Application of a Rotating Magnetic Field for Confinement Enhancement and Control of MHD Instabilities in the FRC

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Outline

Field Reversed Configuration
  Formation
  Equilibrium

Challenges for Future Concept Development
  Stability and Confinement
  Small Scale Fusion

Rotating Magnetic Field (RMF)
  RMF for MHD Stability
  RMF for Enhanced Confinement
FRC General Description

Equilibrium in flux conserver:

\[ \langle \beta \rangle = 1 - \frac{1}{2} x_s^2 \]
\[ B_e = \frac{B_{\text{vac}}}{1 - x_s^2} \]
\[ (x_s = r_s / r_c) \]

Stability: \( s/\varepsilon < 0.5 \) (empirical)
\( s/\varepsilon < 0.2 \) (MHD w Hall)

\[ s = \int_{r_s}^{R} \frac{rdr}{r_s \rho_i} \approx \frac{a_\varphi}{\rho_{\text{io}}} \]
\[ \varepsilon = l_s / r_s \]
• Field-reversed $\theta$ pinch formation is an extremely dynamic process requiring high voltages and excellent uniformity.

• High input powers allow hot (0.1 – 2.0 keV) high density ($\sim 1-5 \times 10^{21} \text{ m}^{-3}$) plasmas to be obtained.

• Best confinement observed with $D_N \sim 4 \text{ m}^2/\text{s}$ with $a_\phi = 0.06 \text{ m}$ (LSX) for $s \leq 4$
Merging of Counter-Helicity Spheromaks to create FRCs  (Y.Ono - University of Tokyo)

Swarthmore Spheromak Experiment (SSX-FRC)

Analytic doublet-CT equilibrium  
(P. Parks, GA)
FRC Produced by a Rotating Magnetic Field (RMF)

Initial axial magnetic field inside flux conserving coil is $B_a$.

RMF antenna coils external to the axial coil are shown below.

$$I_0 \sin(\omega t)$$

$$I_0 \cos(\omega t)$$
Measured FRC Particle Confinement

From past FRC experiments (Hoffman and Slough, 1993):

$$\tau_N = 3.2 \times 10^{-15} \varepsilon^{0.5} x_s^{0.8} r_s^{2.1} n^{0.6} \left( \frac{r_s^2}{\rho_i} \right)$$

For a given plasma energy $E_p = 3/2NkT_{fus}$

$$n\tau \sim r^{2.1} n^{1.6} \sim r^{2.1} (N/r^3)^{1.6} \sim r^{-2.7}$$
Energy requirements vs. plasma density for various Fusion Regimes

(for $n\tau_E = 3 \times 10^{20} \text{ m}^{-3} \text{ sec}, T_i = 10 \text{ keV, and poloidal } \beta \sim 1$)

- Small-scale FRC fusion regime based on confinement scaling observed in previous expts.

- Transport believed to be the result of edge driven microinstability (Lower Hybrid Drift) where $\tau_N \sim R^2/\rho_i$. 
Major Physics Issues for Steady State FRC Reactor

1. Continued stability at larger sizes as represented by the parameter $s$ ($\sim$ # of internal ion gyroradii)

2. Major confinement improvement at low densities

3. Current (or flux) sustainment at $\sim 10$ MA level

4. Technical ability to form larger FRCs

Reactor Issues for Small Scale FRC Fusion

Maintain stability and enhance confinement for higher fusion gain

Lower the required compression field (18 T)

Increased burn time ($\tau_{\text{burn}} \sim \tau_{\text{config}}$)
Confinement Scaling for Various FRC Experiments

Translation and Expansion

Acceleration and Compression
Translation of FRC into TCS

Experimental Measurements

MOQUI Calculated First Reflection Details
Resistive 2D MHD Calculation of FRC with Propagating Magnetic Field (0.4 T)

- FRC confinement scaling ⇒ \( n\tau \approx 10^{19} \text{ m}^{-3}\text{s} \) at \( T > 1 \text{ keV} \) after compression
PHD Burn Parameters for FRC

Assume: \( B_{\text{vac}} = 18 \, \text{T} \) (SCC), \( x_s = r_s/r_c = 0.6 \) \( \varepsilon = l_s/2r_s = 20 \)

\[ \tau_\text{N} = 3.2 \times 10^{-15} \, \varepsilon^{0.5} \, x_s^{0.8} \, r_s^{2.1} \, n^{0.6} \]

For a flux conserving vacuum wall (pipe):

\[ B_e = B_{\text{vac}} / (1-x_s^2), \quad \Rightarrow \quad B_e = 28 \, \text{T} \]

