Plasma 2
Lecture 7: Kinetic Description of Magnetized Plasma Waves

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Waves in a Hot Magnetized Plasma

\[ \mathbf{v} \times \mathbf{B}_0 \cdot \nabla_v f_{s0} = 0 \]  

(10.1.2)

\[ \frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \frac{e_s}{m_s} (\mathbf{v} \times \mathbf{B}_0) \cdot \nabla_v f_s + \frac{e_s}{m_s} [\mathbf{E} + \mathbf{v} \times \mathbf{B}] \cdot \nabla_v f_{s0} = 0, \]  

(10.1.3)
ELECTRON TEMPERATURE MEASUREMENTS IN A DENSE PLASMA USING BERNSTEIN WAVES

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Abstract—Bernstein (electron cyclotron) waves have been successfully excited and detected in a fairly dense r.f. Argon plasma. The use of these waves as a diagnostic tool in studying the thermal properties of the plasma in the direction perpendicular to an applied magnetic field has been investigated. The method could be useful in probing a wide variety of laboratory plasmas.

cussed later. Thus at high values of $\omega_p/\omega_c$ the electron temperature is inferred from essentially a single parameter fit ($V_{\perp}$), giving $T_{\perp} = 2.75$ eV.

Fig. 6.—Comparison of theoretical dispersion curves and experimental data for $B_0 = 357$ G and pressure = 1 m torr.
(a) $\omega_p/\omega_c = 5$ $V_{\perp} = 0.98 \times 10^4$ cm/sec
(b) $\omega_p/\omega_c = 3.25$ $V_{\perp} = 0.98 \times 10^4$ cm/sec.
3.1 Experimental apparatus

The plasma was produced by a 1.2 m long 10 cm in dia. pyrex tube immersed in an axial uniform magnetic field variable between zero and 700 G. r.f. Power from a 8.5 MHz 0.5 kW oscillator was matched into a coil surrounding the plasma and having its axis perpendicular to the axis of the discharge tube Fig. 1. The r.f. power couples into an \( m = 1 \) standing helicon wave (Boswell, 1970) as indicated by measurements of the \( b_0 \) component of the wave field inside the tube. The frequency and wavelength of the helicon waves are determined by the exciting frequency and the dimensions of the exciting coil respectively.
Bernstein Waves in the Io Plasma Torus: A Novel Kind of Electron Temperature Sensor

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During Ulysses passage through the Io plasma torus, along a basically north-to-south trajectory crossing the magnetic equator at \( R \sim 7.8 \, R_J \) from Jupiter, the Unified Radio and Plasma Wave experiment observed weakly banded emissions with well-defined minima at gyroharmonics. These noise bands are interpreted as stable electrostatic fluctuations in Bernstein modes. The finite size of the antenna is shown to produce an apparent polarization depending on the wavelength, so that measuring the spin modulation as a function of frequency yields the gyroradius and thus the local cold electron temperature. This determination is not affected by a very small concentration of suprathermal electrons, is independent of any gain calibration, and does not require an independent magnetic field measurement. We find that the temperature increases with latitude, from \(~1.3 \times 10^5\) K near the magnetic (or centrifugal) equator, to approximately twice this value at \( \pm 10^5 \) latitude (i.e., a distance of \(~1.3 \, R_J\) from the magnetic equatorial plane). As a by-product, we also deduce the magnetic field strength with a few percent error.

Fig. 1. Unified radio and plasma wave dynamic spectrum during encounter displayed as frequency versus time, with relative intensity indicated by the bar chart on the right. The torus traversal took place near 1800-1800 UT. We have superimposed in continuous lines the plasma frequency \( f_p \) deduced from the upper hybrid noise and harmonics of the electron gyrofrequency \( f_e \) (calculated from the data as explained in section 4.3). The distance to Jupiter \( R \) (in Jupiter radii) and magnetic latitude \( \lambda_m \) are given in the middle panel. The dashed line near 1600 UT in the lower panel shows \( f_p/2 \) (see text).
The region, average

Fig. 2. Typical spectra in (a) the torus and (b) its outer fringe. The arrows indicate harmonics of the electron gyrofrequency \( f_g \). The labels correspond to radial distance to Jupiter (R), magnetic latitude \( (\lambda_m) \), and plasma frequency \( (f_p) \).

Fig. 5. Bernstein modes \( (k = k_z) \) drawn as \( \omega/\Omega \) versus \( kp \) for different values of the parameter \( \omega_p/\Omega \) increasing from left to right. (The case \( \omega_p/\Omega = 7.7 \) corresponds to the spectrum of Figure 3a, and its first harmonic band is nearly identical to the limiting curve \( \omega_p/\Omega = \) shown here for comparison.) The results correspond to a Maxwellian plasma but are not significantly changed by a very small proportion of suprathermal electrons.

