

# Application of a Rotating Magnetic Field for Confinement Enhancement and Control of MHD Instabilities in the FRC

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# Outline

Field Reversed Configuration

Formation

Equilibrium

Challenges for Future Concept Development

Stability and Confinement

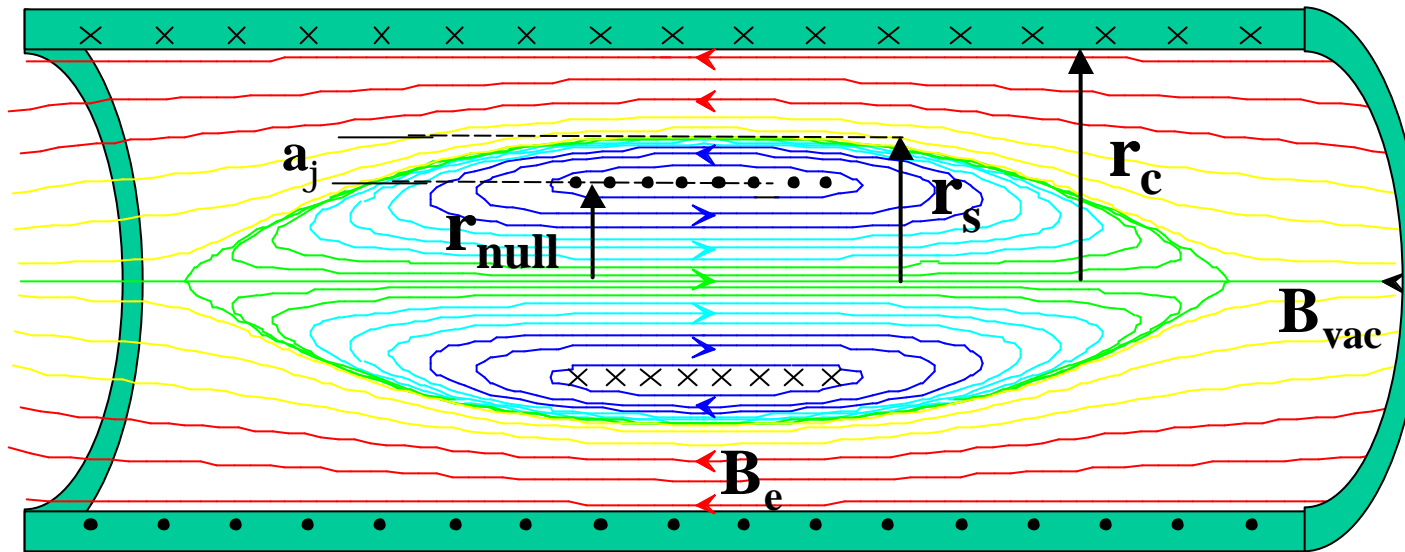
Small Scale Fusion

Rotating Magnetic Field (RMF)

RMF for MHD Stability

RMF for Enhanced Confinement

# FRC General Description



Equilibrium in flux conserver:

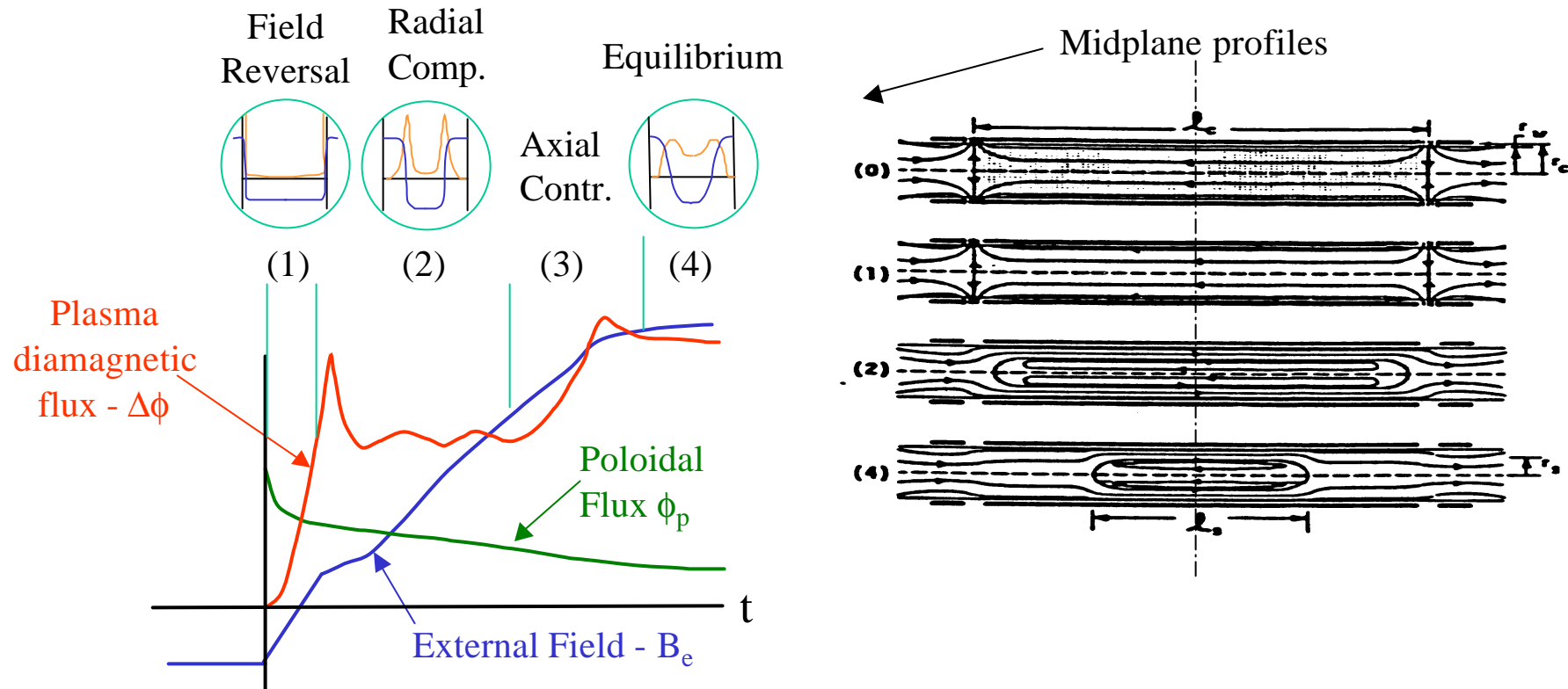
$$\langle \beta \rangle = 1 - \frac{1}{2} x_s^2$$

$$B_e = \frac{B_{\text{vac}}}{1 - x_s^2} \quad (x_s = r_s / r_c)$$

Stability:  $s/\epsilon < 0.5$  (empirical)  
 $s/\epsilon < 0.2$  (MHD w Hall)

$$s = \int_R^{r_s} \frac{r dr}{r_s \rho_i} \approx \frac{a_\phi}{\rho_{i0}} \quad \epsilon = l_s / r_s$$

# Reversed Field $\theta$ - Pinch Formation of FRC

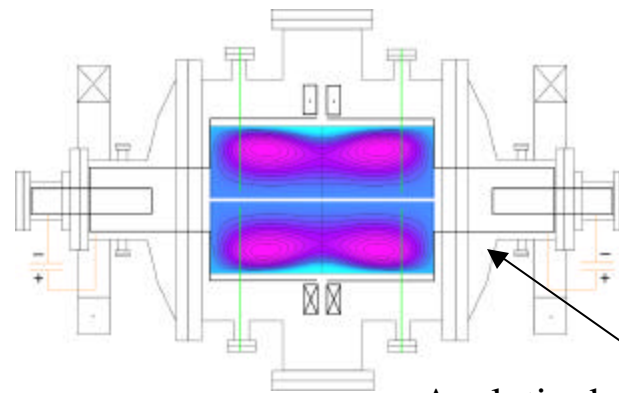
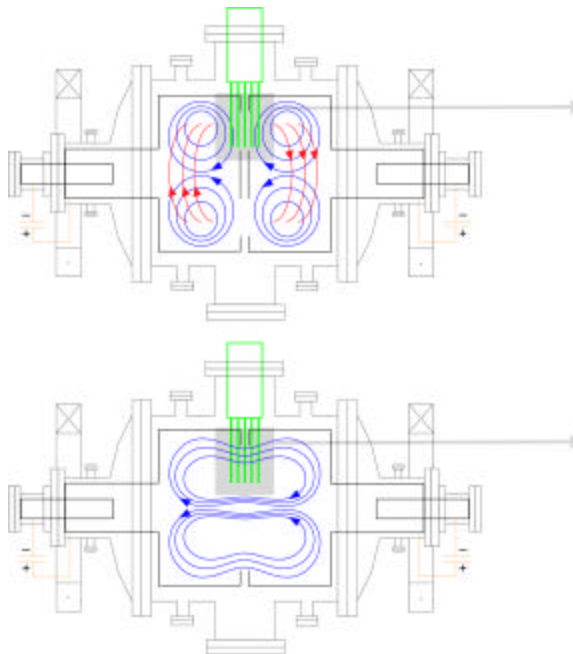


- Field-reversed  $\theta$  pinch formation is an extremely dynamic process requiring high voltages and excellent uniformity.
- High input powers allow hot (**0.1 – 2.0 keV**) high density ( **$\sim 1-5 \times 10^{21} \text{ m}^{-3}$** ) plasmas to be obtained.
- **Best confinement observed with  $D_N \sim 4 \text{ m}^2/\text{s}$  with  $a_f = 0.06 \text{ m}$  (LSX) for  $s \leq 4$**

# Merging of Counter-Helicity Spheromaks to create FRCs (Y.Ono - University of Tokyo)

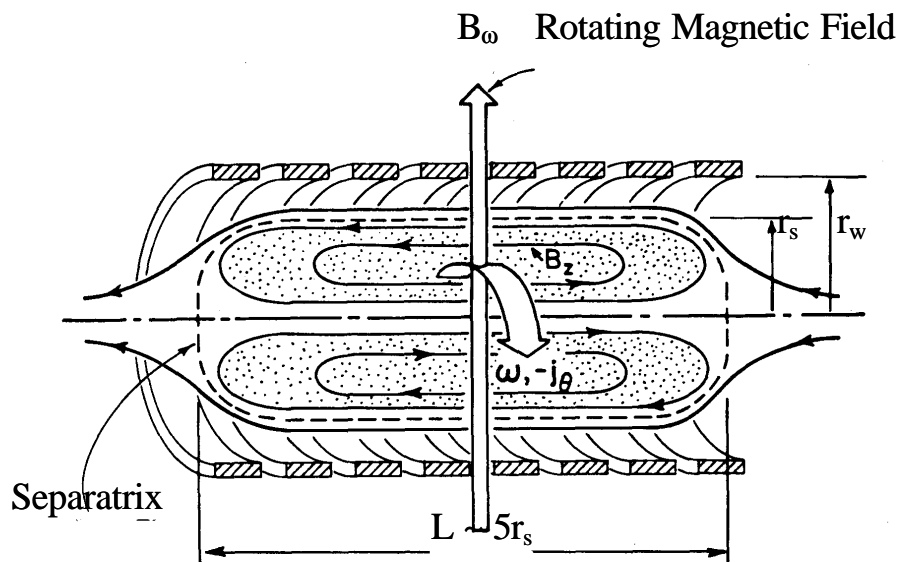


Swarthmore Spheromak Experiment (SSX-FRC)



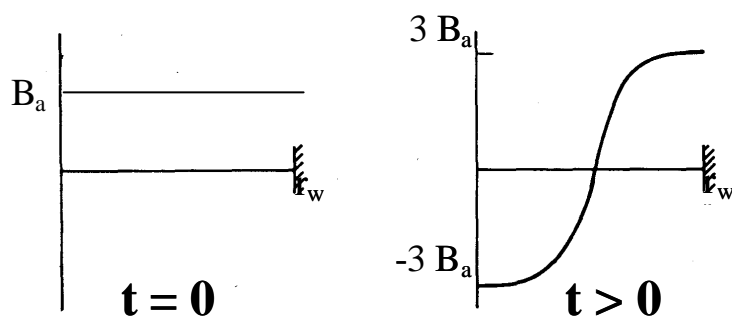
Analytic doublet-CT equilibrium (P. Parks, GA)

# FRC Produced by a Rotating Magnetic Field (RMF)

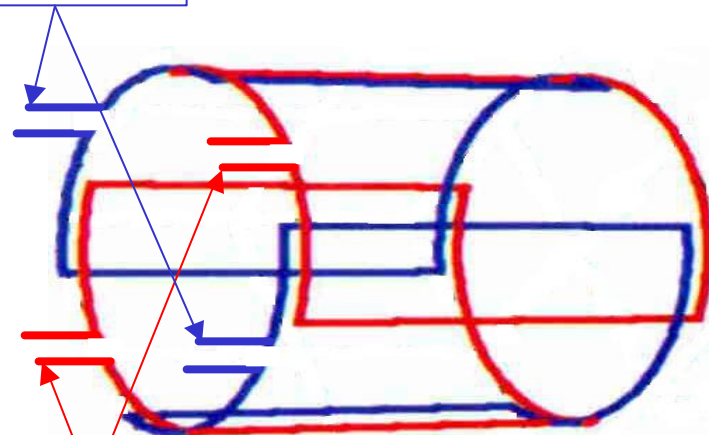


Initial axial magnetic field inside flux conserving coil is  $B_a$ .

RMF antenna coils external to the axial coil are shown below.

