Avoidance of NTMs in High Performance DIII-D Plasmas

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New Understanding of Onset Mechanism Can Lead to Avoidance of NTMs

NTMs are linearly stable but nonlinearly unstable, i.e., metastable.

- Need “seed” or triggering event

Onset mechanism explains two common puzzles:

nth Sawtooth Seeds an NTM? Seedless NTMs?

[Graphs showing frequency vs. time with labeled sawteeth and NTMs]
A Comprehensive Model of NTM Onset Emerges From a Combination of Theories

All three components are necessary to explain the onset and evolution.
All three components are necessary to explain the onset and evolution of NTM (Neutral Mode Turbulence). The model emerges from a combination of theories: Classical Linear Drive, Neoclassical Effects, and Nonlinear Coupling. The time dependence is accurately treated through the free energy $\Delta'$ for mode $rac{d\Delta}{dx}$.

$$\Delta' \equiv \left[ \frac{d\Delta}{dx} \right]$$

$\Delta'$ is the free energy for mode.
Outline

**Thesis:** $\Delta'(t)$ depends strongly on the equilibrium parameters near ideal limit _ key physics for NTM onset in some cases _ **this is a key to avoidance.**

**Island Evolution Equation** : essential physics of nonlinear evolution

**Analytic model** : the basic physics of the time dependence of $\Delta'$

**Sawtooth Seeding of 3/2 NTM** : DIII-D discharge which encompasses nonlinear, neoclassical and classical effects is comprehensively analyzed showing the critical time dependence in $\Delta'$.

**Pole Experiment** : theoretical predictions of the effects of time dependence in $\Delta'$ are compared to results from a DIII-D experiment to isolate these effects.

**Open Questions on Avoidance:** implications on control of NTMs
Island Evolution Equation Captures Essential Physics Necessary to Describe Onset and Early Evolution

Coupling not included
\( \Delta' \) often assumed constant negative

With accurate time dependence and weak coupling, onset and early evolution can be predicted

\[
\frac{dw}{dt} = \frac{\eta^*}{k_0} \left[ \Delta^* + \frac{k_1}{w} \left( D_{nc} + \frac{D_R}{\alpha_s - H} \right) + \frac{D_{pol}}{w^3} \right]
\]

where
\[
\Delta^* = \Delta' \left( \frac{w}{2} \right)^{-2\alpha_l} (-4D_I)^{1/2}
\]
\[
\frac{\eta^*}{k_0} \approx \frac{r_s^2}{\tau_r}
\]
\[
k_1 = \frac{\omega^2}{\omega^2 + \omega_d^2}
\]
\[
D_{nc} = k_2 \sqrt{\epsilon \beta} \frac{L_q}{L_p}
\]
\[
D_{pol} = -D_{nc} \frac{\rho_{i\theta}}{a^2} \frac{L_q}{L_p} g
\]

Classical drive
Neoclassical drive
Polarization drive
Analytic Models of Linear Tearing Stability Show Strong Variation of $\Delta'$ Near Ideal Instability

Small change in equilibrium and/or conducting wall dramatically affects $\Delta'$ near ideal limit due to **POLE DISCONTINUITY** at ideal limit.

The Pole Form is Ubiquitous Among Derivations, Numeric and Analytic

With a conducting wall at a:  
With a vacuum region from a to b:

\[ \Delta' = -2k_x a \cot(k_x a) \]
\[ k_x = \sqrt{j'_o - k_y^2} \]

\[ \Delta' = 2k_x a \frac{E \sin(k_x a) + \cos(k_x a)}{E \cos(k_x a) - \sin(k_x a)} \]
\[ E = \frac{e^{k_x a} - e^{k_x (2b-a)}}{e^{k_x a} + e^{k_x (2b-a)}} \]
\[ k_x = \sqrt{j'_o - k_y^2} \]
Slight Increase in Core Pressure Destabilizes a Seed Island from a Sawtooth and Causes the Onset of a 3/2 NTM

Equilibrium reconstructions between sawteeth

Most pronounced profile variation is core pressure just before onset.
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Point is to answer: why did the last sawtooth set off the 3/2 NTM while the several preceding similar sawteeth did not.
Neoclassical Terms Remain Constant Between Sawtooth Periods, While $\Delta'$ Increases Sharply

Destabilization of seed island and transition to NTM state is driven primarily by change in $\Delta'$

$\beta_N$ approaches no-wall limit at onset of 3/2 NTM $\ldots$ $\Delta'$ pole is underlying mechanism for destabilization
To Test This Proposition Island Evolution is Reconstructed From Experiment and Compared to Theory

\[ w = \left( \frac{16rR_0 \tilde{B}_r}{msB_{\phi o}} \right)^{1/2} \]

\[ |\tilde{B}_r| = \frac{1}{2} \left( \frac{b}{r} \right)^{m+1} |\tilde{B}_\theta| \]

w from Magnetic Probe Signals used to compare \(dw/dt\) from theory and experiment
dw/dt from Island Evolution Equation Agrees with dw/dt from Experimental Data

Island Evolution Equation gives right answer even without nonlinear coupling and with axisymmetric $\Delta'$. 

$D_{\text{pol}}$ determined from $\chi^2$ minimization agrees in sign with analytic value, but is smaller. 

$D_{\text{pol}} = -2.3e-5$
Increase in Core Pressure Causes $\Delta'$ to Sharply Increase Due to Approach of Ideal Limit

Pressure profile at 2990 was gradually modified to approximate profile at 3600, showing transition.

Approach to n=2 ideal limit increases $3/2 \Delta'$.

