Simulating Interchange Turbulence in a Dipole Confined Plasma

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Abstract

The dipole magnetic field is a simple, shear-free configuration. Strong, low-freqency interchange mixing, with $k_{||} = 0$, allows plasma cross-field dynamics to be ‘bounce-averaged’. When dipole confined plasma is produced with ECRH, fast Hot Electron Interchange (HEI) instabilities appear at low densities, and lower-frequency turbulent fluctuations occur at higher densities. The global mode structure of the HEI and centrifugal interchange are understood, with good agreement between laboratory measurements and nonlinear simulations. However, the turbulent fluctuations are much less understood. They exhibit a power-law like spectrum, and require a spatially refined computational grid. To study the interchange turbulence, a fully parallel simulation has been developed to examine these fluctuations. The simulation includes a distributed Fast-Poisson solver for the ion polarization drift, a particle source and sink.

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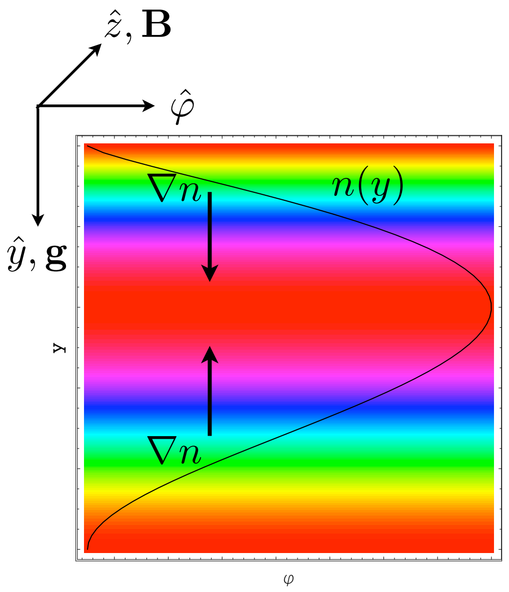
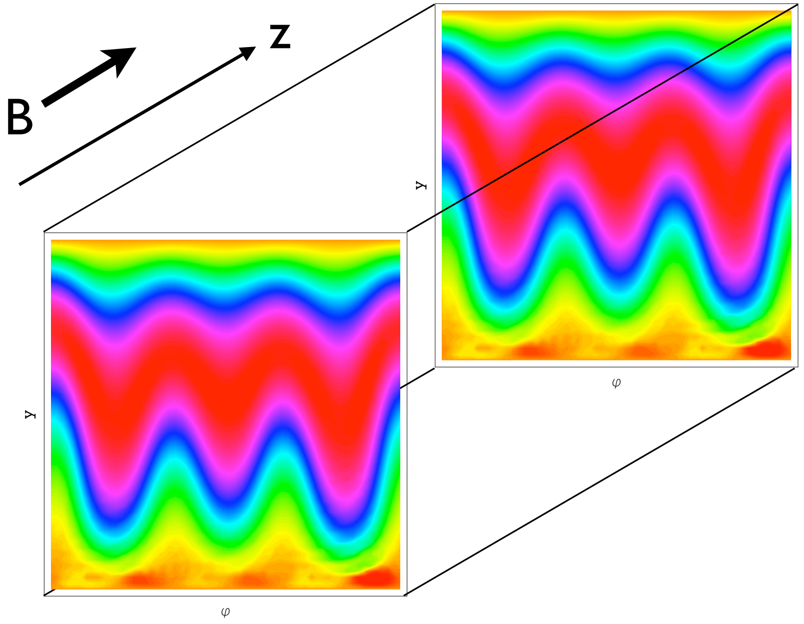
Interchange Modes Exist in a Non-uniform Plasma

- Classic fluid instability, similar to the gravitational Rayleigh-Taylor instability.
- “Fluting” mode, with $k_{||} \approx 0$.





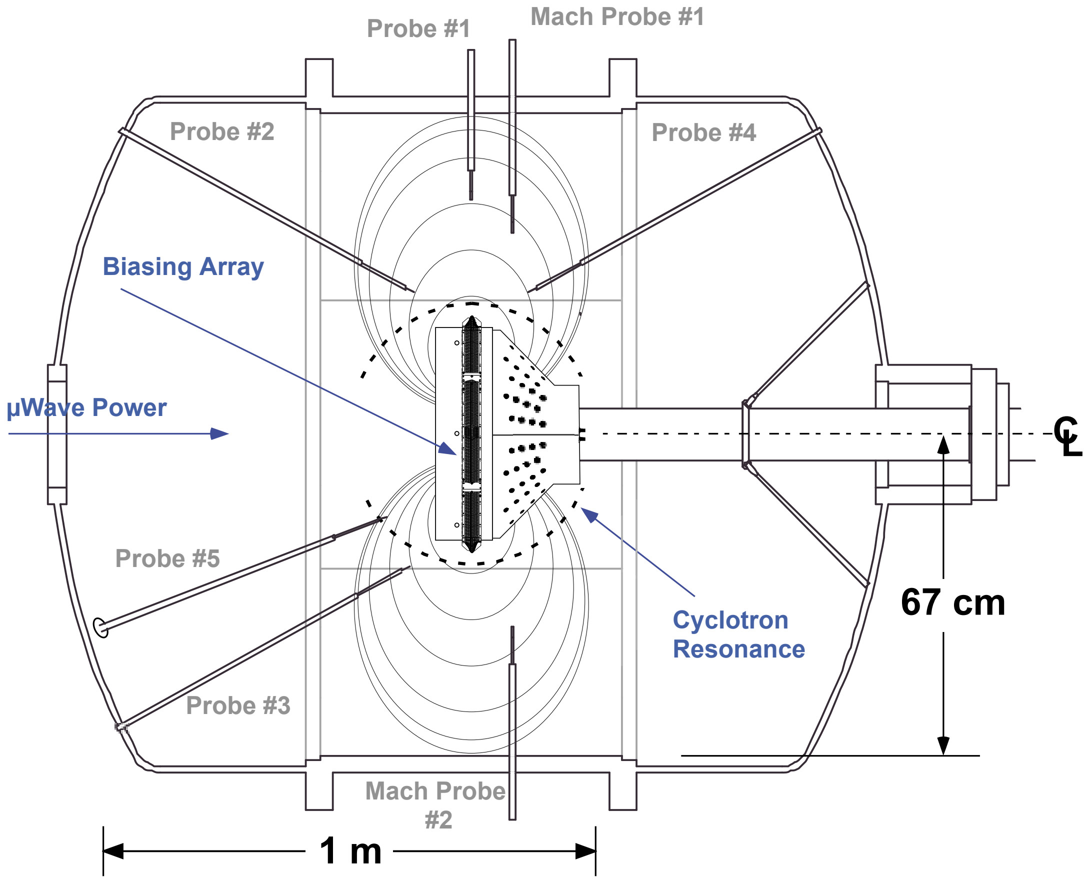
Interchange Instabilities Occur in Fluids and Plasmas

