FEEDBACK AND CONTROL OF LINEAR AND NONLINEAR GLOBAL MHD MODES IN ROTATING PLASMAS

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LANL

OUTLINE

- Resistive wall modes: when MHD modes are wall stabilized, they can persist as *resistive wall modes*. They can be stabilized by rotation, but too much rotation is required.

- Model: reduced resistive MHD in a slab, $0 < x < L_x$, $0 < y < L_y$. Sensor at resistive wall $(y = L_y)$, control at outer wall $y = W$: flux specified.

- Complex gain: $\psi(x, y = W) = -G\psi(x-\delta, y = L_y)$: $Ge^{-ik\delta} = G_r + iG_i$.

Outline, continued

- Equivalence of G_r to a closer outer wall (caveat - single k).
- Equivalence of G_i to rotation of the resistive wall (caveat - single k).
- Nonlinear simulations with G_r, G_i.

Linear stabilization.

Limiting the nonlinear saturation amplitude – if feedback is not quite up to stabilization, or if a low level saturated mode is advantageous, or to reduce required gain (noise.)

MODEL: On $0 < x < L_x$, $0 < y < L_y$

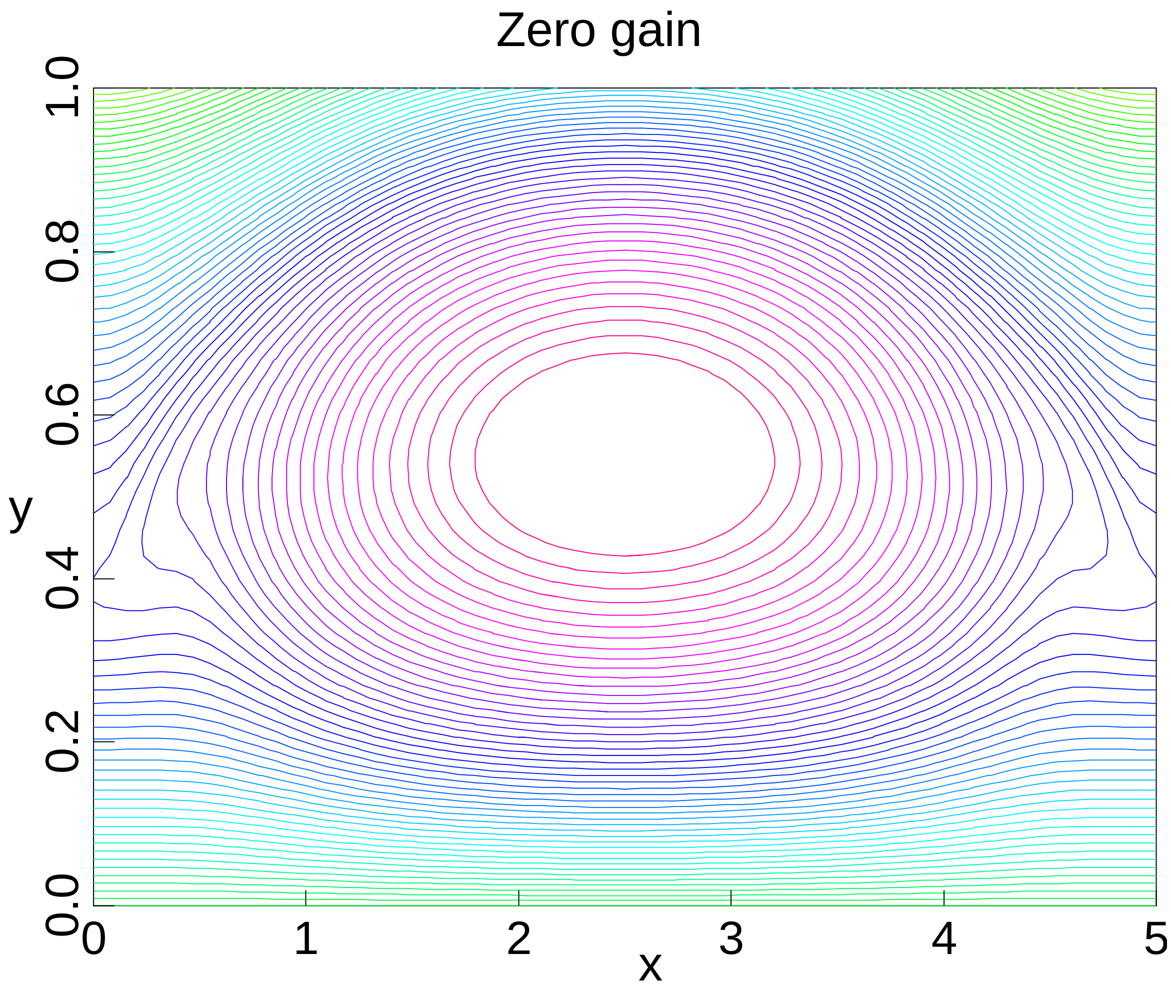

Figure 1: Large amplitude mode with flux in wall.

