Plasma 2
Lecture 2: Nonlinear Landau Damping (Part 1)

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Columbia University

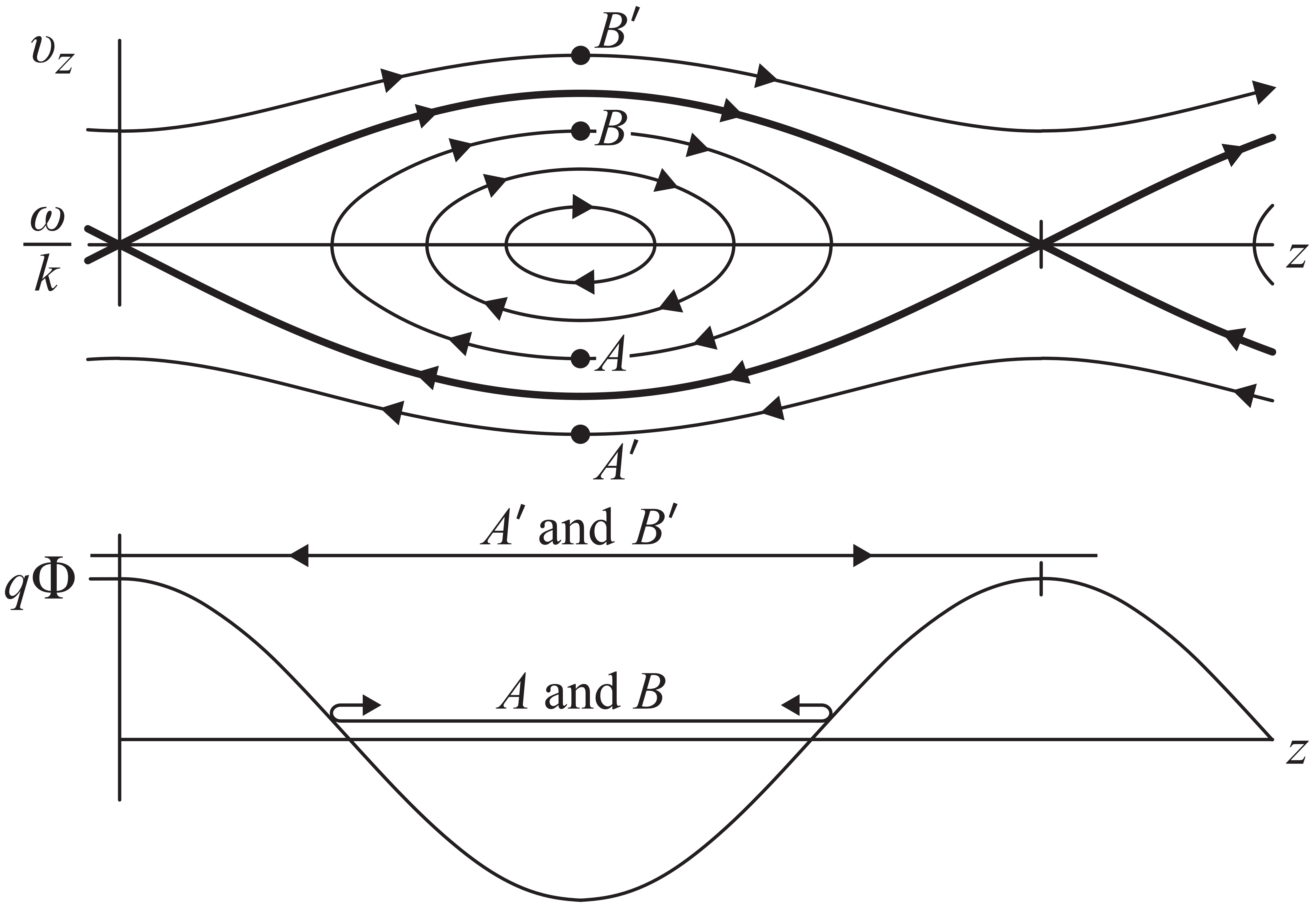

Figure 9.13 Phase-space trajectories in a frame of reference (z, v_z) moving at the phase velocity for a sinusoidal electrostatic potential $\Phi(z) = \Phi_0 \cos kz$.

Bounce (Trapping) Frequency

$$0 < t \ll \omega_{\mathrm{b}}^{-1} = \frac{1}{k}\sqrt{\frac{m}{q\Phi_0}}. \qquad (9.2.53)$$

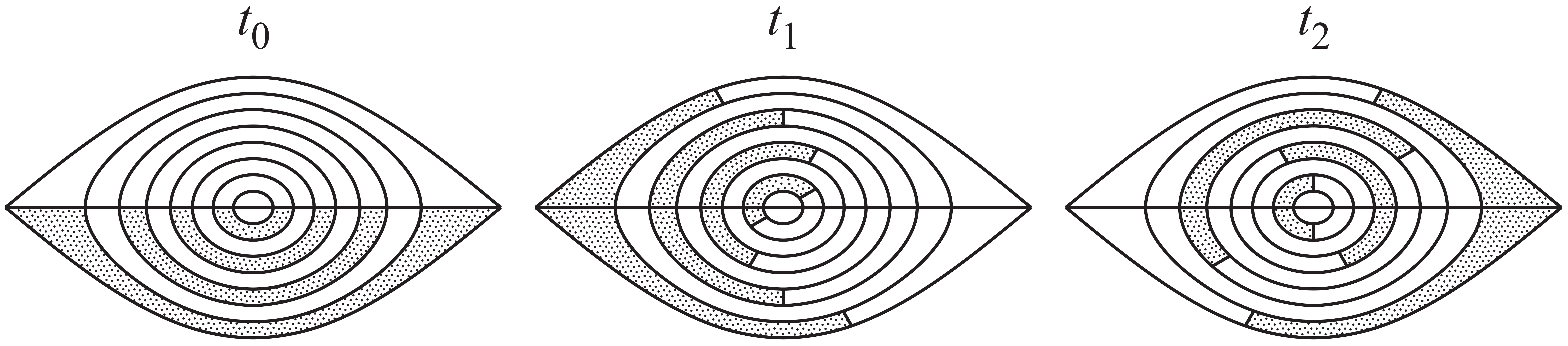

Figure 9.14 The relative phase-space locations of trapped particles at three successive phases of the bounce cycle.

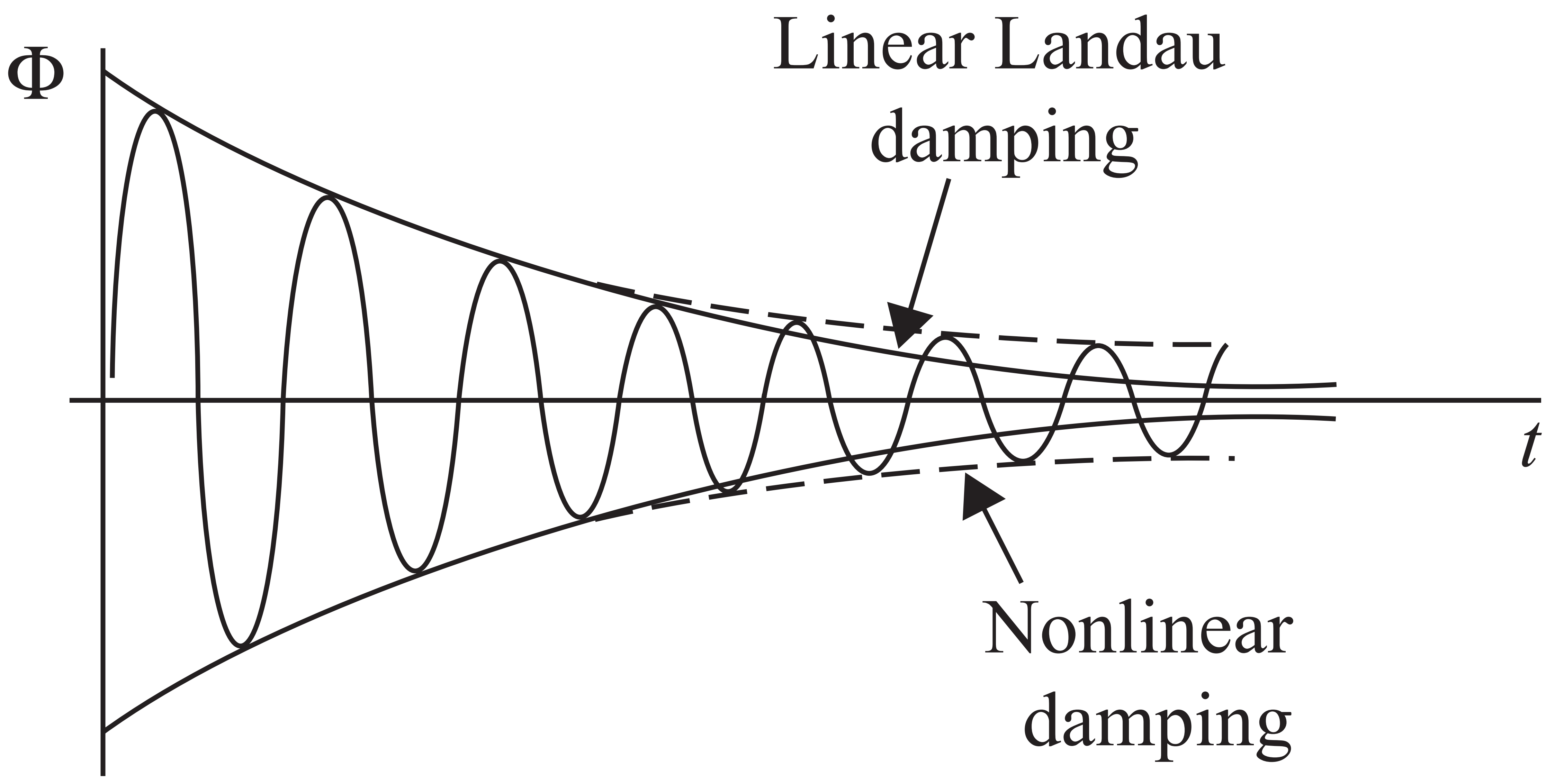

Figure 9.15 The nonlinear effects of particle trapping tend to increase the wave amplitude relative to the predictions of linear Landau damping.