- Interchange instabilities are ubiquitous in fluids and plasmas.
- Rayleigh-Taylor instability in fluids.
- Edge SOL dynamics in tokamak are interchange-like.
- Equatorial spread-F layer and Jupiter's magnetosphere.

Questions This Research Addresses

- What are the characteristics of turbulent interchange modes in a dipole-confined plasma?
- What are the average plasma parameters (n, T, Φ) and their profiles in this turbulent state?
- What is the interplay between the plasma profiles and the interchange turbulence in a dipole plasma?

Experimental Motivation for Simulations



- 1.0kW @2.45GHz ECRH Hydrogen Plasma. B_o=875G
- Vacuum Range ≈ 1-2*10^{-7} Torr
- $\omega_c >> \omega_b >> \omega_d$
- Multiple movable probes for I_{sat}, Φ_{float}, Particle Flux, Mach Number.
- Probe location measured by equatorial 'L'.

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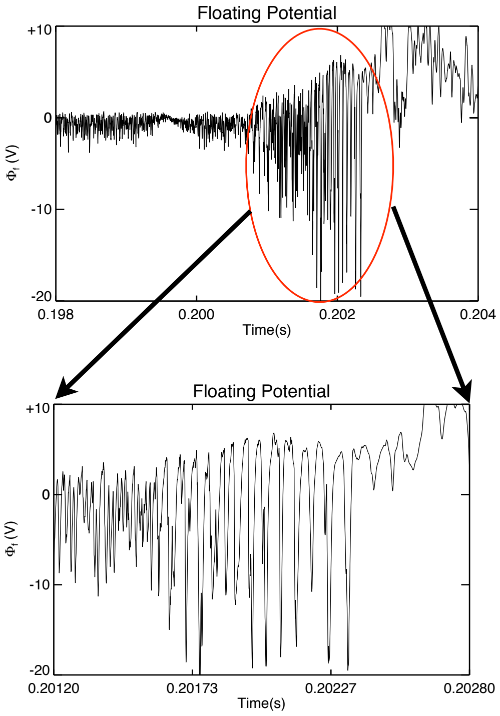
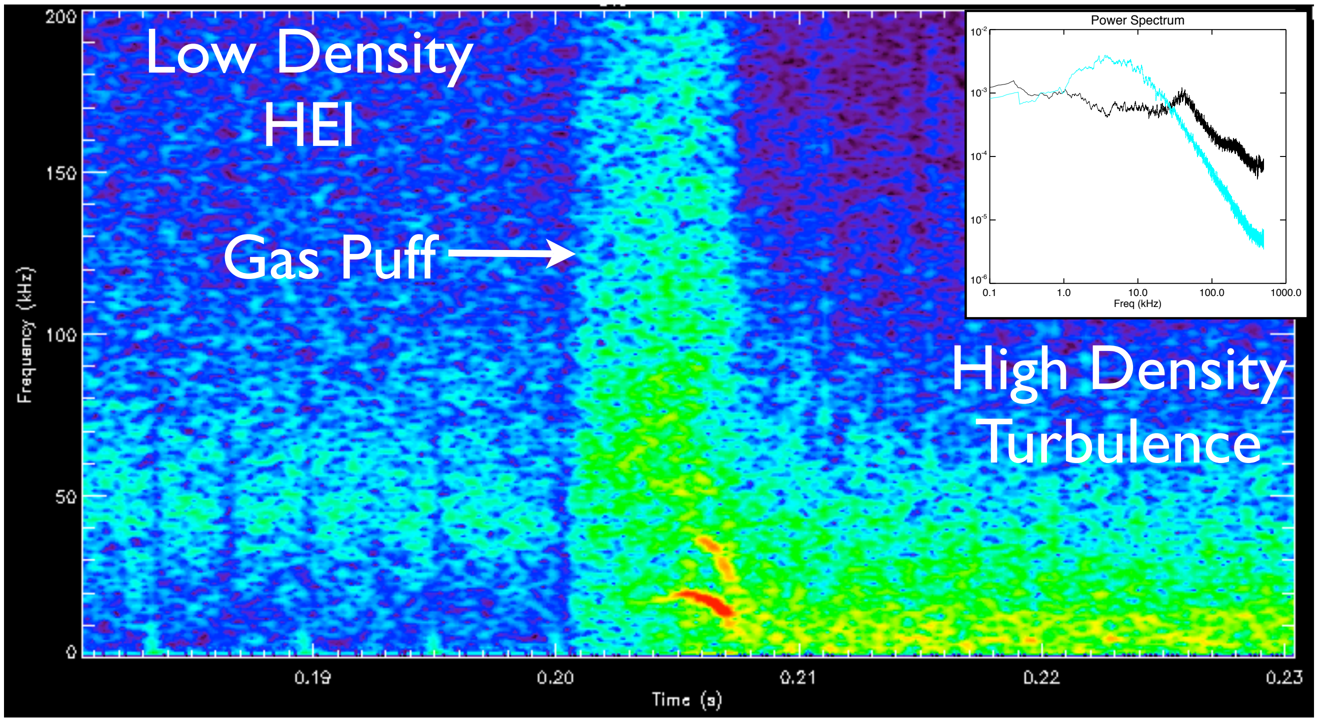
Gas Fueling Causes a Transition To Turbulence





