Empirical Mode Decomposition\(^1\) for High Density, Dipole-Confined Plasmas

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Dipole Density Regimes

A dipole-confined plasma has two distinct density regimes, namely low and high density.

- **Low density**: \( p_n < 10^{-5} \) Torr
- **High Density**: \( p_n \geq 10^{-5.3} \) Torr

The High Density regime displays low frequency (3–8kHz) turbulence.
CTX Dipole

$B_{\text{max}} \sim 2\text{kG}, \ B_{\text{wall}} \sim 50\text{G}$

$L_{\text{Terella}} = 20\text{cm}$

$L_{\text{Chamber}} = 70\text{cm}$

1kW ECRH @2.45GHz

ECRH Resonance at $L=27\text{cm}$
Basic Parameters

Second gas puff causes:
- massive increase in $I_{\text{sat}}$
- drop of x-rays
- increase of photo-emission
Profiles

- No Good Profile measurement, turbulent edge.
- Predicts ExB Sheared Azimuthal Flow
- However, measured fluctuations are drift-resonant with $\omega_d (T_e \sim 10-12$eV)

$$\Phi_f(L) = 0.01L^{-7.5} + 6.6$$
High Density Transition

Large, m=1 Fluctuations

Rigid Rotating ExB ($\Phi_f \sim 1/L$)
Turbulent Time Series

- A turbulent time series displays intermittency.
- A turbulent time series displays a power-law spectra.
- Most data is non-stationary (not strictly periodic), and frequency can change inside of a characteristic period.
- These are problems for Fourier Methods which generally require time series to be:
  a) Linear.
  b) Stationary.
Hilbert Spectrum

- **Hilbert Transform** given by:
  $$ Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{{t-t'}} \, dt' $$

- **Form Analytic Function**:
  $$ Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)} $$

- **Instantaneous Frequency**:
  $$ \omega(t) = \frac{d\theta(t)}{dt} $$

- **Instantaneous Amplitude**:
  $$ a(t) = \sqrt{X(t)^2 + Y(t)^2} $$

- The phase must be `unwrapped' before differentiating.
In order to apply the Hilbert Transform, the time series must be of the class `Intrinsic Mode Functions'.

Envelope functions symmetric about the local zero.

No positive minima or negative maxima.

Same number of zero crossings as extrama, within one.

Formed by `sifting the time series'
Sifting Process$^{1,2}$

- The time series, $S_0(t)$ is to be sifted into many IMFs.

- One spline is fit to all maxima $S_{\text{max}}(t)$, another to all minima $S_{\text{min}}(t)$, then the average is taken $m_{11}=(S_{\text{max}}(t)+S_{\text{min}}(t))/2$.

- Subtract spline average from the original signal $S_0(t)-m_{11}(t)=h_{11}(t)$ and repeat until $h_{1k}=h_{1(k-1)}$, where $k$ is the mean spline subtraction iterate. Then $c_1(t)=h_{1k}$, the first IMF.

- $S_0(t)-c_1(t)=S_1(t)$, and repeat process on $S_1$. 
Sifting $I_{sat}$ Data

- The data is sorted into functions with intrinsic time scales that are inherent to the data.
- Each IMF has a frequency which is approximately half the previous IMF.
The instantaneous phase for each IMF. The frequency regimes are well separated. A linear fit gives the average frequency.

15.5 kHz
7.8 kHz
4.0 kHz
2.0 kHz
Hilbert Spectrum

Intermittent High Energy Bursts

Periods Of Long-Lived Single Modes
While the Hilbert spectrum is qualitative, certain integral quantities are quantitative.

Instantaneous Energy
\[ IE(t) = \int_0^{\omega_N} H^2(t, \omega) d\omega \]

Mean Marginal Spectrum
\[ h(\omega) = \frac{1}{T} \int_0^T H(t, \omega) dt \]
IE and Spectrum

• Instantaneous Energy also records the highly energetic, intermittent bursts of low frequency activity.

• Spectrum also displays a power-law scaling, similar to FFT.
A novel technique has been implemented to study the fluctuations in a turbulent plasma. The time series displays intermittent burst of activity, and non-stationary fluctuations. The method decomposed a turbulent signal into a few mode functions at intrinsic fluctuation time scales. The 4–8kHz frequency range contains most of the power and is most strongly correlated.