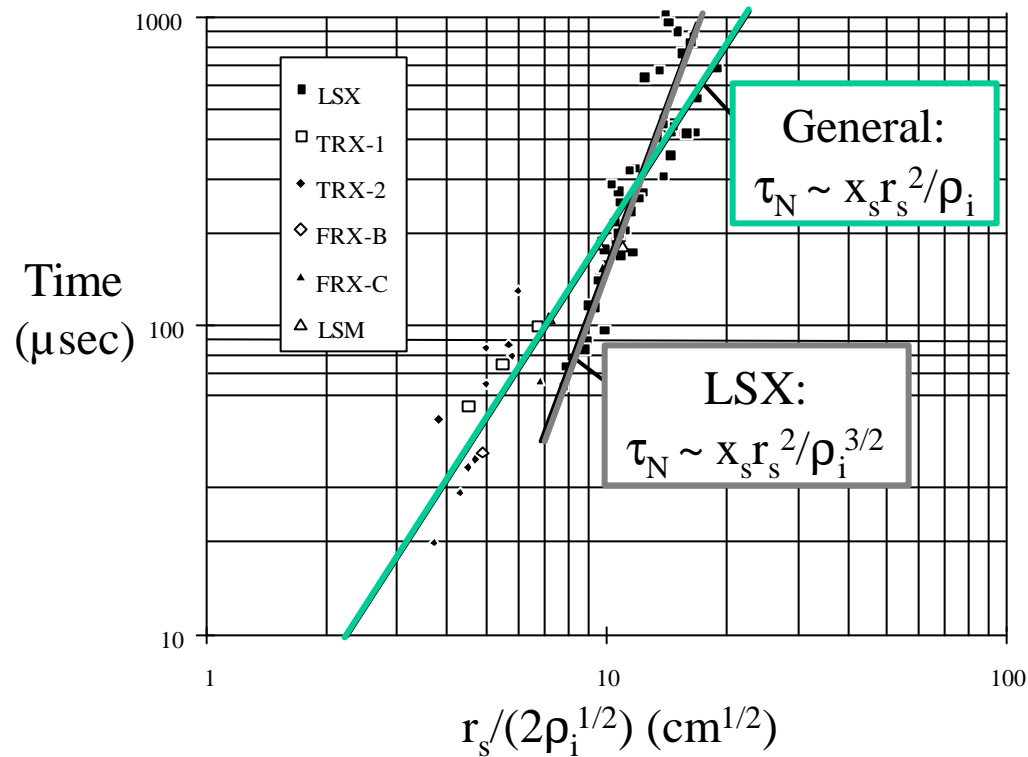


$$I_0 \sin(\omega t)$$



$$I_0 \cos(\omega t)$$

# Measured FRC Particle Confinement



From past FRC experiments (Hoffman and Slough, 1993) :

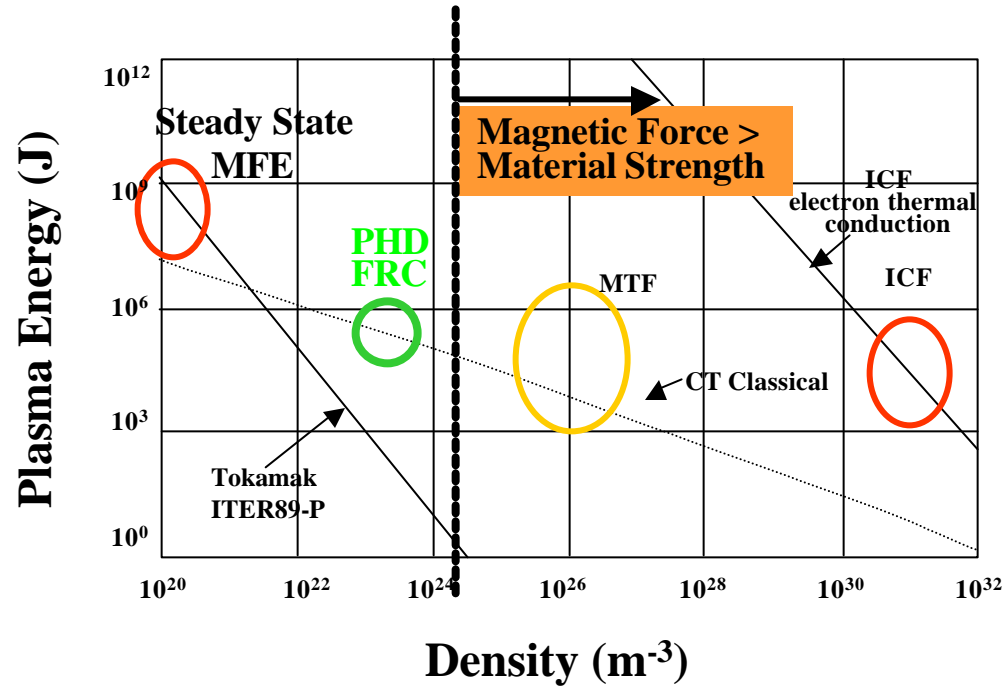
$$\tau_N = 3.2 \times 10^{-15} \epsilon^{0.5} x_s^{0.8} r_s^{2.1} n^{0.6} \left( \sim \frac{r_s^2}{\rho_i} \right)$$

For a given plasma energy  $E_p = 3/2 N k T_{fus}$

$$nt \sim r^{2.1} n^{1.6} \sim r^{2.1} (N/r^3)^{1.6} \sim r^{-2.7}$$

# Energy requirements vs. plasma density for various Fusion Regimes

(for  $nt_E = 3 \times 10^{20} \text{ m}^{-3} \text{ sec}$ ,  $T_i = 10 \text{ keV}$ , and poloidal  $\mathbf{b} \sim 1$ )



- Small-scale FRC fusion regime based on confinement scaling observed in previous expts.
- Transport believed to be the result of edge driven microinstability (Lower Hybrid Drift) where  $\tau_N \sim R^2/\rho_i$ .



# Major Physics Issues for Steady State FRC Reactor

1. Continued stability at larger sizes as represented by the parameter  $s$  (~ # of internal ion gyroradii)
2. Major confinement improvement at low densities
3. Current (or flux) sustainment at ~ 10 MA level
4. Technical ability to form larger FRCs

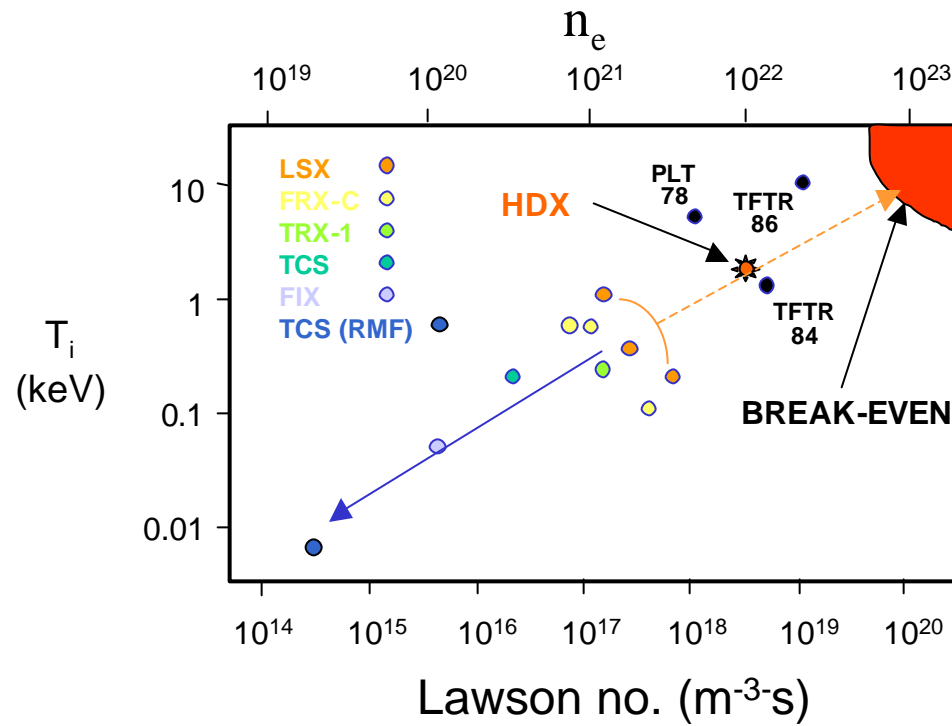
## Reactor Issues for Small Scale FRC Fusion

Maintain stability and enhance confinement for higher fusion gain

Lower the required compression field (18 T)

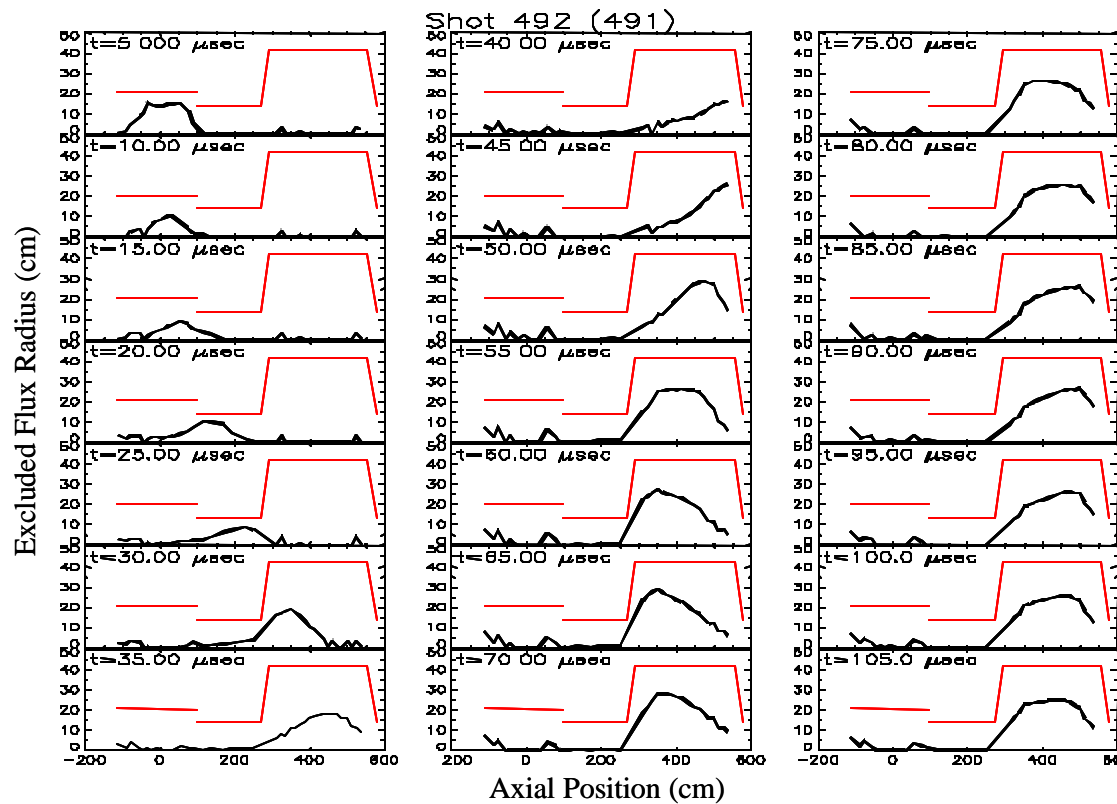
Increased burn time ( $\tau_{\text{burn}} \sim \tau_{\text{config}}$ )

# Confinement Scaling for Various FRC Experiments

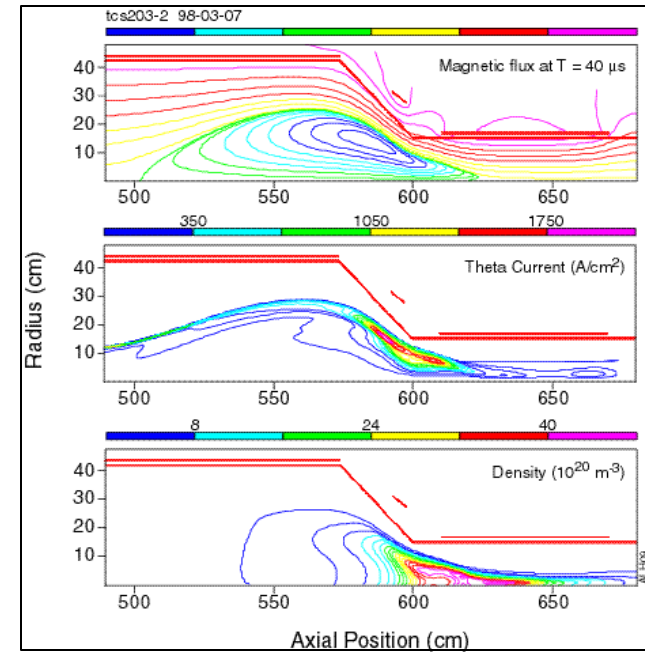


- Translation and Expansion
- Acceleration and Compression

# Translation of FRC into TCS

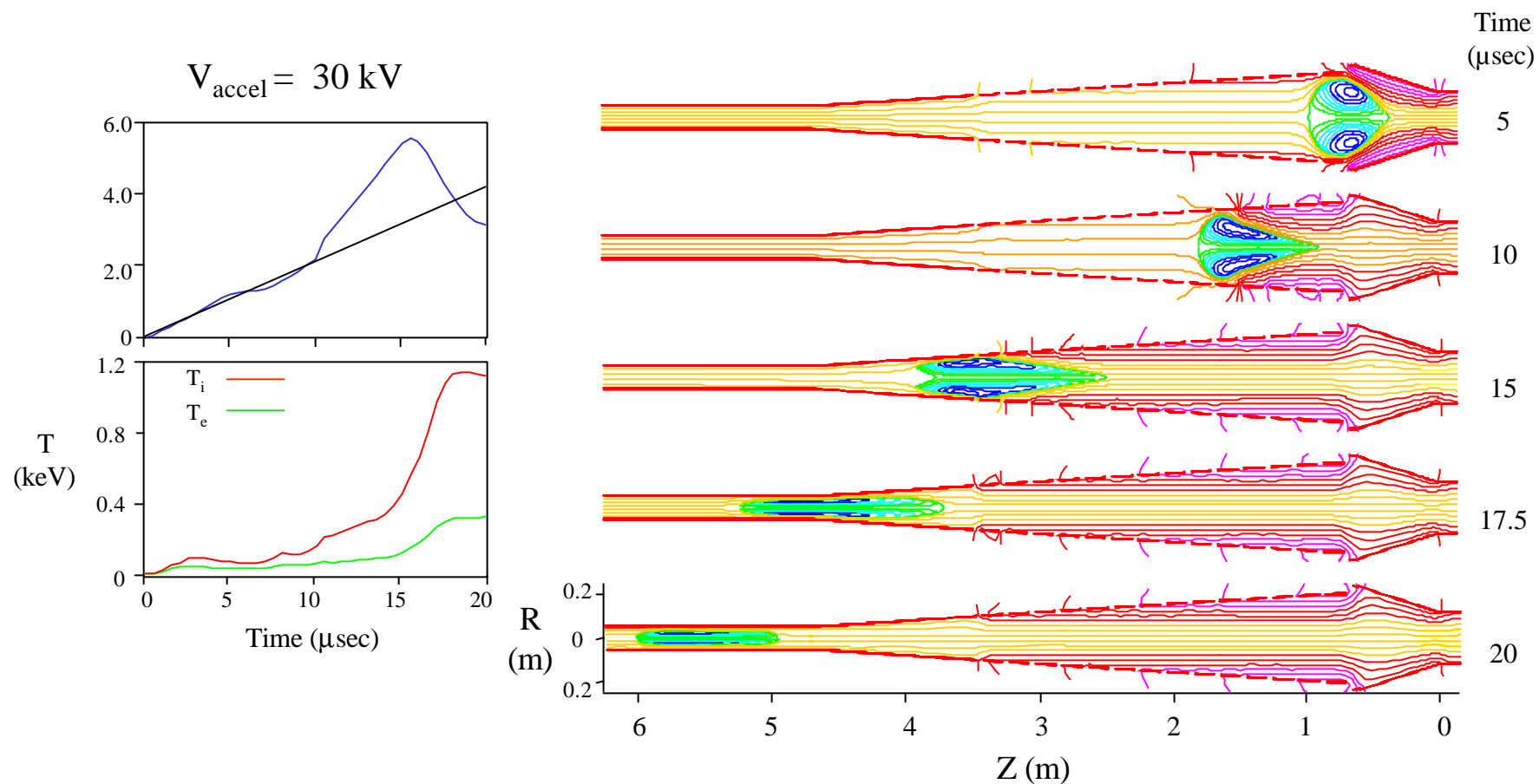


Experimental Measurements



MOQUI Calculated First Reflection Details

# Resistive 2D MHD Calculation of FRC with Propagating Magnetic Field (0.4 T)



- FRC confinement scaling  $\Rightarrow n\tau \sim 10^{19} \text{ m}^{-3}\text{-s}$  at  $T > 1 \text{ keV}$  after compression