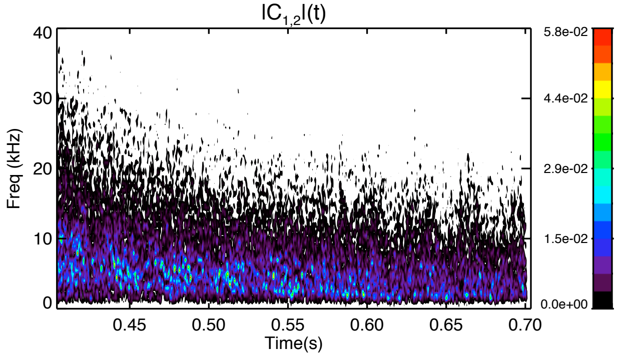
Low Density HEI

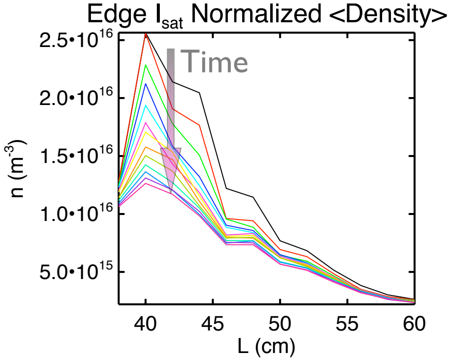
Gas Puff

High Density Turbulence

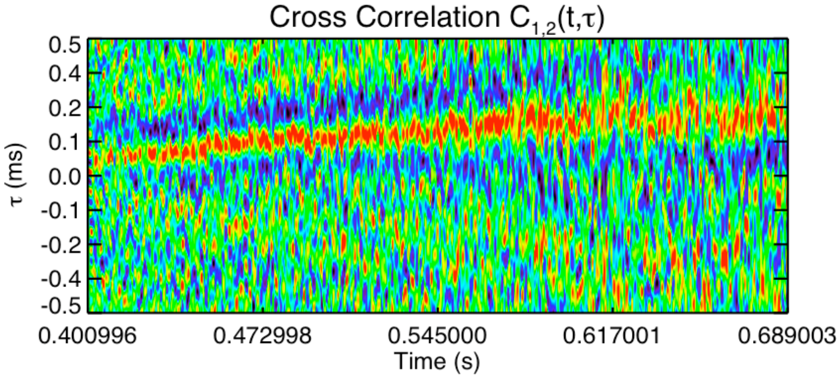
Dipole Interchange Turbulence Characteristics

- Intensity and frequency of fluctuations depend on density profile.

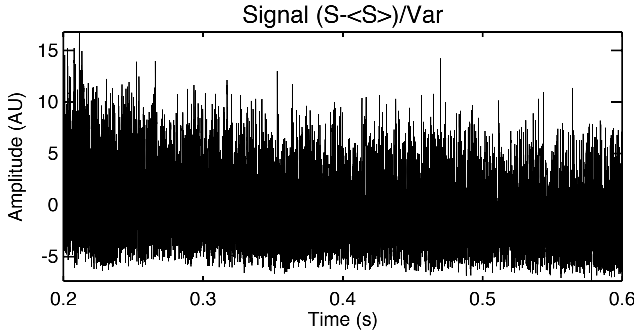


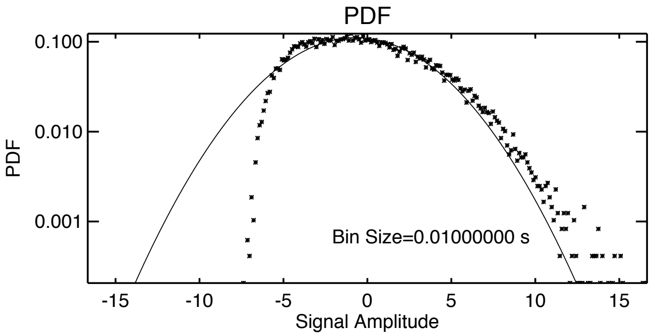


- Intermittency and coherent structures, modes.



- Edge density PDFs have long tails (blobs).