From pressure balance with \( T_e + T_i \sim 10 \, \text{keV} \):

\[ B_e^2/2\mu_0 = n \, k \, (T_e + T_i) \quad \Rightarrow \quad n_{\text{max}} = 2 \times 10^{23} \, \text{m}^{-3} \]

Set \( n\tau \sim 1 \times 10^{20} \, \text{m}^{-2} \, \text{s} \) (Lawson) \( \tau \sim 500 \, \mu\text{sec} \)

From FRC scaling given above we can solve for \( r_s = 2.8 \, \text{cm} \)

Coil Radius \( (x_s = 0.6) \quad \Rightarrow \quad (r_c = 4.7 \, \text{cm}) \quad r_c-r_s \sim 30\rho_{ie} \)

Energy in FRC plasma (ellipsoid)

\[ 3/2 \, <\beta> \, n_{\text{max}} \, kT \, (4/3\pi\varepsilon r_s^3) \quad \Rightarrow \quad E_p = 720 \, \text{kJ} \]
Instabilities Observed in FRCs

- Rotational mode is driven by centrifugal forces from particle loss and/or end-shorting of radial E field.
- Mode limits FRC lifetime to $\sim \frac{1}{2} \tau_N$

- Internal tilt starts out as an axial n=1 shift mode. Only clearly observed in recent expts. (Cothran, Brown, Schaeffer)
- Mode limits FRC lifetime $\sim 3$ axial Alfvén times.
FRC Stability (Theory)

Basic MHD
• Bad curvature produces MHD instability to tilt, interchange and ballooning modes. Compressibility stabilizes interchange modes.

MHD with FLR and Hall (two fluid effects)
• Both effects cause the ion and electron disturbances to get out of phase: Tilt growth rate reduced but not stabilized for FLR (Ishida and Steinhauer, Belova)
• Tilt stability for $s/\varepsilon < 0.2$ from Hall Term (Barnes)

Relaxed States with Flow
• High-beta possible; sheared flow necessary in relaxed states
  – find proper relaxation principle (Steinhauer, Ishida, Mahajan, Yoshida, Dasgupta)

FRC Stability (Experiment)
• FRC tilt stable for $s/\varepsilon < 0.5$
• FRC grossly stable except for rotational modes
FRC Stability in PHD Fusion Regime

\[ s/\varepsilon \text{ parameter} \]

\[ s = \varphi / (2\pi r_s B_e \rho_{ie}) \]

FRC poloidal flux:

\[ \varphi = \pi r_c^2 B_e (x_s/\sqrt{2})^{3.5} \]

\( \varphi = 9.5 \text{ mWb} \)

Empirically from LSX data

\( s/\varepsilon < 0.5 \)

for gross stability AND good confinement

PHD FRC: \( s/\varepsilon = 2.4/20 = 0.12 \)

Recent 3D MHD Results

Internal Tilt Stability

(Barnes and Milroy)

PHD FRC in MHD Stable Regime
FRC Stability at \( s/\varepsilon \sim 0.5 \)

- Much \( n=1 \) activity during formation, but dies away if distortions not too large
- Formation distortions not primarily related to \( s \), but to axial dynamics and low axial viscosity (low \( T_i \))
- Rotational mode (mostly \( n=2 \)) grows from equilibrium when multipole fields not used

Data taken on LSX at \( s \sim 3.5 \)
Stabilization of n=2 Rotational mode
(First demonstrated at U. of Osaka w quadrupole)

- Significant wall contact due to plasma flow onto static multipole field
- Large plasma distortion (quadrupole) from static field created turbulent boundary in LSX at s>2
RMF Current Drive

- ‘Drag’ Electrons Along With Rotating Radial Field
  - Must have $\omega_{ci} < \omega \ll \omega_{ce}$ for electrons, but not ions, to follow rotation
- Electrons Magnetized on Rotating Field Lines ($\omega_{ce}\tau >> 1$)
  - Necessary for efficient current drive
  - Absolutely necessary for rotating field penetration
Control of Diffusive Losses with RMF

Gen. Ohm’s Law (θ comp): \[ E_\theta = \eta j_\theta - \frac{1}{ne} u_r B_z - \frac{1}{ne} \langle j_z B_r \rangle \]

From synchronous electron motion: \[ j_\theta = -ne \omega r \]

From first order term for screening current \( j_z \)

\[ \langle j_z B_r \rangle = \left[ 2 \left( \frac{v_{ci}}{\omega_{ce}} \right) ne \omega r \right] B_\omega \langle \cos^2 \theta \rangle = \eta j_\theta ne \]

This drive term opposes the dissipative \( \eta j_\theta \) term, and stops diffusive losses (\( u_r \sim 0 \)) in steady state (\( E \sim 0 \)): 

