Plasma 2
Lecture 23-RWM:
Kink Mode Modeling with a Resistive Wall

APPH E6102y
Columbia University
2.5 MW/m³ achieved in TFTR!

Establishes basic “scientific feasibility”, but power out ~ power in.

Fusion self-heating, characteristic of a “burning plasma”, to be explored in ITER.

Control instabilities, disruptions & transients still T.B.D.

Steady state, maintainability, high-availability still T.B.D.

The technologies needed for net power still T.B.D.

20 Years Ago: Significant Fusion Power Produced in the Lab

Fusion power development in the D-T campaigns of JET (full and dotted lines) and TFTR (dashed lines), in different regimes:
(Ia) Hot-Ion Mode in limiter plasma; (Ib) Hot-ion H-Mode; (II) Optimized shear; and (III) Steady-state ELMY-H Modes.
IDENTIFICATION OF EXTERNAL KINK MODES IN JET

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ABSTRACT. The ‘outer mode’ is one of the MHD modes that limits the fusion performance of the hot ion H mode discharges in JET. It has previously been proposed that the outer mode is a non-linearly saturated external kink mode. This was based on the localization of the perturbation close to the edge as observed in soft X ray (SRX), electron temperature and electron density measurements. In addition, MHD stability calculations showed that the plasma edge is close to the ideal external kink stability boundary at the time when the outer mode is observed. The SRX data of the outer mode are compared with predictions based on the mode structure of the ideal \( n = 1 \) external kink mode. Excellent agreement is found, confirming the identification of the outer mode as an external kink mode.
FIG. 1. (a) DD reaction rate $R_{DD}$, stored energy $W$, neutral beam injection (NBI) power, $D_\alpha$ signal and central electron temperature $T_{e0}$, showing the effect of the outer mode between 12.8 and 13.1 s. (b) Expanded time trace, showing the outer mode growth on an outboard midplane Mirnov coil.
At intensities above 80 W/m$^2$, determination. The data are sampled at 4.

Section 4, the boundary, was previously used to identify the outer localization of the outer mode close to the plasma is close to the kink stability limit, combined with the parameters of discharge 38 675 is included.

FIG. 3. Edge stability diagram for discharge 38 675, showing the kink and ballooning stability limits in the edge pressure gradient, edge current density plane. The pressure gradient and the current density are taken at the $\psi = 0.97$ flux surface. The time trace of the edge plasma parameters of discharge 38 675 is included.

FIG. 4. JET soft X ray system, showing the 197 lines of sight for cameras A to J and camera V.

ECE and reflectometer signals, are collected and processed through the JET Central Acquisition and Trig-
Identification of a Low Plasma-Rotation Threshold for Stabilization of the Resistive-Wall Mode

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The plasma rotation necessary for stabilization of resistive-wall modes (RWMs) is investigated by controlling the toroidal plasma rotation with external momentum input by injection of tangential neutral beams. The observed threshold is 0.3% of the Alfvén velocity and much smaller than the previous experimental results obtained with magnetic braking. This low critical rotation has a very weak \( \beta \) dependence as the ideal wall limit is approached. These results indicate that for large plasmas such as in future fusion reactors with low rotation, the requirement of the additional feedback control system for stabilizing RWM is much reduced.

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reversed magnetic shear plasma with plasma current with FSTs is equivalent to 2 additional tangential NBs. JT-60U had so far suffered from the large toroidal field ripple. JT-60U vacuum vessel to reduce the toroidal field.

Monte Carlo simulations considering fully 3D magnetic and cover the first wall to the plasma minor radius of 20 cm.

The growth time of NBs (f). The horizontal dotted lines are calculated critical power of counter NBs (d), co NBs (e), and perpendicular and (c) toroidal rotation at q = 0.0006 velocity at /r/0.0136.

FIG. 3 (color online). Waveforms of E46 710(red) and E46 743(blue). (a) β_N, (b) n = 1 radial magnetic fluctuation and (c) toroidal rotation at q = 2 measured by CXRS. The power of counter NBs (d), co NBs (e), and perpendicular NBs (f). The horizontal dotted lines are calculated critical β (a) and critical rotation (c), respectively.

FIG. 2 (color online). The temporal evolution of plasma current and internal inductance (a), and stored energy and injection power of NBs (b) of the plasma for the critical rotation experiment of RWM.