# PHD Burn Parameters for FRC

**Assume:**  $B_{\text{vac}} = 18 \text{ T}$  (SCC),  $x_s = r_s/r_c = 0.6$   $e = l_s/2r_s = 20$

$$t_N = 3.2 \times 10^{-15} e^{0.5} x_s^{0.8} r_s^{2.1} n^{0.6}$$

For a flux conserving vacuum wall (pipe):

$$B_e = B_{\text{vac}} / (1 - x_s^2), \quad \Rightarrow \quad \boxed{B_e = 28 \text{ T}}$$

From pressure balance with  $T_e + T_i \sim 10 \text{ keV}$ :

$$B_e^2 / 2\mu_0 = n k (T_e + T_i) \quad \Rightarrow \quad \boxed{n_{\text{max}} = 2 \times 10^{23} \text{ m}^{-3}}$$

Set  $nt \sim 1 \times 10^{20} \text{ m}^{-2} \text{ s}$  (Lawson)  $t \sim 500 \text{ msec}$

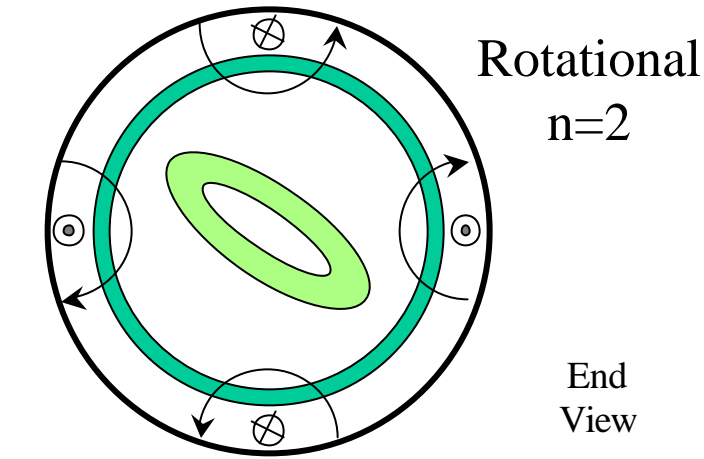
From FRC scaling given above we can solve for  $\boxed{r_s = 2.8 \text{ cm}}$

Coil Radius ( $x_s = 0.6$ )  $\Rightarrow$   $(r_c = 4.7 \text{ cm})$   $r_c - r_s \sim 30r_{ie}$

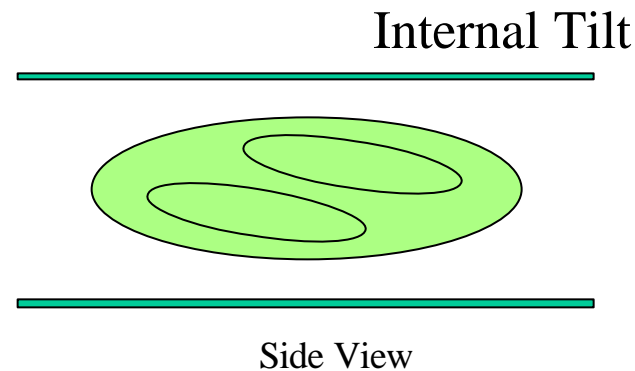
Energy in FRC plasma (ellipsoid)

$$3/2 \langle \beta \rangle n_{\text{max}} kT (4/3 \pi \epsilon r_s^3) \quad \Rightarrow \quad \boxed{E_p = 720 \text{ kJ}}$$

# Instabilities Observed in FRCs



- Rotational mode is driven by centrifugal forces from particle loss and/or end-shortening of radial E field.
- Mode limits FRC lifetime to  $\sim \frac{1}{2} \tau_N$



- ◆ Internal tilt starts out as an axial  $n=1$  shift mode. Only clearly observed in recent expts. (Cothran, Brown, Schaeffer)
- ◆ Mode limits FRC lifetime  $\sim 3$  axial Alfvén times.

# FRC Stability (Theory)

## Basic MHD

- Bad curvature produces MHD instability to **tilt**, **interchange** and **ballooning** modes. Compressibility stabilizes interchange modes.

## MHD with FLR and Hall (two fluid effects)

- Both effects cause the ion and electron disturbances to get out of phase: Tilt growth rate reduced but not stabilized for FLR (Ishida and Steinhauer, Belova)
- Tilt stability for  $s/\epsilon < 0.2$  from Hall Term (Barnes)

## Relaxed States with Flow

- High-beta *possible*; sheared flow *necessary* in relaxed states
  - find proper relaxation principle (Steinhauer, Ishida, Mahajan, Yoshida, Dasgupta)

## FRC Stability (Experiment)

- FRC tilt stable for  $s/\epsilon < 0.5$
- FRC grossly stable except for rotational modes

# FRC Stability in PHD Fusion Regime

## s / e parameter

$$s = \phi / (2\pi r_s B_e \rho_{ie})$$

FRC poloidal flux:

$$\phi = \pi r_c^2 B_e (x_s / \sqrt{2})^{3.5}$$

$$j = 9.5 \text{ mWb}$$

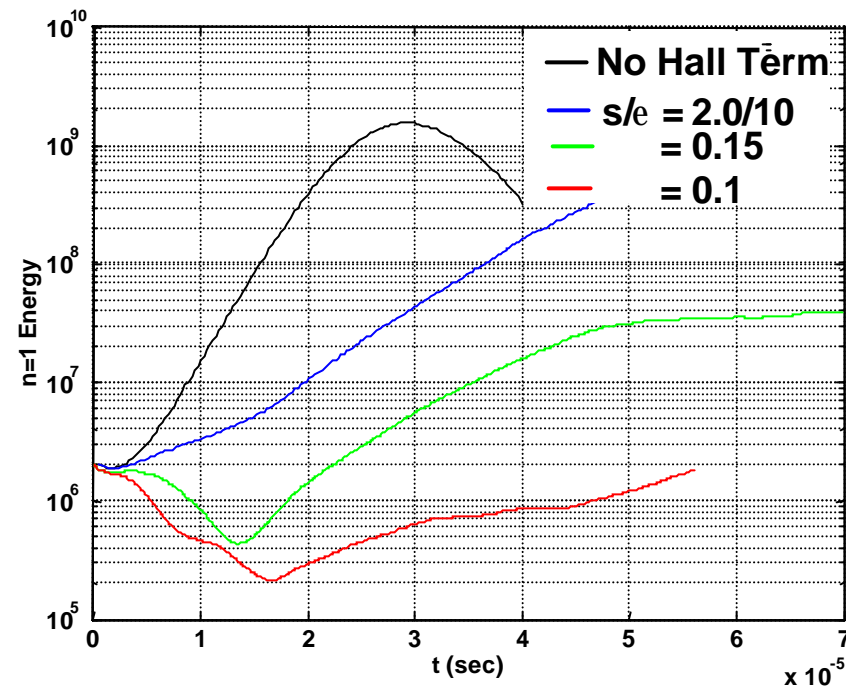
Empirically from LSX data

$$s/\epsilon < 0.5$$

for gross stability **AND** good confinement

$$\text{PHD FRC: } s/e = 2.4/20 = 0.12$$

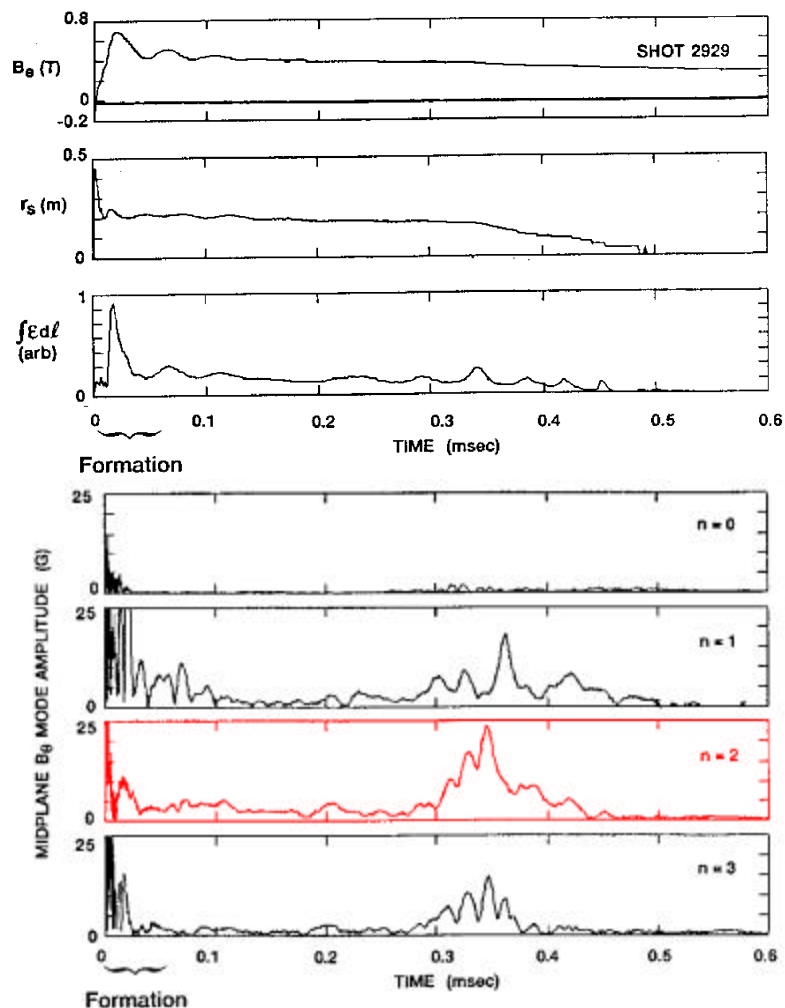
## Recent 3D MHD Results Internal Tilt Stability (Barnes and Milroy)



## PHD FRC in MHD Stable Regime



# FRC Stability at $s/\epsilon \sim 0.5$

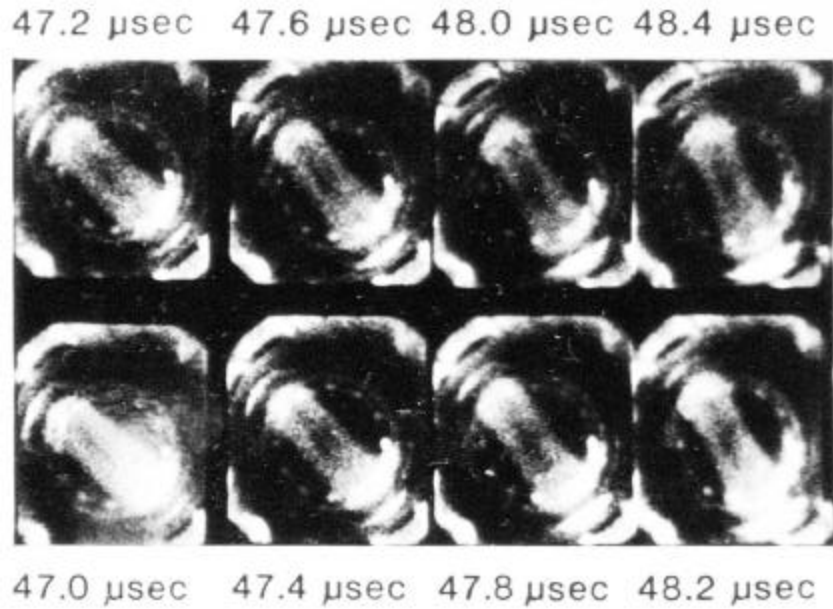


## Data taken on LSX at $s \sim 3.5$

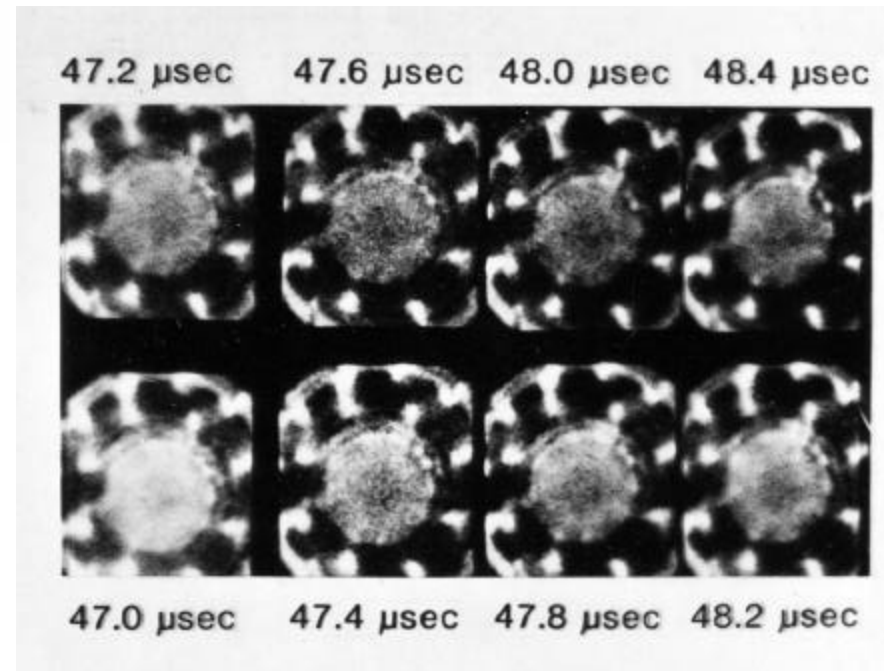
- Much  $n=1$  activity during formation, but dies away if distortions not too large
- Formation distortions not primarily related to  $s$ , but to axial dynamics and low axial viscosity (low  $T_i$ )
- Rotational mode (mostly  $n=2$ ) grows from equilibrium when multipole fields not used

# Stabilization of n=2 Rotational mode

(First demonstrated at U. of Osaka w quadrupole)