![Diagram of electron motion](attachment:image.png)
FRC Confinement by RMF Field

- Axial currents inside FRC limit RMF ~ 2 cm penetration past separatrix
- Strong radial flow $u_r B_z \sim \langle j_z B_r \rangle / \varepsilon n$ is obtained
- Minimum energy and particle loss observed under these conditions
Internal Profile Measurements from STX Experiment

![Graph showing internal profile measurements from STX Experiment.](image)

- **Black** – conventional FRC
- **Red** – RMF driven FRC

- Magnetic Field (G) vs. Radius (cm)
- Density ($10^{18} m^{-3}$) vs. Radius (cm)

Parameters:
- $B_{\text{axial}}$
- $n_e$
- $r_s$
- $r_w$

Additional Information:
- Time = 300
- Shot = 4957
Plasma Density in FRC Jet During Sustainment

Position of triple probe in jet
Internal $B_z$ Profile During RMF (0 to 0.5 ms) on STX-HF

For RMF drive the magnitude of $B_\omega$: 
$$B_e \approx \left( \frac{\mu_0 \omega}{9 \eta} \right)^{1/2} r_c B_\omega$$

In terms of antenna power $P_\omega$: 
$$B_e \sim \left( \frac{P_\omega}{r_c} \right)^{1/2}$$
Density profile from Langmuir probe at $Z= -10 \text{ cm}$
Enhanced particle confinement from RMF

Density profile at $z = -10$ cm from Langmuir probe

For RR profile:

$$\tau = \frac{\mu_0 r_s^2}{16 \eta_\perp}$$

For size ($r_s \sim 0.05$) resistivity inferred for STX-HF:

$$T_e \sim 10 - 30 \text{ eV} \Rightarrow \eta_\perp \sim 60 - 30 \mu\Omega\text{-m}$$

$$\tau = 4 - 8 \mu\text{sec}$$

Ratio of line density:

$$n_L(t = 300) \quad \frac{\square}{n_L(t = 40)} \quad 25 \quad \Rightarrow \quad \tau_n = 25\tau_{\text{transit}} = 120 \mu\text{sec}$$

RMF must provide $>>$ classical confinement
RMF CT Formation

End-on Filtered (510-600 nm) Plasma Emission

Plasma: \( r \approx 0.05 \) m
\( 1 \approx 0.25 \) m

Imacon fast framing camera

\( \sim 5 \) m

2 \( \mu \)sec exp

NO RMF

RMF On at \( t=0 \)
Control of Rotational Modes with RQMF

- Magnetized Cascaded Arc Source
- Flux Conserving Rings
- Solenoidal Magnet
- Boron Nitride Washers
- Molybdenum Washers
- Cathode
- Solenoidal Coil
- Gas Feed
- Plasma Discharge
- Anode
- Rotating Quadrupole Antenna
- 8 Coils per phase
- Copper tubes
- Vacuum wall
- $R_{quad} = .17 \text{ m}$
- $R_{rings} = .14 \text{ m}$
- $R_{wall} = .135 \text{ m}$
Magnetic field lines and density maps from numerical simulations of RMF current drive.

Dipole: $A_{zn} = \sum_n \left\{ \alpha_n \left( \frac{r}{R} \right)^n + \beta_n \left( \frac{r}{R} \right)^{-n} \right\} e^{in\theta}$

Quadrupole: $B_\theta = -\sum_n \left( \frac{\alpha_n n}{R} \left( \frac{r}{R} \right)^{n-1} - \frac{\beta_n n}{R} \left( \frac{r}{R} \right)^{-(n+1)} \right) e^{in\theta}$

$B_r = i \sum_n \left( \frac{\alpha_n n}{R} \left( \frac{r}{R} \right)^{n-1} + \frac{\beta_n n}{R} \left( \frac{r}{R} \right)^{-(n+1)} \right) e^{in\theta}$

The parameter $a_n$ is from external currents, and $\beta_n$ is from internal currents. If there is no plasma, $\beta_n = 0$, and $B_{r,\theta} \sim r^{n-1}$.

