Bounce-Averaged Gyrokinetic Simulations in a Laboratory Magnetosphere

T.M. Roberts†, D.T. Garnier, J. Kesner, M.E. Mauel

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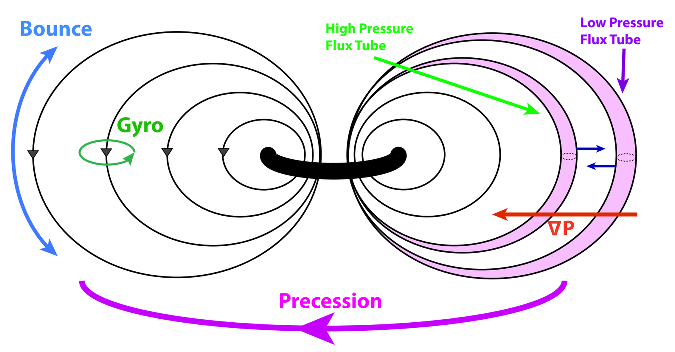
†email: t.maximillian.roberts@gmail.com

Outline

A self-consistent, nonlinear simulation of interchange dynamics including the bounce-averaged gyro-kinetics of deeply trapped electrons[1] was previously used to understand frequency sweeping[2] and the turbulent cascades[3] observed in dipole-confined plasmas. Time stepping is performed in an explicit leap-frog manner and a flux-corrected transport algorithm is implemented. Through adjustment of the particle and heat sources, this code reproduces dynamics that resemble the turbulence measured experimentally, both in spectral power-law trends and in the onset of a steepened density profile. In this presentation, we discuss the physics and numerical methods of the simulations as well as plans for including the effects of a biasing electrode which can collect or inject electrons to compare to recent experiments with current-collection feedback observed to regulate interchange turbulence.

1 Dipole Confined Plasmas

Laboratory dipole-confined plasmas provide a means of studying some of the fundamental physics that govern the dynamics of planetary magnetospheres. Due to the lack of magnetic shear, dipoles can undergo interchange dynamics, driven by the solar in magnetospheres and by steep density profiles in the lab. Magnetospheric convection is often steady, but in the laboratory we can create plasma exhibiting fully developed interchange dynamics.



Early work on the CTX device involved the study of an interchange instability resonant with the drift motion of deeply trapped electrons, known as the Hot Electron Interchange (HEI) instability [2]. A steepened density profile from microwave heating becomes interchange unstable and quasi-periodic bursts characterized by frequency sweeping were observed.

HEIs can be suppressed through sufficient collisionality with neutrals, allowing for transition to a higher density, turbulent plasma characterized by large amplitude fluctuations in potential and density. A balance between plasma sources and sinks creates a stationary turbulent spectrum [3].

Dipole Magnetic Flux Coordinates

Flux coordinates for an ideal dipole are a good approximation for the CTX geometry:

$$B = \nabla\varphi \times \nabla\psi = \nabla\chi$$

where $(\nabla\psi, \nabla\chi, \nabla\varphi)$ is an orthogonal bias in covariant form:

$$\nabla\psi = -\frac{M}{r^2}\sin^2\theta\,\hat{r} - \frac{2M}{r^2}\sin\theta\cos\theta\,\hat{\theta}$$
$$\nabla\chi = -\frac{2M}{r^3}\cos\theta\,\hat{r} - \frac{M}{r^3}\sin\theta\,\hat{\theta}$$
$$\nabla\varphi = -\frac{\hat{\varphi}}{r\sin\theta}$$

We consider two-dimensional dynamics by taking flux-tube averages, eliminating the parallel components. For a quantity A, the flux-tube average is given as:

$$< A > \equiv \delta V^{-1}\int_{-\infty}^{+\infty}\frac{d\chi A}{B^2}$$

We define the flux-tube average of density as $\delta V^{-1}\int_{-\infty}^{+\infty}\frac{d\chi n}{B^2} = \langle n\rangle$, and the number of particles on a field line as $N = \langle n\rangle\delta V$

2 Equations

The self-consistent evolution of interchange dynamics can be described by the motion of kinetic electrons and cold, fluid ions coupled by the bounce-averaged form of Poisson's equation:

$$\frac{\partial F_e}{\partial t} + \nabla\cdot(F_e V_e) = 0 \qquad \frac{\partial n_i}{\partial t} + \nabla\cdot(n_i V_i) = 0 \qquad \nabla^2\Phi = -4\pi\rho$$

The electron distribution function is represented by multiple electron species at different energies, μ. Due to the sufficient separation in frequency of the gyro, bouncing and drift dynamics, we can assume that μ and J are conserved. In addition, it has been observed that the interchange dynamics are flute-like, $k_\parallel \approx 0$ [2], justifying the use of flux-tube averages to reduce the problem's dimensionality.