Equations

$$\overrightarrow{B} = \nabla\psi \times \widehat{z} + B_0\widehat{z}, \qquad \overrightarrow{v} = \nabla\phi \times \widehat{z} + v_{||}\widehat{z}$$

$$\begin{aligned}
(\frac{\partial}{\partial t} + \nabla\phi \times \widehat{z}\cdot\nabla)\omega &= \overrightarrow{B}\cdot\nabla j - \kappa T\partial n/\partial x + \mu\nabla^2\omega \\
\frac{\partial}{\partial t}\psi - \overrightarrow{B}\cdot\nabla\phi &= \eta\nabla^2\psi + E(y) \\
\nabla^2\phi = -\omega, &\qquad j = -\nabla^2\psi \\
(\frac{\partial}{\partial t} + \nabla\phi \times \widehat{z}\cdot\nabla)n &= -n(\overrightarrow{B}/B_0)\cdot\nabla v_{||} + D\nabla^2 n \\
(\frac{\partial}{\partial t} + \nabla\phi \times \widehat{z}\cdot\nabla)v_{||} &= -\frac{c_s^2}{n}(\overrightarrow{B}/B_0)\cdot\nabla n + \mu_{||}\nabla^2 v_{||}
\end{aligned}$$

RESISTIVE WALL BOUNDARY CONDITION AND MATCHING TO VACUUM

$$\frac{\tau}{L_y}\frac{\partial}{\partial t}\psi(x, y = L_y) = \left[\frac{\partial \psi}{\partial y}\right]_{y=L_y}$$

$$\tau = L_y \Delta / \eta_{wall}$$

Thin wall boundary condition.

Vacuum ($\nabla^2 \psi = 0$) for $0 < x < L_x$, $L_y < y < W$

RESISTIVE WALL AND VACUUM

Vacuum ($\widetilde{\psi}_{k,vac} \sim e^{\pm ky}$) and feedback boundary condition:

$$\psi(x, W) = -G\psi(x-\delta, L_y),$$

$$\widetilde{\psi}_k(W) = -Ge^{-ik\delta}\widetilde{\psi}_k(L_y) = -(G_r + iG_i)\widetilde{\psi}_k(L_y) \quad \Rightarrow$$

.

$$\frac{\tau}{L_y}\frac{\partial}{\partial t}\widetilde{\psi}_k(y = L_y) = -k\widetilde{\psi}_k(L_y)\left[\coth k(W - L_y) + \frac{G}{\sinh k(W - L_y)}\right] - (\frac{\partial \widetilde{\psi}_k}{\partial y})_{pl}$$