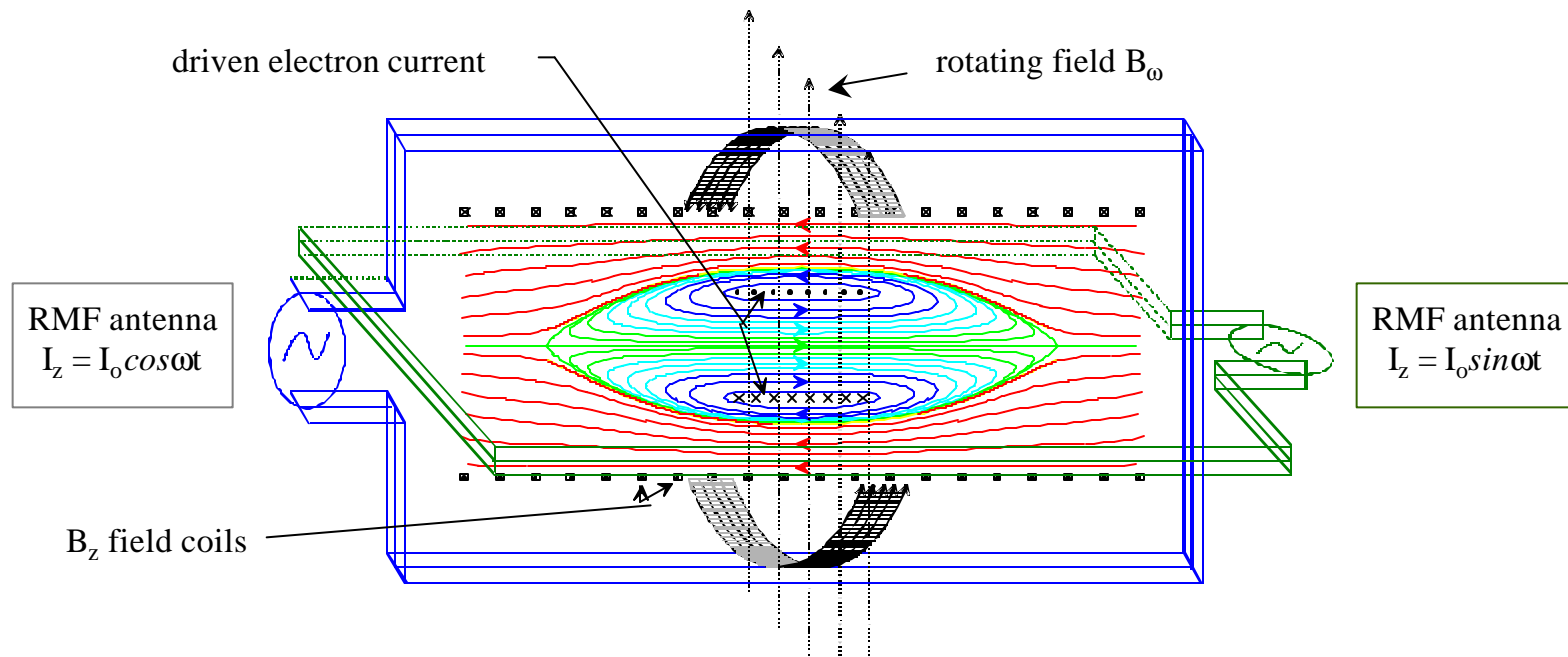
No Stabilization



Octopole Stabilization

- Significant wall contact due to plasma flow onto static multipole field
- Large plasma distortion (quadrupole) from static field created turbulent boundary in LSX at  $s > 2$

# RMF Current Drive



- ‘Drag’ Electrons Along With Rotating Radial Field
  - Must have  $\omega_{ci} < \omega \ll \omega_{ce}$  for electrons, but not ions, to follow rotation
- Electrons Magnetized on Rotating Field Lines ( $\omega_{ce} \tau \gg 1$ )
  - Necessary for efficient current drive
  - Absolutely necessary for rotating field penetration

# Control of Diffusive Losses with RMF

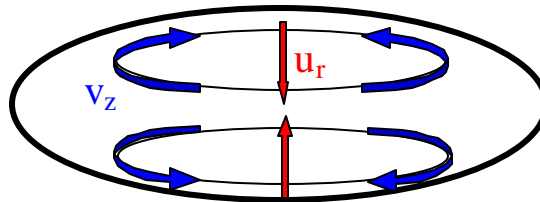
Gen. Ohm's Law ( $\theta$  comp):  $\mathbf{E}_q = \mathbf{h} \mathbf{j}_q - \frac{1}{ne} \mathbf{u}_r B_z - \frac{1}{ne} \langle \mathbf{j}_z \mathbf{B}_r \rangle$

From synchronous electron motion:  $\mathbf{j}_q = -ne \mathbf{w} \mathbf{r}$

From first order term for screening current  $\mathbf{j}_z$

$$\langle \mathbf{j}_z \mathbf{B}_r \rangle = \left[ 2 \left( \frac{n_{ei}}{w_{ce}} \right) ne \mathbf{w} \mathbf{r} \right] B_w \langle \cos^2 \mathbf{q} \rangle = \mathbf{h} \mathbf{j}_q ne$$

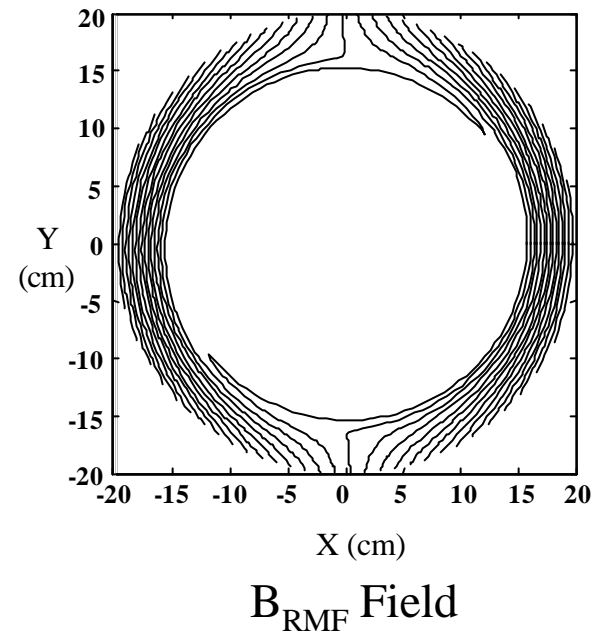
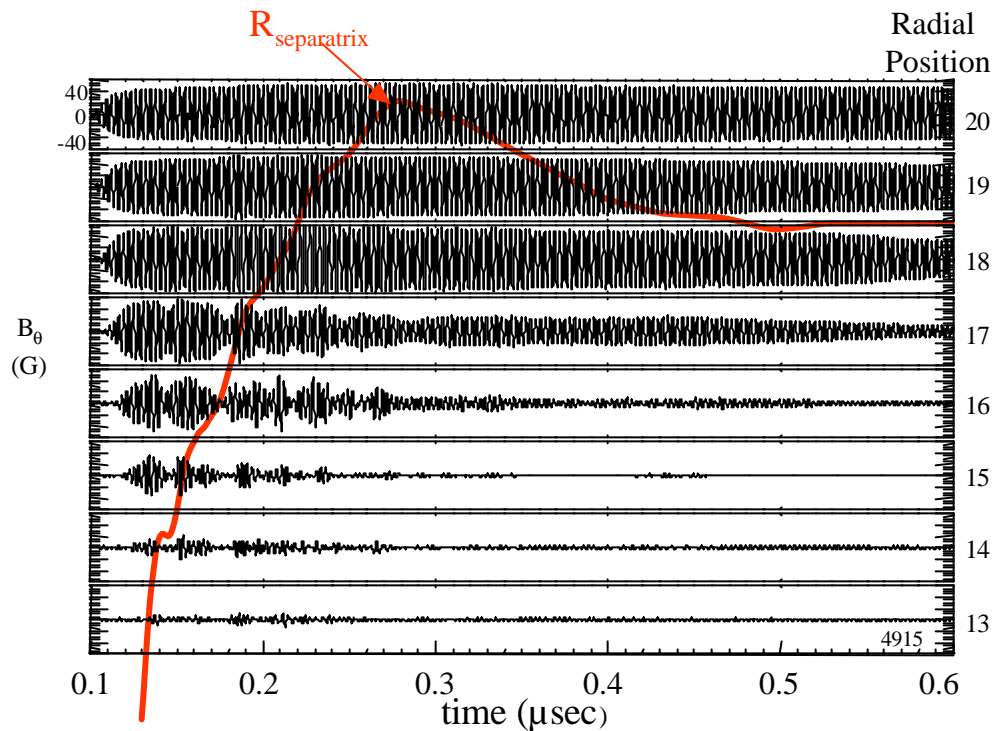
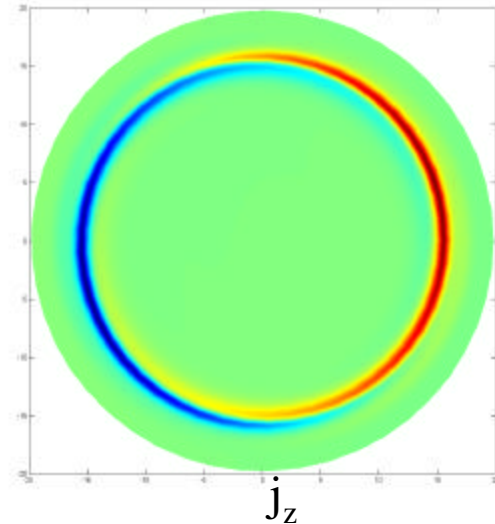
This drive term opposes the dissipative  $\eta \mathbf{j}_\theta$  term, and stops diffusive losses ( $\mathbf{u}_r \sim 0$ ) in steady state ( $\mathbf{E} \sim 0$ ):



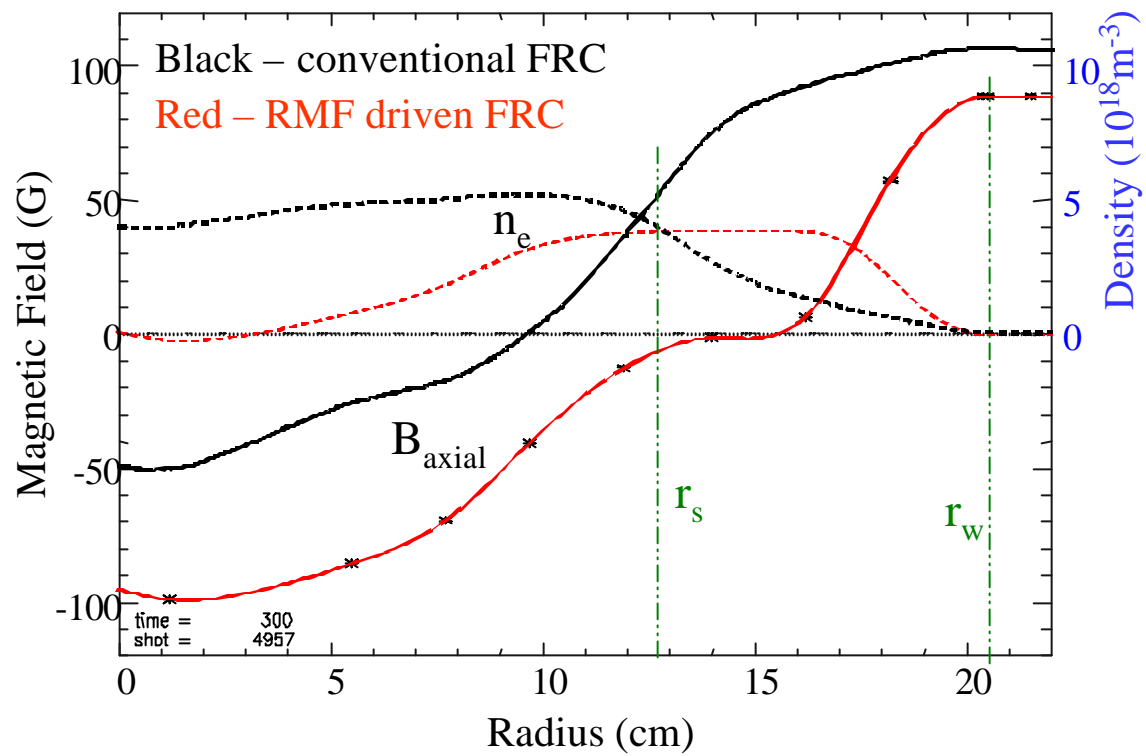
# FRC Confinement by RMF Field

- Axial currents inside FRC limit RMF  $\sim 2$  cm penetration past separatrix
- Strong radial flow  $u_r B_z \sim \langle j_z B_r \rangle / en$  is obtained
- Minimum energy and particle loss observed under these conditions

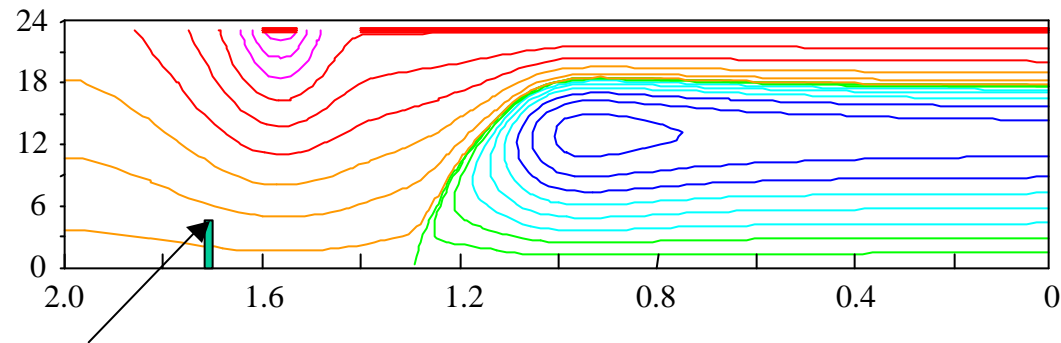
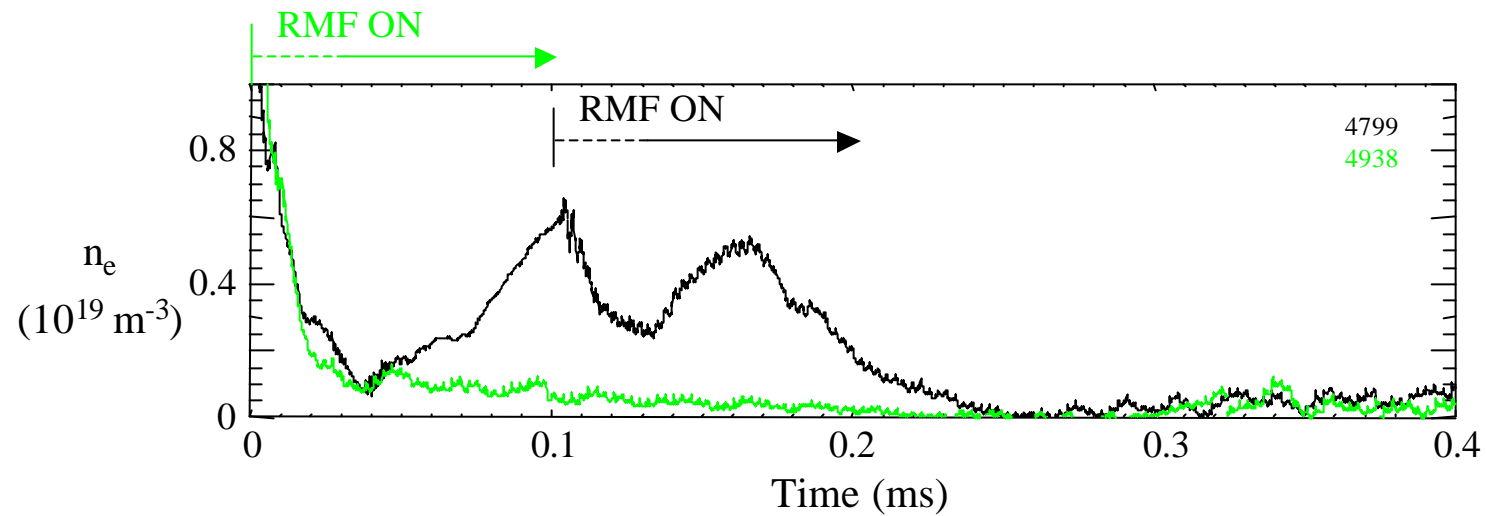
2D MHD Calculation



