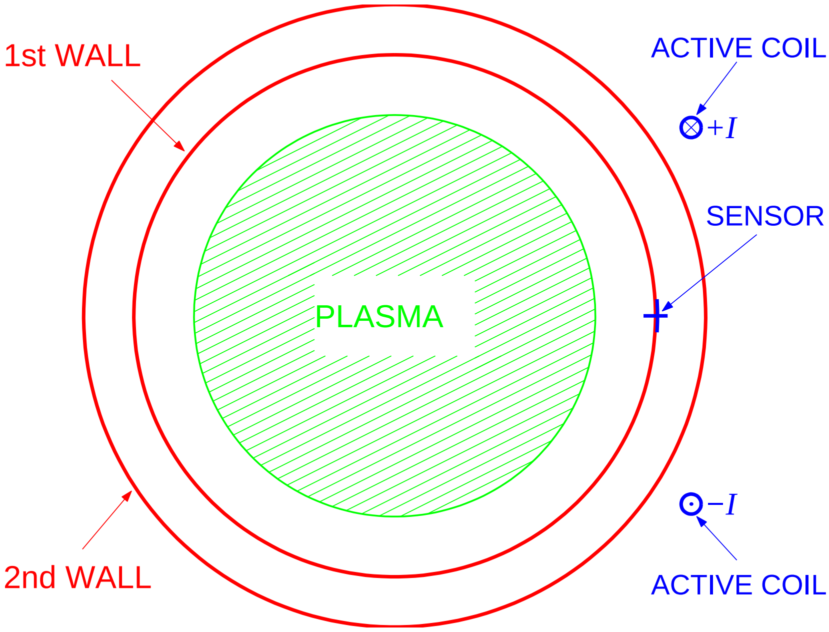
- Voltage control: $V_f = -KV_s$

  Introduce non-dimensional transfer function $P_2(s)$ for the (normalized) loaded self-inductance of the active coils.

  Characteristic equation of closed loop $1+K(s)P_1(s)/[P_2(s)+1/s\tau_f]$, where $\tau_f = L_f/R$.

- Plasma response model $\{P_1(s), P_2(s)\}$ can be constructed analytically for cylindrical equilibria, and computationally for 2D toroidal hight-β equilibria using the MARS-F code.

Cylindrical Plasma Response Model



Assume the equilibrium is ideally unstable for some m without the wall and stable with an ideal wall at $r = r_1$.

At a resistive wall $$\frac{r(b'_{r+} - b'_{r-})}{b_r} = 2s\tau_w$$

Stability index $$\Gamma_m = -\frac{1}{2}\left(\frac{rb'_{rm}}{b_{rm}} + \mu + 1\right), \quad \mu = |m|$$

Outside the wall $$b_{rm} = b_{cm}\left(\frac{r_>}{r_<}\right)^{-\mu}\frac{r_f}{r}I_f + D_m\left(\frac{r}{r_2}\right)^{-\mu-1}$$

direct field from coil wall and plasma

Cylindrical Plasma Response Model

Algebraic equations for vacuum + walls give fields on the first wall

$$\{b_{1m}, b^-_{pm}, b^+_{pm}\} = \{M_{rm}(s), M^-_{pm}(s), M^+_{pm}(s)\} b_{cm}$$

$$= \{1, (2\Gamma_m + \mu)/m, (2\Gamma_m + \mu - 2s\tau_1)/m\} M_{rm}(s) b_{cm}$$

where

$$M_{rm}(s) = \frac{m^2 (r_1/r_f)^{\mu-1}}{s^2 \tau_1 \tau_2 (1 - x^{-2\mu}) - s[\Gamma_m \tau_2 (1 - x^{-2\mu}) - \mu(\tau_1 + \tau_2 x^{-2\mu})] - \mu \Gamma_m}$$

and $x = r_2/r_1$.

Poles for M_{rm} correspond to growth-rates for RWM without feedback.

Single poloidal coil	$b_{cm} = \dfrac{\mu_0 I_f}{2\pi r_f} \sin \mu\theta_c \equiv I_f c_m$
Thin sensors at $\theta = 0$	$b_{\{r,p^\pm\},\mathrm{sens}}(s) = \sum_m b_{\{r,p^\pm\},m}(r_1) = I_f \sum_m M_{\{r,p^\pm\}m}(s) c_m$
Transfer function	$P_{1\{r,p^\pm\}m}(s) = \dfrac{\mu_0}{2\pi r_f b_{sf}} \sum_m M_{\{r,p^\pm\}m}(s) \sin \mu\theta_c$

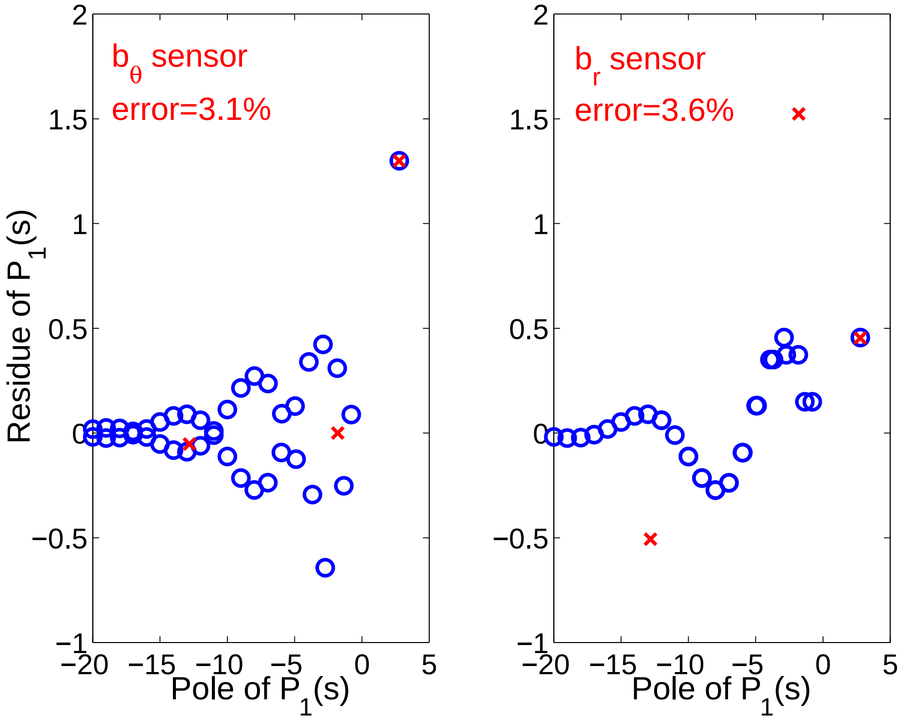
$P_1(s)$ = rational function.

Γ_m can be constructed analytically for Shafranov equilibria.