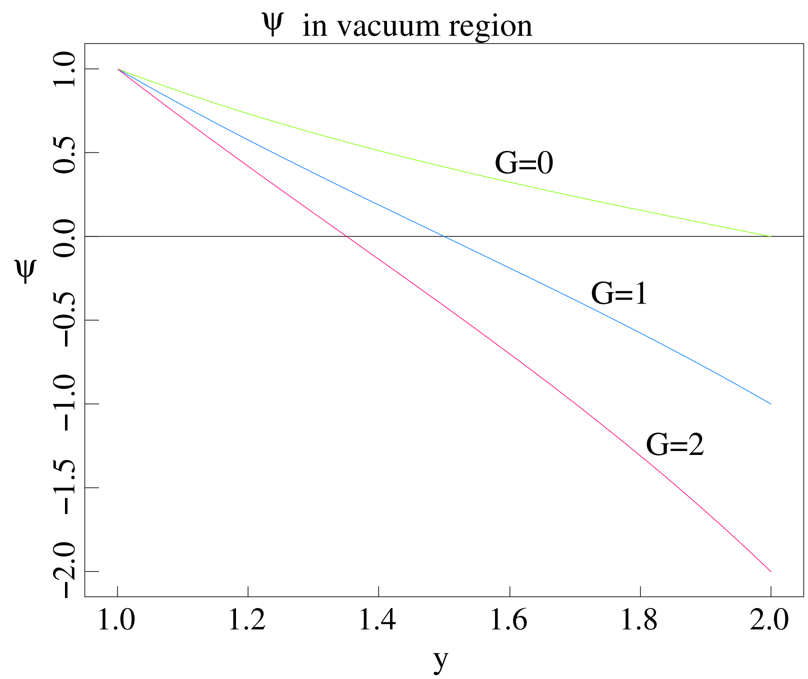

Figure 2: Real gain and equivalent wall position

REAL GAIN

Proportional gain – real G: exactly equivalent to a wall closer, at $y = W'$ for a fixed k:

$$\coth k(W - L_y) + G_r / \sinh k(W - L_y) = \coth k(W' - L_y)$$

for one specific k, i.e. $W' = W'(G_r, W, k)$.

This equivalence works too for nonlinear, except for the spectrum of k.

IMAGINARY GAIN

Stationary resistive wall with imaginary gain:

$$\frac{\tau}{L_y}\frac{\partial}{\partial t}\widetilde{\psi}_k(y=L_y) = -ik\psi_k(L_y)\left[\coth k(W-L_y) + i\frac{G_i}{\sinh k(W-L_y)}\right] - \left(\frac{\partial\widetilde{\psi}_k}{\partial y}\right)_{pl}$$

Rotating wall with no gain:

$$\frac{\tau}{L_y}\frac{\partial}{\partial t}\widetilde{\psi}_k(y=L_y) + \frac{ikv_0\tau}{L_y}\widetilde{\psi}_k(L_y) = -k\widetilde{\psi}_k(L_y)\left[\coth k(W-L_y)\right] - \left(\frac{\partial\widetilde{\psi}_k}{\partial y}\right)_{pl}$$

IMAGINARY GAIN, cont'd

Exact equivalence for single k:

$$\frac{ikv_0\tau}{L_y}\widetilde{\psi}(L_y) = ik\widetilde{\psi}(L_y)\frac{G_i}{\sinh k(W - L_y)}$$

–

$$v_0 = v_0(G_i, W, k) = \frac{G_i}{\sinh k(W - L_y)}$$

This equivalence holds nonlinearly too, except for the spectrum of k.

G_i causes the free flux decay through the resistive wall to propagate. This weakens the coupling with the plasma mode if e.g. the equivalent wall rotation is negative and the plasma rotation is positive or zero. (Mode coupling picture – Finn, Phys. Plasmas 3, 2344 (1996))

Complex gain is equivalent to a closer outside wall *-plus-* rotation of the RW.

But remember, rotational stabilization has hysteresis (locking-unlocking).

PARAMETERS

Equilibrium: Harris sheet – $B_x(y) = \tanh[(y - 1/2)/\lambda]$

$\lambda = 0.5,\ \eta = \mu = D = \mu_{||} = 10^{-3},\ c_s/v_A = 0.25,\ L_y = 1,\ L_x = 5,\ W = 2.5\ \tau = 100$

For $\lambda = 0.5$ and curvature-beta parameter $\kappa\beta = 0$, the mode is unstable for $\tau = 0$ and for $\tau = \infty$. (Finite critical β for perfectly conducting wall and transparent wall.