FIG. 4 (color online). Temporal evolution of rotation profile for E46 710 before disruption. The error in rotation velocity is ±1.5 km/s at q = 2 and is smaller than the open circle.
Overview of JT-60U results towards the establishment of advanced tokamak operation

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Figure 6. Time evolution of magnetic fluctuations for (a) EWM and (b) the RWM precursor. Upper figures in (a) and (b) show integrated mode amplitude of $n = 1$ fluctuation.
Global Kink Eigenmodes

Boundary conditions

What are \((\psi(x), \chi(x))\) inside and outside plasma?

\[\nabla^2 \psi = 0\] (no current) outside plasma

\[\nabla^2 \chi = 0\] (no flow, vorticity, \(\cos kx\))

\[\nabla^2 \psi = 0\] (no currents inside plasma)

\[\nabla^2 \chi = 0\] (no vorticity within plasma)

Features of fields + plasma motion

But no currents or vorticity

\(\psi(x) \sim \left(\frac{a}{x}\right)^m \quad n < m \sim \left(\frac{a}{x}\right)^m \quad n < m\)

\(\sim \left(\frac{a}{x}\right)^m \quad n > m \sim \left(\frac{b}{x}\right)^m \quad n < m\)

\(\frac{b}{a} \sim \frac{a}{b}\)

\(V = \nabla \times \mathbf{A} = \dot{\theta} \cdot \frac{a}{x} \quad \text{inside}\)

\(= -\dot{\theta} \cdot \left(\frac{a}{x}\right)^m \quad \text{outside}\)
Kink Mode

\[ -\omega^2 \hat{\Phi}_a = \frac{B_0}{\mu} \left( m - m_g \right) \hat{X}_a \]

\[ \omega \frac{2X}{2\alpha} \hat{\Phi}_a = \frac{2\mu B_0}{\mu_0 a^2} \hat{\Psi}_a \left[ \frac{-1}{2m/e} \right] \left[ m - m_g \right] \left[ \frac{-1}{2m/e} \right] \left[ 1 \right] \]

\[ \Delta'(a) = -\frac{2m}{\varepsilon} \left( \frac{b/a}{(b/m) - (\varepsilon/e)} \right) \]

\[ \frac{2X}{2\alpha} \hat{\Phi}_a = -\frac{m}{\varepsilon} \hat{\Phi}_a \]

**Global Kink Modes**

\[ \begin{pmatrix} \omega \frac{B_0}{\mu} \left( m - m_g \right) \\ \frac{2\mu B_0}{\mu_0 a^2} \left[ (m - m_g) \frac{\lambda + 1}{2} - 1 \right] \omega \phi \end{pmatrix} \begin{pmatrix} \hat{\Phi}_a \\ \hat{\Psi}_a \end{pmatrix} = 0 \]

\[ \Delta'(a) = -\frac{m}{\varepsilon} (\lambda + 1) \]

**Shapovalov's Formula**

\[ \omega = \frac{1}{c} \sqrt{\frac{m}{m_g} \left( \frac{\lambda^2 + 1}{2} - 1 \right)} \]

For a wall, \( \gamma = b/a \)

\[ \hat{\Phi}_a = \frac{\omega \mu_0 R}{B_0 (m - m_g)} \]

**Eigenvalue**

\[ \omega^2 = 2 \mu B_0 \left( m - m_g \right) \left[ (m - m_g) \frac{\lambda + 1}{2} - 1 \right] \]

\[ b/a \sim 1 \rightarrow \mu \rightarrow \text{unstable} \]

\[ b/a \sim 1 \rightarrow \mu \rightarrow \text{stable} \]
Wesson's Kink Modes

Linearized Equations for Perturbed Stream Function ($\psi$) and Perturbed Poloidal Flux ($\Psi$)

\[ -\rho \omega \nabla_\perp^2 \tilde{\psi} - \frac{m}{\alpha} \tilde{\Psi} + \frac{B_0}{\alpha} (m - m_R) \nabla_\perp^2 \tilde{\psi} = 0 \]  

\[ -\omega \tilde{\Psi} = \frac{B_0}{\alpha} (m - m_R) \tilde{\psi} \]  

(Induction)

Let's take $B$ uniform, with a sharp jump at the plasma edge:

\[ \omega \rho \frac{\partial \tilde{\psi}}{\partial n} \bigg|_{n_0} = \frac{B_0}{\alpha} (m - m_R) \tilde{\psi} \]

With induction equation:

\[ \omega^2 = -\frac{\omega^2_0}{2} (m - m_R) \tilde{\Psi} \tilde{\psi} \frac{\partial \tilde{\psi}}{\partial \Psi} \]

\[ \Delta(t) \]

Perturbed surface current at plasma's edge

\[ \text{Instability requires } \Delta(t) > 0 \]
Wesson's Kink Modes

Since $|\omega| < \omega_A$, the kink mode causes the internal plasma to respond "quickly," so quickly that we can ignore the time it takes to form a distorted, i.e., quasi-equilibrium.

