Issues in neoclassical MHD modeling

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Introduction

- Neoclassical processes have important influences on a number of phenomena in toroidal confinement devices
  - Damping of fluid flows
  - Neoclassical tearing modes
  - Polarization island thresholds
- Theoretical modeling used in numerical simulations
  - Neofar, NIMROD, NFTC, M3D
- Further theoretical developments are under way
  - MHD perturbation induced toroidal viscosity
  - Time dependent viscosities

Neoclassical physics is important in high temperature tokamaks

- Neoclassical theory is usually formulated in the language of viscosities with a small parameter ($\rho_L/a << 1$).
  - To leading order, viscous forces damps flows in the direction of magnetic field asymmetry . Viscous force ~ $\mathbf{v}\cdot \nabla|B|$
  - For axisymmetric equilibrium

$$<\vec{B}\cdot\nabla\cdot\Pi_{\parallel}> = \rho <B^2> \mu \frac{\vec{u}\cdot\nabla\theta}{\vec{B}\cdot\nabla\theta}$$

$$<\vec{B}_T\cdot\nabla\cdot\Pi_{\parallel}> = 0$$

  - μ = flow damping frequency (collisionality dependent), $\mu \sim \varepsilon^{0.5}\nu$ at low collisionality
- Viscous force on ions - damps poloidal ion flow
- Viscous force on electrons - enters through Ohm's law - leads to bootstrap current and neoclassical modification to the plasma resistivity

For numerical simulation, expressions for local viscosities are required

- Neoclassical theory usually used to describe transport processes across flux surfaces - flux surface averaged quantities are evaluated.
  - Requirement for numerical simulations = local quantities in temporal/spatially/topologically evolving magnetic fields.
  - "Heuristic" closure (Gianakon, et al)

$$\nabla \cdot \vec{\Pi}_{\|} = \frac{\vec{e}_\theta}{\vec{B} \cdot \vec{e}_\theta} \vec{B} \cdot \nabla \cdot \vec{\Pi}_{\|}$$

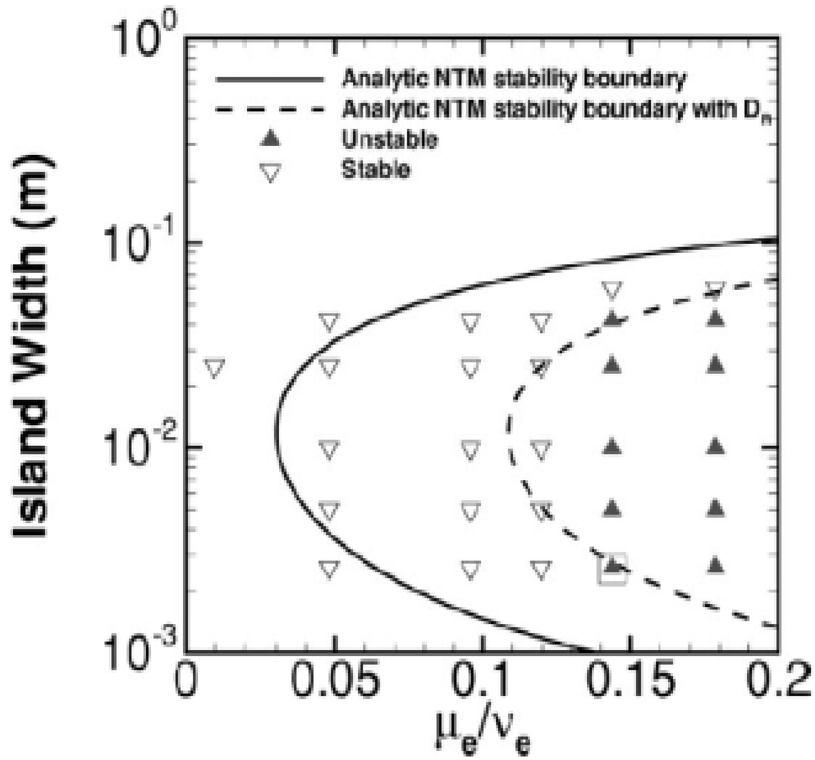
$$\vec{B} \cdot \nabla \cdot \vec{\Pi}_{\|} = \mu \frac{\vec{v} \cdot \nabla \theta}{\vec{B} \cdot \nabla \theta} \rho_M B^2$$