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COLLISIONLESS DAMPING OF ELECTROSTATIC PLASMA WAVES*

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(Received 6 July 1964)

It has been predicted by Landau[1] that electrostatic electron waves in a plasma of finite temperature will be damped, even in the absence of collisions. Landau's theory has been challenged on various grounds[2] and a number of experiments designed to detect the effect for electrostatic electron waves or ion acoustic waves have been reported.[3] The existence of the damping is of interest not only for its own sake, but because the method of calculation has been widely used for related problems. We report here preliminary results of an experiment designed to measure the Landau damping of electrostatic electron waves. We observe heavy damping which exhibits the expected dependence on phase velocity.

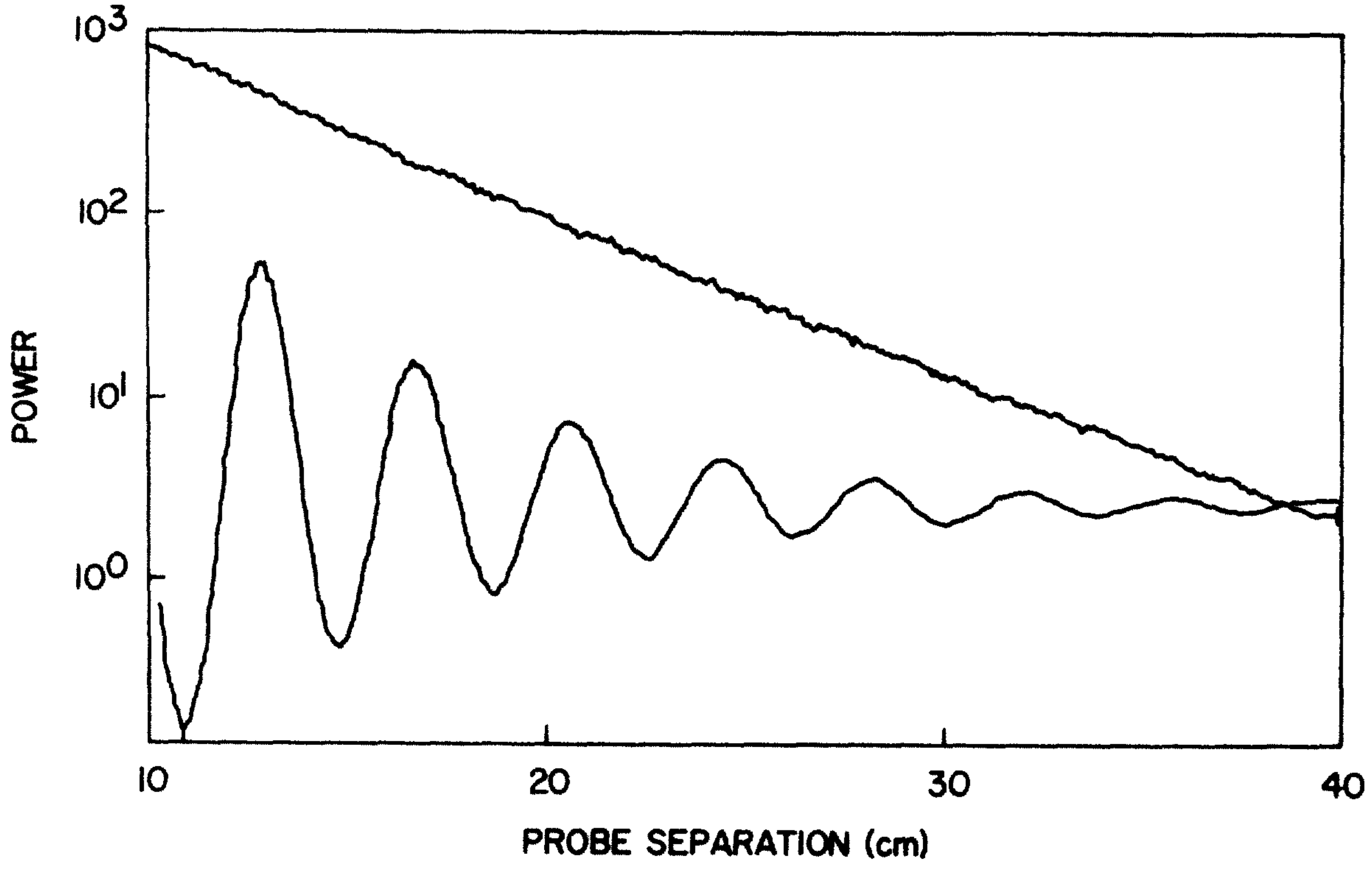
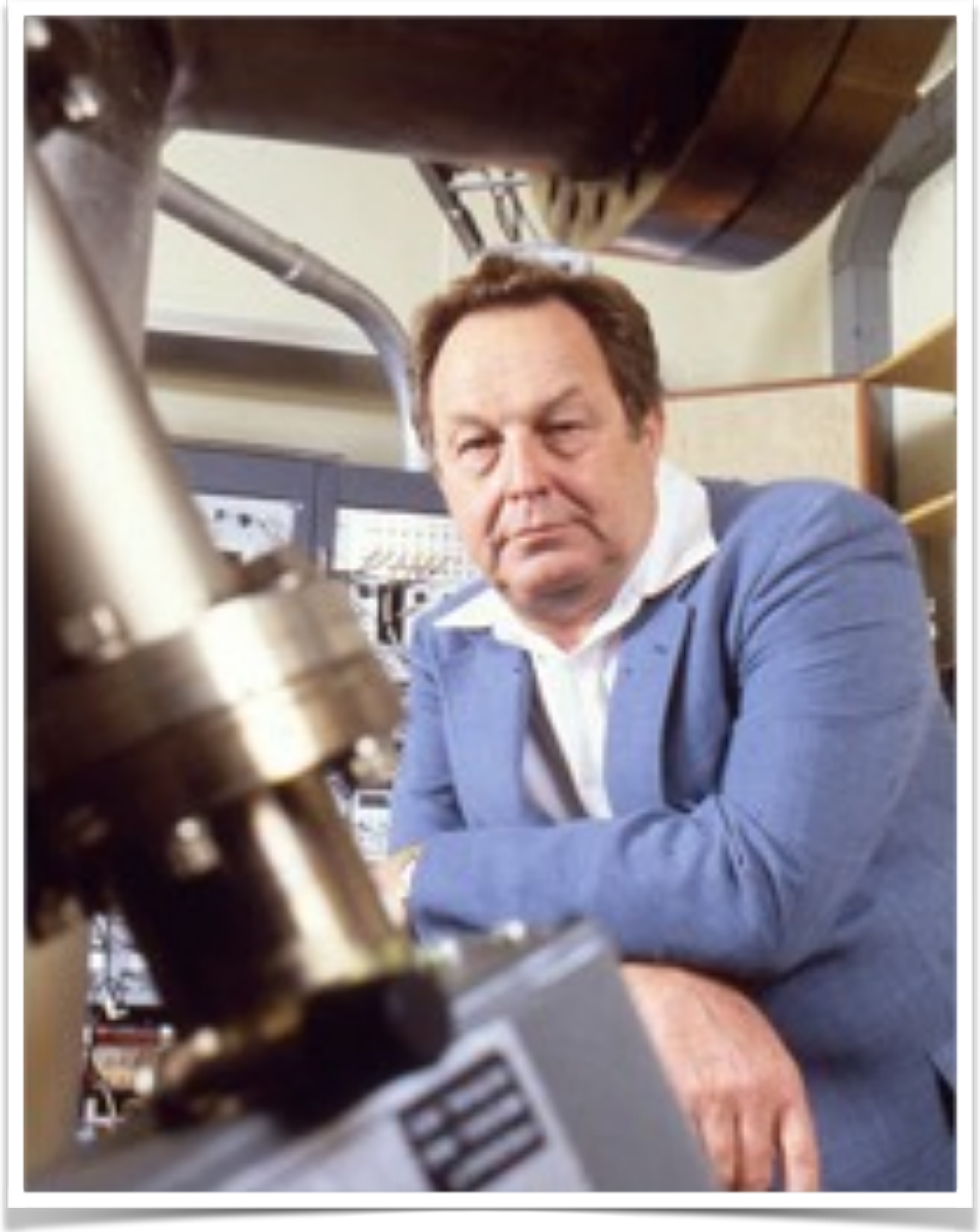


FIG. 1. Raw data. Upper curve is the logarithm of received power. Lower curve is interferometer outpu Abscissa is probe separation.