Inside, the plasma is in a "forced" equilibrium

\[ \frac{\partial \phi}{\partial t} + \frac{B_0}{\mu_0 n} (n - n_0) \nabla^2 \phi = 0 \]

Outside, the response is the "vacuum" response with $J_2(\eta)$, we have to solve for $\phi$ using a computer. (This is very easy for the cylindrical tokamak.)

The surface current "pushes/pulls" plasma, and the "distorted" plasma is measured by $\phi(\rho, \theta, z)$. With a "vacuum" and "perfectly conducting" exterior boundary.
Resistive Wall Modes

- **External Kives with a Perfectly Conducting (Ideal) Wall**
  
  \[ \Phi(b) = 0 \quad \text{i.e.} \quad \mathbf{n} \cdot \mathbf{A} = 0 \]

- With a resistive wall, the normal field diffuses through the wall on a "Long" time scale.

- Imagine two concentric conducting shells with \( q_{12} \ll 1 \)

\[ \Delta' \neq 0 \quad \text{when there is surface current} \]

 currents flow on thin surfaces

\[ \Delta'(a) \hat{\Phi}(a) = M_0 \mathbf{K}_2^{ax}(\theta, \psi) \quad \text{surface current at} \; \eta = a \]

\[ \Delta'(b) \hat{\Phi}(b) = M_0 \mathbf{K}_2^{bx}(\theta, \psi) \quad \text{surface current at} \; \eta = b \]
Cylindrical Form of Perturbed Magnetic Flux

\[ \mathbf{B} = \hat{z} \times \nabla \Phi = \hat{\nu} \frac{2\pi}{\nu} - \hat{3} \pi (\frac{2\pi}{\nu}) = \hat{\nu} \frac{2\pi}{\nu} - \frac{1}{\nu} \frac{\sqrt{\nu}}{\nu} \Phi \quad (\frac{\nu}{\mathbf{r}} < 1) \]

\( \hat{\Phi} = \text{constant across current layer} \Rightarrow \hat{\mathbf{B}} \cdot \hat{n} = 0 \)

\[ \Delta' = \frac{1}{\nu} \left( \frac{2\pi}{\nu} - \frac{2\pi}{\nu} \right) \neq 0 \text{ when there exists a perturbed current} \]

**Note:** This changes inside a plasma.

\[ \Phi(h) = \begin{cases} 
\psi_a \left( \frac{h}{2} \right)^2 & \text{if } a < h \\
\psi_b \left( \frac{h}{2} \right)^2 & \text{if } b > h \\
\left( \frac{\psi_a}{\psi_b} \right) \left( \frac{b}{h} \right)^2 & \text{if } a < h < b 
\end{cases} \]

\[ \Delta' = \frac{1}{\nu} \left( \frac{2\pi}{\nu} - \frac{2\pi}{\nu} \right) = -\frac{2\pi}{\nu} \frac{1}{1-c} + \frac{\sqrt{\nu}}{1-c} \frac{\psi_a}{\psi_b} \]

\[ \Delta' = \frac{1}{\nu} \left( \frac{2\pi}{\nu} - \frac{2\pi}{\nu} \right) = -\frac{2\pi}{\nu} \frac{1}{1-c} + \frac{\sqrt{\nu}}{1-c} \frac{\psi_a}{\psi_b} \]
Cylindrical Form of Perturbed Magnetic Flux

\[ \Delta'(a) = \frac{1}{a^2} \left( \frac{2 \phi}{2n} a^+ - \frac{2 \phi}{2n} a^- \right) \]

\[ \frac{1}{\phi} \frac{2 \phi}{2n} a^+ = \begin{cases} 
- \frac{m}{a} & \text{with no wall} \\
- \frac{m}{a} - \frac{2m}{a} \frac{c}{1-c} + \frac{\phi_0}{\phi} \frac{2m}{a} \frac{\sqrt{c}}{1-c} 
\end{cases} \]