# Internal Profile Measurements from STX Experiment

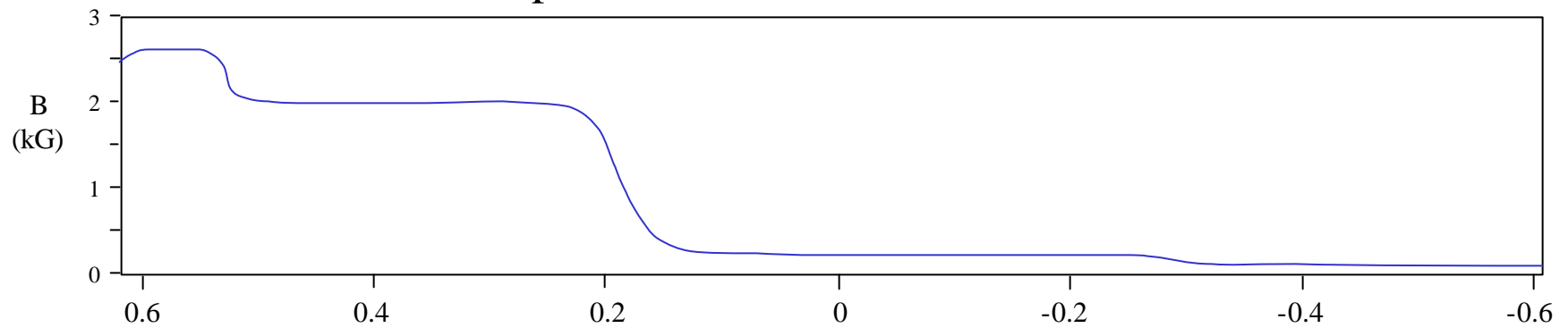
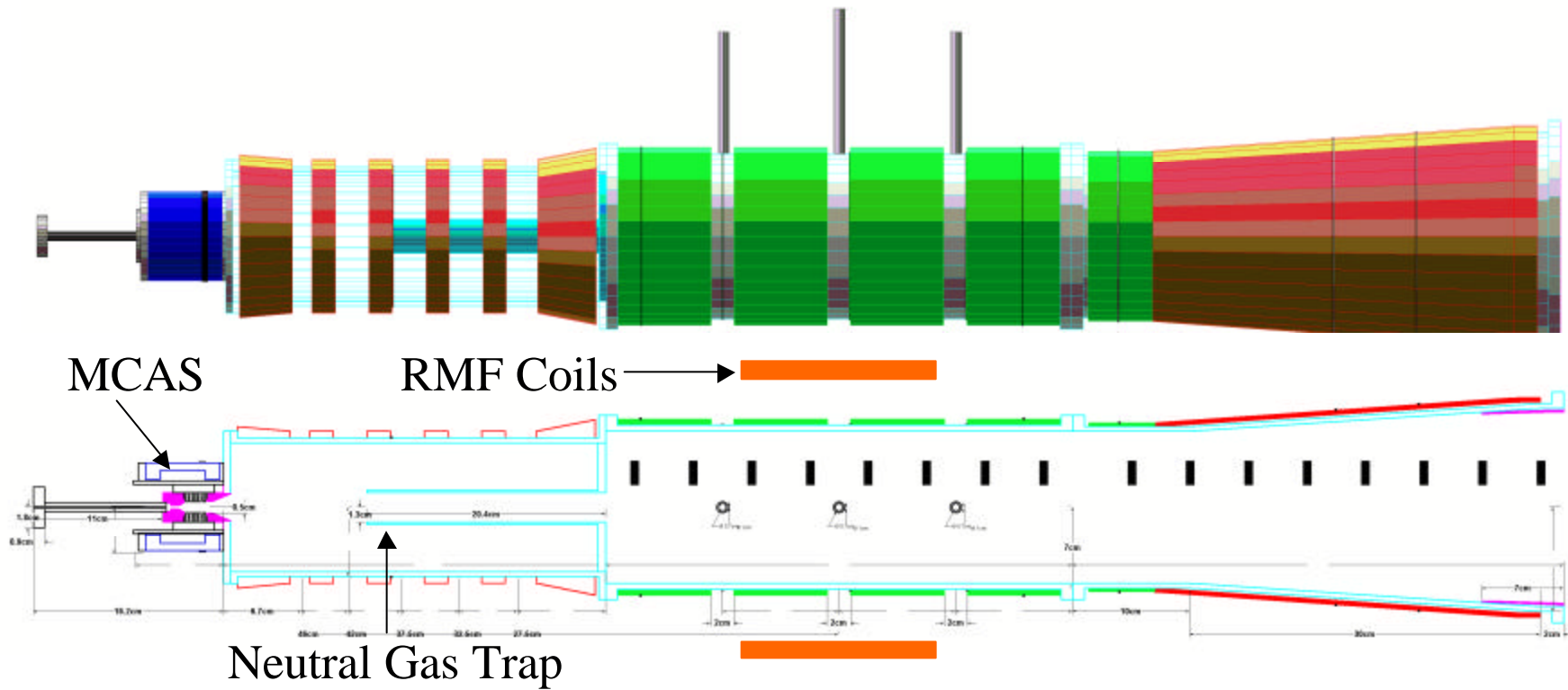


# Plasma Density in FRC Jet During Sustainment



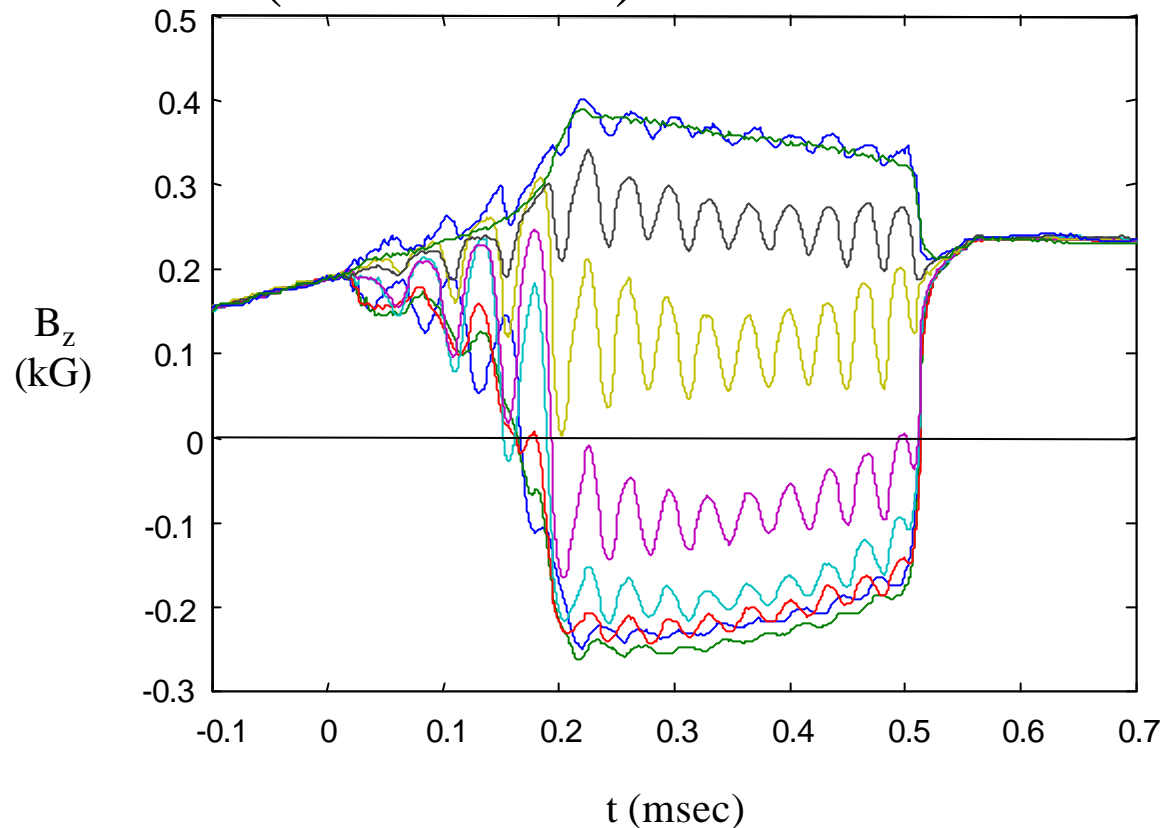
Position of triple probe in jet

# STX-HF Experimental Layout





## Internal $B_z$ Profile During RMF (0 to 0.5 ms) on STX-HF



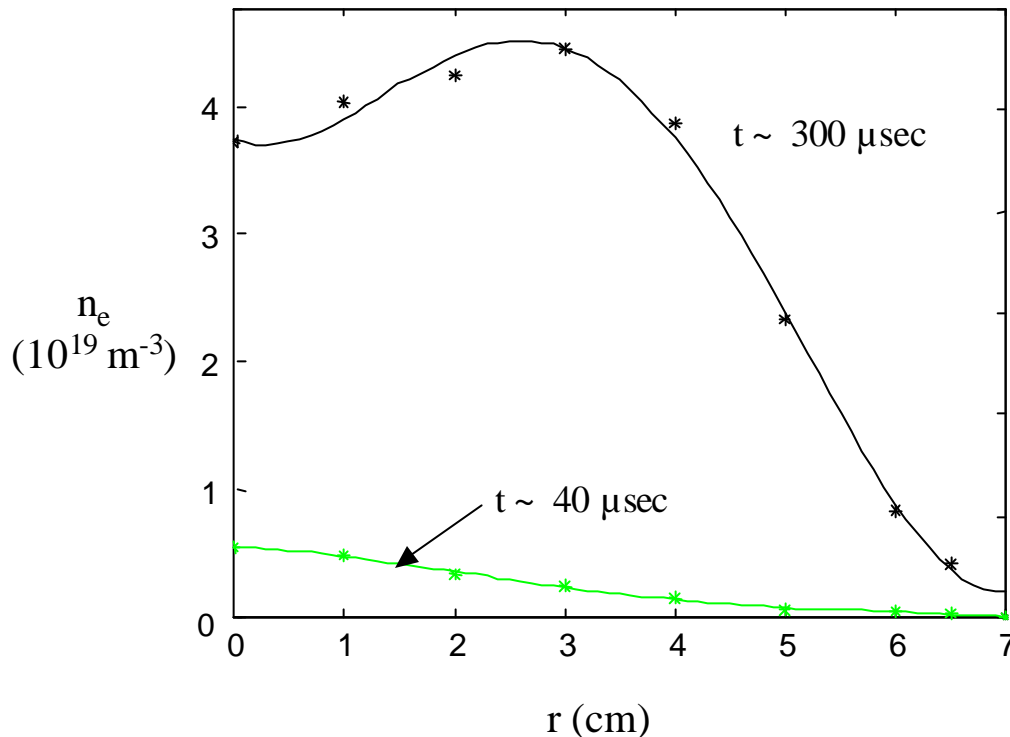
For RMF drive the magnitude of  $B_\omega$ :  $B_e \cong \left( \frac{\mu_0 \omega}{9\eta} \right)^{1/2} r_c B_\omega$

In terms of antenna power  $P_\omega$ :  $B_e \sim \left( \frac{P_\omega}{r_c} \right)^{1/2}$



# Enhanced particle confinement from RMF

Density profile at  $z = -10$  cm from Langmuir probe



For RR profile:  $t = \frac{m_0 r_s^2}{16 h_{\perp}}$

For size ( $r_s \sim 0.05$ ) resistivity inferred for STX-HF:

$T_e \sim 10 - 30$  eV  $\Rightarrow \eta_{\perp} \sim 60 - 30$   $\mu\Omega\text{-m}$

$\tau = 4 - 8$   $\mu\text{sec}$

Ratio of line density:  $(n_L = 2p \int n_e r dr)$

$$\frac{n_L(t = 300)}{n_L(t = 40)} \approx 25 \Rightarrow t_n = 25 t_{\text{transit}} = 120 \text{ msec}$$

**RMF must provide  $\gg$  classical confinement**

# RMF CT Formation

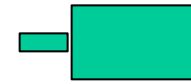
## End-on Filtered (510-600 nm) Plasma Emission