    - Correctly retains all the conservation properties of the actual viscosities,
    - Correctly reproduces the linear and nonlinear properties of NTM physics obtained from kinetic theory predictions

The use of the approximate closure schemes allow Neoclassical Tearing Mode simulations to be performed

- In these simulations, anisotropic thermal Conductions ($\chi_{||}/\chi_{perp}$ ~ 10^{10}) provides a neoclassical tearing mode threshold. (no two fluid effects) $S = 2.7\times10^6$

- Reproduces expected NTM behavior in experimentally relevant parameter ranges



Modifications to neoclassical theory are needed to completely model MHD phenomena in tokamaks

- Toroidal viscosity due to 3-D perturbations (Shaing, et al '02)
  - In the presence of a MHD perturbation, a three-dimensional perturbation is introduced- [B = $B_o(\psi,\theta) + \Sigma_{mn}\, b_{mn}\cos(m\theta - n\zeta)$],
  - For a magnetic island

$$\frac{B}{B_o} = 1 - \frac{r}{R_o}\cos\theta \pm \frac{w}{R_o}\sqrt{\frac{\Psi^* + \cos(m\theta - n\zeta)}{2}}$$

- Time dependent viscosities for "fast" temporally varying phenomena (Garcia-Perciante et al '02)
  - Work in progress

Nonaxisymmetric physics affects toroidal rotation

- Nonaxisymmetric neoclassical fluxes present with nonzero b_{mn}. [$B = B_o(\psi,\theta) + \Sigma_{mn} b_{mn}\cos(m\theta - n\zeta)$], $\omega_{Dr} = T_i/eBRr$
  - Nonresonant (plateau regime) [Shaing, et al PF '86; Smolyakov '95]

$$\vec{B}_T \cdot \nabla \cdot \Pi \cong -\rho_M I \frac{v_{thi}}{qR} \Sigma_{mn} \frac{n^2}{n - m/q} \frac{b_{mn}^2}{B_o^2} [\omega_E - \omega_i^* (1 + k_o \eta_i)]$$

  - Resonant "1/ν" regime [$\nu_i/\varepsilon > (w/R_o)\omega_D$] (Shaing, PRL '01)

$$\vec{B}_T \cdot \nabla \cdot \Pi \cong -\rho_M I \varepsilon^{3/2} \frac{\omega_{Dr}^2 m^2}{\nu_i} \frac{w^2}{R^2} H(\frac{x}{w}) [\omega_E - \omega_i^* (1 + k_1 \eta_i)]$$

  - Resonant "superbanana" [$\nu_i/\varepsilon < (w/R_o)\omega_D$] (Shaing, IAEA '02)

$$\vec{B}_T \cdot \nabla \cdot \Pi \cong -\rho_M I \frac{\upsilon_i}{\sqrt{\varepsilon}} \frac{\omega_{Dr}^2}{\omega_E^2} \frac{w^2}{R^2} G(\frac{x}{w}) [\omega_E - \omega_i^* (1 + k_2 \eta_i)]$$

  - Transition formulae for different "colllisionality regimes" can be generated

The emergence of toroidal viscosity affects a number of important MHD processes

- Interaction of toroidal flow velocity and MHD modes
  - Neoclassical viscosity damps toroidal flow in the presence of 3-D perturbations (Lazzero, et al 'PoP '02; Sabbagh et al '02; LaHaye et al '01) -Different mechanism than slowing due to localized torques due
- Effect of neoclassical polarization currents for NTM physics
  - Enhances polarization current from Wilson, et al calculation
  - Introduces dissipation - damps polarization currents - contributes to the determination of the mode frequency

Toroidal flow evolution determined by a number of effects

- Parallel momentum balance

$$< \rho \frac{d\vec{v} \cdot \vec{B}_T}{dt} > = < \vec{B}_T \cdot \delta\vec{J} \times \delta\vec{B} > - < \vec{B}_T \cdot \nabla \cdot \Pi_{||} > + \nabla_\perp \cdot (\rho\mu_\perp \nabla_\perp \vec{v} \cdot \vec{B}_T)$$