Poisson's Equation

We derive Poisson's equation in dipole coordinates starting with the microscopic version of Gauss's Law:

$$\nabla\cdot E = -\nabla\cdot\nabla\Phi = 4\pi\rho$$

where ρ is the total charge. In covariant notation, the Laplacian is:

$$\nabla^2\Phi = |B|^2\frac{\partial}{\partial\psi}\left(\frac{|\nabla\psi|^2}{|B|^2}\frac{\partial\Phi}{\partial\psi}\right) + |B|^2\frac{\partial}{\partial\chi}\left(\frac{|\nabla\chi|^2}{|B|^2}\frac{\partial\Phi}{\partial\chi}\right) + |B|^2\frac{\partial}{\partial\varphi}\left(\frac{|\nabla\varphi|^2}{|B|^2}\frac{\partial\Phi}{\partial\varphi}\right)$$

Taking a flux-tube average (killing the $\frac{\partial}{\partial\chi}$ term):

$$< \nabla^2\Phi > = \int_{-\infty}^{+\infty}\frac{\partial}{\partial\psi}\left(\frac{d\chi}{|\nabla\varphi|^2}\frac{\partial\Phi}{\partial\psi}\right) + \int_{-\infty}^{+\infty}\frac{\partial}{\partial\varphi}\left(\frac{d\chi}{|\nabla\psi|^2}\frac{\partial\Phi}{\partial\varphi}\right)$$
$$= h_\varphi\frac{\partial^2\Phi}{\partial\psi^2} + h_\psi\frac{\partial^2\Phi}{\partial\varphi^2} = \int_{-\infty}^{+\infty}(-4\pi\rho)\frac{d\chi}{B^2} = -4\pi e(N_i - N_e)$$

where h_ψ and h_φ are geometric terms given as:

$$h_\psi = \int_{-\infty}^{+\infty}\frac{d\chi}{|\nabla\varphi|^2} = 4M \qquad h_\varphi = \int_{-\infty}^{+\infty}\frac{d\chi}{|\nabla\psi|^2} = 2\frac{M}{\psi^2}$$

Cold Ion Fluid

Cold ions in a dipole magnetic field move under the influence of $E\times B$ and polarization drifts. The electric field in terms of potential given as:

$$E = -\nabla\Phi = -\frac{\partial\Phi}{\partial u^i}\nabla u^i = -\frac{\partial\Phi}{\partial\psi}\nabla\psi - \frac{\partial\Phi}{\partial\varphi}\nabla\varphi$$

So our expressions for the $E\times B$ and polarization drifts become:

$$\vec{v}_E = c\frac{\vec{E}\times\vec{B}}{B^2} = -c\frac{\partial\Phi}{\partial\psi}\frac{\nabla\varphi}{|\nabla\varphi|^2} + c\frac{\partial\Phi}{\partial\varphi}\frac{\nabla\psi}{|\nabla\psi|^2}$$
$$\vec{v}_P = \frac{c}{\omega_{ci}B}\frac{dE}{dt} = -c\frac{M_i}{eB^2}\frac{d\nabla\Phi}{dt} = -c\frac{M_i}{eB^2}\frac{d}{dt}\left[\frac{\partial\Phi}{\partial\psi}\nabla\psi + \frac{\partial\Phi}{\partial\varphi}\nabla\varphi\right]$$

In the absence of ion sources/sinks, we transform the ion continuity equation into dipole coordinates and plug in our velocities:

$$-\frac{\partial n_i}{\partial t} = B^2\frac{\partial}{\partial\psi}\left[c\frac{n_i}{B^2}\left(\frac{\partial\Phi}{\partial\varphi} - \frac{M_i}{eB^2}|\nabla\psi|^2\frac{d}{dt}\frac{\partial\Phi}{\partial\psi}\right)\right] + B^2\frac{\partial}{\partial\varphi}\left[c\frac{n_i}{B^2}\left(-\frac{\partial\Phi}{\partial\psi} - \frac{M_i}{eB^2}|\nabla\varphi|^2\frac{d}{dt}\frac{\partial\Phi}{\partial\varphi}\right)\right]$$

