COMPARISON OF SENSORS FOR RWM CONTROL IN A SIMPLIFIED ANALYTIC MODEL

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A simplified version of analytic RWM feedback models

- Purpose: exploration of key qualitative features of feedback models
  - Impact of the choice of RWM detection scheme
  - Dependence on sensor location

- Geometry-dependent quantities (self and mutual inductances) do not appear explicitly
  - Variables are perturbed magnetic flux only
  - All flux amplitudes are evaluated at the resistive wall
  - Frequencies and growth rates are scaled by $\tau_{\text{wall}}$

- Quantitative results would require explicit evaluation of the geometric quantities

This work is based on the model described in
NON-AXISYMMETRIC “C-COIL” IS USED FOR ERROR FIELD CORRECTION AND RWM FEEDBACK CONTROL

- Six midplane coils (C–coil) connected in three pairs for n=1 control
- External and internal saddle loops measure $\delta B_r$
- Poloidal field probes measure $\delta B_p$ with reduced coupling to the control coils
Feedback configuration (schematic)

- **Br sensors**: induced wall current opposes the driving field
  - coupled to control coils (normally)
  - decoupled from control coils (by analog or digital compensation)
- **Bp sensors**: induced wall current reinforces the plasma field for sensor inside wall
  - decoupled from control coils (midplane coils ⇒ purely radial field)
  - coupled to control coils (helical control coils)
  - outside the wall: flux from induced wall currents changes sign

\[
B_r, \Phi_r \sim e^{(\gamma + i\omega)t}
\]
Analytic feedback model can be reduced to simple form

\[ s - \gamma_0 + G(s) F(s) = 0 \]

Dispersion relation

\[ \Phi = \Phi_p + \Phi_{pw} + \Phi_c + \Phi_{cw} \]

Contributions to perturbed flux at the wall

\[ \Phi_{pw, cw} = - \Phi_{p,c} \frac{s}{1+s} \]

Flux from induced wall currents

\[ \Phi_p = (1+\gamma_0) \Phi \]

Plasma response model

\[ \Phi_c = -G(s) \Phi_s \]

Feedback model

\[ \Phi_s = F(s) \Phi \]

Sensor model

\[ s = \gamma + i\omega = \text{complex growth rate (in units of the inverse wall time)} \]

\[ \gamma_0 = \text{growth rate without feedback (in units of the inverse wall time)} \]

\[ G(s) = \text{gain function for amplifier-coil system} \]

\[ F(s) = \text{transfer function for sensors} \]

\[ \Phi = \text{total perturbed flux (all perturbed fluxes are defined at the wall)} \]

\[ \Phi_p = \text{perturbed flux due to plasma} \]

\[ \Phi_c = \text{perturbed flux due to control coils} \]

\[ \Phi_{pw, cw} = \text{perturbed flux due to wall currents induced by plasma, coils} \]
Sensors are defined in terms of coupling to the fluxes

Idealized Mode Detection \( \Phi_S = \Phi_P \)

Br Sensor: Smart Shell \( \Phi_S = \Phi_P + \Phi_{PW} + \Phi_C + \Phi_{CW} \)

Br Sensor: DC compensation \( \Phi_S = \Phi_P + \Phi_{PW} + \Phi_{CW} \)

Br Sensor: AC compensation \( \Phi_S = \Phi_P + \Phi_{PW} \)

– or decoupled Bp sensor outside the wall

Bp Sensor: midplane coils (decoupled) \( \Phi_S = \Phi_P - \Phi_{PW} \)

Bp Sensor: helical coils (coupled) \( \Phi_S = \Phi_P - \Phi_{PW} + \Phi_C + \Phi_{CW} \)

• Bp sensor is defined in terms of the perturbed radial flux (with 90 degree phase shift from the actual measured poloidal perturbation)

• Sensor transfer function \( F(s) = \Phi_S / \Phi \) is obtained by combining these sensor definitions with the rest of the model.
Critical gain for stability ($\gamma < 0$) depends on type of sensor