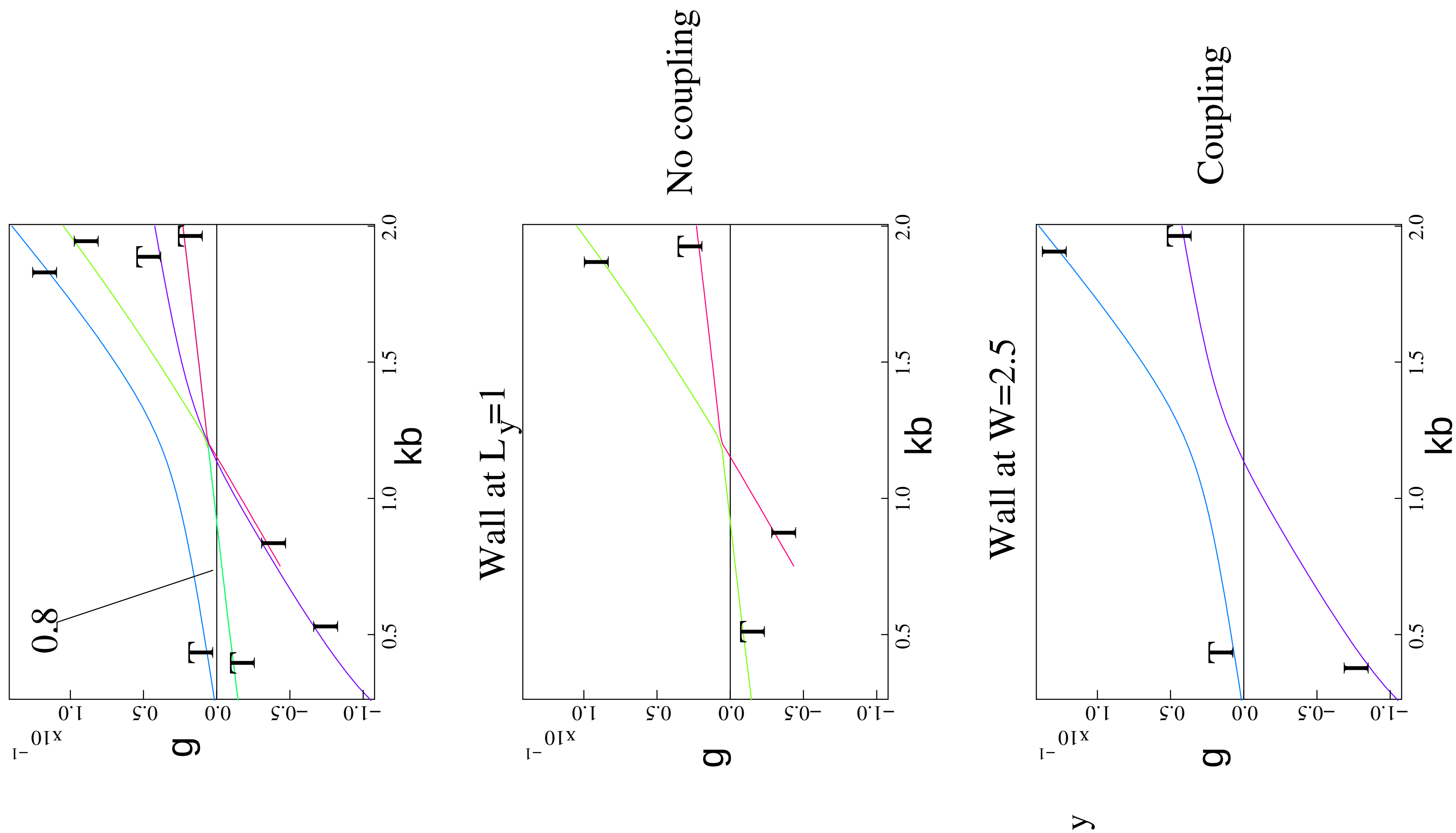

Figure 3: Growth rate of tearing and interchange modes vs $\kappa\beta$. Resistive plasma - ideal wall mode cannot be stabilized by rotation! Ideal plasma - resistive wall mode AND resistive plasma - resistive wall mode must be stabilized. [Region III of Finn, Phys Plasmas 2, 3782 (1995).]