Unstable when

$$m - 1 < nq_0 < nq_a < m$$

Cylindrical Plasma Response Model



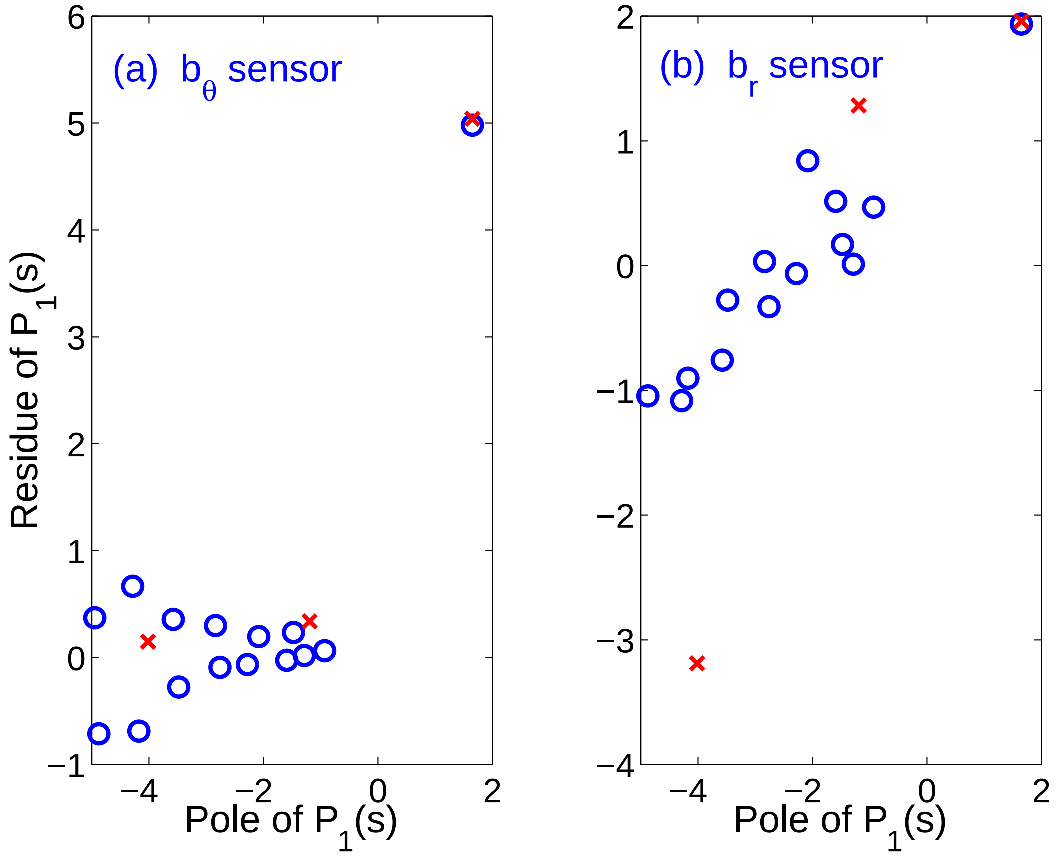
Poles and residues for cylinder with poloidal and radial sensors.

- For radial sensors, $\pm m$ modes add constructively to P_1. Convergence is slow and the stable modes can add to change the sign of $P_1(0)$.
- For poloidal sensors $\pm m$ almost cancel in P_1, which is less influenced by other m's.
- Related to mutual inductances between sensor and feedback coils.
- Result: for radial sensors, P_1 is much more influenced by the stable modes $\Rightarrow$ difficulties to control with radial sensors.

Toroidal Plasma Response Model

$$P_1(s) = \sum_{i=0}^{\infty} \frac{R_i}{s-\gamma_i}$$

$$K(s) = -1/P_1(s) \Rightarrow R_i = -ds_i/dK|_{K=0}$$



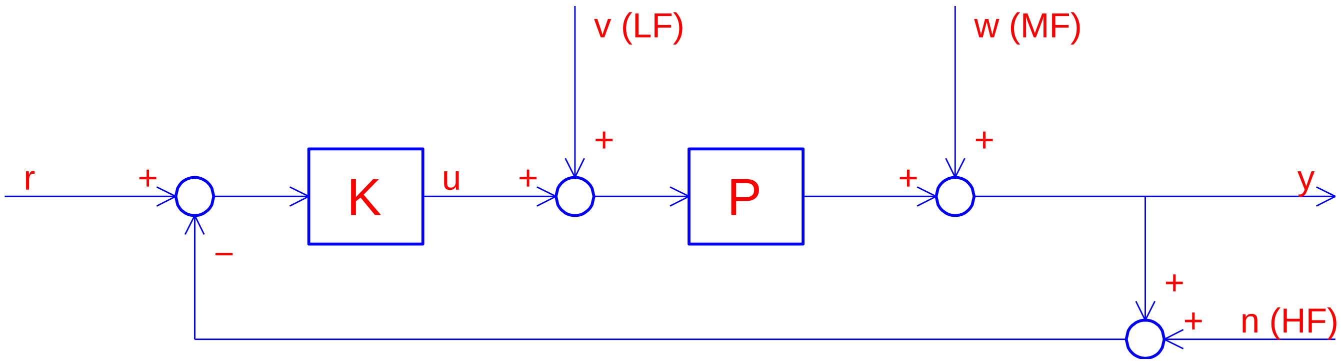
Poles and residues for high-beta tokamak.

'o' - true toroidal modes, 'x' - third order Padé.

Outline

1. Plasma Response Model
   - Cylindrical Model
   - Toroidal Model
2. SISO Control and Robust Control
3. MIMO Control
4. MISO Control
5. RWM Control in ITER
6. Conclusions

SISO Controller Design



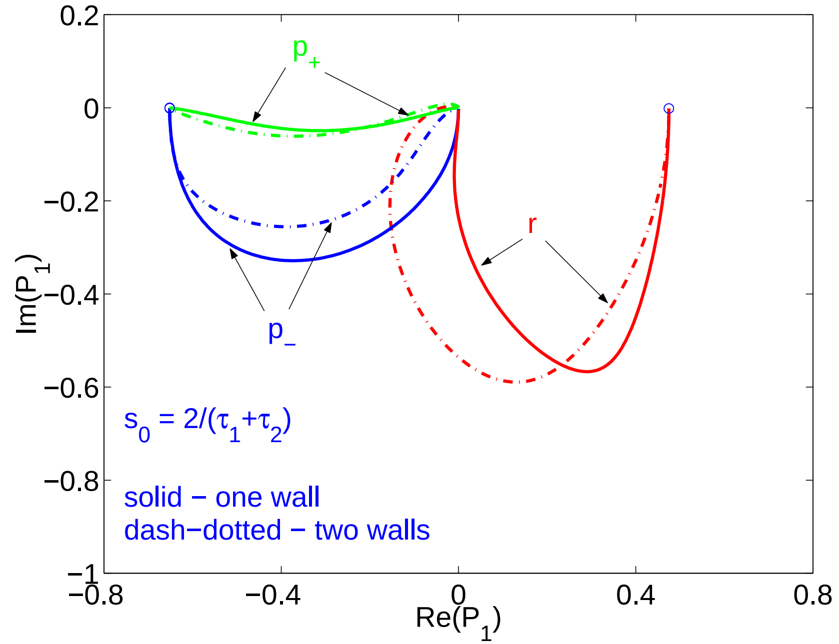
- Controller design as optimization problem with constraints.