- Assuming constant proportional gain $G$:
  - Idealized Mode Detection \[ G > \frac{\gamma_0}{1+\gamma_0} \]
  - Br Sensor: Smart Shell \[ G > \gamma_0 \]
  - Br Sensor: DC compensation \[ G > \frac{\gamma_0}{1+\gamma_0} \text{ and } G < 1 \]
  - Br Sensor: AC compensation \[ G > \frac{\gamma_0}{1+\gamma_0} \text{ and } \gamma_0 < 1 \]
  - Bp Sensor: midplane coils \[ G > \frac{\gamma_0}{1+\gamma_0} \]
  - Bp Sensor: helical coils \[ G > \gamma_0 \]

- Decoupled Bp sensor (midplane coils) is equivalent to ideal sensor – can stabilize arbitrarily large $\gamma_0$ with $G \sim 1$.
- Smart shell Br sensor and coupled Bp sensor (helical coils) are equivalent – can stabilize arbitrarily large $\gamma_0$, but requires large gain as $\gamma_0$ increases.
- DC compensated Br sensor is not robust – narrow range of stable gain.
- AC compensated Br sensor (and external Bp) can only control weakly unstable modes.
Critical gain for stability ($\gamma<0$) depends on type of sensor

- Ideal sensor
- Decoupled $B_p$ sensor
- Smart Shell $B_r$ sensor
- Coupled $B_p$ sensor
- DC compensated $B_r$ sensor
- AC compensated $B_r$ sensor

Graphs showing the stability regions for different types of sensors.
Sensor’s effectiveness is related to its coupling to plasma perturbation

Idealized Mode Detection  \( \Phi_S = \Phi_P \)  

Smart Shell  \( \Phi_S = \Phi_P / (1+\gamma_0) \)  

Br Sensor: DC Compensated  \( \Phi_S = \Phi_P [1 - s / (1+\gamma_0)] \)  

Br Sensor: AC Compensated  \( \Phi_S = \Phi_P / (1+s) \)  

– or external Bp sensor

Bp Sensor: midplane coil  \( \Phi_S = \Phi_P [1 + s / (1+s)] \)  

Bp Sensor: helical coil  \( \Phi_S = \Phi_P [1/(1+\gamma_0) + s/(1+s)] \)  

Sensor response in terms of the plasma perturbation, assuming that the control coil current is determined by the feedback model.
INTERNAL $B_p$ SENSORS IMPROVE
ACTIVE CONTROL OF THE RWM

- Stable duration and $\beta/\beta_{\text{no-wall}}$ increase with internal $B_p$ sensors
- Internal $B_p$ sensors stabilize RWM with larger open-loop growth rate $\gamma_0$
- Measured open-loop growth rate is consistent with VALEN prediction

![Graphs showing $\beta_T$ and $f_{\text{rot}}$ over time, and $\gamma_0 \tau_{\text{wall}}$ vs. $\beta_N/\ell_i$.]
Finite amplifier bandwidth

- Add a single-pole high frequency cutoff to the gain function:

\[ G(s) = G \frac{\Omega_0}{s + \Omega_0} \]

- Previous results are essentially unchanged, except for an additional constraint on the maximum growth rate that can be stabilized:

\[ \gamma_0 < \Omega_0 \]

Ideal sensor,

Br Sensor: Smart Shell

\[ \gamma_0 < \Omega_0 + \frac{1}{2} \]

Bp Sensor: midplane coils

- With proportional gain only, feedback cannot stabilize a mode with a growth rate that is significantly faster than the cutoff frequency.
  - Bp sensor’s high frequency enhancement gives a modest extension of \( \gamma_0 \).
Sensor displaced away from the wall

- Assume small displacement of the sensor from the wall:
  \[ \Phi_i \propto (1 \pm \delta), \quad \delta << 1 \]
  Slab model: \[ \Phi(x) \propto \exp(\pm kx), \quad \delta = k \delta \]
  - sign for each \( \Phi_i \) depends on whether sensor moves closer or farther from the source