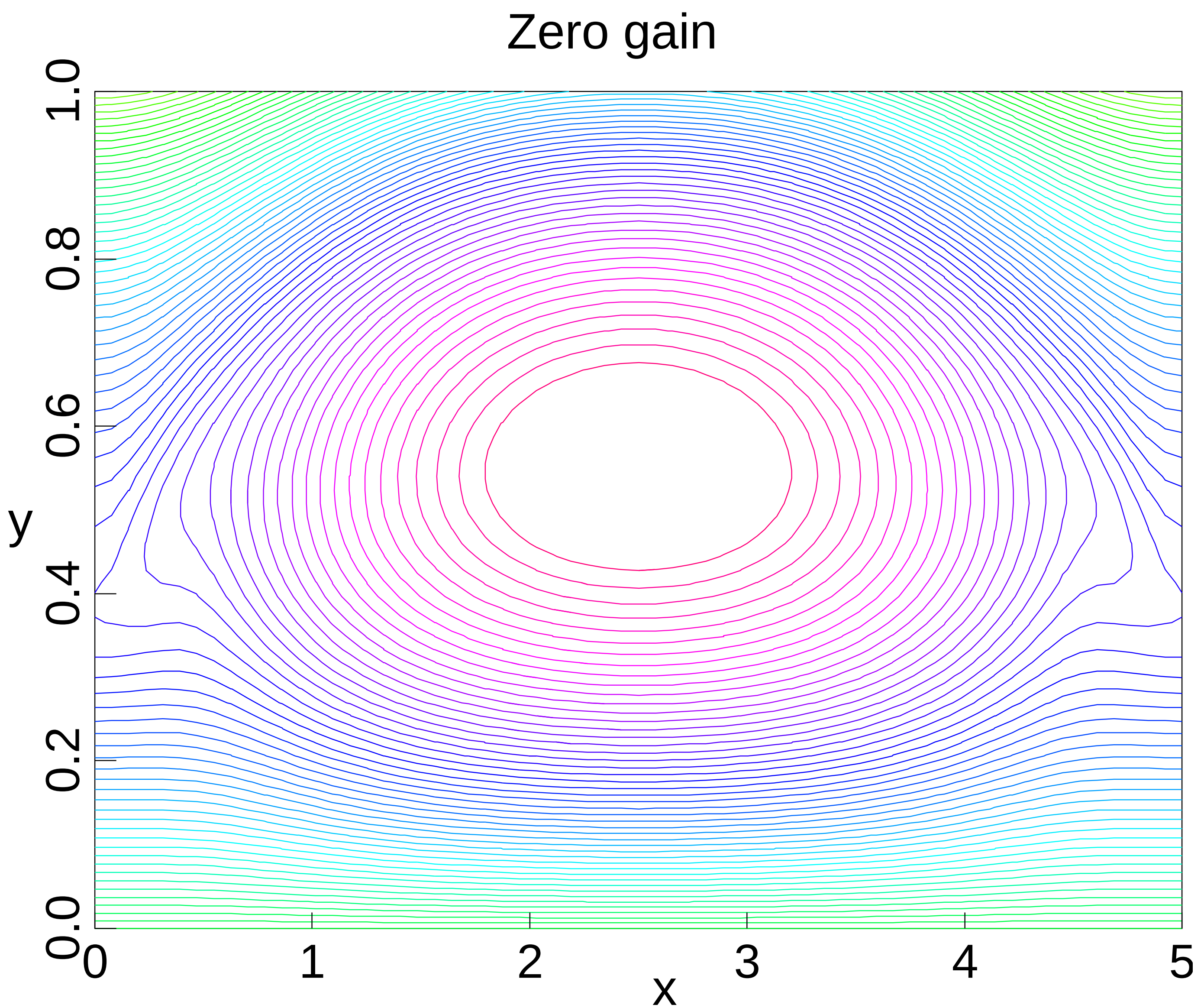

Figure 4: The case with zero gain is mixed tearing-interchange and has a large island at saturation