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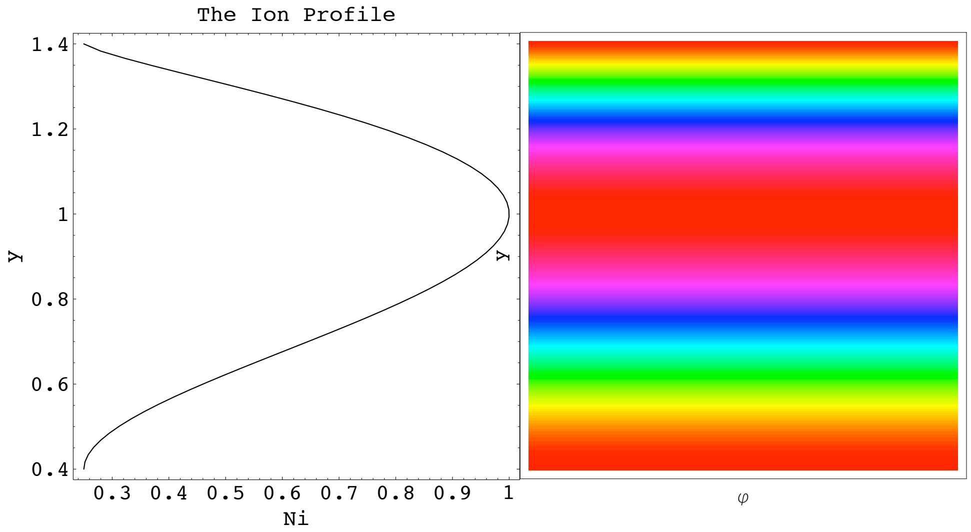
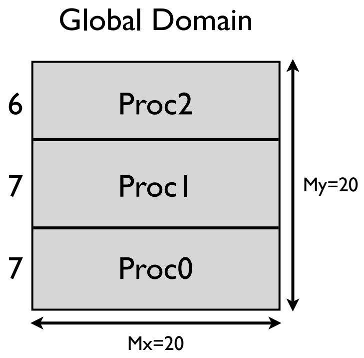
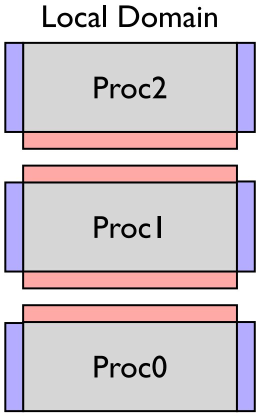
Adaptation of Initial Value HEI Simulation For Steady State Turbulence

- Time-explicit Leapfrog method in 2D.
- Includes ion polarization drift.
- Fully parallelized using the PETSc package for distributed computation.
- Requires parallel solver developed in collaboration with ANL (H. Zhang).

The Computational Domain







Square 2D domain in azimuthal angle φ and radial coordinate normalized flux $y=\psi/\psi_0$.

Periodicity in φ motivates spectral methods, and distributed array geometry into ‘horizontal bands’.

Calculating The Ion Polarization Drift in a Loop

$$\frac{\partial}{\partial t}\nabla\cdot\epsilon\nabla\hat{\Phi} = -\frac{\partial\hat{\rho}}{\partial t} = \nabla\cdot\hat{\mathbf{J}}$$

Poisson's Equation

$$\frac{2}{y^2}\frac{\partial}{\partial\varphi}\epsilon_\varphi(y)\frac{\partial\hat{\Phi}'}{\partial\varphi} + 4\frac{\partial}{\partial y}\epsilon_y(y)\frac{\partial\hat{\Phi}'}{\partial y} = -0.91\frac{L_0^2}{\lambda_{D0}^2}\Delta\tilde{\rho}$$

In dipole magnetic flux coordinates

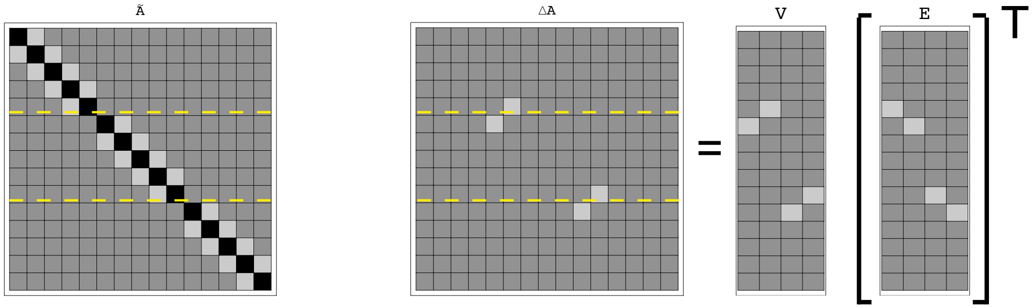
Resulting non-symmetric tri-diagonal system.

$$b(l)\hat{\Phi}'(m,l-1) + a(l)\hat{\Phi}'(m,l) + c(l)\hat{\Phi}'(m,l+1) = R(l)$$

Parallel Partition Method[1] Solver: Ax=b

$$
\begin{aligned}
\mathcal{A}\mathbf{x} &= \mathbf{b} \\
\mathcal{A} &= \tilde{\mathcal{A}} + \Delta\mathcal{A} \\
&= \tilde{\mathcal{A}} + \mathcal{V}\mathcal{E}^T \\
\mathbf{x} &= \mathcal{A}^{-1}\mathbf{b} \\
&= (\tilde{\mathcal{A}} + \mathcal{V}\mathcal{E}^T)^{-1}\mathbf{b} \\
&= \tilde{\mathcal{A}}^{-1}\mathbf{b} - \tilde{\mathcal{A}}^{-1}\mathcal{V}(\mathcal{I} + \mathcal{E}^T\tilde{\mathcal{A}}^{-1}\mathcal{V})^{-1}\mathcal{E}^T\tilde{\mathcal{A}}^{-1}\mathbf{b}
\end{aligned}
$$

The matrix A is decomposed based on the array distribution.