John Malmberg and Chuck Wharton

The first experimental measurement of Landau Damping



John Malmberg

(obit, Nov 1992)

Prof. Malmberg joined UCSD from General Atomics in 1969 as a professor of physics. Much of his work revolved around theoretical and experimental investigations of fully ionized gases or plasmas. The field could offer insights into how stars work and how to ignite and control thermonuclear reactions to produce fusion energy--the power that drives the sun.

A plasma is the fourth state of matter, with solids, liquids and gases making up the other three. Most of the matter in the Universe is in the plasma state; for example, the matter of stars is composed of plasmas.

In recent years, Prof. Malmberg had been experimenting with pure electron plasmas that were trapped in a magnetic bottle. By contrast with electrically neutral plasmas that contain an equal number of positive and negative electrons, pure electron plasmas are rare in nature.

Before joining UCSD, Prof. Malmberg was director of the Plasma Turbulence group at General Atomics, where he carried out some of the first and most important experiments to test the basic principals of plasma physics. **Perhaps his most important experiment involved the confirmation of the phenomenon called "Landau damping," where electrons surf on a plasma wave, stealing energy from the wave and causing it to damp (decrease in amplitude).**

For his pioneering work in testing the basic principals of plasma, and for his more recent work with electron plasmas, Prof. Malmberg was named the recipient of the American Physical Society's James Clerk Maxwell Prize in Plasma Physics in 1985.

Chuck Wharton

(emeritus, Cornell)

Professor Wharton was a staff member at the University of California Lawrence Radiation Laboratory at Livermore and Berkeley, California from 1950-1962. While in this position, he spent a year (1959-60) as engineer-scientist at the Max-Planck Institute for Physics in Munich, Germany, and also as a lecturer at the International Summer Course in Plasma Physics at Riso, Denmark. For the next five years he was a staff member of the Experimental Physics Group at General Atomics in San Diego, California. He joined the EE faculty as a full professor in 1967.

In 1973 he received the Humboldt Prize awarded by the Alexander von Humboldt Foundation. He was elected a fellow of the American Physical Society in 1973. In 1976 he was elected a fellow of the IEEE "in recognition of contributions to the understanding of plasmas and to the development of plasma diagnostic techniques." In 1979 he was given the award, Socio Onorario, by the International School of Plasma Physics (Milan, Italy).

Charles (Chuck) taught undergraduate courses in electromagnetic theory, plasma physics, and electrical sciences laboratory. His research was primarily in the area of plasma-physics diagnostics, in which he is a recognized world authority, and in plasma interactions and heating with waves and beams with applications to controlled thermonuclear fusion.

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DISPERSION OF ELECTRON PLASMA WAVES*

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(Received 31 May 1966)

Description of the Experimental Device

The machine which produces the plasma has been described in detail elsewhere.[4] The plasma is produced in a duoplasmatron-type hydrogen arc source and drifts from it into a long uniform magnetic field of a few hundred gauss. The entire machine is steady state. The resulting plasma has, in a typical case, a radius of 7 mm, a length of 230 cm, a density of 5×10^8 electrons/cm^3, and a temperature of 12 ± 3 eV as measured by Langmuir probes. The background pressure is 1.7×10^{-5} Torr (mostly H_2). Hence, the Debye length is about 1 mm, the electron mean free path for electron-ion collisions is of the order of 1000 meters and for electron-neutral collisions is about 40 meters. The plasma is surrounded by a stainless steel tube 3.8 cm in radius which acts as a waveguide beyond cutoff to reduce electromagnetic coupling between probes. The plasma density depends somewhat on distance from the source.

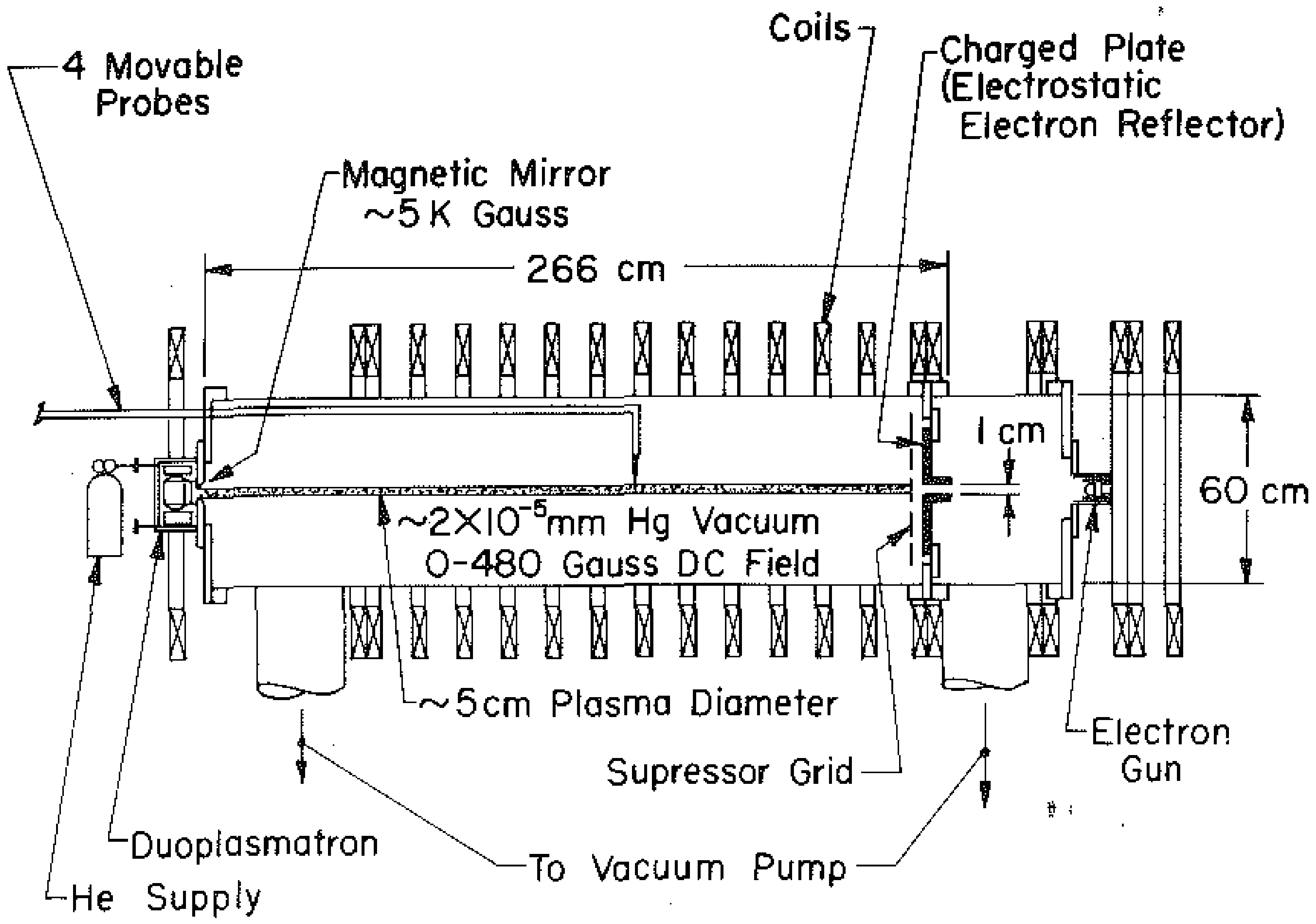


FIGURE 10.8.4
Schematic of experiment used to investigate plasma wave echoes. [*After J. H. Malmberg, et al., Proceedings of Conference on Phénomènes d'ionization dans les Gaz*, 4; 229 (1963).]