$\Uparrow$ Plasma inertia

$\Uparrow$ EM torques due to interaction with resistive walls, and field errors

$\Uparrow$ Toroidal Neoclassical viscosity

$\Uparrow$ Cross-field viscosity

- Unlike the EM torques which are localized to the vicinity of the rational surfaces, the toroidal neoclassical viscosity describes damping throughout the plasma cross-section

Heuristic model for neoclassical physics can be expanded to include toroidal viscosities

- A phenomenological fluid-like model is suggested to describe the toroidal viscosity modification with viscous damping frequencies ($\mu_{\|}^{\theta}$, $\mu_{\|}^{\zeta}$, μ_{T}^{ζ}, υ_{T}^{θ}, etc)

$$\nabla\cdot\Pi_i = \frac{\nabla\theta}{\vec{B}\cdot\nabla\theta}[\vec{B}\cdot\nabla\cdot\Pi_i - \vec{B}_T\cdot\nabla\cdot\Pi_i] + \frac{\nabla\zeta}{\vec{B}\cdot\nabla\zeta}\vec{B}_T\cdot\nabla\cdot\Pi_i$$

$$\frac{\vec{B}\cdot\nabla\cdot\Pi_i}{\rho_M < B^2 >} \cong \mu_{\|}^{\theta}\frac{\vec{u}\cdot\nabla\theta}{\vec{B}\cdot\nabla\theta} + \mu_{\|}^{\zeta}\frac{\vec{u}\cdot\nabla\zeta}{\vec{B}\cdot\nabla\zeta} + \mu_{\|q}^{\theta}\frac{\vec{q}\cdot\nabla\theta}{p_i\vec{B}\cdot\nabla\theta} + \mu_{\|q}^{\zeta}\frac{\vec{q}\cdot\nabla\zeta}{p_i\vec{B}\cdot\nabla\zeta}$$

$$\frac{\vec{B}_T\cdot\nabla\cdot\Pi_i}{\rho_M < B^2 >} \cong \mu_{T}^{\theta}\frac{\vec{u}\cdot\nabla\theta}{\vec{B}\cdot\nabla\theta} + \mu_{T}^{\zeta}\frac{\vec{u}\cdot\nabla\zeta}{\vec{B}\cdot\nabla\zeta} + \mu_{Tq}^{\theta}\frac{\vec{q}\cdot\nabla\theta}{p_i\vec{B}\cdot\nabla\theta} + \mu_{Tq}^{\zeta}\frac{\vec{q}\cdot\nabla\zeta}{p_i\vec{B}\cdot\nabla\zeta}$$

Toroidal viscosity modifies the neoclassical polarization drift

- Neoclassical effects dominate MHD polarization drifts in the quasineutrality equation of high temperature tokamaks.

$$\vec{B}\cdot\nabla\frac{J_{\|}}{B} = -\nabla\cdot\frac{\vec{B}\times\nabla\cdot\Pi_i}{B^2} = -\frac{\partial}{\partial\psi}\left[\frac{\vec{B}_T\cdot\nabla\cdot\Pi}{q\vec{B}\cdot\nabla\theta} - \frac{(1-\delta)}{q\vec{B}\cdot\nabla\theta}\vec{B}\cdot\nabla\cdot\Pi\right]$$

$$\nabla\cdot\frac{\vec{B}\times\nabla\cdot\Pi}{B^2} \cong \frac{\partial}{\partial\psi}\frac{\rho B^2}{q^2(\vec{B}\cdot\nabla\theta)^2}\left[\frac{\partial\varphi}{\partial\psi} + \frac{T_i}{ne}\frac{\partial n}{\partial\psi}(1+k\eta_i)\right](-\mu_D + i\omega' f)$$

- New element brought in by toroidal viscosity is a damping of the neoclassical polarization. (for $\mu_{\|}^{\zeta} = \mu_T^{\theta} = 0$, $\mu_{\|}^{\theta} > \omega$)

$$\nabla\cdot\frac{\vec{B}\times\nabla\cdot\Pi}{B^2} \cong \frac{\partial}{\partial\psi}K^2\left[\frac{\partial\varphi}{\partial\psi} + \frac{T}{nq}\frac{\partial n}{\partial\psi}(1+k\eta_i)\right](-\mu_T^{\zeta} + i\omega' f)$$

$$K^2 = \frac{\rho B^2}{q^2(\vec{B}\cdot\nabla\theta)^2},\ f = (1-\delta)^2 + \frac{\delta\mu_T^{\zeta}}{\mu_{\|}^{\theta}})$$