Fig. 9. Measured frequencies of the minima of the spectral shape (dots), together with the gyrofrequency and its harmonics computed by averaging the gyrofrequencies deduced from these minima (continuous lines). The gyrofrequency calculated from the magnetometer data (64-s averages; courtesy of A. Balogh) is shown for comparison as a dashed line.

Fig. 8. Temperature (15 min averaged) as a function of time, deduced from the spin modulation of Bernstein waves from \(-11^\circ\) to \(-11^\circ\) magnetic latitude along Ulysses trajectory in the torus (at \(-8 R_J\) from Jupiter). The shaded region sketches the maximum error bars, whose present large values are due to the very simple method used here.
Waves in a Hot Magnetized Plasma

\[ \mathbf{v} \times \mathbf{B}_0 \cdot \nabla_v f_{s0} = 0 \]  \hspace{1cm} (10.1.2)

\[ \frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \frac{e_s}{m_s} (\mathbf{v} \times \mathbf{B}_0) \cdot \nabla_v f_s + \frac{e_s}{m_s} [\mathbf{E} + \mathbf{v} \times \mathbf{B}] \cdot \nabla_v f_{s0} = 0, \]  \hspace{1cm} (10.1.3)
“Cylindrical Velocity-Space Coordinates”

Figure 10.1 Cylindrical velocity space coordinates $v_\perp$, $v_\parallel$, and $\phi$.

\[
(-i\omega + ik \cdot v)\tilde{f}_s - \omega_{cs} \frac{\partial \tilde{f}_s}{\partial \phi} + \frac{e_s}{m_s} [\tilde{E} + v \times \tilde{B}] \cdot \nabla_v f_{s0} = 0. \tag{10.1.10}
\]

\[
k \cdot v = k_\parallel v_\parallel + k_\perp v_\perp \cos \phi. \tag{10.1.12}
\]
The first-order magnetic field can be eliminated by using Faraday's law, which gives

\[ \tilde{f}_s = \frac{e}{m_s \omega_{cs}} \left[ \tilde{E} + \mathbf{v} \times \left( \frac{\mathbf{k} \times \tilde{E}}{\omega} \right) \right] \cdot \nabla_v f_{s0}, \quad (10.1.13) \]

where to simplify the notation we have introduced the definitions

\[ \alpha_s = \frac{k_{||} \mathbf{v}_{||} - \omega}{\omega_{cs}} \quad \text{and} \quad \beta_s = \frac{k_{\perp} \mathbf{v}_{\perp}}{\omega_{cs}}. \quad (10.1.14) \]
10.2.1 The Harris Dispersion Relation

\[ \tilde{E} = -i k \tilde{\Phi} \quad \tilde{B} = k \times \tilde{E} = 0 \]

\[
\frac{\partial \tilde{f}_s}{\partial \phi} - i(\alpha_s + \beta_s \cos \phi) \tilde{f}_s = \frac{-i e_s}{m_s \omega_{cs}} \tilde{\Phi} k \cdot \nabla_v f_{s0}. \quad (10.2.1)
\]

\[
\frac{df}{dx} + P(x)f = Q(x), \quad (10.2.2)
\]

\[
k^2 \tilde{\Phi} = \sum_s \frac{e_s}{\epsilon_0} \int_{-\infty}^{\infty} \tilde{f}_s d^3v. \quad (10.2.7)
\]
The electrostatic waves propagating in a hot magnetized plasma and is valid for phase shift, consider an applied electric field of the form of the phase shift is illustrated in Figure 10.3. To understand the significance of the Harris dispersion relation gives a very general result for small-amplitude \footnote{10.2 Electrostatic Waves} of the phase shift is illustrated in Figure 10.3. To understand the significance of the Harris dispersion relation gives a very general result for small-amplitude electric field by the zero-order cyclotron motion of the particle. The origin derived this result. The parameter any direction of propagation and any choice of plasma parameters. The origin derived this result.

\[ \tilde{E}_0 e^{i(k_\perp x - \omega t)} = \tilde{E}_0 e^{i\beta \sin \omega_c t} e^{-i\omega t} = \tilde{E}_0 \sum_{n=-\infty}^{\infty} J_n(\beta) e^{i n \omega_c t} e^{-i\omega t} \]

\[ x = \rho_c \sin \omega_c t, \]

\[ \beta \equiv k_\perp \rho \]

\[ \beta_s = k_\perp v_\perp / \omega_{cs} \]

Figure 10.3 For a finite cyclotron radius, \( \rho_c \), a phase shift is introduced by the cyclotron motion of the particle around the magnetic field. This phase shift is responsible for the resonances at \( n \omega_c \).