taking a flux tube average,

$$-\frac{\partial N_i}{\partial t} = \frac{\partial}{\partial\psi}\left[cN_i\left(\frac{\partial\Phi}{\partial\varphi} - \varepsilon_\psi\frac{d}{dt}\frac{\partial\Phi}{\partial\psi}\right)\right] - \frac{\partial}{\partial\varphi}\left[cN_i\left(\frac{\partial\Phi}{\partial\psi} + \varepsilon_\varphi\frac{d}{dt}\frac{\partial\Phi}{\partial\varphi}\right)\right]$$

where N is the flux-tube averaged density. The density weighted, flux-tube averaged coefficients for the polarization drifts are found by assuming a $\sin\theta$ distribution of density along a field line:

$$\varepsilon_\varphi = \frac{1}{\delta V}\int_{-\infty}^{+\infty}\frac{d\chi}{B^2}\frac{n_iM_i}{e\langle n_i\rangle B^2}|\nabla\varphi|^2 \approx 0.66\frac{M^2B_0}{\psi^4\omega_{ci0}}$$
$$\varepsilon_\psi = \frac{1}{\delta V}\int_{-\infty}^{+\infty}\frac{d\chi}{B^2}\frac{n_iM_i}{e\langle n_i\rangle B^2}|\nabla\psi|^2 \approx 0.77\frac{M^2B_0}{\psi^2\omega_{ci0}}$$

Kinetic Electron Dynamics

The motion of deeply trapped ($J\approx 0$) electrons in a curl free magnetic field is given by the guiding center drift Hamiltonian [4] with Hamilton's equations:

$$H = \frac{\mu cB}{e} - c\Phi \qquad \dot{\varphi} = \frac{\partial H}{\partial\psi} = \frac{\mu c\partial B}{e\,\partial\psi} - c\frac{\partial\Phi}{\partial\psi} \qquad \dot{\psi} = -\frac{\partial H}{\partial\varphi} = c\frac{\partial\Phi}{\partial\varphi}$$

where $\mu = m_ev^2/2B$.

For time scales significantly slower than the gyration and bounce periods, μ and J are preserved quantities as an electron moves across field lines. The Vlasov equation for the electron distribution function is given as:

$$\frac{dF_e}{dt} = \frac{\partial F_e}{\partial t} + \frac{\partial\vec{x}}{\partial t}\cdot\frac{\partial F_e}{\partial\vec{x}} = \frac{\partial F_e}{\partial t} + \frac{\partial}{\partial\varphi}(\dot{\varphi}F_e) + \frac{\partial}{\partial\psi}(\dot{\psi}F_e) = 0$$
$$= \frac{\partial F_e}{\partial t} + \frac{\partial}{\partial\varphi}\left[\left(\frac{\mu c\partial B}{e\,\partial\psi} - c\frac{\partial\Phi}{\partial\psi}\right)F_e\right] + \frac{\partial}{\partial\psi}\left[c\frac{\partial\Phi}{\partial\varphi}F_e\right] = 0$$

We use a bounce-averaged distribution, $F_e = F_e(\mu, J, \psi, \varphi, t)$, where F_e is given as the sum of a cold and hot electron population.

$$F_e = N_{i0}(\psi)\Big([1-\alpha(\psi)]\delta(\mu)\delta(J) + \alpha(\psi)G(\mu)\delta(J)\Big), \quad G(\mu) = \frac{\mu^{l-1}l^l}{\mu_0^l\Gamma(\mu)}e^{-\mu l/\mu_0}$$

$G(\mu)$ is the electron distribution in μ. Integration over velocity space then returns the number of electrons per unit flux, $N_e = \int d\mu dJ F_e$. The first azimuthal term represents the hot electron drift frequency $\omega_d \equiv \frac{\mu c\partial B}{e\,\partial\psi} = \frac{\mu c}{e}\frac{3B}{\psi}$.

Normalization

We make these equations dimensionless by normalizing them with parameters evaluated at the profile peak (heating resonance). Starting with Poisson's equation, if we normalize the following quantities as:

$$\frac{\psi}{\psi_0} \equiv y, \qquad \frac{N_i}{N_i0} \equiv \hat{N}_i \qquad \frac{e\Phi}{\mu_0 B_0} \equiv \hat{\Phi}, \qquad \omega_{dh0}t \equiv \hat{t} \qquad \frac{\mu}{\mu_0} \equiv \hat{\mu}$$

where the x_0 quantities are defined at the profile peak. We multiply Poisson's equation through by $e\psi^2/M\mu_0B_0$:

$$\frac{2}{y^2}\frac{\partial^2\hat{\Phi}}{\partial\varphi^2} + 4\frac{\partial^2\hat{\Phi}}{\partial y^2} = -4\pi e\frac{eL_0}{\mu_0}\frac{N_{i0}}{N_{i0}}(N_i - N_e) = -.91\frac{L_0^2}{\lambda_{D0}^2}(\hat{N}_i - \hat{N}_e)$$

where $\lambda_{D0}^2 = \mu_0B_0/4\pi e^2\langle n_{i0}\rangle$.
We normalize the ion continuity equation, dividing by $\omega_{dh0}N_{i0}$,

$$\frac{\partial\hat{N}_i}{\partial\hat{t}} + \frac{\partial}{\partial y}\left[\hat{N}_i\left(\frac{1}{3}\frac{\partial\hat{\Phi}}{\partial\varphi} - \hat{\varepsilon}_y\frac{d}{d\hat{t}}\frac{\partial\hat{\Phi}}{\partial y}\right)\right] - \frac{\partial}{\partial\varphi}\left[\hat{N}_i\left(\frac{1}{3}\frac{\partial\hat{\Phi}}{\partial y} + \hat{\varepsilon}_\varphi\frac{d}{d\hat{t}}\frac{\partial\hat{\Phi}}{\partial\varphi}\right)\right] = 0$$

$$\hat{\varepsilon}_\varphi = \frac{0.66}{3y^4}\frac{\omega_{dh0}}{\omega_{ci0}} \qquad \hat{\varepsilon}_y = \frac{0.77}{3y^2}\frac{\omega_{dh0}}{\omega_{ci0}}$$

Finally, if we divide the electron equation by ω_{dh0}, with the same normalization for $E\times B$ drift:

$$\frac{\partial F_e}{\partial\hat{t}} + \frac{\partial}{\partial\varphi}\left[\left(\frac{\omega_{dh}}{\omega_{dh0}} - \frac{\partial\hat{\Phi}}{\partial\psi}\right)F_e\right] + \frac{\partial}{\partial y}\left[\frac{\partial\hat{\Phi}}{\partial\varphi}F_e\right] = 0$$

where $\frac{\omega_{dh}}{\omega_{dh0}} = \hat{\mu}y^2$.
This normalization shows that the simulation is only dependent on two parameters, the ratio of L_0 to λ_{D0} and hot electron drift frequency over the ion cyclotron frequency.

3 Numerical Techniques

The simulation solves the finite-difference approximations to the coupled equations given above. The potential is advanced with a nonlinear solve of the equation for charge continuity. Due to the periodic boundary condition in the φ direction, employing a pseudo-spectral method effectively reduces the dimensionality of the finite difference problem to be solved. We implement a numerical stepping scheme for advancing the ion and electron populations based on a flux conserving method by Zalesak [5].

Trapezoidal Leap-Frog Method

A second-order trapezoidal leap-frog algorithm for explicit time integration is implemented to advance the ion and electron populations and potential. First, the ion and electron populations are advanced a half-step in time by using the existing potential. At this half-step, a new potential and charge density are calculated using the intermediate values of F_e and N_e, and with these we find the intermediate fluxes. With the intermediate fluxes, we advance the populations a full trapezoidal time step, then recompute the potential and charge density. These two steps are identical aside from the time-base of the terms being advanced. If we have a population, $f(t)$, with a rate of change $\dot{f} = -\nabla\cdot\Gamma$, we can describe this method as:

$$f_{t-\Delta t/2} = \frac{1}{2}(f_t + f_{t-\Delta t})$$
$$f_{t+\Delta t/2} = f_{t-\Delta t/2} - \Delta t\Delta\Gamma_t$$
$$f_{t+\Delta t} = f_t - \Delta t\Delta\Gamma_{t/2}$$

where $\Gamma = \Gamma\big(f(t), \Phi(t), \dot{\Phi}(t)\big)$ is the flux calculated at each half-step, which includes solving for the potential. In this approach, the populations are advanced by fluxes which are always a half-step off in time.