Solution

$$\tilde{\mathcal{A}}\tilde{\mathbf{x}} = \mathbf{b}$$
$$\tilde{\mathcal{A}}\mathcal{Y} = \mathcal{V}$$

Concurrent Solves

$$\mathbf{h} = \mathcal{E}^T\tilde{\mathbf{x}}$$
$$\mathcal{Z} = \mathcal{I} + \mathcal{E}^T\mathcal{Y}$$
$$\mathcal{Z}\mathbf{y} = \mathbf{h}$$

Dense Solve

$$\Delta\mathbf{x} = \mathcal{Y}\mathbf{h}$$
$$\mathbf{x} = \tilde{\mathbf{x}} - \Delta\mathbf{x}$$

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[1] Xian-He Sun, Hong Zhang, and Lionel M. Ni, Efficient Tridiagonal Solvers on Multicomputers, IEEE Transactions on Computers Vol. 41, No. 3, March 1992.

Particle Source and Sink Are Required For Steady State Turbulence

A diffusive particle flux, as well as a particle source is added to the continuity equation.

$$\frac{\partial \hat{N}_i}{\partial t} + \nabla \cdot (\hat{N}_i \hat{\mathbf{V}}_i) + \nabla \cdot \mathbf{\Gamma}_D = S$$

The diffusion is modeled as cross-field only, with a radial functional form. The source is also axisymmetric. The parameter $\hat{D}$ is the strength of the particle ‘recycling’.

$$\begin{aligned} \mathbf{\Gamma}_D &= -\mathcal{D} \cdot \nabla \hat{N}_i \\ \mathcal{D} &= D(y)(\mathcal{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}) \\ D(y) &= \hat{D} h_D(y) \end{aligned}$$

$$S = D_S \hat{D} h_S(y)$$

The continuity equation in normalized magnetic flux coordinates.

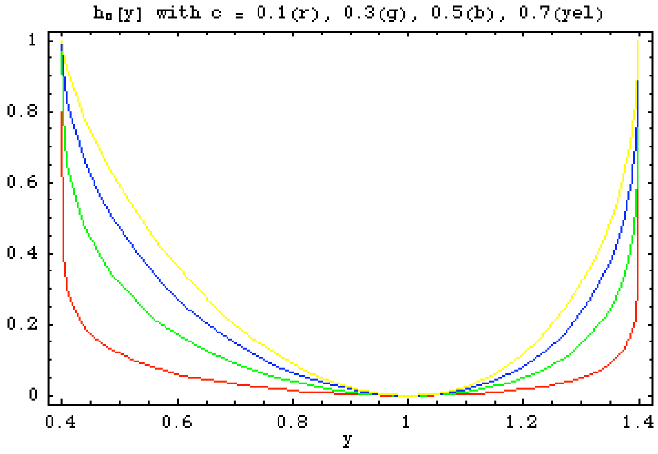
$$\begin{aligned} \frac{\partial \hat{N}}{\partial \hat{t}} &+ \frac{\partial}{\partial \varphi}(\hat{N}_i V_{i,\varphi}) + \frac{\partial}{\partial y}(\hat{N}_i V_{i,y}) \\ &- 1.8 \hat{D} h_D y^2 \frac{\partial^2 \hat{N}_i}{\partial \varphi^2} - 3.2 \frac{\partial}{\partial y}\left[\hat{D} h_D \frac{\partial}{\partial y}(y^4 \hat{N}_i)\right] \\ &= \hat{D} D_S h_S \end{aligned}$$

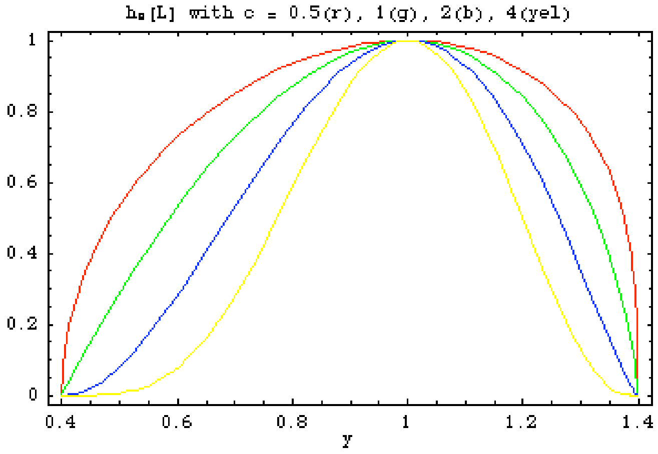
The integrated diffusive particle loss provides the particle conserving source coefficient.

$$D_S = \frac{-\int 1.8 h_D y^2 \frac{\partial^2 \hat{N}_i}{\partial \varphi^2} + 3.2 \frac{\partial}{\partial y}(h_D \frac{\partial}{\partial y}(y^4 \hat{N}_i)) d^2\mathbf{x}}{\int h_S d^2\mathbf{x}}$$