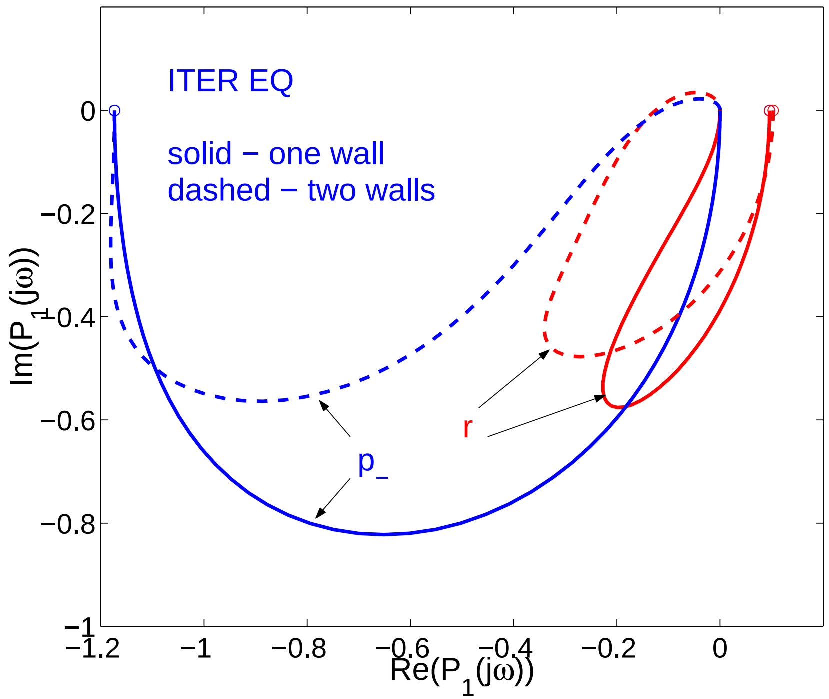
- Guarantee good control performance is by constraining the stability margins $J_S \equiv ||S||_\infty \equiv \sup_{\omega\in\Re} |S(j\omega)| \leq c_S$ and $J_T \equiv ||1-S||_\infty \leq c_T$.
  $S \equiv 1/(1+KP)$ is the sensitivity to disturbances at the output and T is the sensitivity to measurement errors.

- We minimize either the control activity $J_u \equiv ||KS||_\infty$, or the maximum voltage $V_f^{\max}$ of the amplifier time response, typically with $||S||_\infty < 2.5$ and $||T||_\infty < 2.5$.

- Can optimize, e.g., the parameters of a PID controller $K_{PID} = (K_p + K_i/s)(1+T_d s)/(1+T_d s/\xi)$.

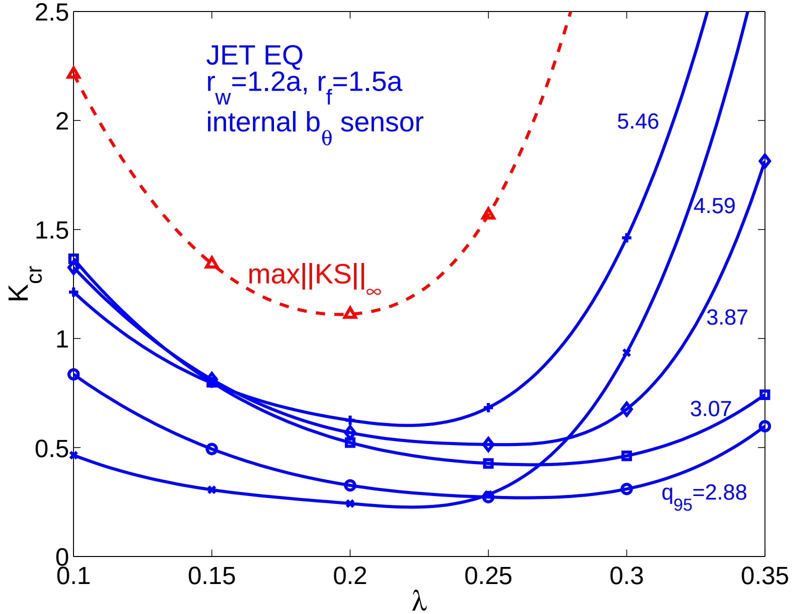
SISO System With Poloidal & Radial Sensors

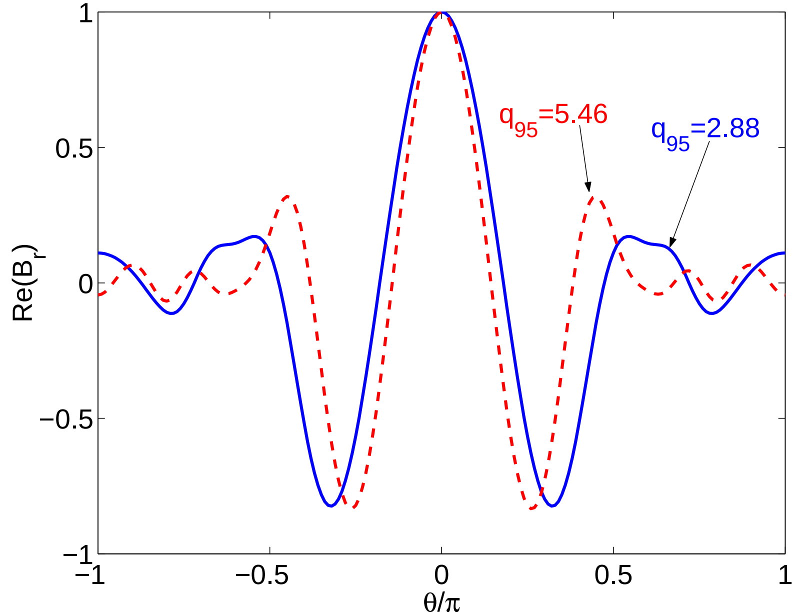




- Internal poloidal sensors give superior performance to radial sensors.
- External poloidal sensors have large phase lag, derivative action needed to achieve good control.
- Double wall also increases the phase lag, especially at high-frequency.