This is always stabilizing since \( \Delta'(a) > 0 \) for instability.
Resistive Relaxation of Wall Eddy Currents

\[ \frac{2\overline{b}}{2t} = -\nabla \times \overline{E} \]
\[ \overline{D} = \overline{\varepsilon} \times \overline{\psi} \]
\[ \overline{E} = \eta \overline{J} \]

**Resistivity of Wall**

\[ \nabla \times (\varepsilon \frac{2\overline{b}}{2t}) = \nabla \times \eta \overline{J} \]
\[ \nabla \times \psi = -\sigma \nabla \psi \]

**Surface with Current**

\[ \frac{2\overline{b}}{2t} = \eta \frac{\delta W}{\delta t} = \frac{\partial}{\partial t} \frac{K_2}{\delta W} \]
\[ = \frac{\mu_0}{\delta W} \Delta \left( b \right) \frac{\Phi}{(b)} \]
\[ = \frac{\mu_0}{\delta W} \left[ -\frac{2\overline{b}}{b} \frac{1-c}{1-c} \frac{\Phi}{b} + \frac{\overline{Vc}}{1-c} \frac{2\overline{b}}{b} \frac{\Phi}{b} \right] \]

**On**

\[ \frac{\partial \overline{\psi}}{\partial t} + \frac{\delta W}{1-c} \frac{\overline{\psi}}{b} = \frac{\gamma_0 \overline{Vc}}{1-c} \overline{\psi} \]

\[ \delta W \equiv \frac{\mu_0 2\overline{b}}{b} \frac{\delta W}{\overline{\psi}} \]

**Wall Time**

\[ \overline{\psi} = \frac{\mu_0 2\overline{b}}{b} \frac{\delta W}{\overline{\psi}} \]

**Examples:**

1. If \( \overline{\psi}_a = 0 \), then \( \overline{\psi}_b = \frac{\gamma_0 \overline{Vc}}{1-c} \overline{\psi}_a \)
2. If \( \overline{\psi}_b = \gamma_0 \overline{\psi}_0 \), then \( \overline{\psi}_b = \gamma_0 \overline{\psi}_0 \)
External Kink and Resistive Wall Mode

Shaframian Equilibrium

$\frac{d^2 \tilde{\Phi}_e}{dt^2} = \frac{\beta_0}{2} \frac{e}{m} (m-n_0) \tilde{\Phi}_e$

$\frac{d \tilde{\Phi}_e}{dt} = \epsilon \frac{m_0 \alpha}{\epsilon^2} \left[ (m-n_0) \frac{\tilde{A}'(\alpha)}{2\epsilon \alpha} + 1 \right] \tilde{\Phi}_e$

$\frac{d^2 \tilde{\Phi}_a}{dt^2} = -2 \Omega_d^2 (m-n_0) \left[ (m-n_0) \frac{\tilde{A}'(\alpha)}{2\epsilon \alpha} + 1 \right] \tilde{\Phi}_a$

$\tilde{A}'(\alpha) \tilde{\Phi}_a = \frac{1}{1-c} \tilde{\Phi}_e + \frac{\epsilon c}{1-c} \tilde{\Phi}_b$

Plasma dynamics

$\frac{d^2 \tilde{\Phi}_a}{dt^2} = -2 \Omega_d^2 (m-n_0)^2 \frac{c}{1-c} \left[ \frac{\tilde{\Phi}_b}{\epsilon^2} + \left( \frac{1-c}{c(m-n_0)} - \frac{1}{c} \right) \tilde{\Phi}_e \right]$

With coupling
**External Kink and Resistive Wall Mode**

\[ \omega^2 = -\frac{W_x}{2} (m - m_W^2)^2 \Delta'(a) \frac{X_n}{2a} \]

\textit{Equilibrium}

\[ \text{Stability requires } \Delta'(a) > 0 \]