Plasma:  $r \sim 0.05$  m  
 $l \sim 0.25$  m



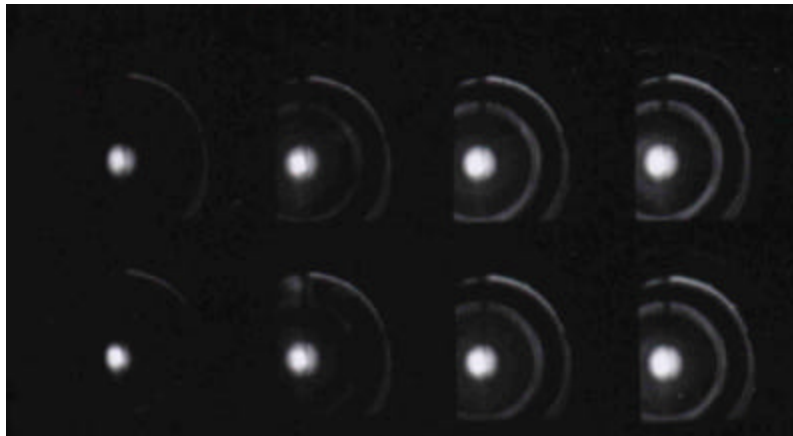
$\sim 5$  m

Imacon fast framing camera



2  $\mu$ sec exp

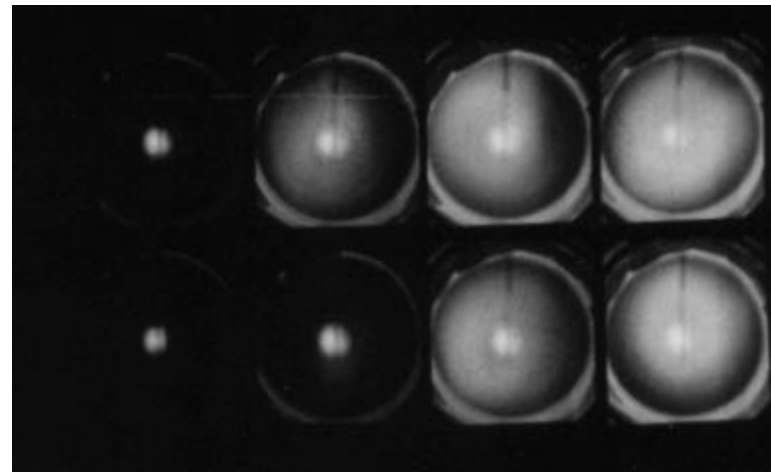
30 50 70 90



20 40 60 80

NO RMF

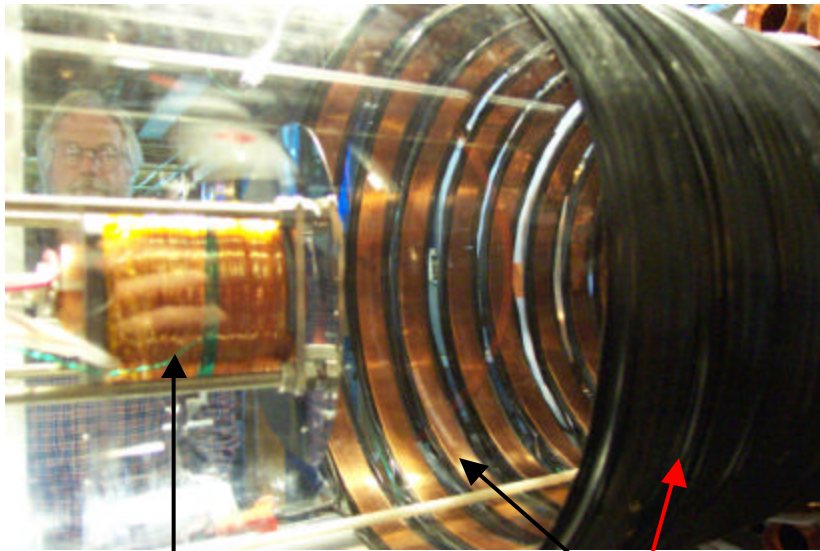
30 50 70 90



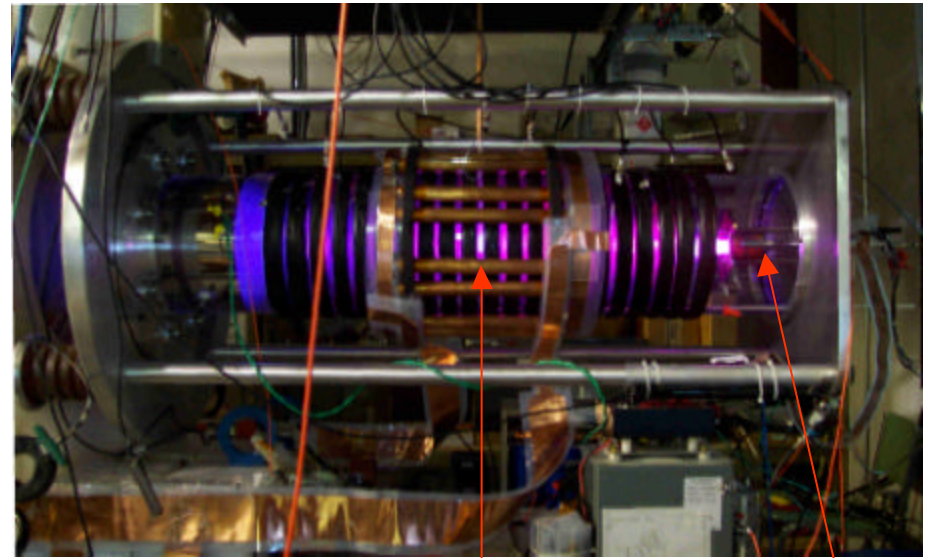
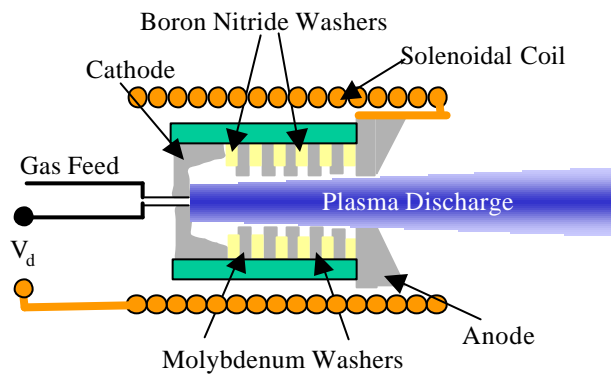
20 40 60 80

RMF On at  $t=0$

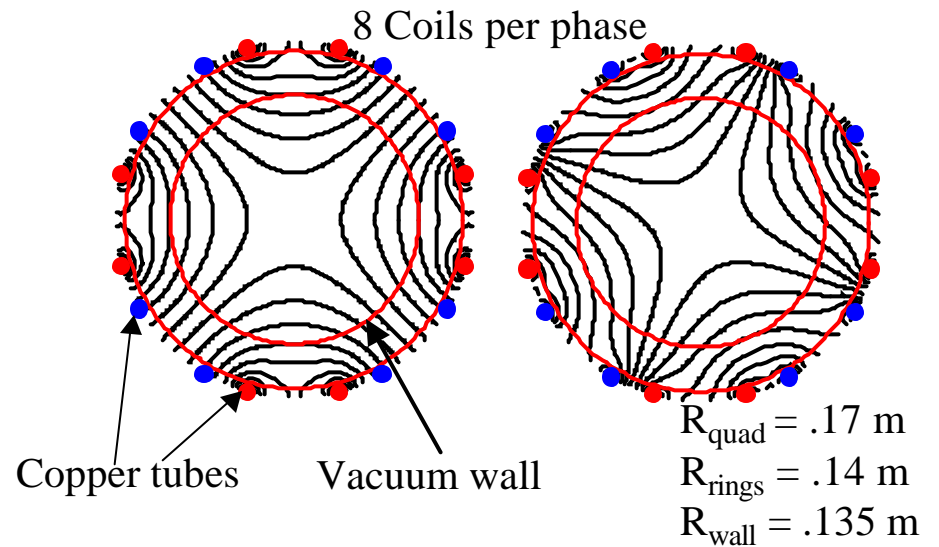
# Control of Rotational Modes with RQMF



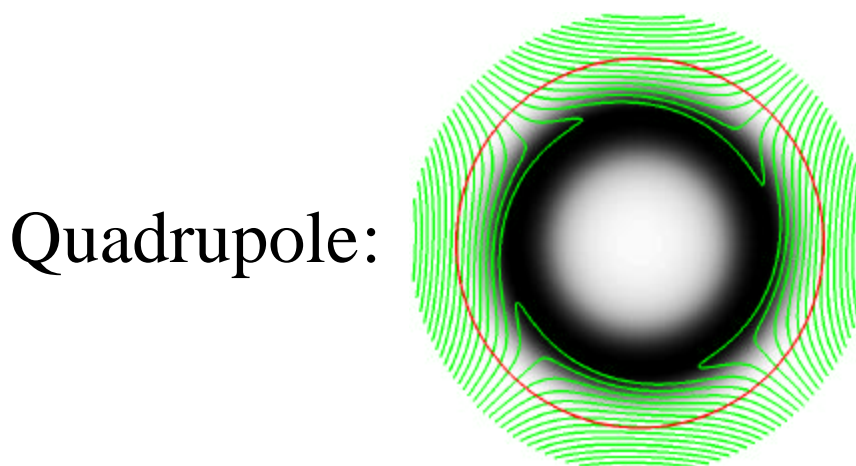
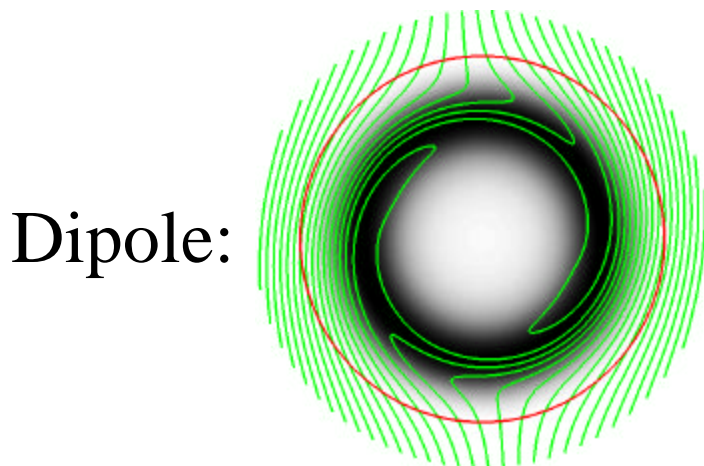
Magnetized Cascaded Arc Source  
Flux Conserving Rings  
Solenoidal Magnet



Rotating Quadrupole Antenna  
Source



# Magnetic field lines and density maps from numerical simulations of RMF current drive.



$$A_{zn} = \sum_n \left\{ \mathbf{a}_n \left( \frac{r}{R} \right)^n + \mathbf{b}_n \left( \frac{r}{R} \right)^{-n} \right\} e^{inq}$$

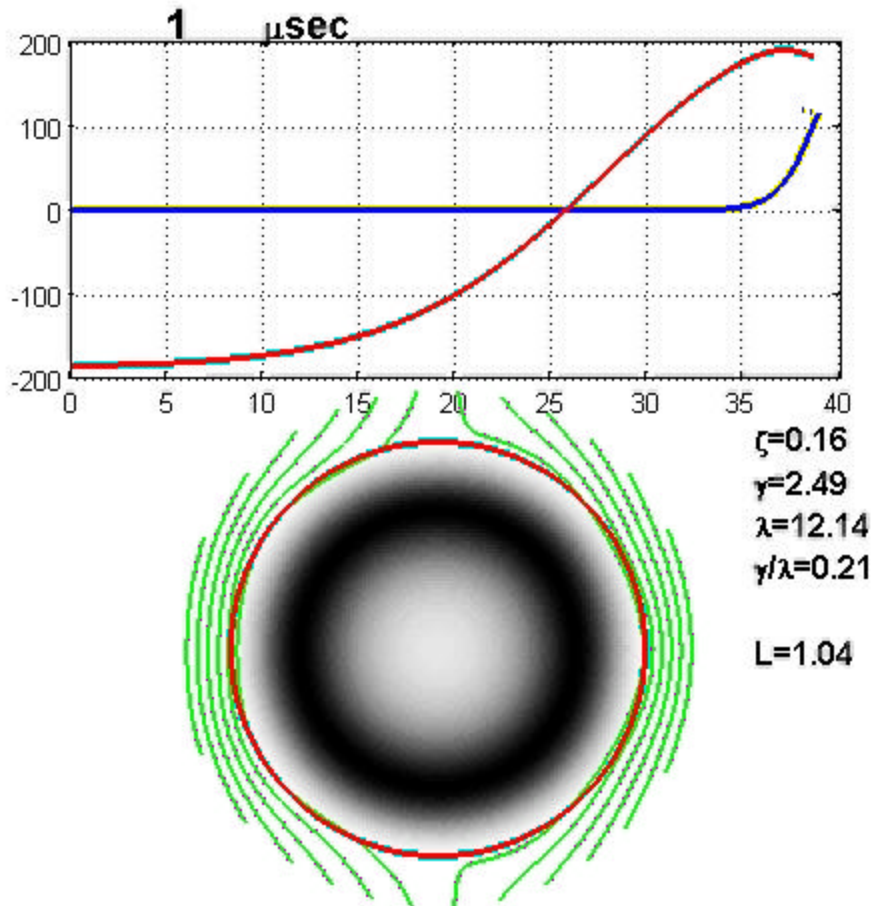
$$B_q = - \sum_n \left( \frac{\mathbf{a}_n n}{R} \left( \frac{r}{R} \right)^{n-1} - \frac{\mathbf{b}_n n}{R} \left( \frac{r}{R} \right)^{-(n+1)} \right) e^{inq}$$

$$B_r = i \sum_n \left( \frac{\mathbf{a}_n n}{R} \left( \frac{r}{R} \right)^{n-1} + \frac{\mathbf{b}_n n}{R} \left( \frac{r}{R} \right)^{-(n+1)} \right) e^{inq}$$

The parameter  $a_n$  is from external currents, and  $\beta_n$  is from internal currents. If there is no plasma,  $\beta_n=0$ , and  $\mathbf{B}_{r,\theta} \sim r^{n-1}$ .

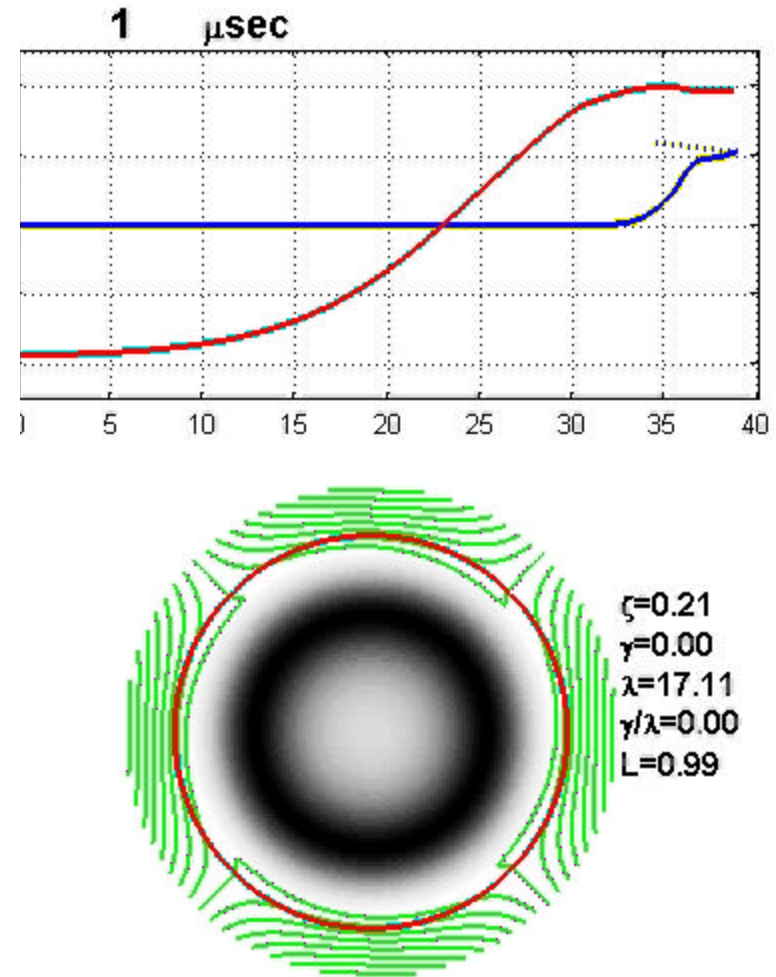
- n=1 → rotating dipole (normal RMF)
- n=2 → rotating quadrupole (current expt.)
- n=3 → octopole field (may be explored)