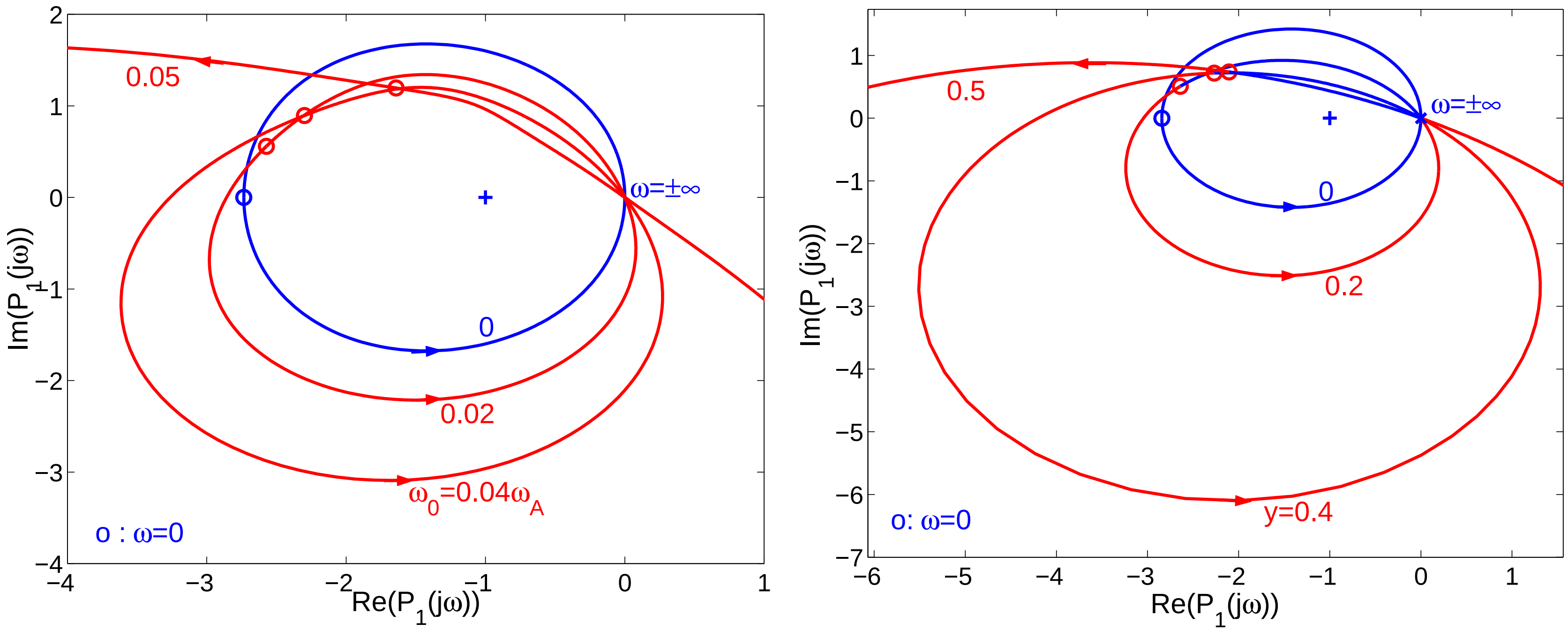
Robust Control w.r.t. Plasma Current Variation





- RWM can be stabilized for a wide range of plasma current by:
  - Single feedback coil placed at the outboard midplane
  - Internal poloidal sensor
  - Optimal coil width about 20% of total poloidal circumference, i.e. $\lambda_{opt} \simeq 0.2$.
- Reason: similarity of mode structures for different plasma currents – strongly ballooning.

Robust Control w.r.t. Toroidal Flow

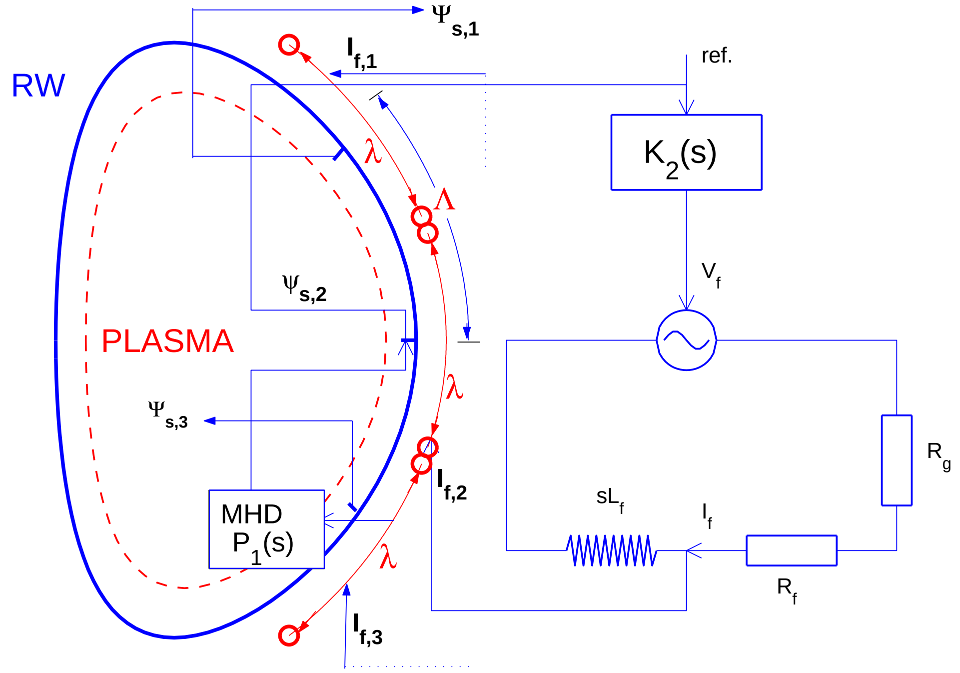


- $\omega_0/\omega_A = 0, 0.02, 0.04, 0.05 \Longrightarrow \arg(K_p^{\text{opt}}) = 0^o, -20^o, -31^o, -51^o$.
- Strong synergy when rotation and feedback push RWM in the same direction.
- A simplified cylindrical theory (single harmonic) with feedback $+$ rotation shows very similar results.