Nonlinear Solve for $\frac{\partial\hat{\Phi}}{\partial t}$

To advance the potential at each half-step, we can consider the rate of change of charge density. We can find this by equating the time derivative of Poisson's equation to the combination the ion and electron dynamic equations:

$$\frac{2}{y^2}\frac{\partial^2\dot{\hat{\Phi}}}{\partial\varphi^2} + 4\frac{\partial^2\dot{\hat{\Phi}}}{\partial y^2} = -.91\frac{L_0^2}{\lambda_{D0}^2}\frac{\partial\hat{\rho}}{\partial\hat{t}} = -.91\frac{L_0^2}{\lambda_{D0}^2}\left(\frac{\partial\hat{N}_i}{\partial\hat{t}} - \frac{\partial\hat{N}_e}{\partial\hat{t}}\right)$$
$$= .91\frac{L_0^2}{\lambda_{D0}^2}\left(\frac{\partial}{\partial\varphi}\left[\hat{\rho}\frac{\partial\hat{\Phi}}{\partial y} + \hat{N}_i\hat{\varepsilon}_\varphi\frac{\partial\dot{\hat{\Phi}}}{\partial\varphi} + \Sigma\hat{\mu}y^2F_{e,u}\right] - \frac{\partial}{\partial y}\left[\hat{\rho}\frac{\partial\hat{\Phi}}{\partial\varphi} - \hat{N}_i\hat{\varepsilon}_y\frac{\partial\dot{\hat{\Phi}}}{\partial y}\right]\right)$$

where $\hat{\rho}$ is the difference of the normalized flux-tube averaged ion density and the electron distribution function integrated over velocity space.

We note the $\dot{\hat{\Phi}}$ terms on both sides of our rate equation. To efficiently implement a pseudo-spectral technique, we bring the azimuthally symmetric part of the polarization terms ($\bar{N}_i = \hat{N}_i - \tilde{N}_i$) to the LHS. We rewrite the equations with an effective dielectric:

$$\frac{2}{y^2}\frac{\partial}{\partial\varphi}\hat{\epsilon}_\varphi(y)\frac{\partial\dot{\hat{\Phi}}}{\partial\varphi} + 4\frac{\partial}{\partial\psi}\hat{\epsilon}_y(y)\frac{\partial\dot{\hat{\Phi}}}{\partial y}$$
$$= .91\frac{L_0^2}{\lambda_{D0}^2}\left(\frac{\partial}{\partial\varphi}\left[\hat{\rho}\frac{\partial\hat{\Phi}}{\partial y} + \tilde{N}_i\hat{\varepsilon}_\varphi\frac{\partial\dot{\hat{\Phi}}}{\partial\varphi} + \Sigma\hat{\mu}y^2F_{e,u}\right] - \frac{\partial}{\partial y}\left[\hat{\rho}\frac{\partial\hat{\Phi}}{\partial\varphi} - \tilde{N}_i\hat{\varepsilon}_y\frac{\partial\dot{\hat{\Phi}}}{\partial y}\right]\right)$$

where

$$\hat{\epsilon}_\varphi \approx 1 + \frac{\bar{N}_i}{9.99y^2}\frac{L_0^2}{\lambda_{D0}^2}\frac{\omega_{dh0}}{\omega_{ci0}} \qquad \hat{\epsilon}_y \approx 1 + \frac{\bar{N}_i}{17.1y^2}\frac{L_0^2}{\lambda_{D0}^2}\frac{\omega_{dh0}}{\omega_{ci0}}$$

The above equation is iteratively solved for the time rate of change in potential at the half and full leap-frog steps. This must be done before advancing the ions as the polarization velocity depends on the rate of change in potential.

Flux-Corrected Transport Algorithm

The $\Delta\Gamma$ expression in the Leap-Frog section is an operator representing the fourth order FCT process. "High-order" fluxes improve the spatial resolution of the time step, but can cause numerical oscillations and lead to instability. To limit these oscillations, "low-order" fluxes are used as an artificial diffusion, specifically chosen to prevents numerical artifacts from high-order methods to develop. The process at each time step is:

1. Find the ion and electron velocities.
2. Find the ion and electron low and high order fluxes, F^L and F^H.
3. Define the anti-diffusive flux, $A \equiv F^H - F^L$
4. Limit A so as not to produce or enhance extrema in step 5.
5. Find time advanced populations with the limited A.