Pole causes rapid change in $\Delta'(\beta)$ as $\beta$ changes slightly

Nonlinear coupling between n=1 and n=2 modes not addressed by this model

How will this affect the stability and evolution?
All three components are necessary to explain the onset and evolution
A Comprehensive Model of NTM Onset Emerges From a Combination of Theories

Classical Linear Drive

Neoclassical Effects

Nonlinear Coupling

Time Dependence
Accurate Treatment

$\Delta' = \frac{d\phi}{dx}$

$\Delta'$ is Free energy for mode

Comprehensive NTM Onset and Evolution Model

NIMROD Simulations

All three components are necessary to explain the onset and evolution
Studying the structure and nonlinear evolution of these modes with NIMROD can lead to new intuition and new physics.
NIMROD Simulations of two discharge times show unstable \( n=1 \) mode and driven \( n=2 \) mode, in agreement with experiment.

\[ t_{\text{exp}} = 2990 \]

Stable \( n=2 \) decays after \( n=1 \) drive begins to reduce

\[ t_{\text{exp}} = 3600 \]

Unstable \( n=2 \) grows after \( n=1 \) drive begins to reduce

\[ S = \frac{\tau_R}{\tau_A} = 2.3 \times 10^6 \]

approaching realistic conditions
At later times, after saturation of $n=1$, the $m=3$ component of $n=2$ is dominant.

$3/2$ decays in simulation of $2990\text{ms}$

$3/2$ is larger and grows to saturation in simulation of $3600\text{ms}$

Eigenfunctions show $m=3$ to be dominant $n=2$ component.

$m=1$ is dominant $n=1$. 
Dynamics of NIMROD Simulations Indicate That Linear $\Delta'$ Drive Affects Islands Of Finite Width

$m/n=3/2$ is unstable for 3600ms, when $n=1$ mode decays.

Nonlinear coupling effects do not prevent later seed island growth.

Equilibrium at 3600ms is closer to ideal limit and has larger linear drive.

The pole in $\Delta'$ effects the stability of NTM seed islands
DIII-D Experiment was Designed and Performed to Determine Effect of Poles in \( \Delta' \) on Tearing Stability

Experiment designed to isolate \( \Delta' \) pole mechanism

- avoid other modes
- vary \( d\beta_N/dt \) on approach to onset of 2/1

Prediction was made that for spontaneous NTMs, evolution should depend on rate of approach to ideal boundary.

Measure \( \beta_N \) at point where \( w \approx w_d \) as a function of \( d\beta_N/dt \)

Forced reconnection gives random \( w \) at random \( t \), and should not show a correlated function like \( \Delta'(\beta) \)
Spontaneous/Seedless NTMs were Generated to Isolate Effect of $\Delta'$ Pole Onset Mechanism

Neutral beam injection applied at increasing rates up to $\beta$ limit for 2/1 NTM onset.

Experimental data indicates spontaneous onset in many cases.
Spontaneous/Seedless NTMs were Generated to Isolate Effect of $\Delta^\prime$ Pole Onset Mechanism

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Experimental Data Confirms New Theoretical Prediction from $\Delta'$ Pole Model

Model uses $\beta_N(t)$ from experiment and $\Delta'(\beta_N)$ in island evolution equation.

Pressure profiles modified in model equilibria and $\Delta'$ at 2/1 surface calculated. Result is a function $\Delta'(\beta_N)$.

$D_{pol}$ is single constant free parameter fit to find $\beta_N$ vs. $d\beta_N/dt$ at mode onset.

Results support hypothesis that $\Delta'$ is increasing rapidly in time, consistent with theoretical pole model.
Modeling of $\Delta'$ Pole Experiment Reproduces $dw/dt$ Lull Shortly After Onset

$D_{pol}$ term negligible at small island width. A model for $D_{pol}$ is assumed which decays at $w<w_b$.

$$\frac{dw}{dt} = \frac{\eta^*}{k_0} \left( \Delta^* [\beta_N(t)] + \frac{w D_{tot}}{w_d^2 + w^2} + \frac{w D_{pol}}{w_b^4 + w^4} \right)$$

Resulting phase space plots have a Dip in growth rates.

$\beta_N(t)$ driving function reproduces Lull in growth
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Resulting phase space plots have a Dip in growth rates.

$\beta_N(t)$ driving function reproduces Lull in growth.
Onset Model Reproduces Trace Elements and $dw/dt$ Lull Shortly After Onset

Model reproduces Lull in growth and Trace Elements
Summary of Results

- Increasing $\beta$ sharply increases $\Delta'$ near ideal limit, due to approaching a pole, destabilizing NTMs
  - In a sawtooth seeding a NTM case, eventual onset was caused by increased $\Delta'$ due to approach of a pole
  - Nonlinear NIMROD simulations confirm that axisymmetric $\Delta'$ is meaningful for nonlinearly coupled finite islands

- DIII-D experiment confirmed effect of poles in $\Delta'$ on NTM stability with varying rates of $d\beta/dt$ causing spontaneous NTMs
  - Data from early evolution are in agreement with prediction, both as function of $d\beta_N/dt$ and detailed time evolution.
Future Plans and Open Questions

- Predict the onset of spontaneous NTMs in the presence of an ECCD current peak
  - Address the important physics model for $D_{ECCD}$ and $D_{pol}$ at $w<w_b$
- Can the lull phase and the trace elements be used as real-time indicators for avoidance of NTM transition
  - Size of trace elements, slight $\beta$ reduction during Lull phase
  - Experiment on DIII-D
  - Burning plasma projection
- Seeded NTMs as ideal limit is approached, determine smaller threshold in burning plasma model
  - Seeding in burning plasmas less sensitive to changes in $\Delta$?