Description of the Experimental Device

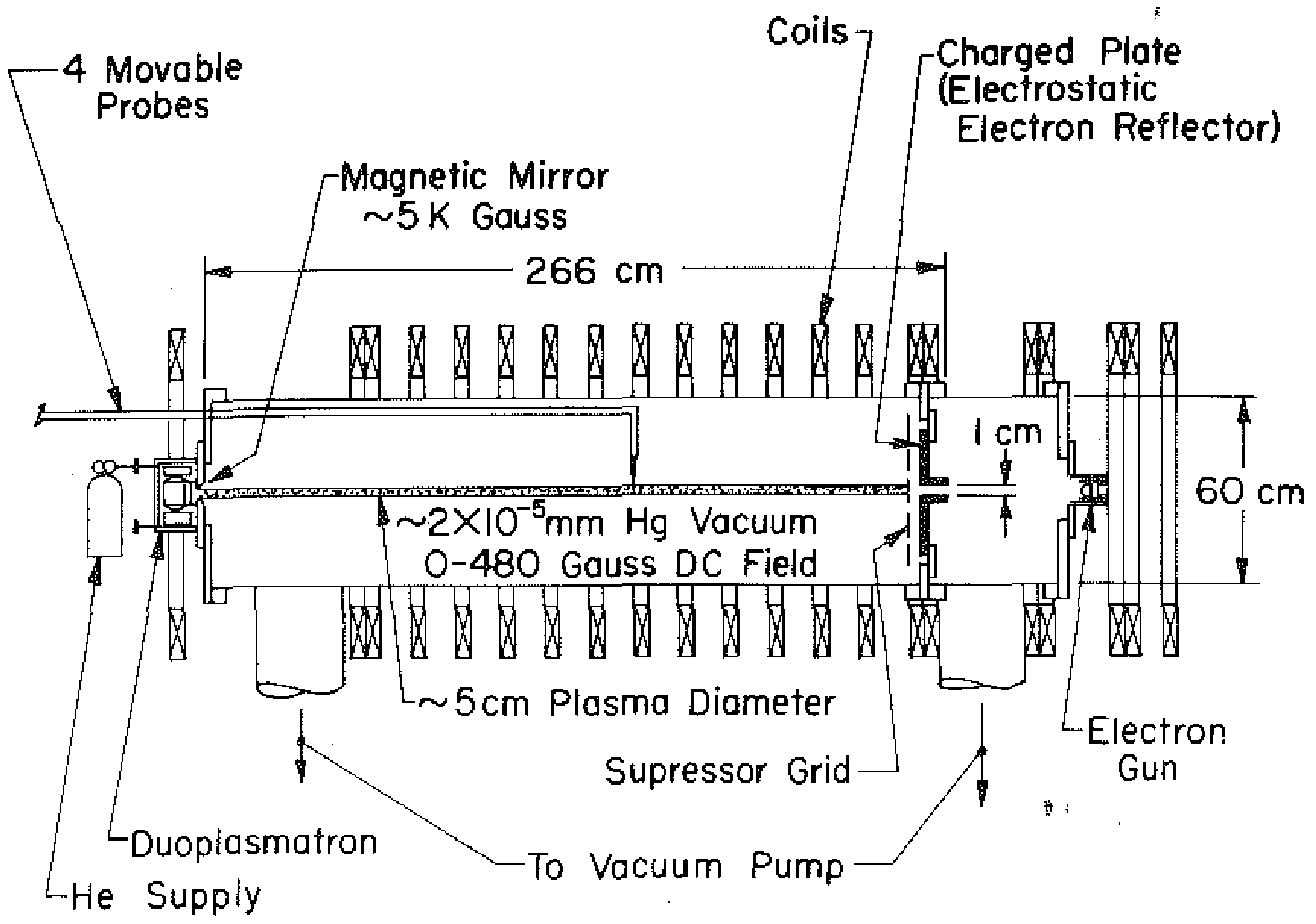


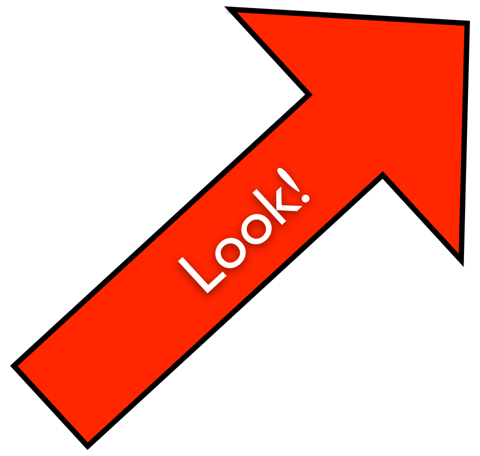
FIGURE 10.8.4
Schematic of experiment used to investigate plasma wave echoes. [*After J. H. Malmberg, et al., Proceedings of Conference on Phénomènes d'ionization dans les Gaz*, 4; 229 (1963).]

If L is large compared with the Landau damping length, and if $\omega_2/(\omega_2 - \omega_1)$ is of order unity, this third electric field, which is the spatial plasma echo, appears at a position well separated from the first two electric field excitation positions. The experiment used by Malmberg *et al.*[1] to study the spatial plasma wave echoes is depicted schematically in Fig. 10.8.4. The plasma column is 180 cm long and 5 cm in diameter, with a central density of 1.5×10^8 cm^{-3}. The axial magnetic field is 300 G and can be regarded as infinite for the purposes of the experiment. The plasma has a temperature of 9.4 eV and a Debye length of 2 mm. The electron mean free path is 10^5 cm for electron-ion collisions and 4×10^4 cm for electron-neutral collisions. The plasma column is surrounded by a 5.2-cm-radius cylinder that acts as a waveguide beyond cutoff and reduces the stray electromagnetic coupling between the excitation and detection probes.

A plasma wave echo obtained with this experiment is shown in the lower trace of Fig. 10.8.5. The upper trace is the spatial distribution of the 120-MHz signal in the vicinity of the excitation probe at $x = 0$. The middle trace is the spatial distribution of the 130-MHz signal in the vicinity of the second probe at

[1] J. H. Malmberg, C. B. Wharton, R. W. Gould, and T. M. O'Neil, *Phys. Fluids*, **11**: 1147 (1968).

JOURNAL OF APPLIED PHYSICS VOLUME 30, NUMBER 11 NOVEMBER, 1959



Space Charge Waves in Cylindrical Plasma Columns*

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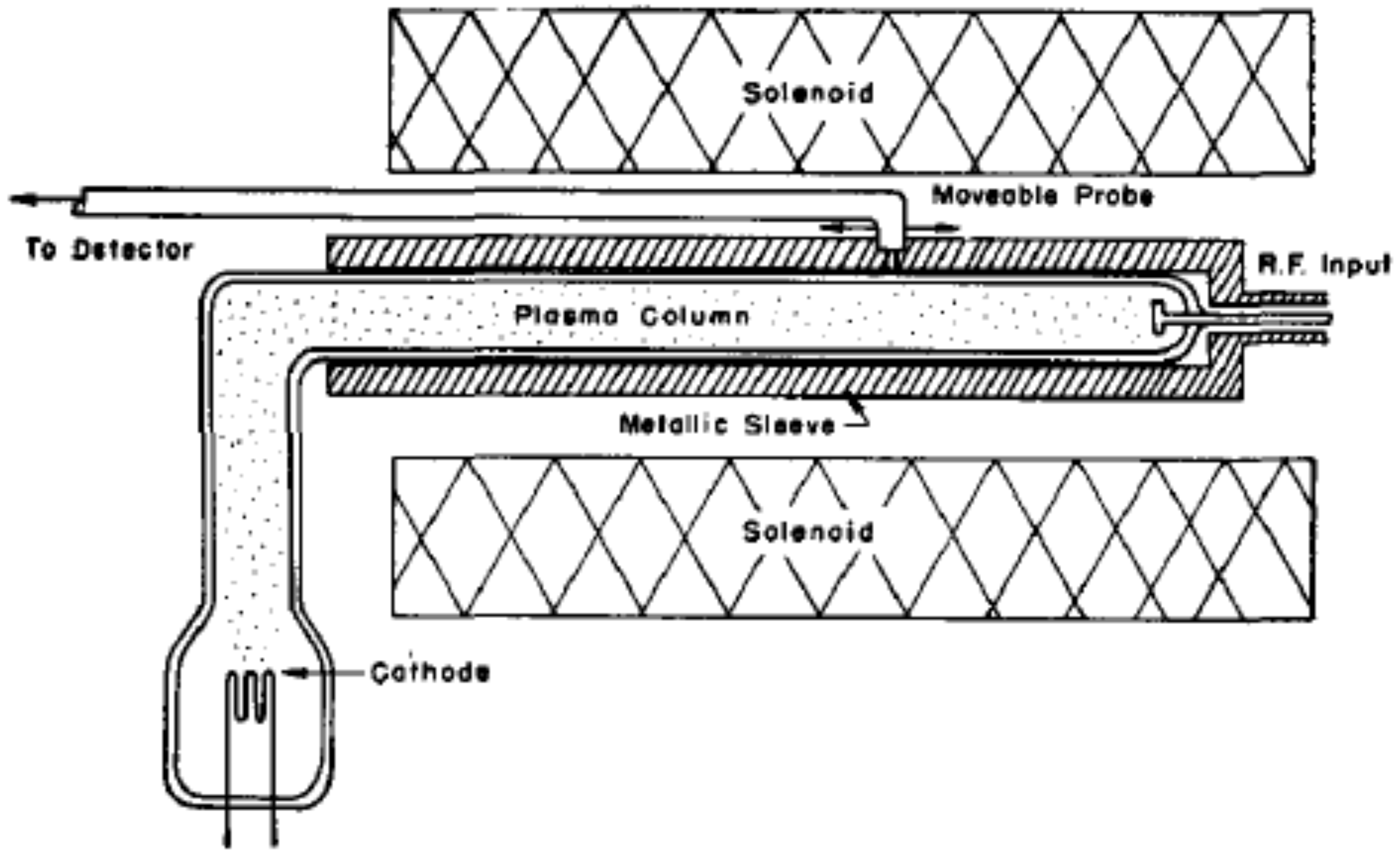


Fig. 10. Schematic of apparatus to measure phase and attenuation characteristics of space charge waves in a plasma.

Table I. Pertinent dimensions of experimental apparatus used in space charge wave propagation experiment.