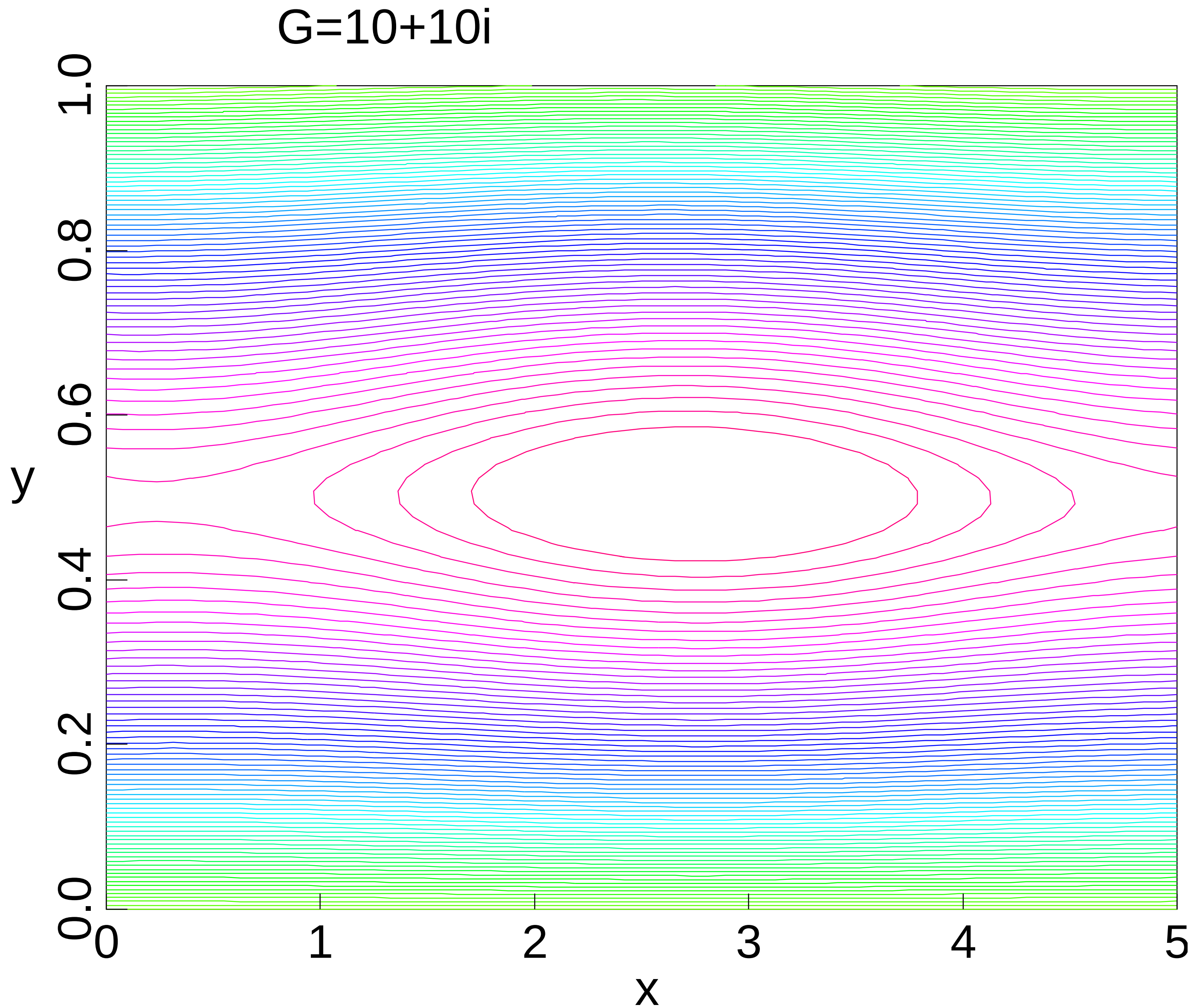

Figure 5: $G_r = G_i = 15$ (N.B.) case is below the value required for linear stabilization