Figure 10.4 Plots of the zero-order, first-order, and second-order Bessel functions, \( J_0(\beta_s), J_1(\beta_s), \) and \( J_2(\beta_s) \).
\[
\tilde{f}_s = \frac{-ie_s n_s \tilde{\Phi}}{m_s \omega_{cs}} e^{i(\alpha_s \phi + \beta_s \sin \phi)} \int_0^\phi k \cdot \nabla_v F_{s0} e^{-i(\alpha_s \phi' + \beta_s \sin \phi')} \, d\phi',
\]

\[
D(k, \omega) = 1 + \sum_s \frac{\omega_{ps}^2}{k^2 \omega_{cs}} \int e^{i(\alpha_s \phi + \beta_s \sin \phi)}
\]

\[
\times \int_0^\phi i k \cdot \nabla_v F_{s0} e^{-i(\alpha_s \phi' + \beta_s \sin \phi')} \, d\phi' \, d\phi \, d\nu_\perp \, d\nu_\perp \, d\nu_\parallel = 0,
\]
\[
e^{-i\beta_s \sin \phi'} = \sum_{n=-\infty}^{\infty} J_n(\beta_s) e^{-i n \phi'}, \tag{10.2.11}
\]

\[
\int_{\phi}^{\phi} e^{-i(\alpha_s \phi' + \beta_s \sin \phi')} \, d\phi' = \sum_{n} J_n(\beta_s) \int_{\phi}^{\phi} e^{-i(\alpha_s + n) \phi'} \, d\phi'
= i \sum_{n} \frac{J_n(\beta_s)}{\alpha_s + n} e^{-i(\alpha_s + n) \phi}. \tag{10.2.12}
\]

\[
\int_{\phi}^{\phi} (e^{i \phi'} + e^{-i \phi'}) e^{-i(\alpha_s \phi' + \beta_s \sin \phi')} \, d\phi'
= \sum_{n} J_n(\beta_s) \int_{\phi}^{\phi} \left[ e^{-i(\alpha_s - 1 + n) \phi'} + e^{-i(\alpha_s + 1 + n) \phi'} \right] \, d\phi'
= i \sum_{n} J_n(\beta_s) \left[ \frac{e^{-i(\alpha_s - 1 + n) \phi}}{\alpha_s - 1 + n} + \frac{e^{-i(\alpha_s + 1 + n) \phi}}{\alpha_s + 1 + n} \right]. \tag{10.2.13}
\]
where the first integral on the right-hand side of Eq. (10.2.10) becomes

\[
\int_{\phi} \cdots d\phi' = -k_\parallel \frac{\partial F_{s0}}{\partial u_\parallel} \sum_{n,m} J_m J_n \left[ \frac{e^{i(m-n)\phi}}{\alpha_s + n} \right]
\]

\[
- \frac{1}{2} k_\perp \frac{\partial F_{s0}}{\partial u_\perp} \sum_{n,m} J_m J_n \left[ \frac{e^{i(m-n+1)\phi}}{\alpha_s + n - 1} + \frac{e^{i(m-n-1)\phi}}{\alpha_s + n + 1} \right].
\]

(10.2.15)

In the second summation, the index can be relabeled to give

\[
\sum_{n,m} J_m J_{n+1} \frac{e^{i(m-n)\phi}}{\alpha_s + n} + J_m J_{n-1} \frac{e^{i(m-n)\phi}}{\alpha_s + n} = \sum_{n,m} J_m [J_{n+1} + J_{n-1}] \frac{e^{i(m-n)\phi}}{\alpha_s + n}.
\]

(10.2.16)

Using the Bessel function recursion formula, \(J_{n+1} + J_{n-1} = (2n/\beta_s)J_n\), this sum can be written in the more compact form

\[
\sum_{m,n} \frac{2n J_m J_n}{\beta_s (\alpha_s + n)} e^{i(m-n)\phi}.
\]