Diameter of plasma	0.328 in.
Diameter of tube containing plasma	0.410 in.
Diameter of slotted wave guide	0.410 and 0.750 in.
Length of plasma column	25 cm
Signal frequency range	10 to 4000 mc
Cyclotron frequency range	0 to 5000 mc
Plasma frequency range	500 to 5000 mc
Temperature of mercury in tube	300±0.1°K
Empty wave guide cutoff frequency (approx)	25 000 mc
Pressure of mercury at 300°K (approx)	2 microns
Mean free path of plasma electrons (approx)	5 cm

Assuming wave solutions $\exp[j(\omega t-\beta z)]$

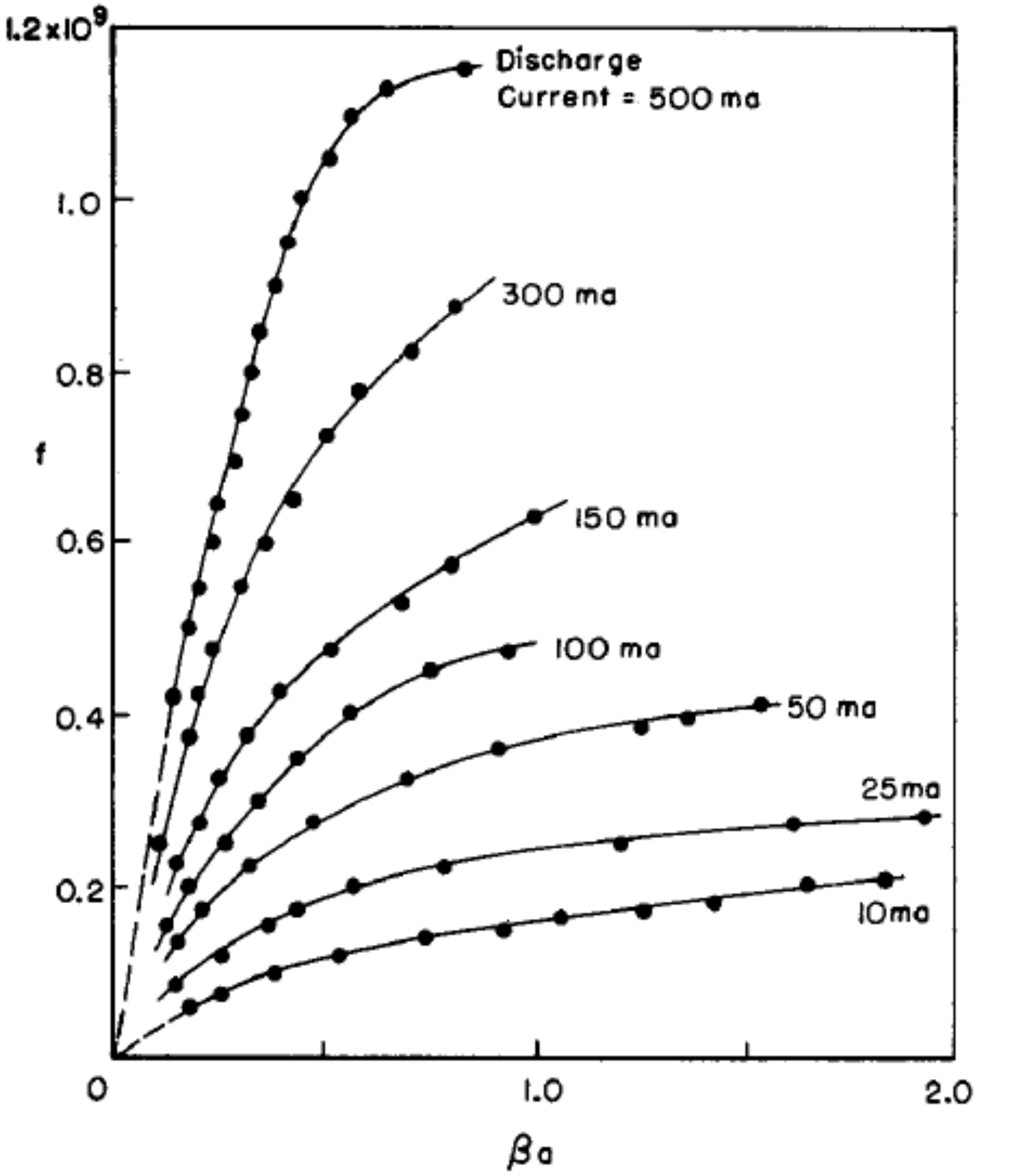


Fig. 13. Measured phase characteristics of plasma space charge waves for no magnetic field for $a=0.52$ cm, $b=0.62$ cm, $K=4.6$.

TG Modes: Low Frequency Surface Waves

Inside plasma ($r < a$): $\bar{E} = -\nabla \Phi$ $\quad \nabla\cdot(\epsilon_0 K \bar{E}) = 0$ $\quad K = 1 - \frac{\omega_p^2}{\omega^2}$

$$\Rightarrow \nabla^2 \Phi = 0$$

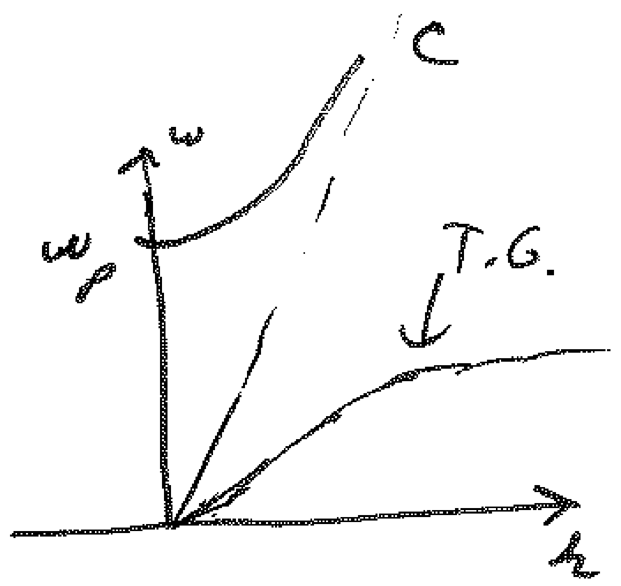
Outside plasma ($r > a$): $\bar{E} = -\nabla \Phi$ $\quad \nabla\cdot(\epsilon_0 \bar{E}) = 0$

$$\nabla^2 \Phi = 0$$

Boundary conditions ($r = a$):

$$\hat{n}\cdot(\epsilon_0 K \bar{E}_{in} - \epsilon_0 \bar{E}_{out}) = 0$$

$$\Phi_{in} = \Phi_{out}$$



$$\nabla^2 \Phi = 0 = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \Phi}{\partial r}\right) - k^2 \Phi = 0$$

$$\Phi(r, z, t) \propto e^{-j(\omega t - kz)}$$

$$\left(1 - \frac{\omega_p^2}{\omega^2}\right) I_0' C_1 = K_0' C_2$$

$\Phi(r) \sim I_0(kr)$ inside, $K_0(kr)$ outside $\Big\}$ $r = a$

$$I_0 C_1 = K_0 C_2$$

$$1 - \frac{\omega_p^2}{\omega^2} = \frac{I_0(ka) K_0'(ka)}{I_0'(ka) K_0(ka)}$$

Raw Data

Two probes, each consisting of a 0.2-mm diameter radial tungsten wire, are placed in the plasma. One probe is connected by coaxial cable to a chopped signal generator. The other probe is connected to a receiver which includes a sharp high-frequency filter, a string of broad-band amplifiers, an rf detector, a video amplifier, and a coherent detector operated at the transmitter chopping frequency. Provision is made to add a reference signal from the transmitter to the receiver rf signal, i.e., we may use the system as an interferometer. The transmitter is set at a series of fixed frequencies, and at each, the receiving probe is moved longitudinally. The position of the receiving probe, which is transduced, is applied to the x axis of an x-y recorder, and the interferometer output or the logarithm of the received power is applied to the y axis.

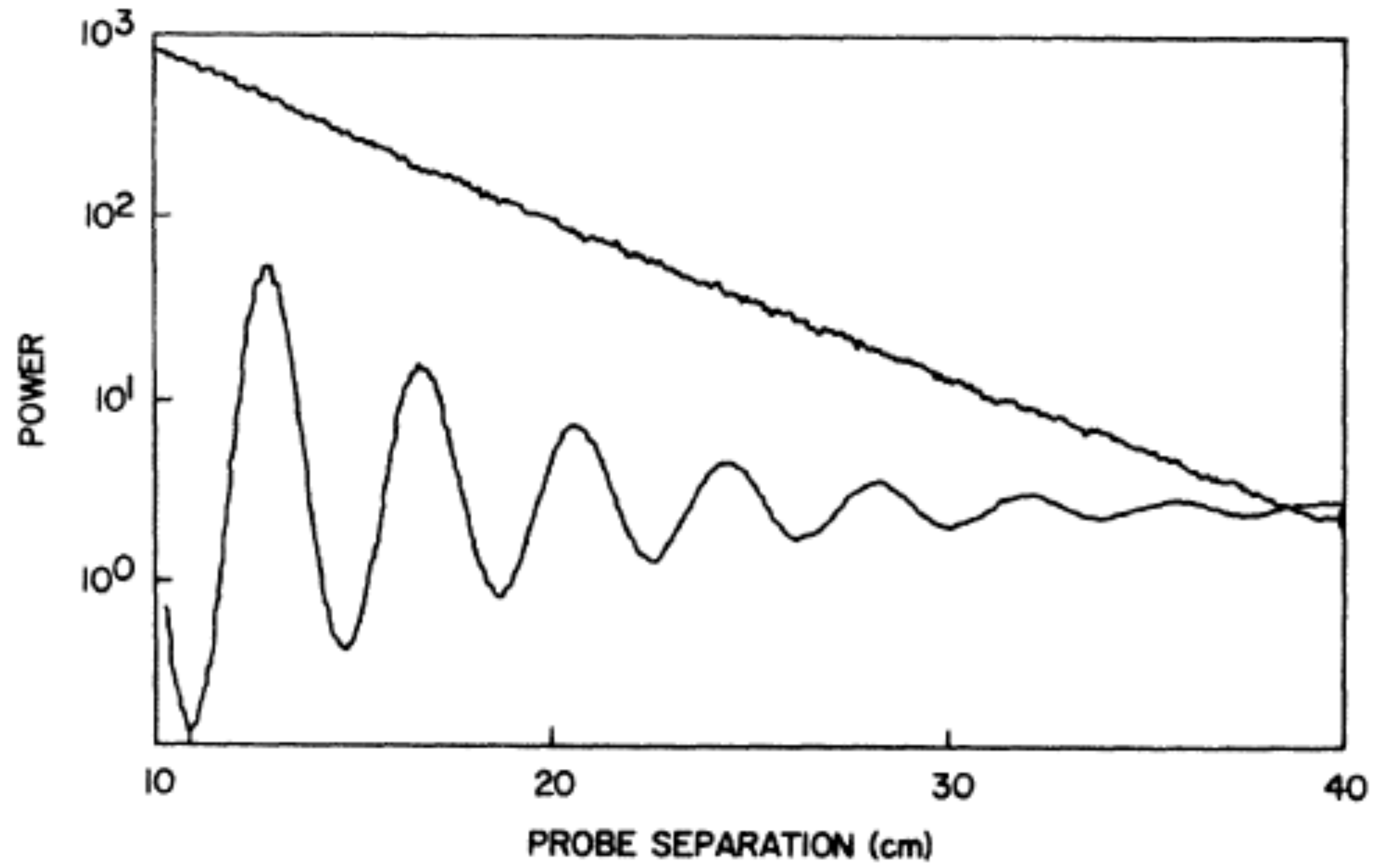


FIG. 1. Raw data. Upper curve is the logarithm of received power. Lower curve is interferometer output. Abscissa is probe separation.