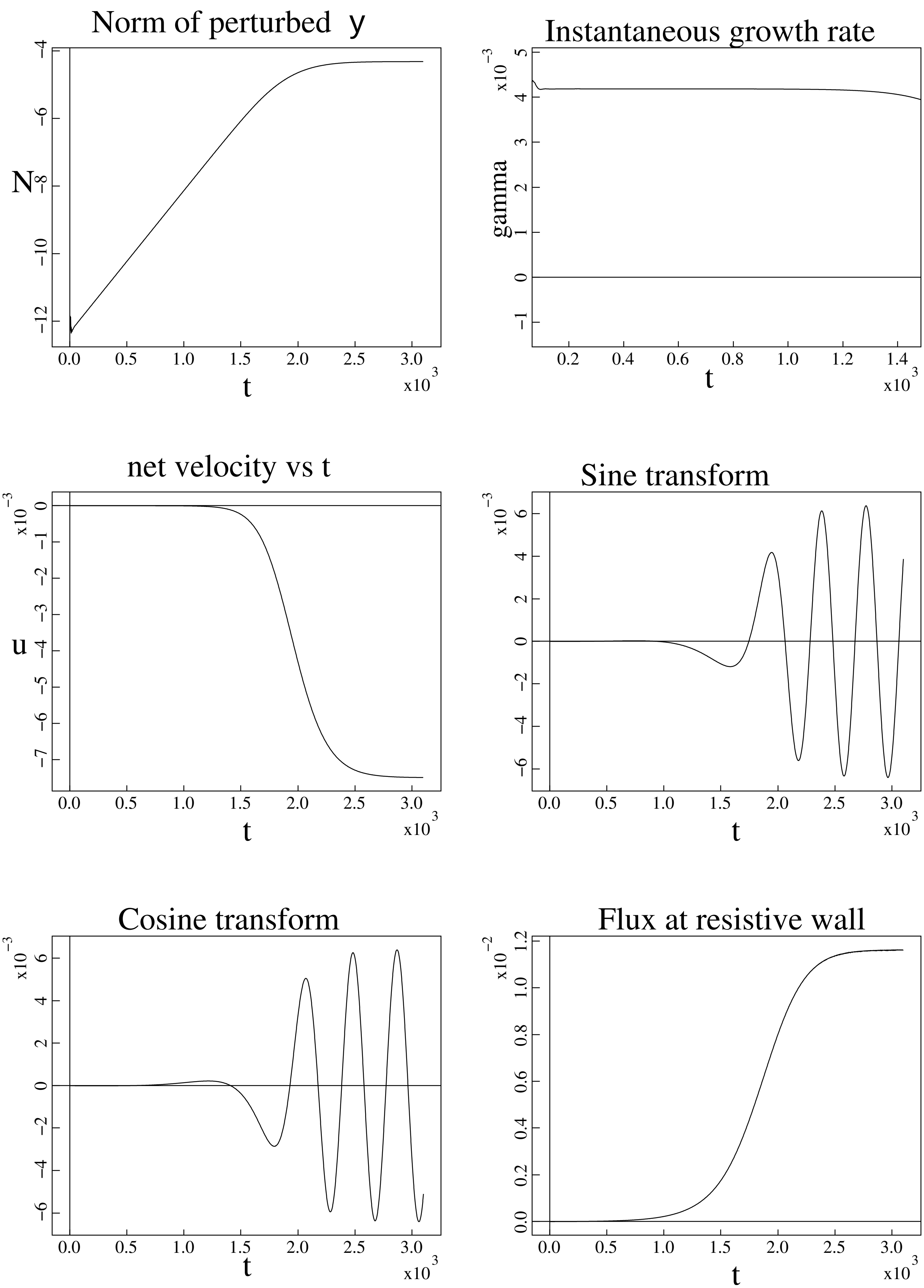

Figure 6: Nonlinear cases with $G_r = 10$, $G_i = 10$. Note propagation, net plasma velocity, and flux at resistive wall

Sample nonlinear cases

(G_r, G_i)	γ	$Width$	$\|\widetilde{\psi}\|$	$4\sqrt{\left(\|\widetilde{\psi}\|\right)}$
0,0	$\mathbf{1.2 \times 10^{-2}}$	0.71	0.40	0.80
3,0	$\mathbf{9.5 \times 10^{-3}}$	0.65	0.31	0.71
10,0	$\mathbf{5.5 \times 10^{-3}}$	0.49	0.018	0.53
7.5,7.5	$\mathbf{5.4 \times 10^{-3}}$	0.49	0.018	0.53
10,10	$\mathbf{4.0 \times 10^{-3}}$	0.44	0.013	0.46
12.5,12.5	$\mathbf{2.9 \times 10^{-3}}$	0.37	0.010	0.40
15,15	$\mathbf{2.1 \times 10^{-3}}$	0.34	0.0073	0.34
17.5,17.5	$\mathbf{1.5 \times 10^{-3}}$	0.29	0.0059	0.31
20,20	$\mathbf{1.2 \times 10^{-3}}$	0.26	0.0047	0.27
25,25	$\mathbf{3.9 \times 10^{-4}}$	0.21	0.0034	0.23
29,29	$\mathbf{1.9 \times 10^{-5}}$	small	small	small
29.5,29.5	$\mathbf{-2.0 \times 10^{-5}}$	small	small	small

$4\|\widetilde{\psi}\|$ is an approximation to the island width.

CONCLUSIONS

- Real (proportional) gain is equivalent to a closer perfectly conducting wall for each k.

- Imaginary gain is equivalent to rotation of the resistive wall, which is equivalent to rotating the plasma in the opposite direction.

- Rotational stabilization (G_i) has hysteresis, which might be dangerous, i.e. allow locking for finite perturbation even ir RWM is linearly stable.

- β must be below the resistive-plasma, ideal wall marginal point: the resistive-plasma, ideal wall mode cannot be stabilized by rotation. Need to compute the resistive-plasma, ideal wall β limit (including differential rotation between mode rational sufraces) for the external kink in toroidal geometry. 'Tearing' and 'interchange' cross near marginal stability.

CONCLUSIONS, cont'd

- Nonlinear simulations show that real and imaginary gain can stabilize or just control the nonlinear saturation of an ideal or resistive plasma mode.

- We explored a range of parameters for which large gain is required for linear stabilization, but smaller gain is required for benign saturation.