Time dependent viscosity

- Conventional neoclassical theory calculates in the long time asymptotic regimes ($t >> 1/\nu_i$)
- Some problems require understanding timescales short of the collision time
  - Finite frequency
  - NTM seed island formation problem
    - seeding often associated with a “fast” MHD event, e. g., sawtooth
    - temporal behavior of the polarization threshold.

Formulation of the problem is developing

- Chapman-Enskog distribution function

  f(**x**,**v**,t) = f_M(**x**,**v**,t) + F(**x**,**v**,t)

  – f_M: flow shifted Maxwellian - n(**x**,t), **V**(**x**,t), T(**x**,t)
  – F: kinetic distortion; does not contribute to density, momentum and energy moments

- Time dependent drift kinetic equation

$$\frac{\partial F}{\partial t} + v_{\|}\vec{b}\cdot\nabla[F + v_{\|}B\frac{m}{T}f_M U] - \frac{1}{2}\nu_{\perp}(v)L(f) = v_{\|}[\nu_o + \frac{\vec{b}\cdot\nabla\cdot\Pi}{p}]f_M$$

  ν_O related to the collisional momentum restoring term

$$\int d^3v\nu_o = -\frac{m}{p}\int d^3v\frac{\nu_{\perp}}{2}v_{\|}L_0^{3/2}L(F)$$

  – Lowest order solution (bounce time)

$$F_0(v,\mu,\psi,t) = v_{\|}B\frac{m}{T}U(\psi,t)f_M + g(v,\mu,\psi,t)$$

Perturbed distribution satisfies a partial differential equation involving time and pitch angle scattering

- Bounce averaging the next order solution yields an equation for g

$$\frac{\partial}{\partial t}[-<B^2>\frac{m}{T}f_M U+<\frac{B}{v_{\|}}>g]-\frac{1}{v}(v_{\perp}\frac{\partial}{\partial\lambda}\lambda<\frac{v_{\|}}{v}>\frac{\partial g}{\partial\lambda})$$

$$=\frac{v_{\perp}}{v_{th}^2}<B^2>[\frac{V}{n}+U(1-\frac{2m}{3T}v^2)]f_M+\frac{1}{p}<\vec{B}\cdot\nabla\cdot\Pi>$$

Where $V=\int_0^{\lambda}d\lambda g$

  - Solution of g in terms of time dependent sources U, <**B**·Π>

- Similar procedure developed for describing neoclassical heat flux with multiple magnetic asymmetries lengthscales (Held, et al Phys. Plasmas 2001)

Solution can be obtained using eigenfunctions of scattering operator

- The equation can be solved by using an expansion in eigenfunctions of the homogeneous equation in order to separate the speed and pitch angle dependencies

$$g(v,\mu,\psi,t) = \sum_{n=1}^{\infty} Y_n(v,\psi)\Lambda_n(\psi,\lambda)$$

- Where the eigenfunction equation satisfies

$$\frac{1}{v}\frac{\partial}{\partial\lambda}\lambda < \frac{v_\|}{v} > \frac{\partial\Lambda_n}{\partial\lambda} = \kappa_n < \frac{B}{v_\|} > \Lambda_n$$

And satisfies orthogonality conditions

$$\int_0^{\lambda} d\lambda < \frac{B}{v_\|} > \Lambda_n\Lambda_m = \delta_{n,m}$$

Summary

- Parallel neoclassical viscosities - poloidal flow damping, bootstrap currents, neoclassical modification to resistivity
  - Neoclassical tearing mode physics included in present day simulation tools
- Extensions of neoclassical theory are required to improve our understanding of various MHD phenomena
  - Torodial viscosities
    - Effect on toroidal flow evolution in the presence of 3-D perturbations
    - Neoclassical polarization physics
  - Time dependent viscosities
    - For use in time scales short of collision frequencies ($\varepsilon\omega > \nu_i$)
    - Seed island formation for NTM physics