## Dipole Calculation



Transport inhibited by  
lack of penetration control

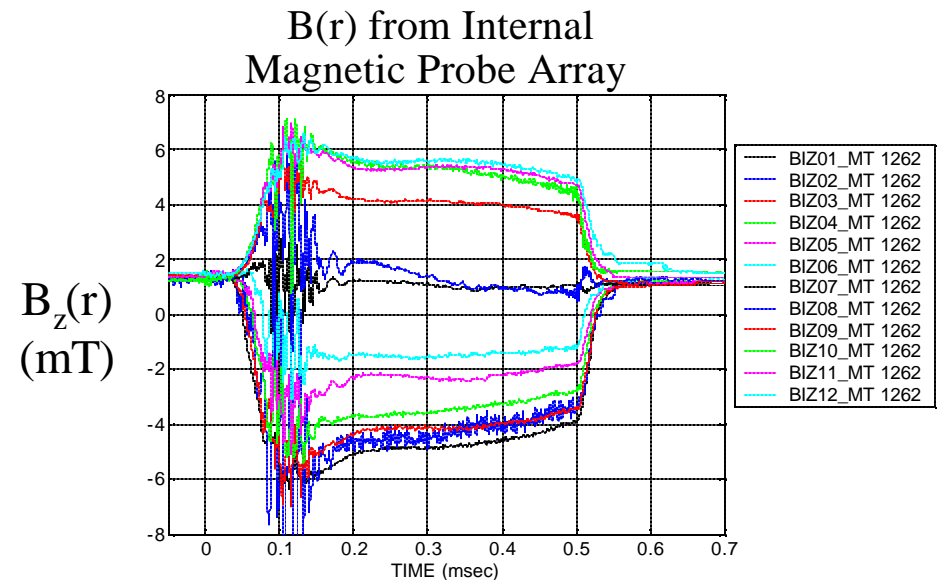
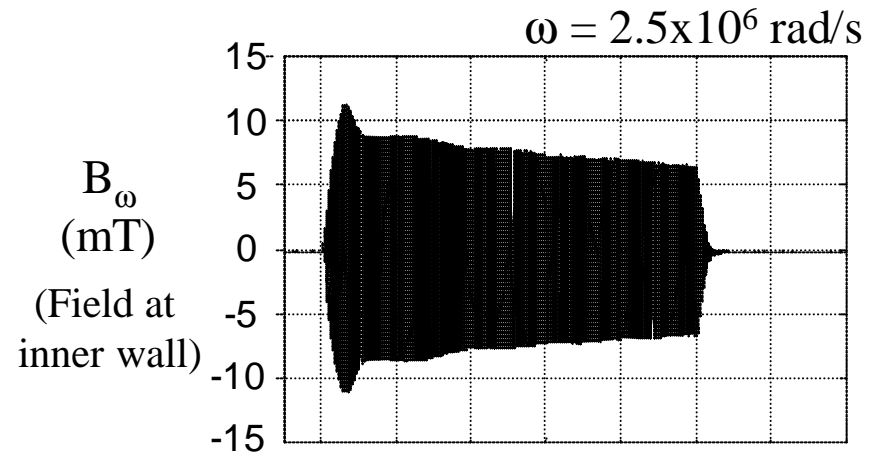
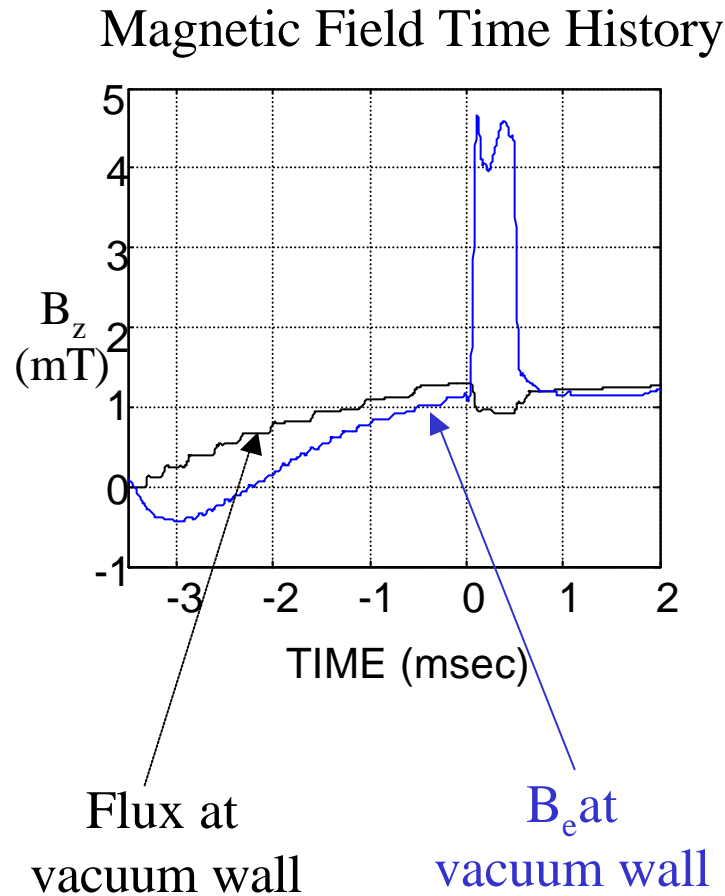
## Quadrupole Calculation



Radial penetration naturally limited  
By  $1/r$ . Can also be controlled  
by amplitude feedback

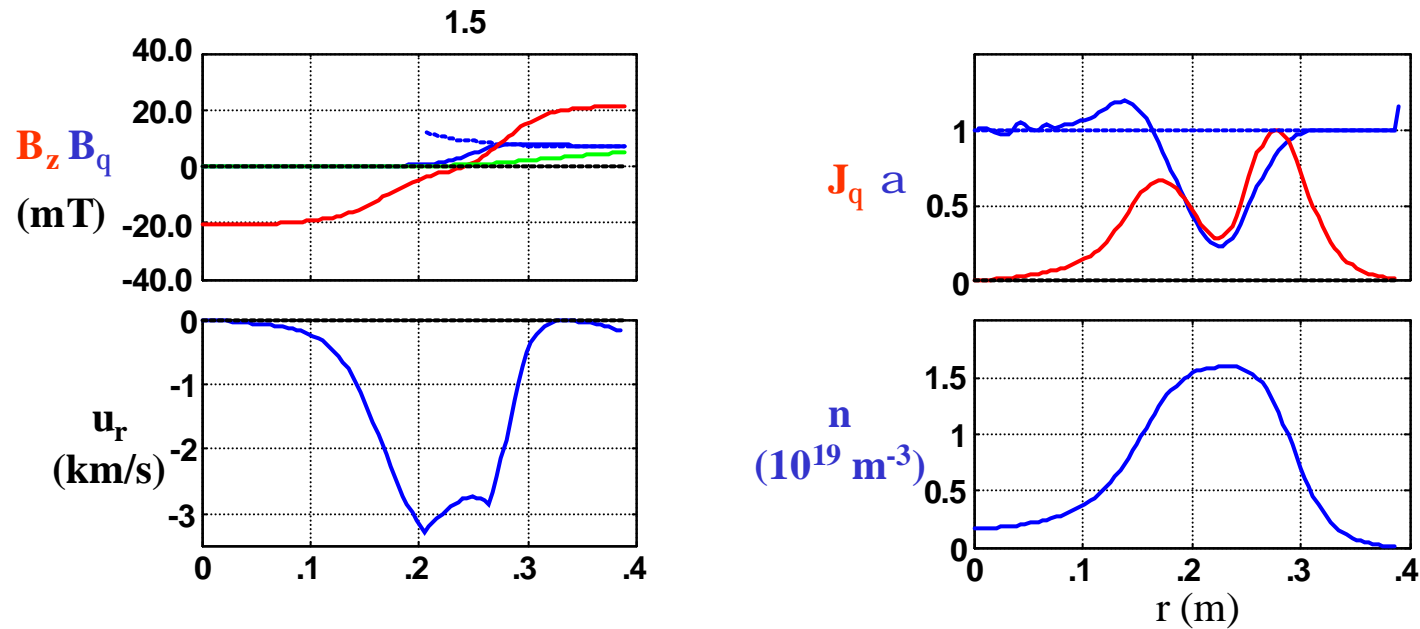
# Initial Experiments with Quadrupole Current Drive

## Magnetic Field Time Histories

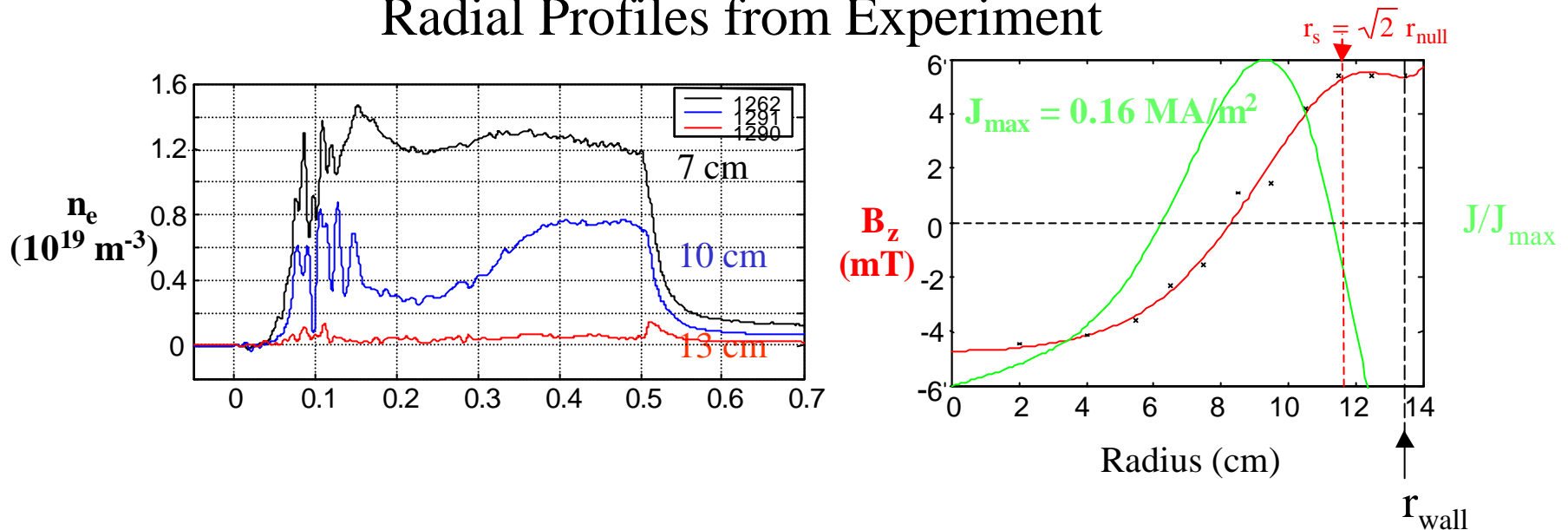




## Radial Profiles (2D Quadrupole Calculation)



## Radial Profiles from Experiment



# Summary

- Small-scale FRC fusion reduces stability and confinement issues to those observed in past FRC experiments
- Rotating Quadrupole field Stabilizes FRC to rotational modes
  - Does not allow for plasma flow to wall
  - Does not distort FRC or drive edge turbulence
- Rotating magnetic field enhances FRC confinement
  - Drives an inward diffusion that opposes the normal diffusive decay
  - Reduces density gradient at separatrix eliminating the drive from microinstabilities (LHD)

# RMF Induced Ion Rotation

Ion Spin-up Time:  $\tau_s = \frac{m_i}{m_e} \frac{1}{v_{ei}} = \frac{m_i}{ne^2 \eta_{\perp}}$

For  $n$  ( $10^{19} \text{ m}^{-3}$ ),  $\eta$  ( $\mu\Omega\text{-m}$ ),  $m$  ( $m_H$ )  $t_s = \frac{12.5}{n_{19} h_{m\Omega}} \text{ msec}$

STX-HF:  $n_{19} \sim 5$  (10),  $\eta \sim 60$  (30),  $\Rightarrow \tau_s \sim 40 \mu\text{sec}$

Ion spin-up momentum is lost in  $\tau_N \sim 100 \mu\text{sec}$ . The fraction of spin-up,  $\alpha$  ( $= u_{i\theta}/u_{e\theta}$ ):

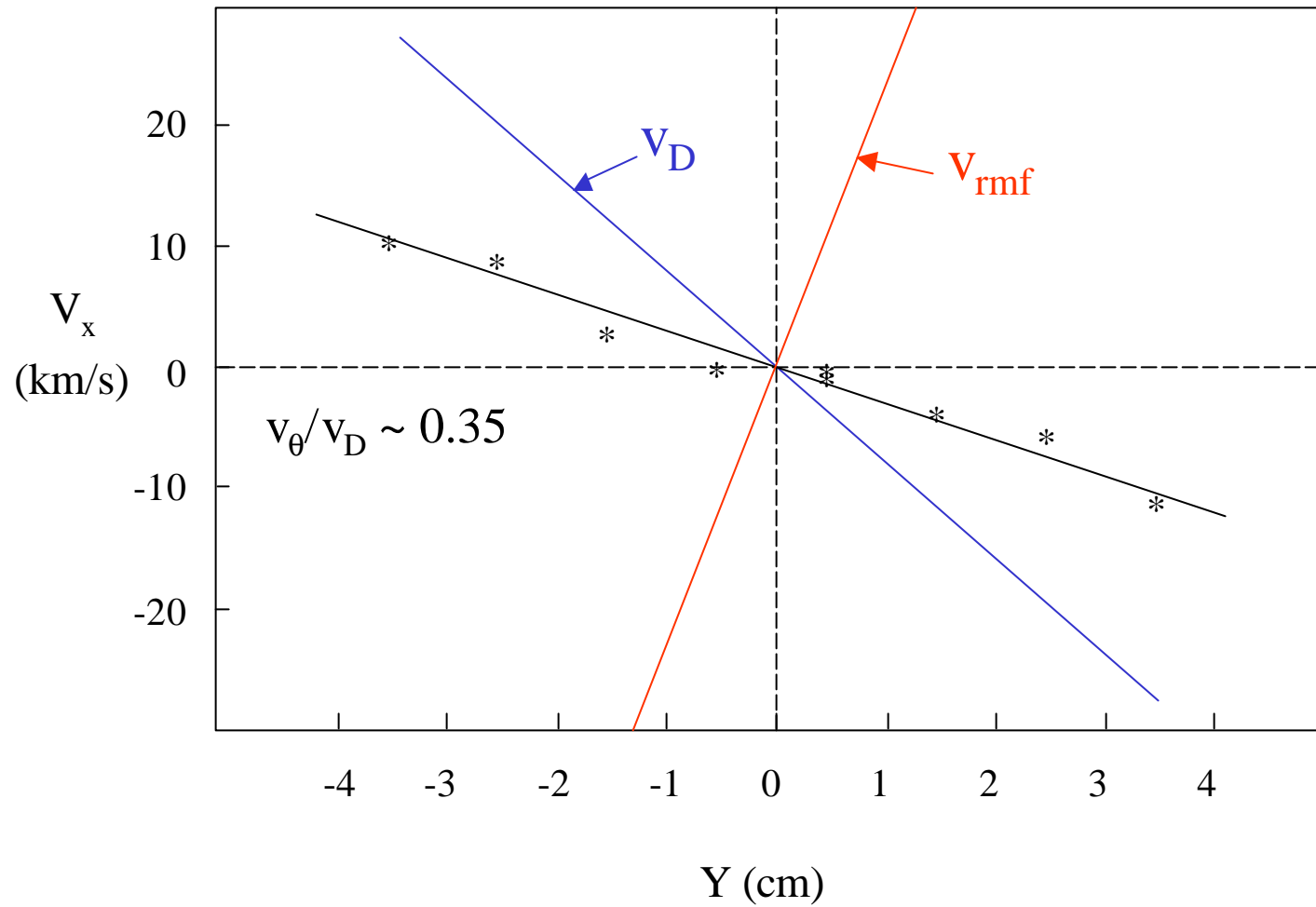
$$a = \left[ 1 + \frac{t_s}{t_N} \right]^{-1} = 0.7$$