Landau Damping: The Measurement

Important key observation…

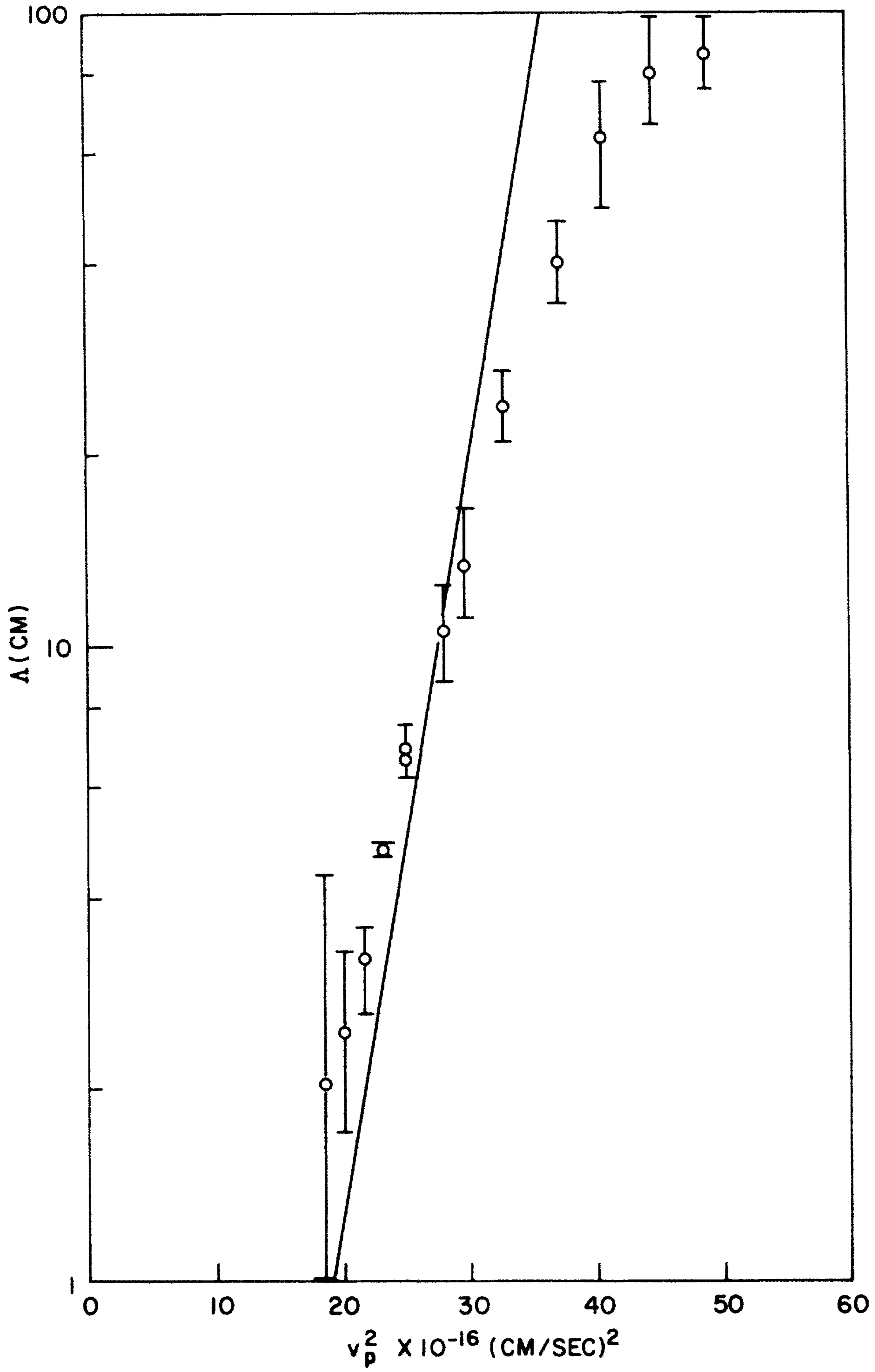


FIG. 3. Logarithm of damping length vs phase velocity squared. The solid curve is theory of Landau for a Maxwellian distribution with a temperature of 10.5 eV.

LANDAU WAVES: AN EXPERIMENTAL FACT*

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(Received 4 April 1966)

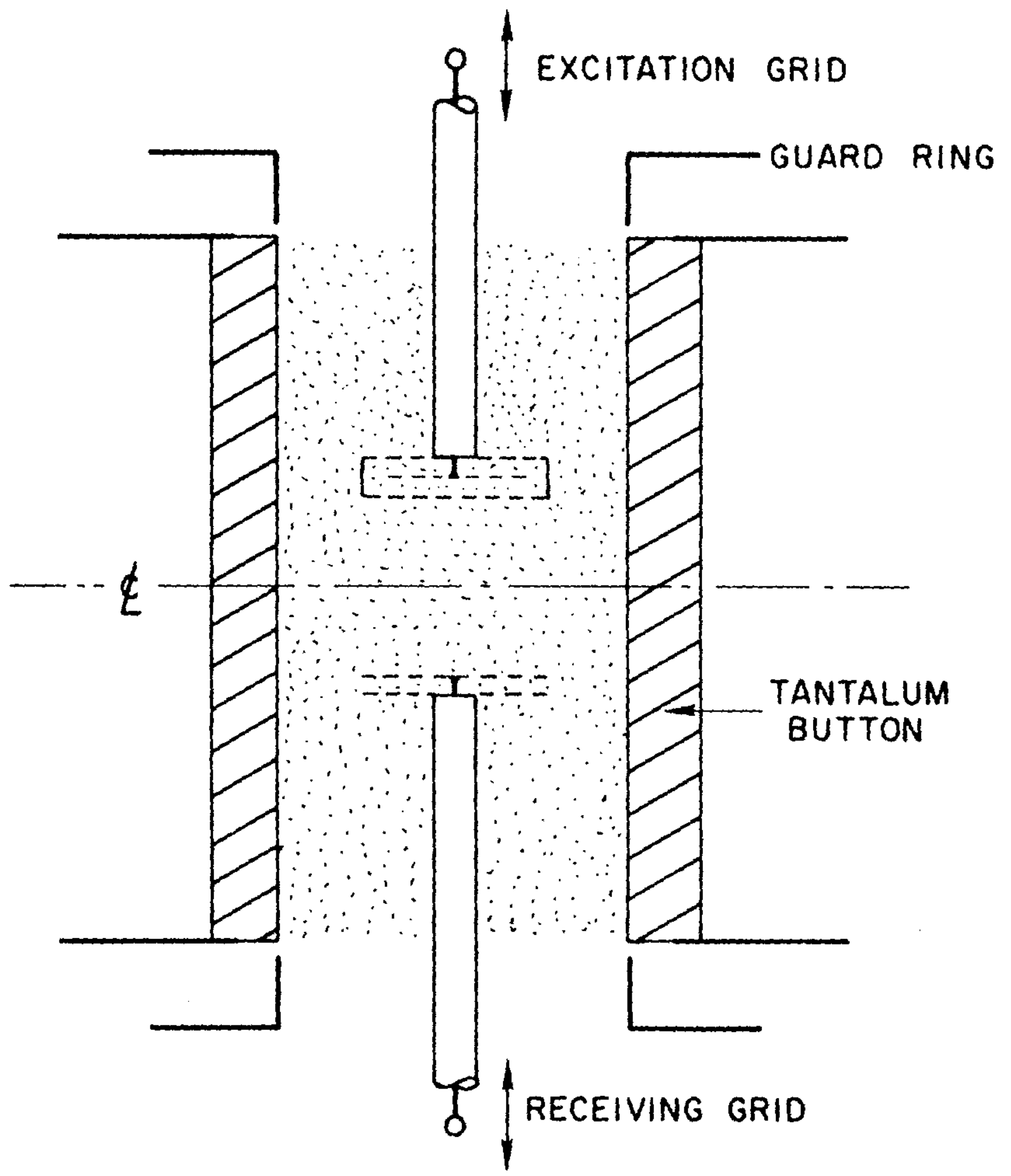

FIG. 1. Diagram of the sodium plasma tube showing the probe arrangement.