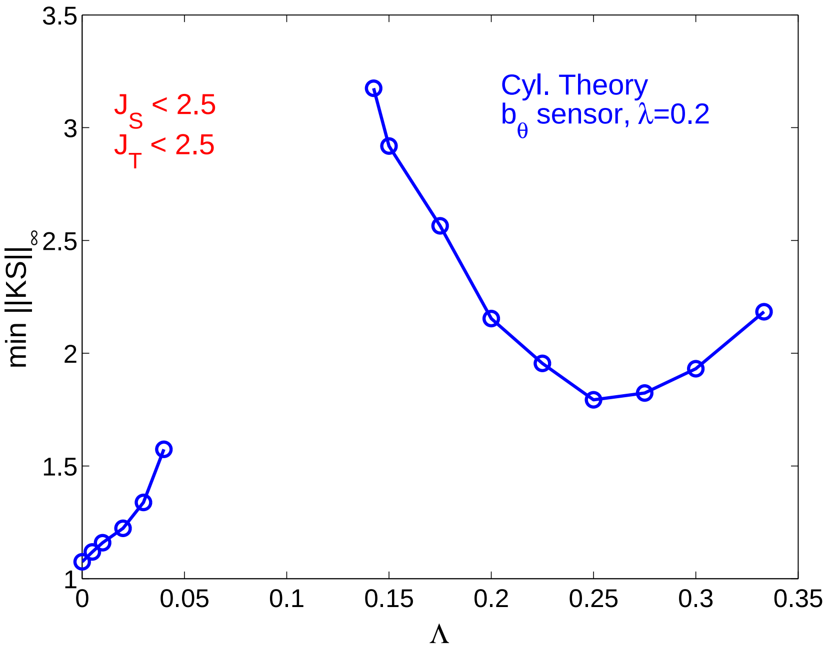
Outline

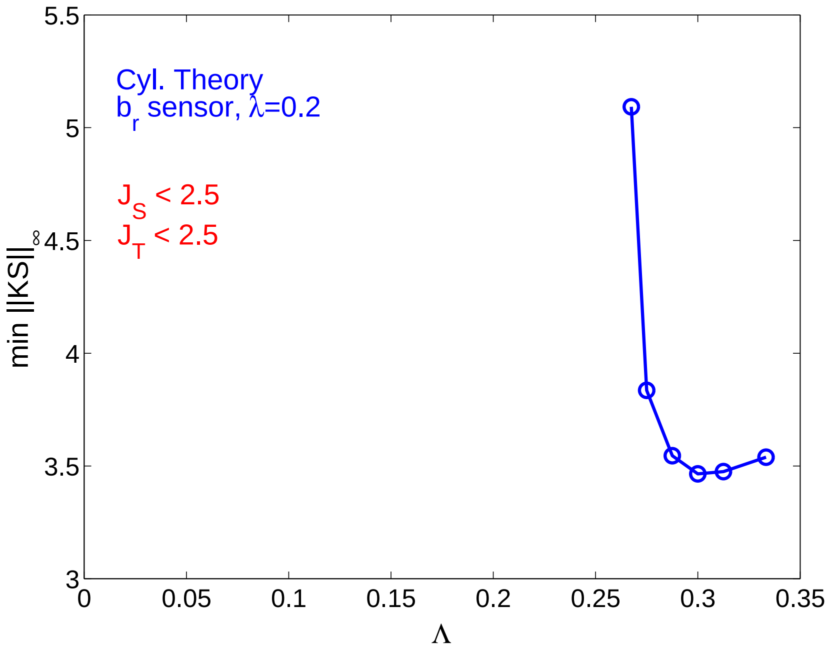
1. Plasma Response Model
   - Cylindrical Model
   - Toroidal Model
2. SISO Control and Robust Control
3. MIMO Control
4. MISO Control
5. RWM Control in ITER
6. Conclusions

MIMO Control Diagram



- In a MIMO (Multiple Input Multiple Output) system, several pairs of active and sensor coils are placed along the poloidal angle. Each pair is connected by an independent controller.
- Consider three identical controllers with PID structure $\Longrightarrow$ diagonal controller matrix.
- $\Lambda \equiv$ poloidal distance between centers of two neighboring coils.
  $\Lambda > \lambda \rightarrow$ gap between coils; $\Lambda < \lambda \rightarrow$ coils overlap; $\Lambda = 0 \rightarrow$ SISO system.





- Cylindrical theory with multiple harmonics & multiple coils.
- With poloidal sensors, single coil configuration ($\Lambda = 0$) works better than multiple coils ($\Lambda > 0$).
- With radial sensors, MIMO system improves feedback control. Good results obtained when three active coils are well separated ($\Lambda > \lambda$) $\Longrightarrow$ reduced coil coupling.

- Toroidal plasma response model for MIMO system (transfer function matrix) can also be constructed from MARS results: $\mathbf{P} = [P_{jk}(s)]_{j=1,\ldots,3}^{k=1,\ldots,3}$

$$P_{jk}(s) = \sum_i \frac{a_{ji} b_{ik}}{s - s_i}$$

- Controller optimization performed for a JET-shaped equilibrium.
- With poloidal sensors, SISO control outperforms MIMO control.

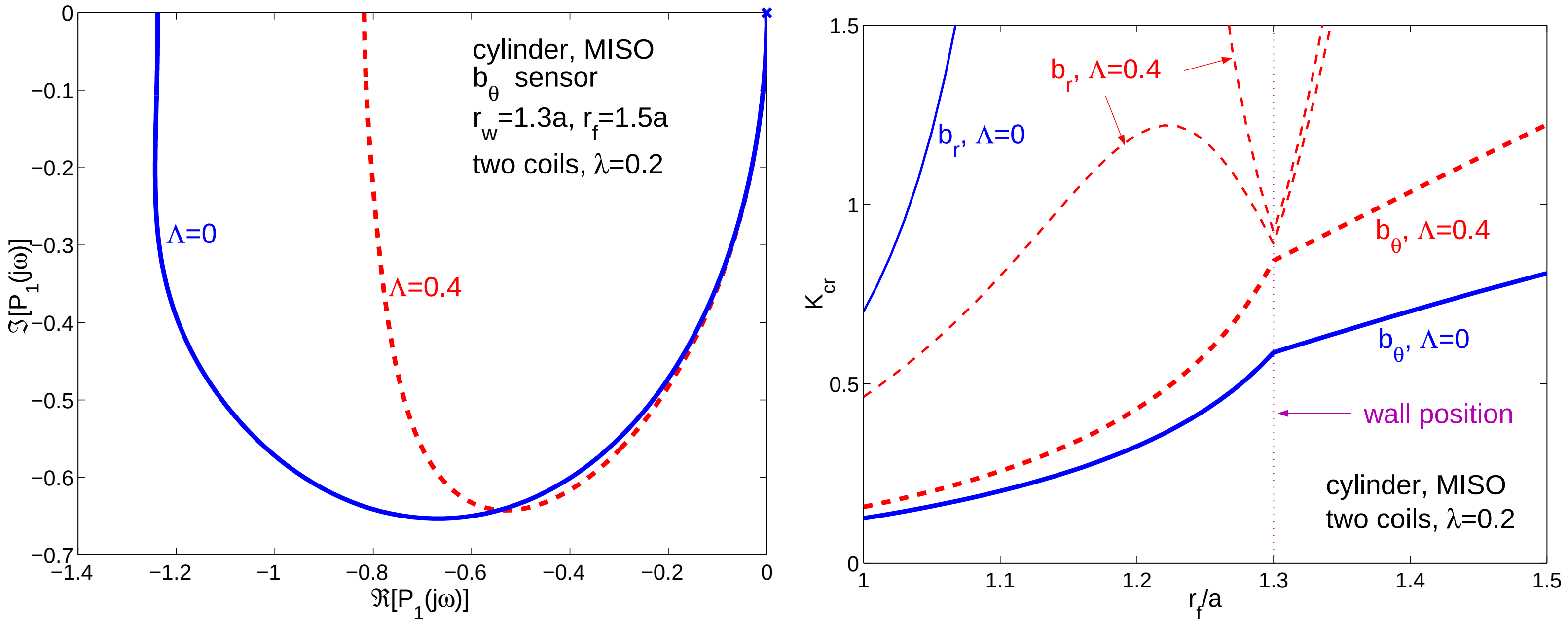
	k_p	T_d	ξ	J_S	J_T	J_u
MIMO	0.62	1.17	1.43	2.11	2.50	1.32
SISO	1.35	0.62	0.73	1.00	1.73	0.98