The low order flux is given by an "upwind" differencing scheme and an ad-hoc diffusion:

$$\Gamma_\varphi^L(l, k+1/2) = \frac{1}{2}[v_\varphi(l, k+1) + v_\varphi(l, k)]F^{DC}(l, k+1/2) - \frac{1}{8}\frac{\Delta\varphi}{\Delta t}[F^0(l, k+1) - F^0(l, k)]$$
$$\Gamma_y^L(l+1/2, k) = \frac{1}{2}[v_y(l+1, k) + v_y(l, k)]F^{DC}(l+1/2, k) - \frac{1}{8}\frac{\Delta y}{\Delta t}[F^0(l+1, k) - F^0(l, k)]$$

where F^{DC} represents the flux from an "upwind" donor cell. The high-order flux is a fourth order finite differencing known as "ZIP" form:

$$\Gamma_\varphi^H(l, k+1/2) = \frac{2}{3}[v_\varphi(l,k)F(l,k+1) + v_\varphi(l,k+1)F(l,k)] - \frac{1}{12}[v_\varphi(l,k)F(l,k+2) + v_\varphi(l,k+2)F(l,k) + v_\varphi(l,k-1)F(l,k+1) + v_\varphi(l,k+1)F(l,k-1)]$$
$$\Gamma_y^H(l+1/2, k) = \frac{2}{3}[v_y(l,k)F(l+1,k) + v_y(l+1,k)F(l,k)] - \frac{1}{12}[v_y(l,k)F(l+2,k) + v_y(l+2,k)F(l,k) + v_y(l-1,k)F(l+1,k) + v_y(l+1,k)F(l-1,k)]$$

With these two fluxes, we define $A(l,k) = \Gamma^H - \Gamma^L$, and then limit this flux as described in [5] to prevent the formation of new extrema, or the enhancement of existing extrema. We use a centered differencing method to represent the convection velocities of the ions and electrons.

Numerical Dissipation in Potential

To prevent numerical instability in advancing the potential, dissipation is added. This also acts as the physical nonresonant dissipation which limits the frequency sweeping observed in HEIs. We advance the potential in the leap-frog manner as:

$$\Phi_{t-\Delta t/2} = \frac{1}{2}(\Phi_t + \Phi_{t-\Delta t})$$
$$\Phi_{t+\Delta t/2} = \Phi_{t-\Delta t/2} + \Delta t\dot{\Phi}_t - (-1)^k\Delta t\nu\nabla^{2k}\Phi_t$$
$$\Phi_{t+\Delta t} = \Phi_t + \Delta t\dot{\Phi}_{t+t/2} - (-1)^k\Delta t\nu\nabla^{2k}\Phi_{t+t/2}$$

k sets the dissipation length scale, and adjustment of ν sets the nonresonant dissipation for limiting the frequency sweeping.

4 Results

This simulation reproduces many of the experimentally observed phenomena in CTX. Unstable initial density profiles are set, and an initial perturbed potential is made of randomly phased sinusoidal oscillations.

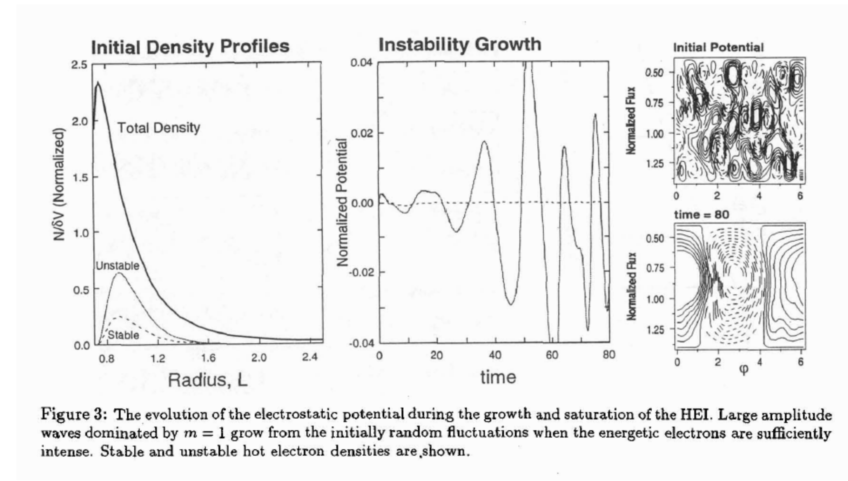


Figure 3: The evolution of the electrostatic potential during the growth and saturation of the HEI. Large amplitude waves dominated by $m = 1$ grow from the initially random fluctuations when the energetic electrons are sufficiently intense. Stable and unstable hot electron densities are shown.

From these initial conditions, large amplitude waves develop. These modes are radially broad and dominantly m = 1 in structure. The evolution of the individual electron populations displays the propagation of phase-space "holes", starting with the lower energy populations and moving inward with the resonance as the mode frequency increases.