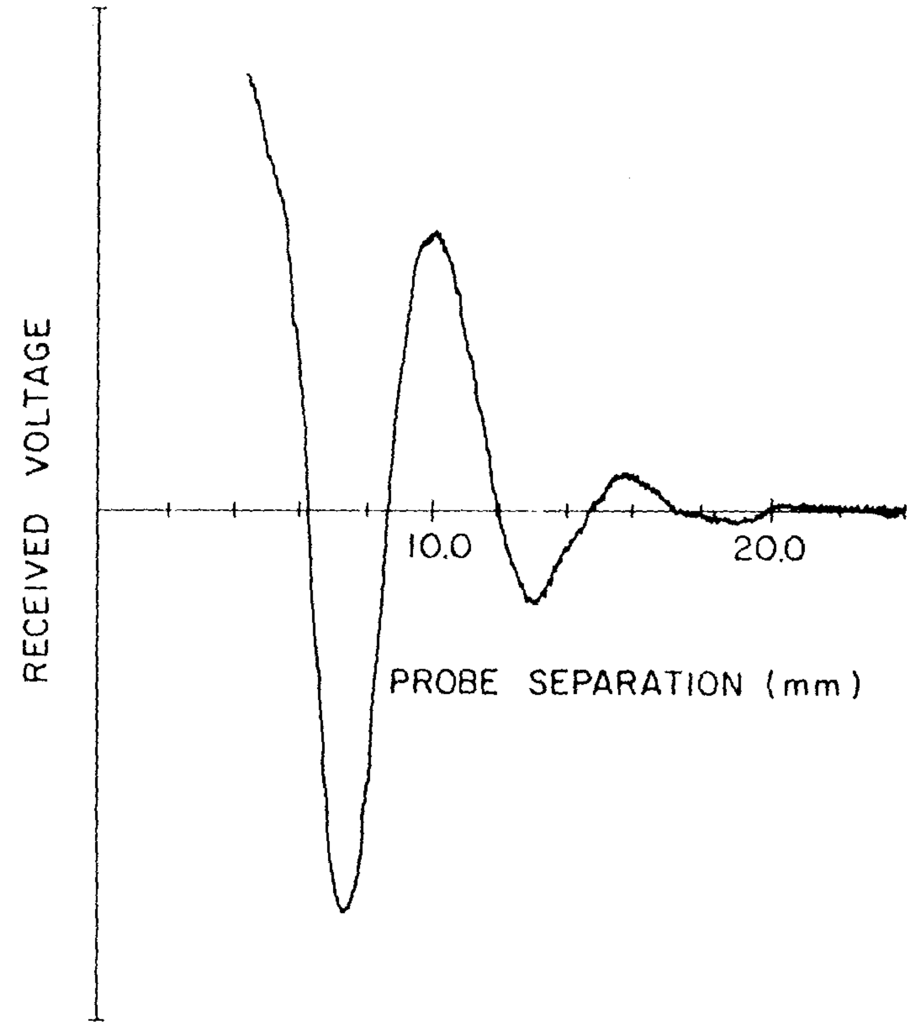

FIG. 2. Interferometer output at 95 Mc/sec as a function of grid separation.

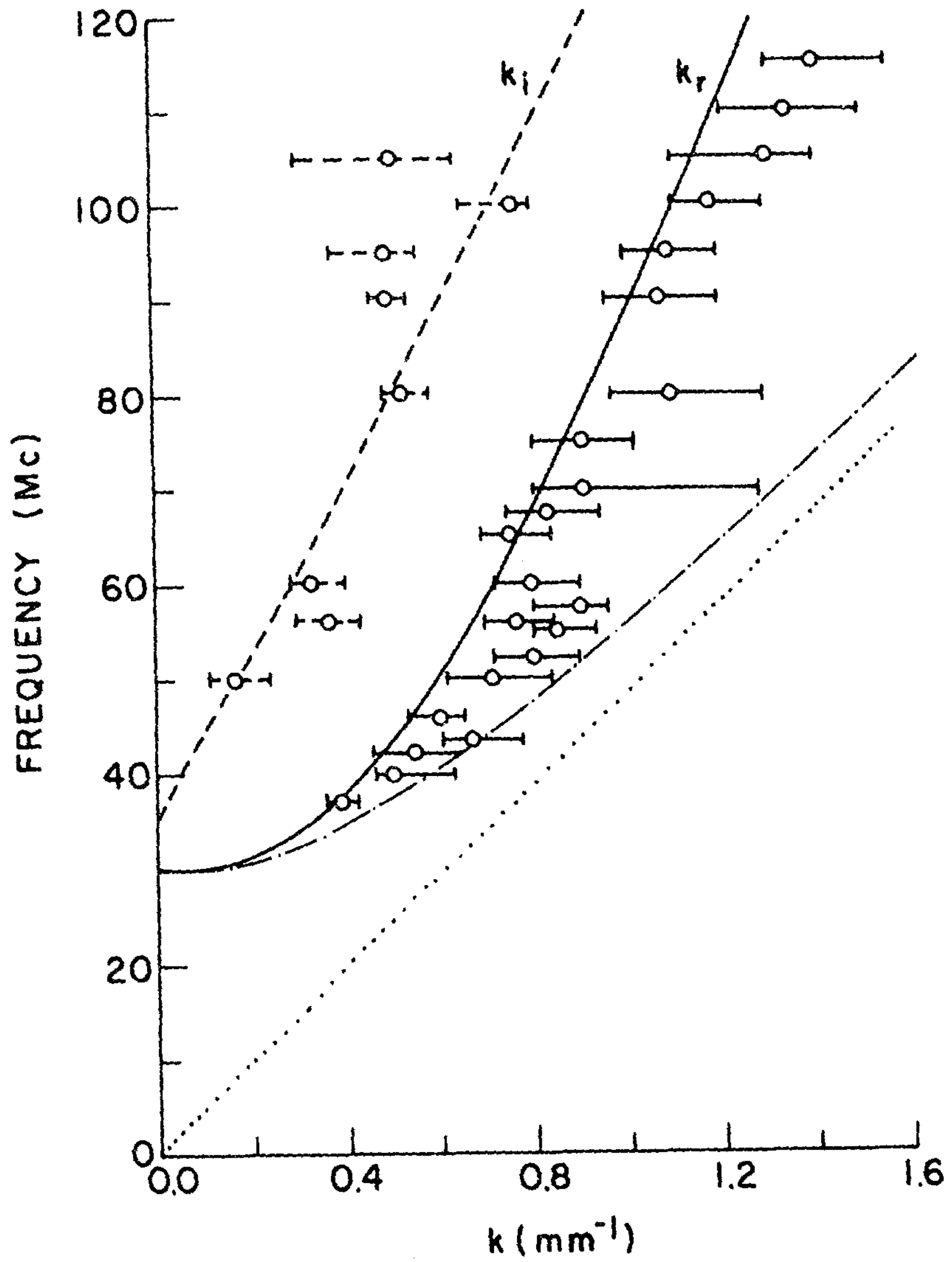

FIG. 3. Dispersion diagram: Circles with bars through them, experimental measurements; k_r and k_i Landau Eq. (1); dotted curves, thermal speed $\omega/k = \sqrt{3}\,\kappa T/m$; and dash-dotted curves, Bohm and Gross, Eq. (2).

Electrostatic Dispersion Relation for a Multi-Component Plasma

$$Đ(k,p) = 1 - \sum_s \frac{\omega_{ps}^2}{k^2} \int_C \frac{\partial F_{s0}/\partial v_z}{v_z - \mathrm{i}p/k}\, \mathrm{d}v_z = 0, \qquad (9.4.1)$$

$$Đ(k,\omega) = 1 - \frac{\omega_p^2}{k^2} \int_{-\infty}^{\infty} \frac{\partial F_0/\partial v_z}{(v_z - \omega/k)}\, \mathrm{d}v_z = 0. \qquad (9.1.13)$$