**Recall**

\[ \Delta'(a) = \frac{1}{\Phi_s} \left( \frac{2\psi}{\delta n_c} \right) \]

\[ \text{Depends upon plasma response} \]

\[ \frac{\psi}{\delta n_c} \text{ depends on vacuum plasma response} \]

\[ \frac{1}{\psi} \frac{2\psi}{\delta n_c} \sim -\frac{m}{a} - \frac{2m}{a} \frac{C}{1 - C} + \frac{q_0}{a} \frac{2m}{a} \frac{C}{1 - C} \]

\[ \text{Unstable without wall} \]

\[ \Delta' > 0 \]

\[ \Delta' < 0 \text{ with wall} \]
General RWM Dynamics w No Flow/No Dissipation

\[ \frac{d^2 \psi_a}{dt^2} + \gamma_{mho}^2 \left(1 - \frac{1}{c^2}\right) \psi_a = \frac{\gamma_{mho}^2}{c^2} \psi_b \]

\[ \frac{d \psi_b}{dt} + \frac{\gamma_{w}}{1 - c} \psi_b = \frac{\gamma_{w}}{1 - c} \psi_a \]

**Three Roots!**

Root 1 = STABLE ALFVEN WAVE

Root 2 = EXTERNAL KINK MODE - COUPLED TO WALL

Root 3 = WALL MODE

2 Eigen Vectors
Usual "RWM Limit"...

\[
\frac{d}{dt} \sim \gamma_w < \gamma_{\text{max}}
\]

Then: "Plasma inertia" is important

\[
(1-\delta) \hat{\Psi}_a = \frac{\hat{\psi}_b}{\nu_c}
\]

\[
\frac{d\hat{\psi}_b}{dt} + \frac{\gamma_w}{1-c} \hat{\psi}_b = \gamma_w \frac{\nu_c}{1-c} \hat{\psi}_a = \gamma_w \frac{\hat{\psi}_b}{(1-c)(1-\delta)}
\]

or

\[
\frac{d\hat{\Psi}}{dt} + \frac{\gamma_w}{1-c} \left(1 - \frac{1}{1-\delta} \right) \hat{\Psi} = 0
\]

Flux rate

\[
\text{Flux rate} = \gamma_w \frac{\delta}{(1-\delta)(1-c)} \quad \text{(valid for small $$\delta$$)}
\]
Simplified forms of the dynamical equations for the perturbed flux at the plasma edge, $\psi_a$, and the conducting wall, $\psi_w$, were summarized by Fitzpatrick [3] who also included the perturbed flux at the wall driven by control coils, $\psi_c$. The perturbed flux is related to perturbed radial magnetic field as

$$b_r(r, \theta, \phi, t) = \Re \left\{ \frac{m}{r} \psi(r)e^{i(m\theta-\phi)+\gamma t} \right\}.$$  \hspace{1cm} (1)

The three model equations describe an external kink mode coupled to a surrounding wall, the resistive dissipation of wall eddy currents, and toroidal torque balance. These equations are:

$$\frac{d^2 \psi_a}{dt^2} + (\nu^* - 2i\Omega_\phi) \frac{d\psi_a}{dt} + \left[ \gamma_{MHD}^2 (1 - \tilde{s}) - \Omega_\phi^2 - i\nu^* \Omega_\phi \right] \psi_a = \gamma_{MHD}^2 \frac{\psi_w}{\sqrt{c}}$$  \hspace{1cm} (2)

$$\frac{d\psi_w}{dt} + \frac{\gamma_w}{1 - c} (\psi_w - \sqrt{c}\psi_a) = \gamma_w \psi_c \hspace{1cm} (3)$$

$$\frac{d\Omega_\phi}{dt} + \nu^* (\Omega_\phi - \Omega_\phi^{(0)}) = -\frac{1}{2} \left[ \frac{1}{1 - c} \frac{L_p}{\delta M_a} \right] \frac{\psi_a^2}{c} \hspace{1cm} (4)$$

where $c$ is a coupling coefficient between the wall and the plasma, $\tilde{s} \equiv s/s_{crit} = s(1 - c)/c$ is the normalized Boozer stability parameter, $\Omega_\phi$ is the plasma rotation at the edge, $\nu^*$ is the (anomalous) viscosity, $\delta M_a$ is the effective mass of the inertial layer, and $L_p$ is the effective inductance of the perturbed plasma skin current. For a complete cylindrical wall, $c \approx (a/b)^{2m}$. VALEN [9] can be used to calculate $c$ and $\gamma_w$ for an incomplete wall, since $c$ is related to the ideal wall stability limit through the definition $s_{crit} = c/(1 - c)$ and since the resistive wall mode growth rate is proportional to $\gamma_w$ when $0 < \tilde{s} \ll 1$. 

Fitzpatrick–Aydemir Model 
(Nuc Fusion 36 (1996) pp. 11)

Further Reading…