Not ignorable  
For STX-HF

From radial component of gen. Ohm's law:  $u_q = -\frac{E_r}{B_z} + \frac{1}{neB_z} \frac{dp_i}{dr}$

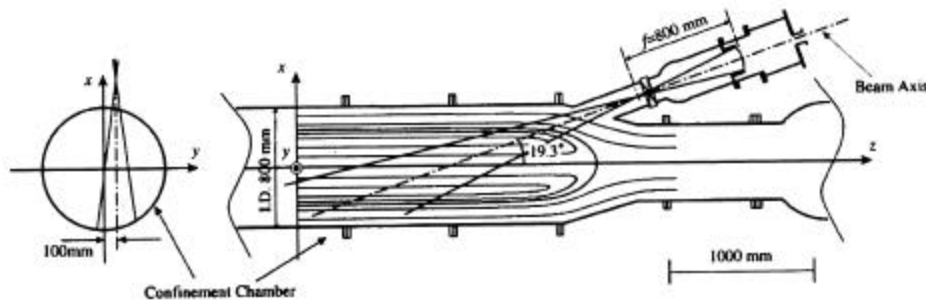
Ion diamagnetic drift  
Opposite dir. to spin-up

# Shift of He II Line Center as a Function of Impact Parameter

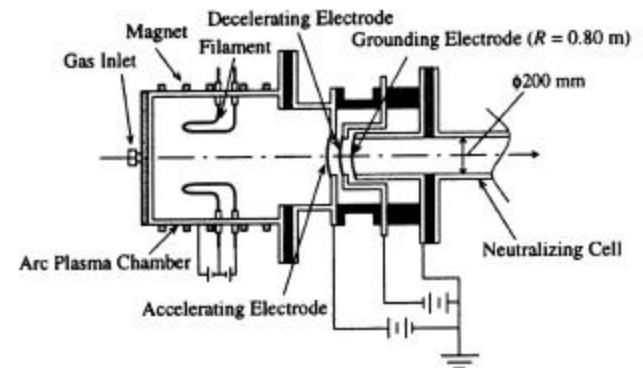


# Enhanced Confinement Using Neutral Beams on FIX

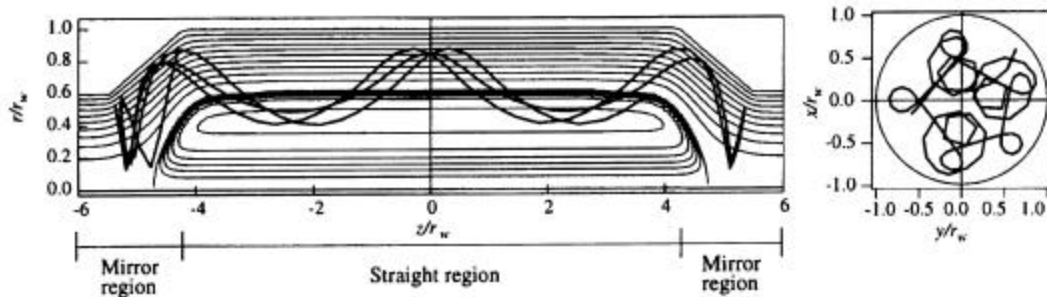
NB injection geometry on 80-cm diameter FIX confinement chamber



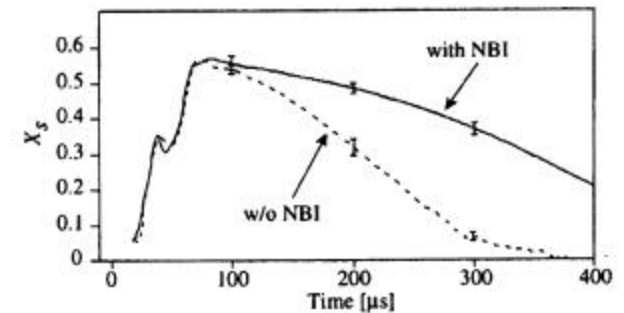
Simple 15-25 kV NB Injector



Resultant ion orbits in low flux FIX FRC

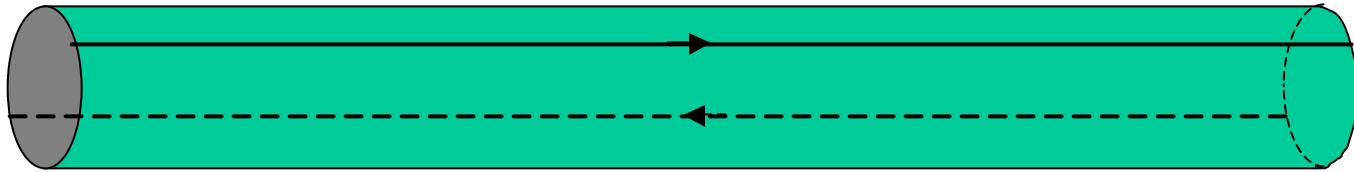


Measured enhanced lifetime

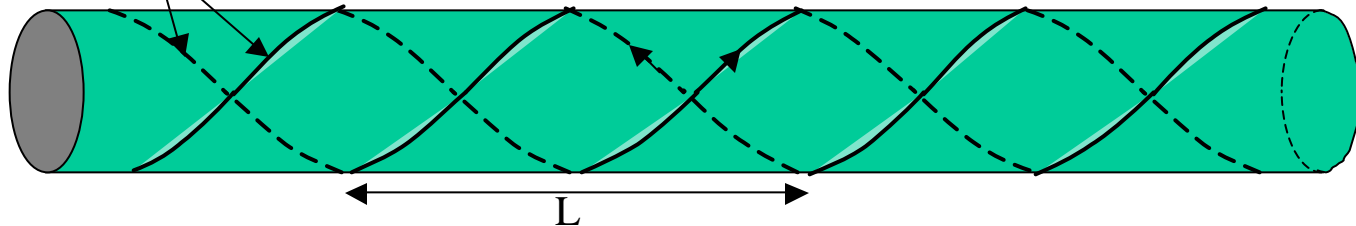


# RMF Confinement and Drive from Steady Coils

One of two loops for RMF in stationary FRC where  $B(t) \sim \sin\omega t$



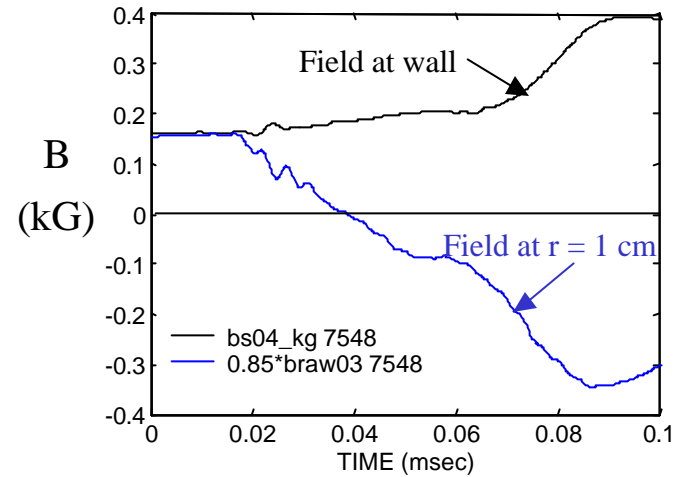
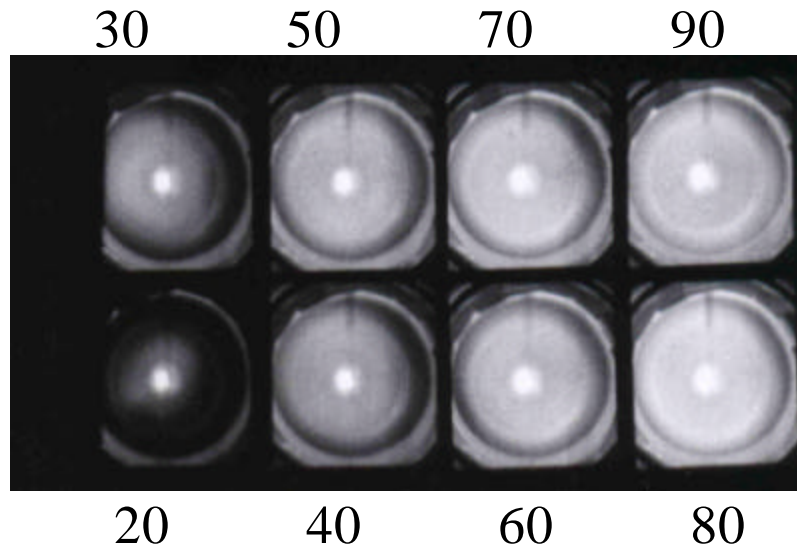
One of several pair of current loops where  $B(t) = \text{const.}$



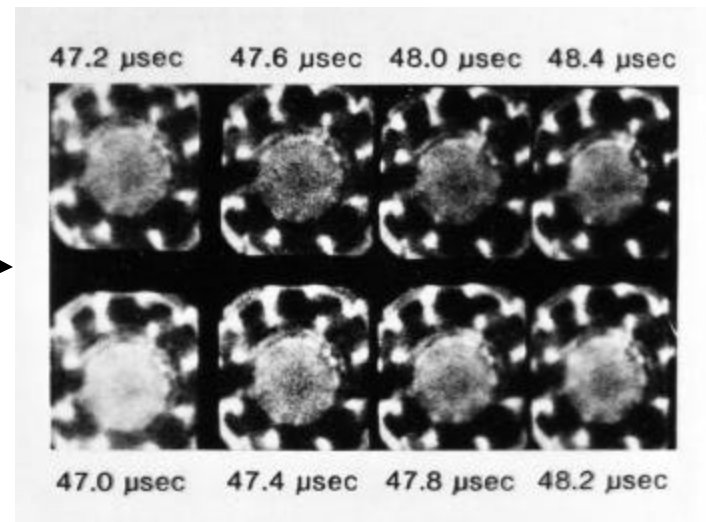
In frame of FRC traveling at velocity  $v_{\text{FRC}}$ , the transverse field rotates at a frequency  $\omega = 2\pi v_{\text{FRC}}/L$ :

$$\omega = 9 \times 10^6 \text{ and } L = 0.10 \text{ m} \Rightarrow v_{\text{FRC}} = 1.4 \times 10^5 \text{ m/s}$$

# Wall Protection from RMF Induced Flow



“Conventional” CT  
With Steady Transverse  
(Octopole) Magnetic Field

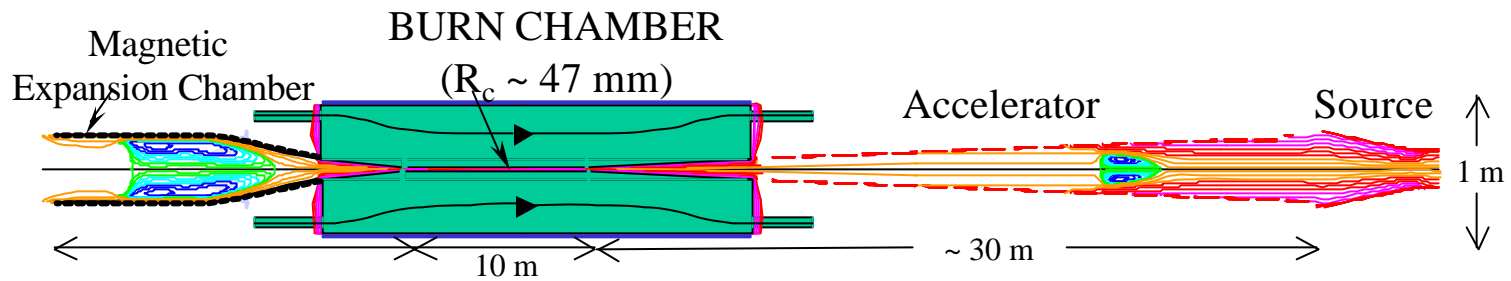


# PHD Fusion Reactor Summary

- Straight forward extrapolation of demonstrated FRC formation and acceleration techniques
- Simple linear system that could employ **superconducting magnets**
- Easily varied fusion output power (10s of MW **not multi-GW**)
- Low cost development path to demonstration ~ **10 M\$ not 10 G\$**
- Converter, burn chamber, accelerator and formation sections
- well separated.
- **Direct electric power conversion** possible with flux compression from expansion of fusion heated FRC



# PHD Accelerator Parameters



From energy conservation:

$$v_{\text{FRC}} = (5kT_{\text{fus}}/m_{\text{DT}})^{1/2} \sim 1 \times 10^6 \text{ m/s}$$

For constant acceleration  $a$  ( $2 \times 10^{10} \text{ m/s}^2$ ), accelerator length  $L_{\text{acc}}$ :

$$L_{\text{acc}} = v_{\text{FRC}}^2 / 2a \sim 25 \text{ m} \quad t_{\text{acc}} \sim 50 \text{ } \mu\text{sec}$$

Burn chamber length:  $L_{\text{burn}} \leq v_b \tau_E \sim (2 \times 10^4 \text{ m/s})(5 \times 10^{-4} \text{ s})$

$$L_{\text{burn}} \sim 10 \text{ m}$$

Fusion Power:  $P_{\text{fus}} = G \cdot E_p \cdot \text{rep rate} = (10)(700 \text{ kJ}) \cdot (10)$

$$P_{\text{fus}} \sim 70 \text{ MW} \quad @ 10 \text{ s}^{-1} \text{ (duty cycle } \sim 0.5\%)$$