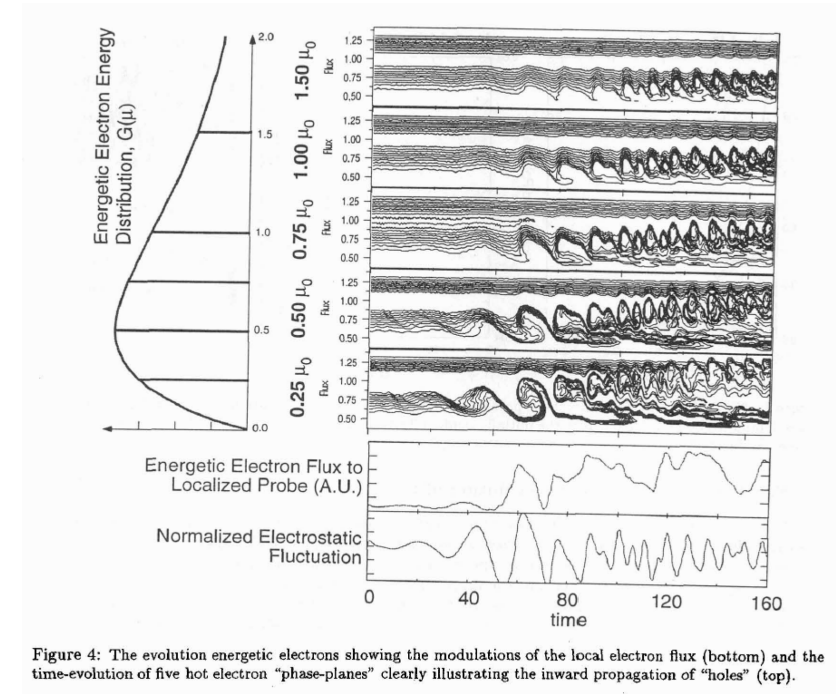
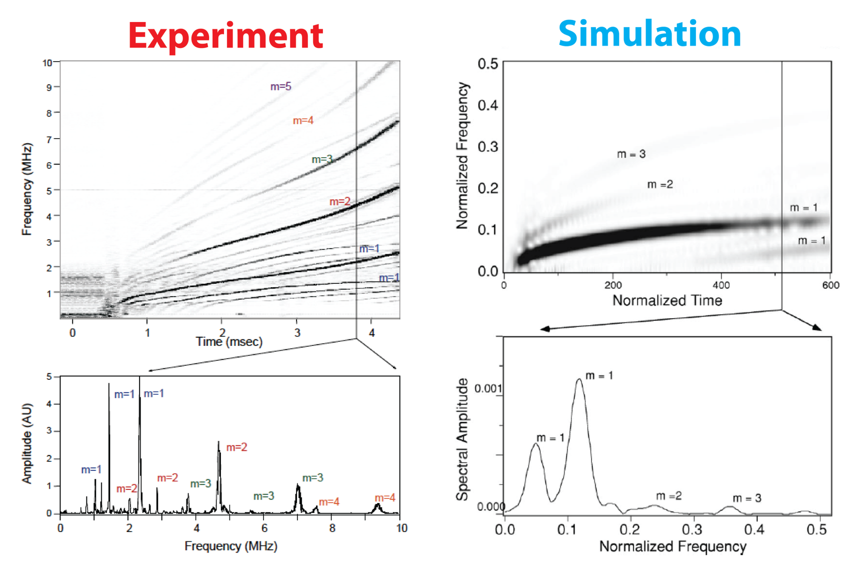


Figure 4: The evolution energetic electrons showing the modulations of the local electron flux (bottom) and the time-evolution of five hot electron "phase-planes" clearly illustrating the inward propagation of "holes" (top).

This phase-space evolution produces frequency-sweeping dynamics similar to the experiment. This is comparing by looking at the spectral content of the potential fluctuation in time.



5 Particle Conserving Source/Sink

To reproduce dynamics related to driven turbulence, we require a mechanism to maintain the unstable profile. Grierson implemented a conservative source and sink of particles and electron energy[6]. In normalized magnetic coordinates, the particle continuity equation becomes:

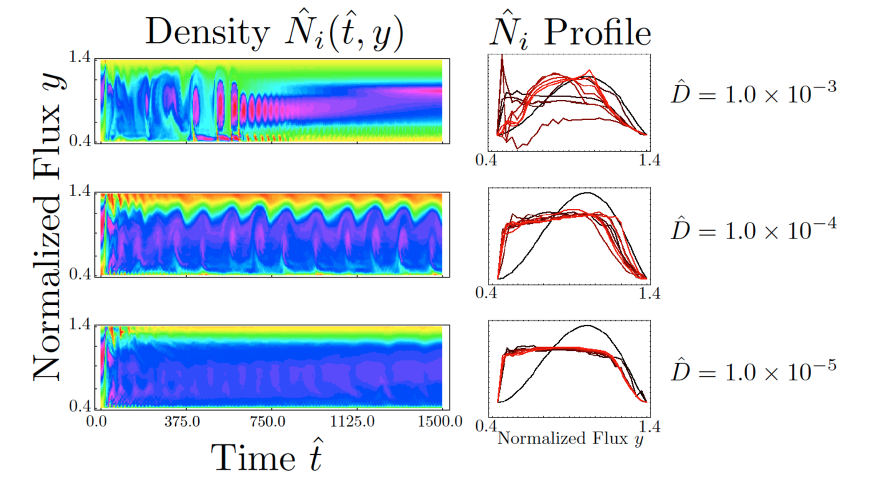
$$\frac{\partial\hat{N}}{\partial\hat{t}} + \frac{\partial}{\partial\varphi}(\hat{N}v_\varphi) + \frac{\partial}{\partial y}(\hat{N}v_y) = 1.8\hat{D}h_Dy^2\frac{\partial^2\hat{N}}{\partial\varphi^2} + 3.2\hat{D}\frac{\partial}{\partial y}\left[h_D\frac{\partial}{\partial y}(y^4\hat{N})\right] + \hat{D}D_sh_s$$

h_s and h_D are the normalized source and diffusion profiles. Conservation requires the number of particles in the volume to be fixed, therefore the volume integral of the above equation yields:

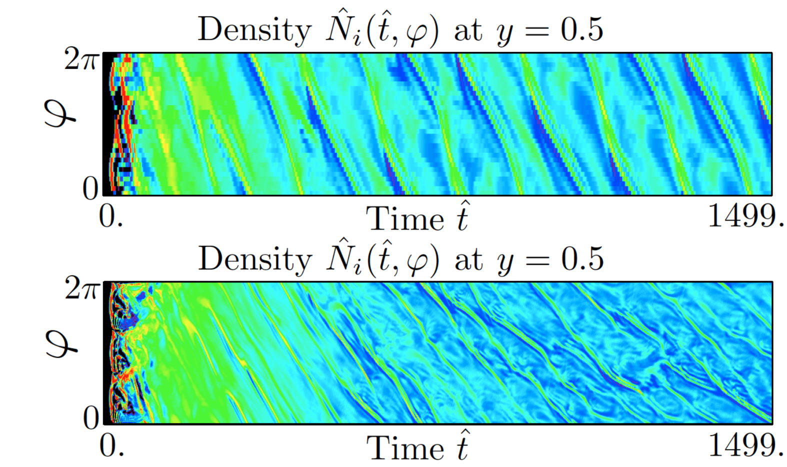
$$D_s = \frac{-\int 1.8h_Dy^2\frac{\partial^2\hat{N}}{\partial\varphi^2} - 3.2\frac{\partial}{\partial y}\left[h_D\frac{\partial}{\partial y}(y^4\hat{N})\right]d^2x}{\int h_sd^2x}$$

At every time step D_s is calculated, and the ion and electron grids are updated with a source/sink grid that sums to zero. For the electrons we are also conserving energy, so the grid integrated over μ sums to zero as well.

Notice that $\hat{D}$ is on all the new terms, so the effect of this source/sink can be tuned simply by varying this parameter. A value for $\hat{D}$ is found that maintains a slightly unstable density profile and results in the onset of radially broad convective cells.



Large quasi-coherent structures are observed to form and rotate azimuthally in the electron magnetic drift direction. Further study of the simulation including this source/sink is required.



Current-Collection Feedback

Recent experiments have shown that the application of current-collection feedback in interchange turbulent plasmas can locally amplify or suppress broadband fluctuations (*NI2.00001 9:30 AM, Wednesday, Bissonet*). We are extending this simulation to include the effects of current collection.

In this feedback, the potential to a capacitively coupled biasing electrode is varied in proportion to the floating potential fluctuations measured by a sensor. The current driven by this system is given as the difference between the applied bias and the local potential fluctuations, divided by the sheath resistance:

$$I \equiv \frac{\tilde{V}_A - \tilde{V}_P}{R_S} \approx \frac{\left(|G|e^{i\theta} - 1\right)\tilde{V}_P}{R_S}$$

If the bias to the electrode is far from the plasma potential, probe theory tells us varying the bias will only vary the number of electrons collected. We propose including a point source/sink of electrons proportional to the difference in applied bias and local potential. This work is currently underway.

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