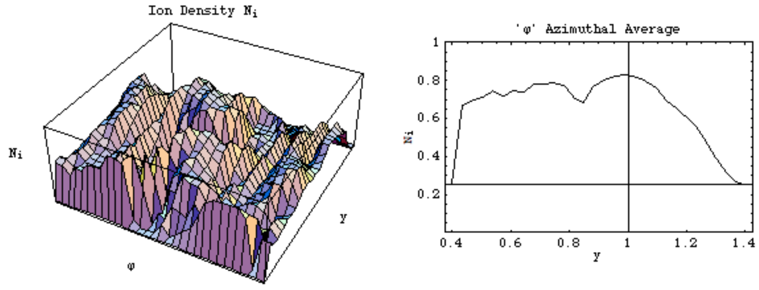
Diffusion and Source Profiles

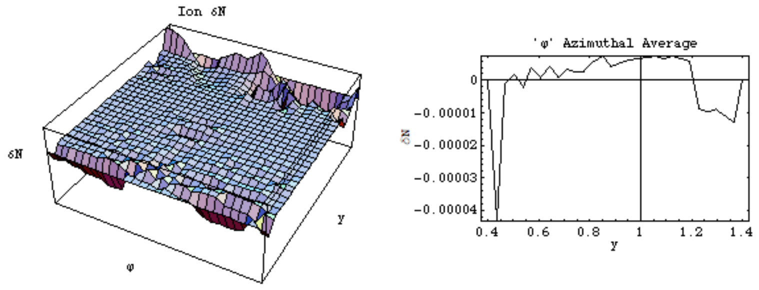
- Strong edge diffusion is used.
- Ions and electrons are added at the experimental ECRH resonance.





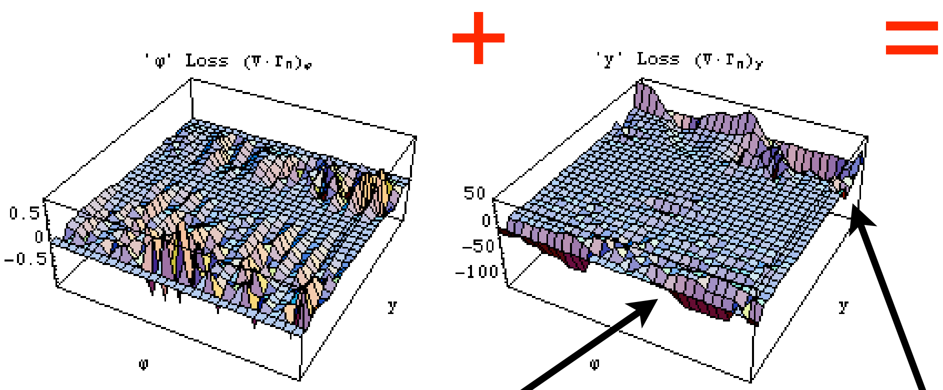
Example N_i and diffusive δN_i.

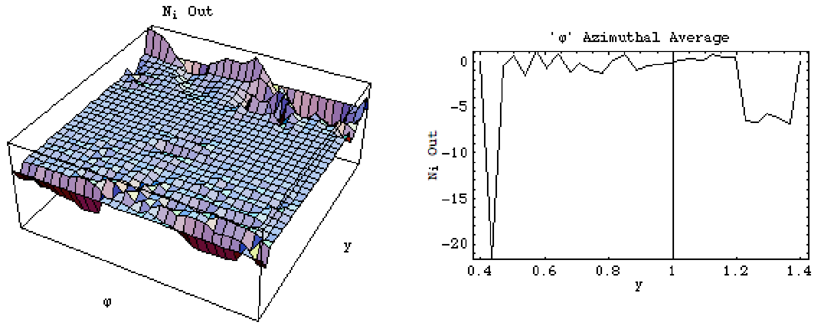




Add axi-symmetric source $h_S(y)$ and scale by diffusion coefficient.

Given Ni and profiles $h_D(y)$, $h_S(y)$, compute 'φ' and 'y' losses.





Loss dominated by the divergence of the 'y' diffusive flux.

Steep $h_D(y)$ gradient.

Steep $h_D(y)$ gradient + y^4 term.

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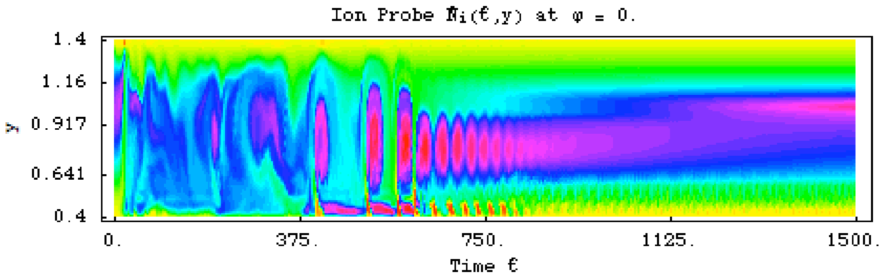
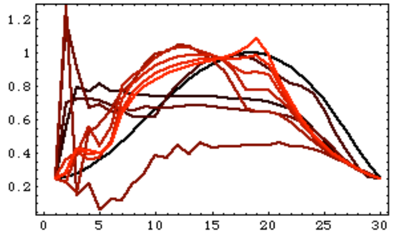
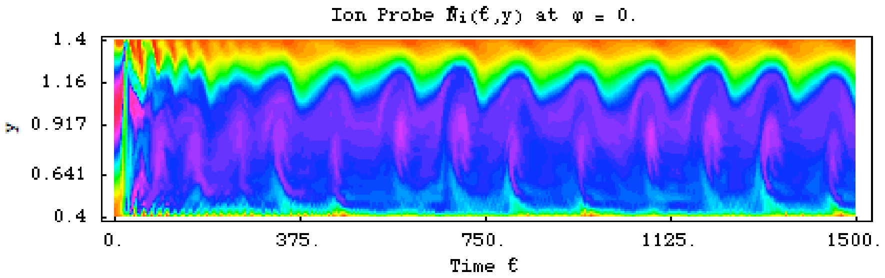
Steady State Turbulence is Achieved for the First Time in a Dipole Simulation