- $n=1 \rightarrow$ rotating dipole (normal RMF)
- $n=2 \rightarrow$ rotating quadrupole (current expt.)
- $n=3 \rightarrow$ octopole field (may be explored)
Transport inhibited by lack of penetration control

Radial penetration naturally limited by $1/r$. Can also be controlled by amplitude feedback
Initial Experiments with Quadrupole Current Drive

Magnetic Field Time Histories

- Flux at vacuum wall
- \( B_z \) (mT)
- \( B_z \) (r) (mT)
- \( B_\omega \) (mT)
- Field at inner wall

\( \omega = 2.5 \times 10^6 \text{ rad/s} \)

B(r) from Internal Magnetic Probe Array

- Initial Experiments with Quadrupole Current Drive
- Magnetic Field Time Histories
Summary

• Small-scale FRC fusion reduces stability and confinement issues to those observed in past FRC experiments

• Rotating Quadrupole field Stabilizes FRC to rotational modes
  ➢ Does not allow for plasma flow to wall
  ➢ Does not distort FRC or drive edge turbulence

• Rotating magnetic field enhances FRC confinement
  ➢ Drives an inward diffusion that opposes the normal diffusive decay
  ➢ Reduces density gradient at separatrix eliminating the drive from microinstabilities (LHD)
RMF Induced Ion Rotation

Ion Spin-up Time: \[ \tau_s = \frac{m_i}{m_e v_{ei}} = \frac{m_i}{n e^2 \eta_\perp} \]

For \( n \ (10^{19} \text{ m}^{-3}) \), \( \eta \ (\mu \Omega \text{-m}) \), \( m \ (m_H) \)

\[ \tau_s = \frac{12.5}{n_{19} \eta_{\mu\Omega}} \text{ msec} \]

STX-HF: \( n_{19} \sim 5 \ (10), \ \eta \sim 60 \ (30), \ \Rightarrow \ \tau_s \sim 40 \mu\text{sec} \)

Ion spin-up momentum is lost in \( \tau_N \sim 100 \mu\text{sec} \). The fraction of spin-up, \( \alpha \ (= u_{i\theta}/u_{e\theta}) \):

\[ \alpha = \left[ 1 + \frac{\tau_s}{\tau_N} \right]^{-1} = 0.7 \]

From radial component of gen. Ohm’s law:

\[ u_\theta = -\frac{E_r}{B_z} + \frac{1}{n e B_z} \frac{dp_i}{dr} \]

Ion diamagnetic drift

Opposite dir. to spin-up
Shift of He II Line Center as a Function of Impact Parameter

\[
\frac{v_\theta}{v_D} \sim 0.35
\]
Enhanced Confinement Using Neutral Beams on FIX

NB injection geometry on 80-cm diameter FIX confinement chamber

Simple 15-25 kV NB Injector

Resultant ion orbits in low flux FIX FRC

Measured enhanced lifetime
RMF Confinement and Drive from Steady Coils

One of two loops for RMF in stationary FRC where $B(t) \sim \sin \omega t$

One of several pair of current loops where $B(t) = \text{const.}$

In frame of FRC traveling at velocity $v_{FRC}$, the transverse field rotates at a frequency $\omega = 2\pi v_{FRC}/L$:

$$\omega = 9 \times 10^6 \text{ and } L = 0.10 \text{ m} \Rightarrow v_{FRC} = 1.4 \times 10^5 \text{ m/s}$$
Wall Protection from RMF Induced Flow

“Conventional” CT
With Steady Transverse (Octopole) Magnetic Field
PHD Fusion Reactor Summary

• Straight forward extrapolation of demonstrated FRC formation and acceleration techniques
• Simple linear system that could employ superconducting magnets
• Easily varied fusion output power (10s of MW not multi-GW)
• Low cost development path to demonstration ~ 10 M$ not 10 G$
• Converter, burn chamber, accelerator and formation sections
  • well separated.

• Direct electric power conversion possible with flux compression from expansion of fusion heated FRC
From energy conservation:

\[ v_{\text{FRC}} = \left( \frac{5kT_{\text{fus}}}{m_{\text{DT}}} \right)^{1/2} \sim 1 \times 10^6 \text{ m/s} \]

For constant acceleration \( a \) (\( 2 \times 10^{10} \text{ m/s}^2 \)), accelerator length \( L_{\text{acc}} \):

\[ L_{\text{acc}} = \frac{v_{\text{FRC}}^2}{2a} \sim 25 \text{ m} \quad \tau_{\text{acc}} \sim 50 \mu\text{sec} \]

Burn chamber length:

\[ L_{\text{burn}} \leq v_b \tau_E \sim (2 \times 10^4 \text{ m/s})(5 \times 10^{-4} \text{ s}) \]

\[ L_{\text{burn}} \sim 10 \text{ m} \]

Fusion Power:

\[ P_{\text{fus}} = G \cdot E_p \cdot \text{rep rate} = (10)(700 \text{ kJ}) \cdot (10) \]

\[ P_{\text{fus}} \sim 70 \text{ MW} \quad @ 10 \text{ s}^{-1} \text{ (duty cycle } \sim 0.5\%) \]