- With radial sensors, no controllers satisfying performance criteria were found for both SISO and MIMO systems.

Outline

1. Plasma Response Model
   - Cylindrical Model
   - Toroidal Model
2. SISO Control and Robust Control
3. MIMO Control
4. MISO Control
5. RWM Control in ITER
6. Conclusions

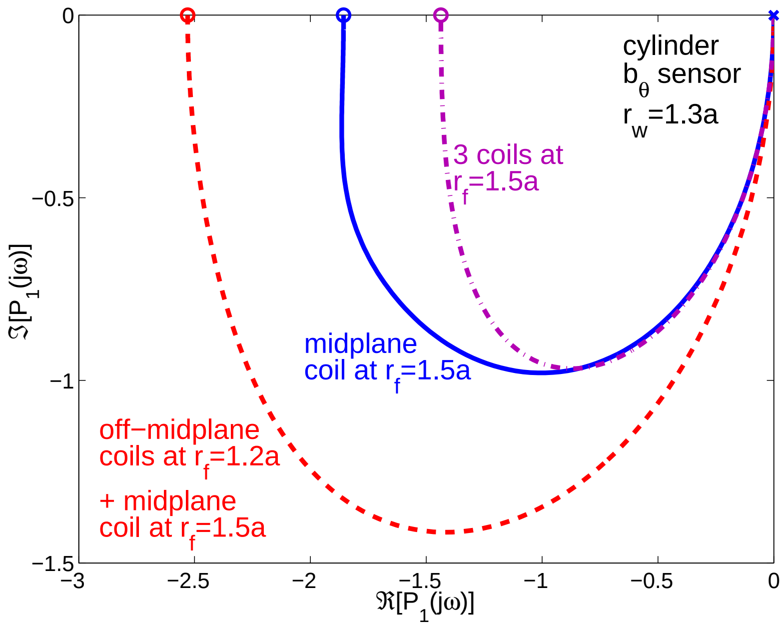
MISO Control for Cylindrical Plasmas

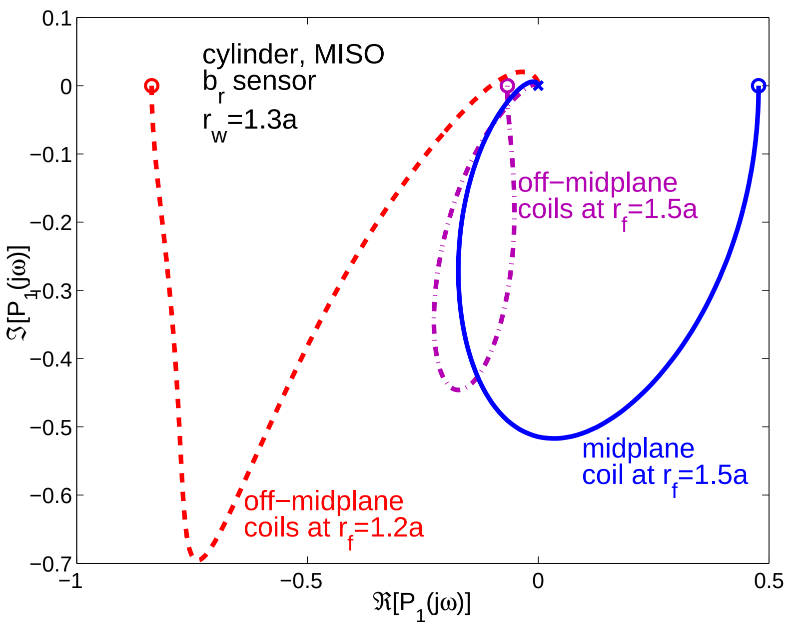


- In a MISO (Multiple Input Single Output) system, several active coils along the poloidal angle are connected to a single sensor loop at the midplane. We consider simple cases, where all the controllers are identical.
- With internal poloidal sensors, a SISO system ($\Lambda = 0$) with a single coil array at midplane out-performs MISO ($\Lambda = 0.4$) with two off-midplane coil arrays.
- With radial sensors, both MISO and SISO work only when the active coils are close to the plasma surface, and MISO works better.

MISO Control for Cylindrical Plasmas

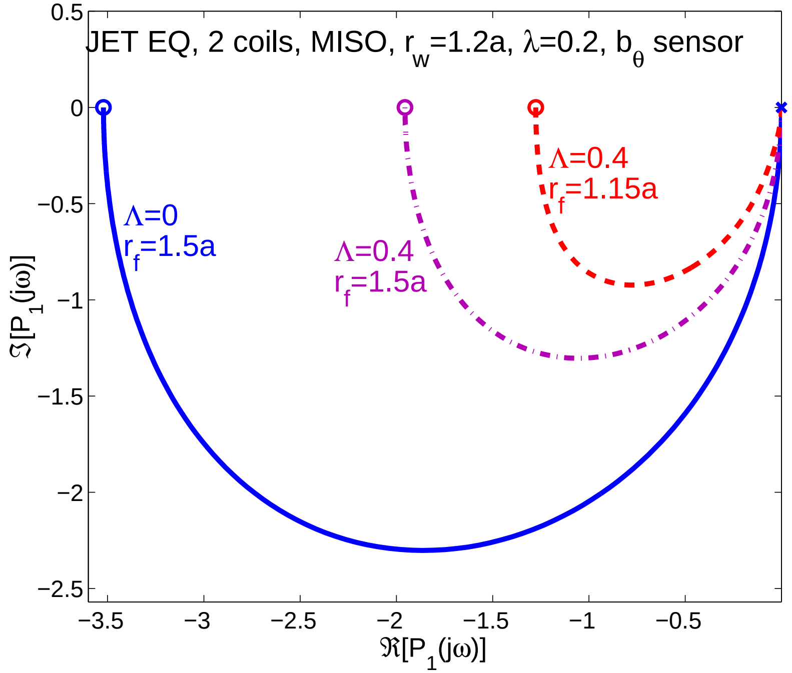
- Various configurations of MISO control studied. For all cases, sensor loops placed just inside/on the wall at the poloidal midplane.