(10.2.17)
\[ D(k, \omega) = 1 - \sum_s \frac{\omega_{ps}^2}{k^2 \omega_{cs}} \sum_{n,m} \int_{-\infty}^{\infty} \int_0^\infty v_\perp \, dv_\perp \, dv_\parallel \, \frac{J_m J_n}{\alpha_s + n} \]

\[ \times \left[ k_\parallel \frac{\partial F_{s0}}{\partial v_\parallel} + \frac{n \omega_{cs}}{v_\perp} \frac{\partial F_{s0}}{\partial v_\perp} \right] \int_0^{2\pi} e^{i(m-n)\phi} \, d\phi = 0. \quad (10.2.19) \]
\[
D(k, \omega) = 1 - \sum_s \frac{\omega_{ps}^2}{k^2} \sum_n \int_{-\infty}^{\infty} \int_0^\infty \frac{J_n^2(k_n u_\perp / \omega_{cs})}{k_n u_\parallel - \omega + n \omega_{cs}} \left[ k_\parallel \frac{\partial F_{s0}}{\partial u_\parallel} + \frac{n \omega_{cs}}{u_\perp} \frac{\partial F_{s0}}{\partial u_\perp} \right] \\
\times 2\pi u_\perp du_\perp du_\parallel = 0. \tag{10.2.20}
\]

This equation is called the Harris dispersion relation after Harris (1959), who first derived this result.
10.2.2 The Low-temperature, Long-Wavelength Limit

\[ D(k, \omega) = 1 - \sum_s \frac{\omega_{ps}^2}{k^2} \sum_n \int_{-\infty}^{\infty} \int_0^\infty \frac{J_n^2(k_\perp v_\perp / \omega_{cs})}{k_\parallel v_\parallel - \omega + n \omega_{cs}} \left[ k_\parallel \frac{\partial F_{s0}}{\partial v_\parallel} + \frac{n \omega_{cs}}{v_\perp} \frac{\partial F_{s0}}{\partial v_\perp} \right] \]

\[ \times 2\pi v_\perp dv_\perp dv_\parallel = 0. \quad (10.2.20) \]

\[ D_0(k, \omega) = \left[ 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \right] \cos^2 \theta + \left[ 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2 - \omega_{cs}^2} \right] \sin^2 \theta = 0, \quad (10.2.22) \]
10.2.3 The Bernstein Modes

\[ D(k_\perp, \omega) = 1 - \frac{\omega_p^2}{k_\perp^2} \sum_{n=-\infty}^{\infty} \frac{n \omega_c}{-\omega + n \omega_c} \]

\[ \times \int_{-\infty}^{\infty} \int_{0}^{\infty} j_n^2 \left( \frac{k_\perp v_\perp}{\omega_c} \right) \frac{\partial F_0}{\partial v_\perp} 2\pi dv_\perp dv_\parallel = 0, \quad (10.2.24) \]

\[ \frac{n \omega_c}{-\omega + n \omega_c} + \frac{(-n)\omega_c}{-\omega + (-n)\omega_c} = \frac{-2n^2 \omega_c^2}{\omega^2 - n^2 \omega_c^2}, \quad (10.2.25) \]

\[ D(k_\perp, \omega) = 1 + \frac{2\omega_p^2}{k_\perp^2} \sum_{n=1}^{\infty} \frac{n^2 \omega_c^2}{\omega^2 - n^2 \omega_c^2} \]

\[ \times \int_{-\infty}^{\infty} \int_{0}^{\infty} j_n^2 \left( \frac{k_\perp v_\perp}{\omega_c} \right) \frac{\partial F_0}{\partial v_\perp} 2\pi dv_\perp dv_\parallel = 0, \quad (10.2.26) \]
10.2.3 The Bernstein Modes

\[ D(k_\perp, \omega) = 1 - \sum_{n=1}^{\infty} \frac{2\omega_p^2}{\beta_c^2 \omega_c^2} \frac{\Gamma_n(\beta_c)}{(\omega/n\omega_c)^2 - 1} = 0, \]  
\[ (10.2.29) \]

\[ \Gamma_n(\beta_c) = e^{-\beta_c^2} I_n(\beta_c^2), \]  
\[ (10.2.30) \]

Figure 10.6 Plots of \( \Gamma_n(\beta_c) \) for \( n = 1, 2, \) and 3.
10.2.3 The Bernstein Modes

\[ D(k_\perp, \omega) = 1 - \sum_{n=1}^{\infty} \frac{2\omega_p^2}{\beta_c^2 \omega_c^2} \frac{\Gamma_n(\beta_c)}{(\omega/n \omega_c)^2 - 1} = 0, \]  

(10.2.29)

Figure 10.8 A plot of the solutions of \( D(k_\perp, \omega) = 0 \) as a function of the perpendicular wave number, \( k_\perp \). These solutions are called the Bernstein modes.
Figure 10.25 A loss-cone electron velocity distribution showing the cyclotron resonance velocity, \( v_{\parallel \text{Res}} \), for a whistler-mode wave propagating in the \( k_{\parallel} \) direction.