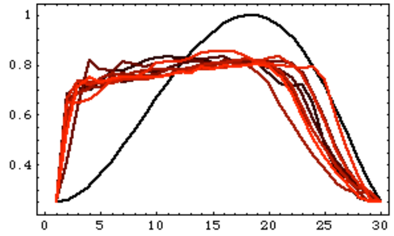
D=0.001



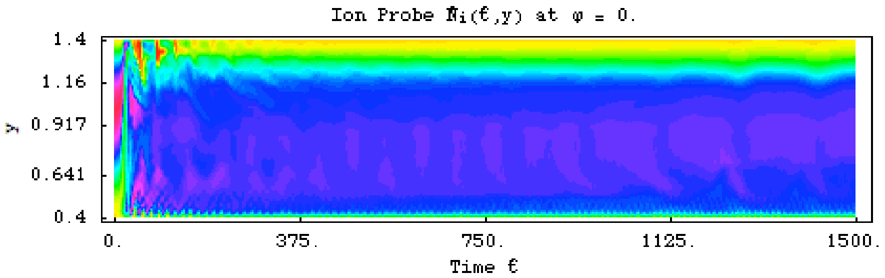
Diffusion too strong. Steep edge gradients and charge densities.



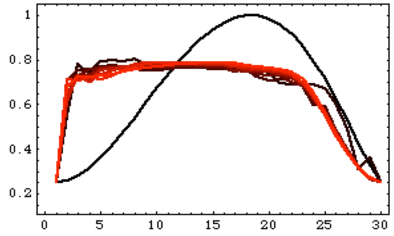
D=0.0001



Diffusion just right. Steady-state turbulence achieved.

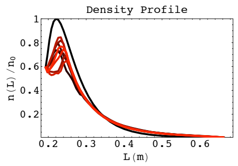


D=0.00001



Diffusion too weak. Not enough edge loss to facilitate recycling.

Note: When $\hat{N}(\psi)=\text{cnst}$, $n(L) \sim 1/L^4$



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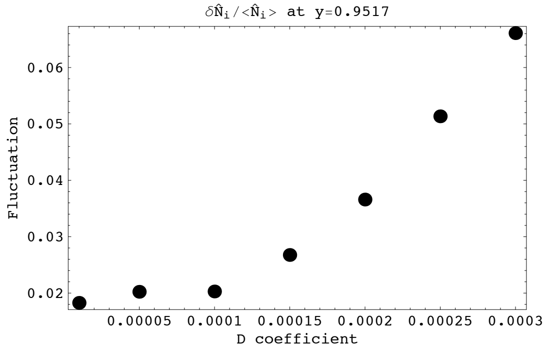
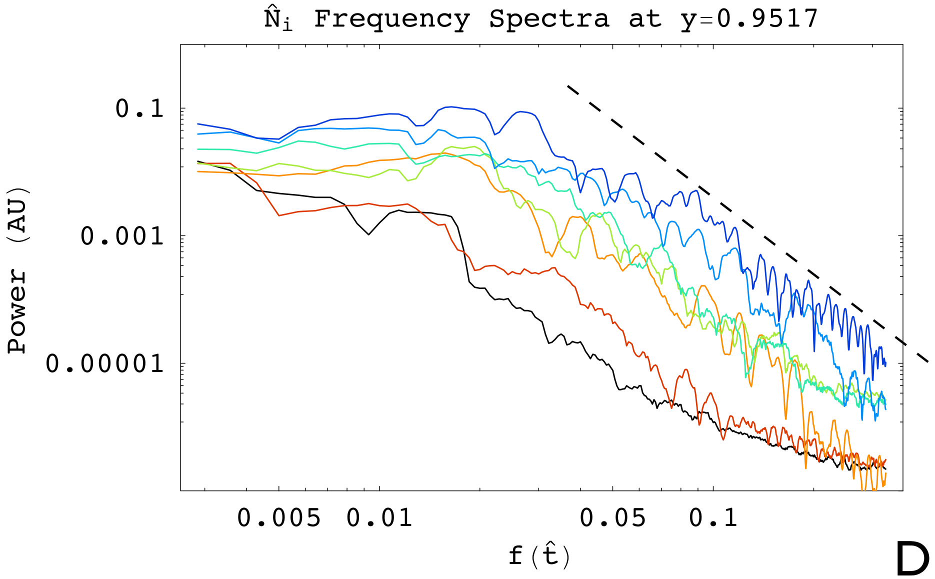
Power-law Spectrum Reproduced

As recycling is increased, the fluctuation amplitude increases.



The density spectrum takes on a power law slope.