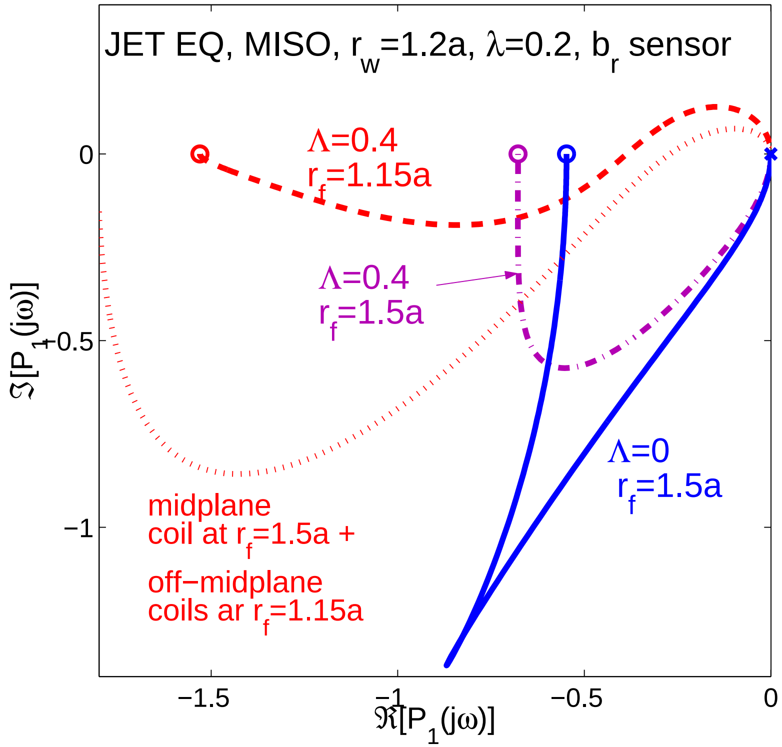




- Internal poloidal sensors give feedback system which is not sensitive to the MISO coil configurations.
- Radial sensors give better control if two off-midplane coils placed inside the wall.

MISO Control for Toroidal Plasmas



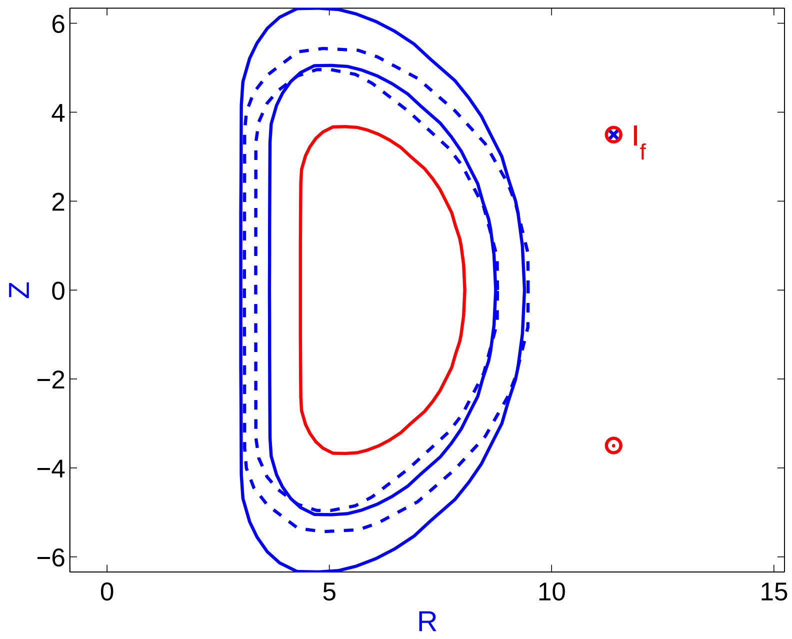


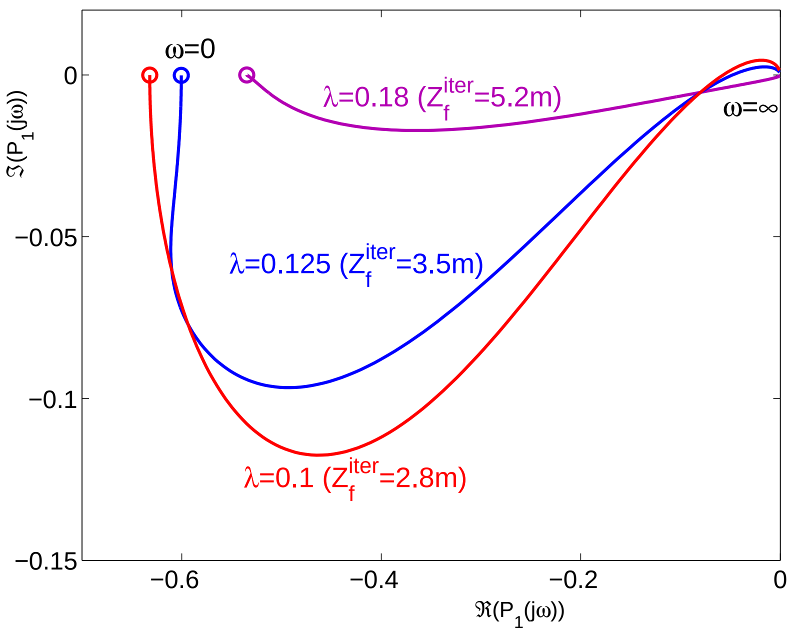
- Toroidal computations for a JET-shaped advanced equilibrium show similar results.
- Poloidal sensors work well for all configurations, but SISO system requires less total gain.
- With radial sensors, two internal off-midplane coils + one external midplane coil give stabilization with reasonable performance.