- Conditions for stability become
  \[
  \begin{align*}
  &\text{Br Sensor: Smart Shell (} \delta < 0 \text{)} \quad G > \gamma_0 / \left[ 1 + |\delta| (1 + 2 \gamma_0) \right] \\
  &\text{Br Sensor: Smart Shell (} \delta > 0 \text{)} \quad G > \gamma_0 / \left[ 1 - |\delta| (1 + 2 \gamma_0) \right] \quad \text{and} \quad \gamma_0 < (1 - \delta) / 2 \delta \\
  &\text{Bp Sensor: midplane coils (} \delta < 0 \text{)} \quad G > \gamma_0 / \left[ (1 + \gamma_0) (1 + |\delta|) \right]
  \end{align*}
  \]

- Br sensor inside the wall can stabilize an arbitrarily large growth rate with finite (but large) gain, \( G = 1 / 2|\delta| \)

- Br sensor outside the wall acquires an upper limit to the growth rate that can be stabilized.

- Bp sensor inside the wall has only weak dependence on sensor position
Error field amplification

- Add a static perturbation $\Phi_0$ to the model (equation is no longer homogenous).

\[ s\Phi - \gamma_0 \Phi + G(s) F(s) \Phi - \Phi_0 = 0 \]

- Consider the limits of static response ($s = 0$) and large gain ($G \to \infty$). Assume the RWM is stable ($-1 < \gamma_0 < 0$). The model becomes

\[ \Phi_s = 0 \quad \text{Ideal control in limit of high gain} \]

\[ \Phi = \Phi_p + \Phi_c + \Phi_0 \quad \text{Perturbed flux, without induced currents} \]

\[ \Phi_p = (1+\gamma_0) \Phi \quad \text{Plasma response model ($\gamma_0 \to -1$ if no plasma)} \]

- Assume the static error field is not detected directly (reference level for detection is taken after vacuum fields are established).

\[ \Phi_s = \Phi_p \quad \text{Bp Sensor: midplane coils} \]

\[ \Phi_s = \Phi_p + \Phi_c \quad \text{Br Sensor: Smart Shell} \]
Suppression of resonant field amplification depends on type of sensor

- Assuming RWM is stable (−1 < γ₀ < 0), solving for the plasma perturbation yields

  \[ \Phi_P = \Phi_0 \left(1 + \gamma_0\right) / |\gamma_0| \]
  \[ \Phi_P = 0 \]
  \[ \Phi_P = \Phi_0 \left(1 + \gamma_0\right) \]

  No feedback (Φₛ ≠ 0, Φₛ = 0)
  Bp Sensor: midplane coils
  Br Sensor: Smart Shell

- No-feedback case shows resonant behavior as \( \gamma_0 \to 0 \).
- Bp sensor reduces the plasma perturbation to zero.
- Br sensor can only reduce the plasma perturbation to a level comparable to the external error field as the plasma approaches marginal stability (\( \gamma_0 \to 0 \)).
  – could still allow significant drag on the rotation
Summary

- Simplified analytic model allows qualitative study of RWM feedback

- Radial field sensors with “smart shell” control can stabilize arbitrary growth rates.
  - Br sensors are sensitive to radial position (for better or worse performance)
  - Br sensors with compensation for coupling to control coils perform poorly.

- Internal poloidal field sensors can stabilize arbitrary growth rates with finite gain.
  Two features are essential to this superior performance:
  - Fast time response
    \(\text{(compare external } B_p: \text{ decoupled from coils but slow } \Rightarrow \text{ poor performance)}\)
  - No coupling to control coils
    \(\text{(compare coupled } B_p: \text{ fast response to plasma but performance similar to } Br)\)

- Amplifier bandwidth sets an upper limit to the growth rate that can be stabilized.

- Poloidal field sensors that are decoupled from the control coils may be superior also for feedback-controlled error correction.