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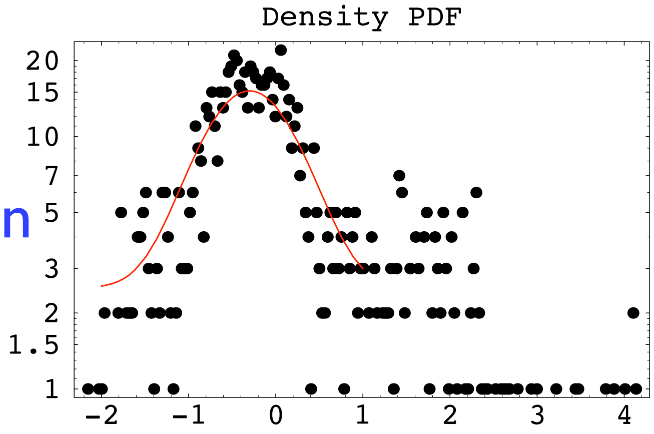
Non-Symmetric PDF is Reproduced

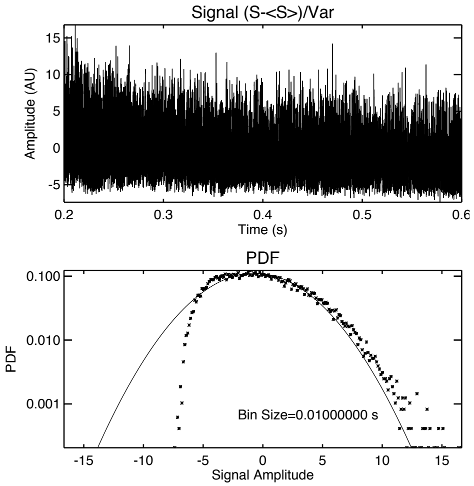
The simulation also reproduces the PDF with a strong positive tail.

This is indicative of intermittent `blob' transport events.

Simulation



Experiment



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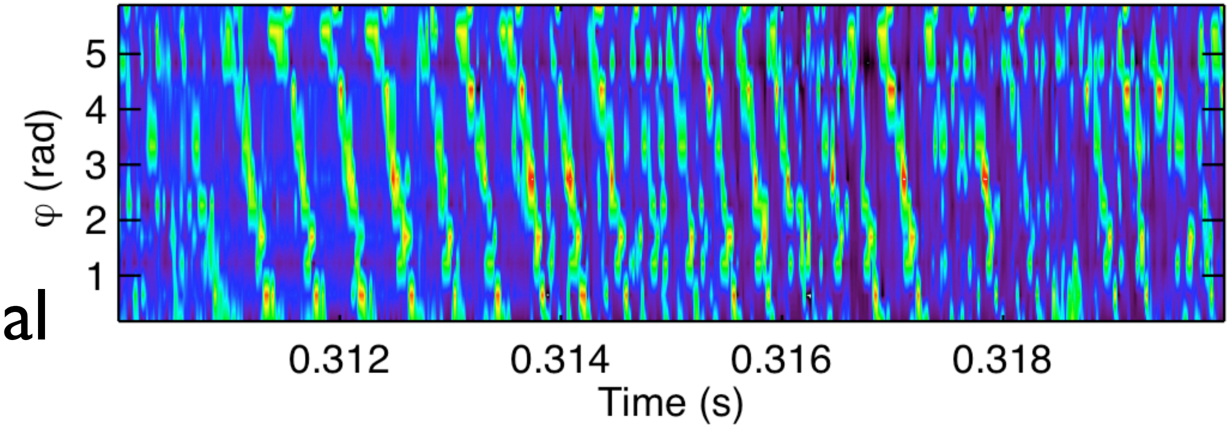
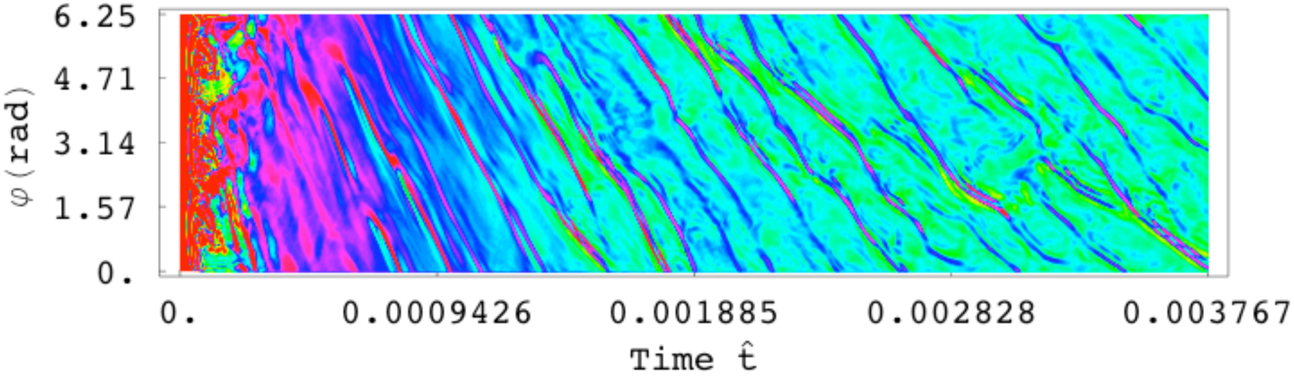
Coherent Structure is Observed and Simulated

Observed and simulated azimuthal mode structure is dominated by low m-number.


Polar Loss in azimuthal angle, L=0.40m


Ion Probe $\hat{N}_i(\hat{t},\varphi)$ at y = 0.641 -> L = 0.421 m

Summary

- The basic characteristics of dipole interchange turbulence have been reproduced by simulation.
- The particle source/sink drive determines the intensity and spectra of the turbulent fluctuations.
- A power-law spectra is seen in the simulation results.

Future Work

- Simulate dipole interchange turbulence on a more refined computational grid, and on a massively parallel architecture.
- Vary the source profiles, diffusion profiles, and 'heating' electron energy.
- Further comparison between experiments, observations, and simulation.