Outline

1. Plasma Response Model
   - Cylindrical Model
   - Toroidal Model
2. SISO Control and Robust Control
3. MIMO Control
4. MISO Control
5. RWM Control in ITER
6. Conclusions

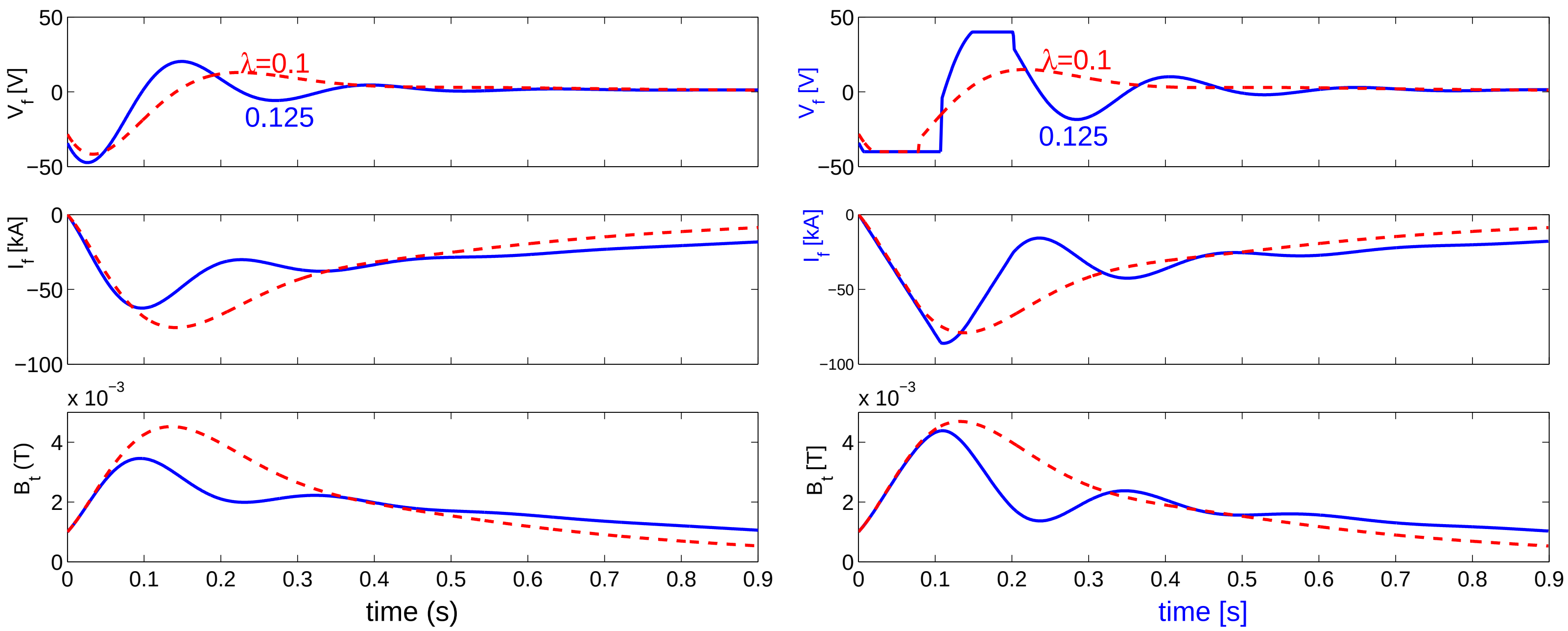
RWM Control for ITER Advanced Scenario





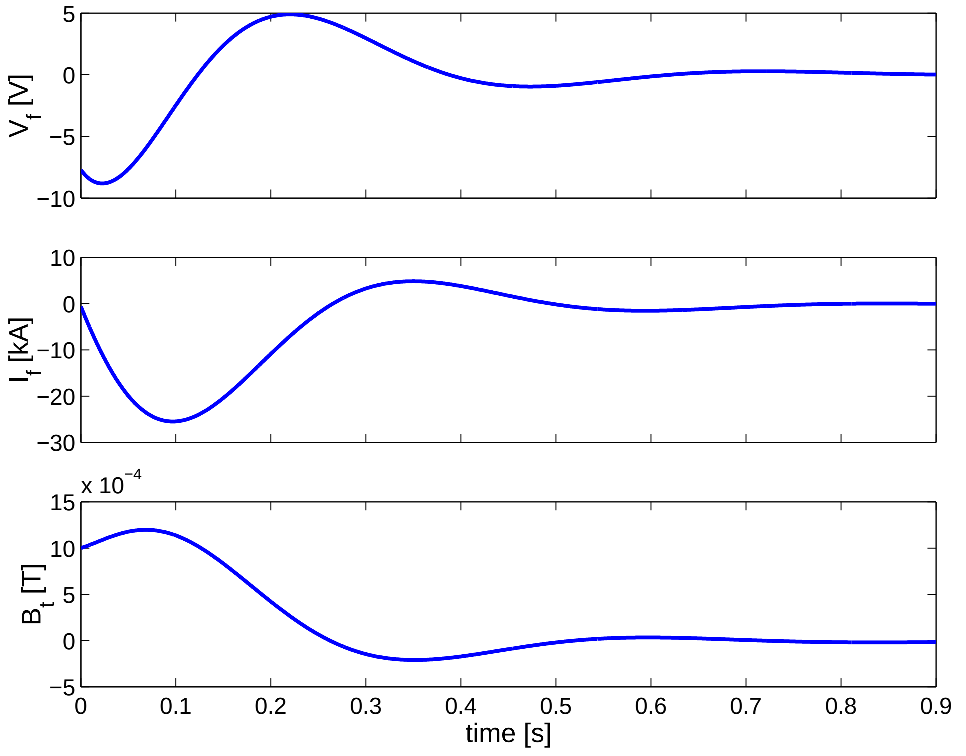
- ITER steady state Scenario 4 with 9MA current
- Up-down symmetrized equilibrium & conformal walls (solid lines)
- β_N 15% above no-wall limit ($\sim$ half way between no-wall & ideal wall limits), $r_1 = 1.375a, r_2 = 1.725a, \tau_1 = \tau_2 = 0.15$[s], $r_f = 3.0a$, design coil width $\lambda = 0.125$
- Present design works with poloidal sensors. Slightly smaller coil ($\lambda = 0.1$) gives better control.

Time Response of Feedback Controlled RWM in ITER



- With internal poloidal sensor, RWM in ITER is controlled with stability margin $J_S = 5$.
- RWM is stabilized with voltage saturation level at 40 V/turn and detection limit at 1mT.
- Faster controller (i.e. smaller J_S) gives worse control with voltage saturation.

Possible Improvement of Feedback Design



- Place the active coil closer to the outer wall and use coil with larger width λ.
- In the simulation: $r_1 = 1.3a, r_2 = 1.55a, r_f = 1.75a, \lambda = 0.2$
- Optimal controller with good performance requires less than 10 V/turn.

Conclusions

- Large gain in $n = 1$ ideal-MHD beta limit with SISO control is possible.
- SISO control with internal poloidal sensors is robust with respect to plasma pressure, current and toroidal rotation. Dynamic tuning is not necessary.
- Multiple coils along poloidal direction (MIMO/MISO) improve performance for radial sensors, but not for internal poloidal sensors.
- PID voltage control can handle RWM in present ITER advanced scenario. Improvement can be achieved by moving active coil closer to the wall or reducing its size.