Electrostatic Dispersion Relation for IAW

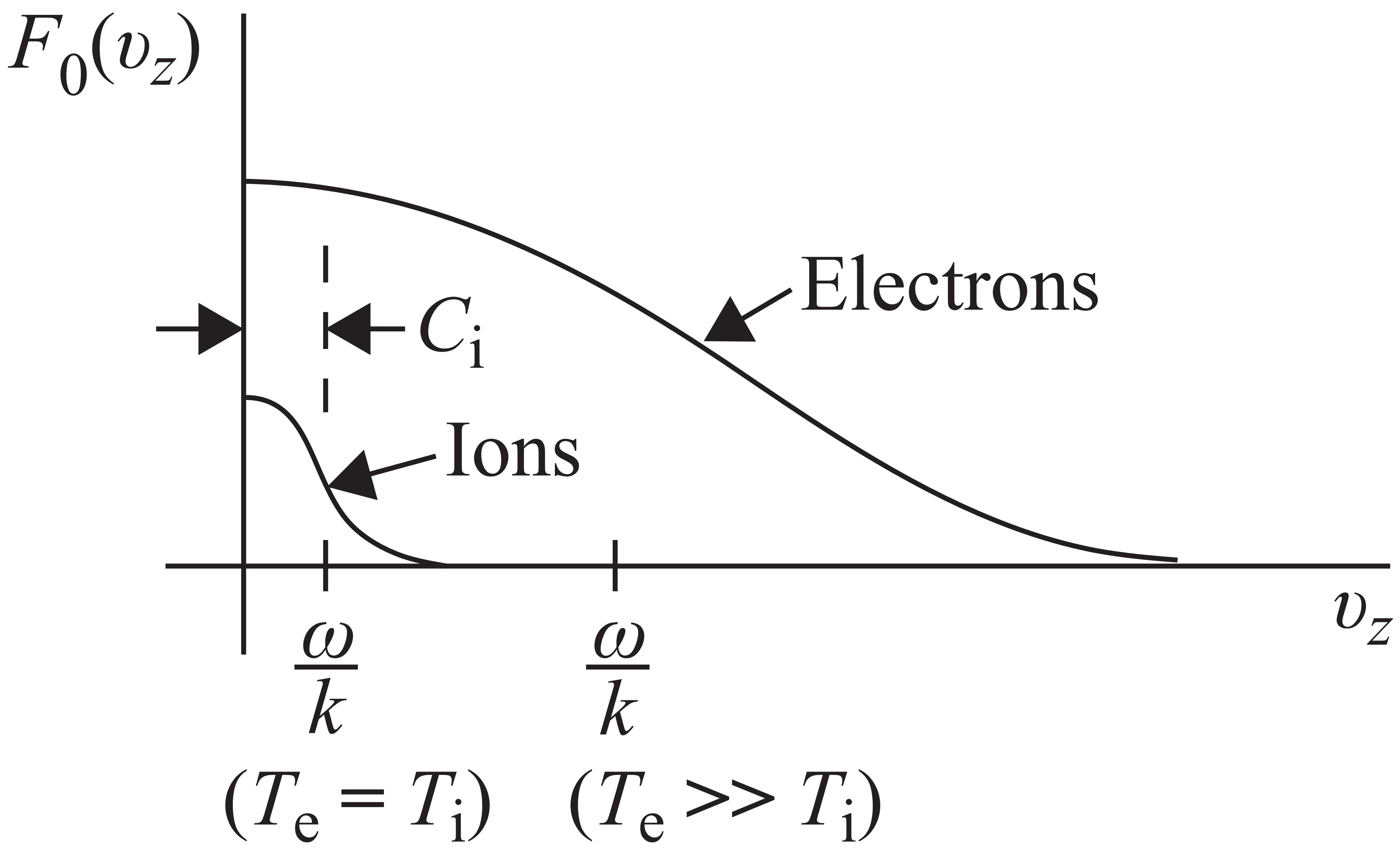

Figure 9.17 When $T_e \gg T_i$, the ion acoustic wave is weakly damped because the phase velocity, ω/k, is much greater than the ion thermal speed, C_i. When $T_e = T_i$, the damping is very large because $\omega/k \simeq C_i$.

Electrostatic Dispersion Relation for IAW

$$Đ(k,p) = 1 + \frac{1}{(k\lambda_{\mathrm{De}})^2}\left\{1 - \frac{1}{2}\left(\frac{T_{\mathrm{e}}}{T_{\mathrm{i}}}\right)\frac{1}{x_{\mathrm{i}}^2}\left(1 - \mathrm{i}\frac{2y_{\mathrm{i}}}{x_{\mathrm{i}}}\right)\right.$$

$$\left. + \mathrm{i}\frac{k}{|k|}\sqrt{\pi}x_{\mathrm{i}}\left[\sqrt{\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}}\sqrt{\frac{T_{\mathrm{i}}}{T_{\mathrm{e}}}} + \left(\frac{T_{\mathrm{e}}}{T_{\mathrm{i}}}\right)\mathrm{e}^{-x_{\mathrm{i}}^2}\right]\right\} = 0, \qquad (9.4.23)$$

$$\frac{\omega}{k} = \pm\sqrt{\frac{\kappa T_{\mathrm{e}}}{m_{\mathrm{i}}}}\frac{1}{(1 + k^2\lambda_{\mathrm{De}}^2)^{1/2}} \qquad (9.4.24)$$

and

$$\frac{\gamma}{\omega} = -\sqrt{\frac{\pi}{8}}\left[\sqrt{\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}} + \left(\frac{T_{\mathrm{e}}}{T_{\mathrm{i}}}\right)^{3/2}\exp\left(-\frac{T_{\mathrm{e}}}{2T_{\mathrm{i}}}\frac{1}{(1 + k^2\lambda_{\mathrm{De}}^2)}\right)\right]\frac{1}{(1 + k^2\lambda_{\mathrm{De}}^2)^{3/2}}. \qquad (9.4.25)$$

Next Week: Ch. 11

11.1.1 The Quasi-linear Diffusion Equation

Next, we develop an equation called the quasi-linear diffusion equation, which describes the time evolution of the average distribution function. This equation is

$$\frac{\partial}{\partial t}\langle f_s\rangle = \frac{e_s}{m_s}\frac{\partial}{\partial \upsilon_z}\left\langle f_{s1}\frac{\partial \Phi_1}{\partial z}\right\rangle. \qquad (11.1.15)$$

Experimental Test of Quasilinear Theory*

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(Received 5 November 1970)

The shape and amplitude of the electron-plasma wave spectrum resulting from a "gentle bump" on the tail of the electron velocity distribution of a plasma is measured and found to be in good agreement with quasilinear theory.

In this Letter we report an experiment designed to test the validity of this theory by measuring the electron-plasma wave spectrum resulting from the injection of an electron beam of sufficiently low density and large velocity spread to satisfy the assumptions of quasilinear theory. In prior beam-plasma experiments the initial velocity spread of the beam electrons was not sufficient to meet the requirements.[3-5]

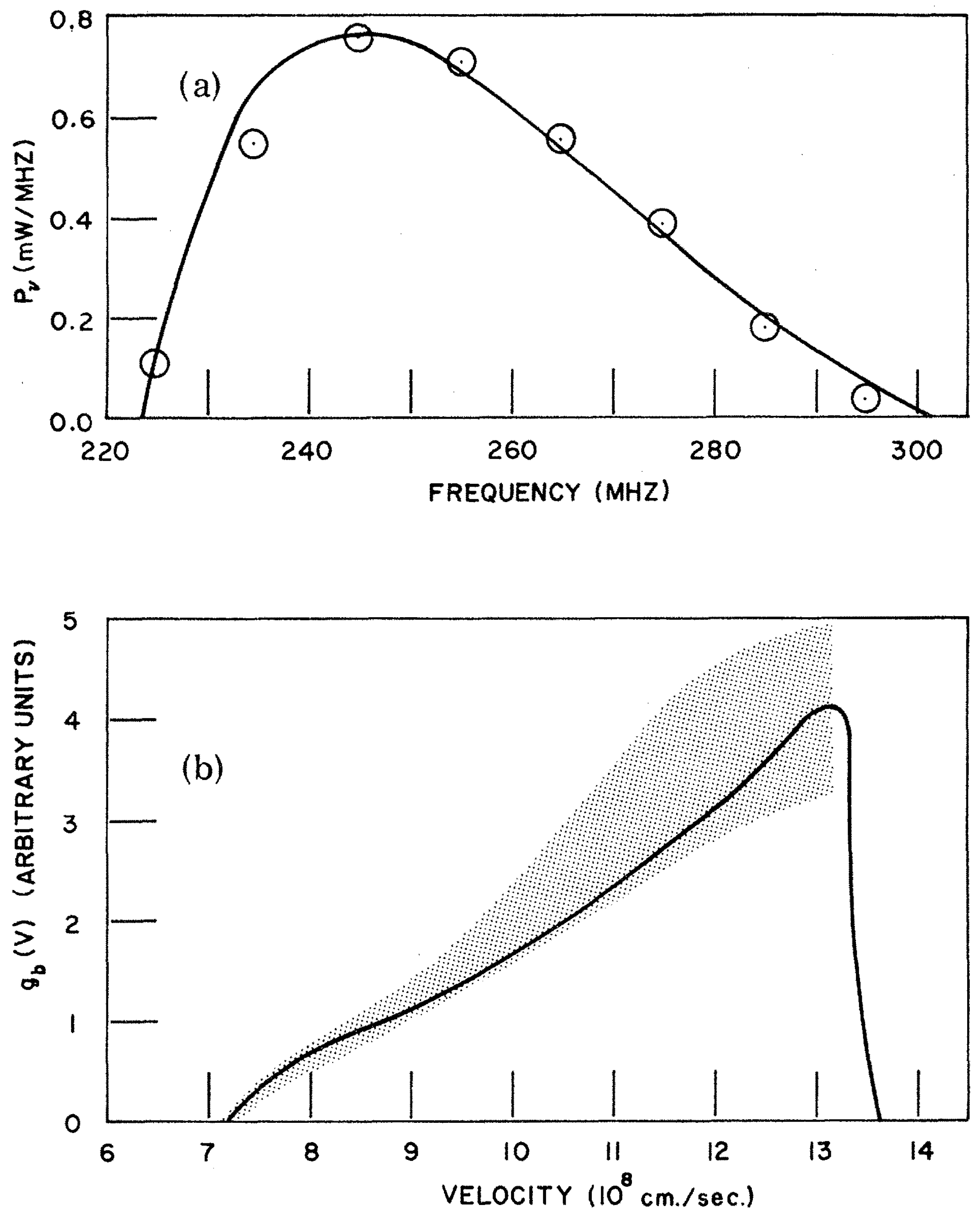

FIG. 3. (a) Shape of wave spectrum. Solid line is theory, circles are experimental values. Beam current is 2 mA. (b) Inferred beam-velocity distribution versus energy analyzer distribution. The solid curve is obtained from an electronically differentiated output of the analyzer.