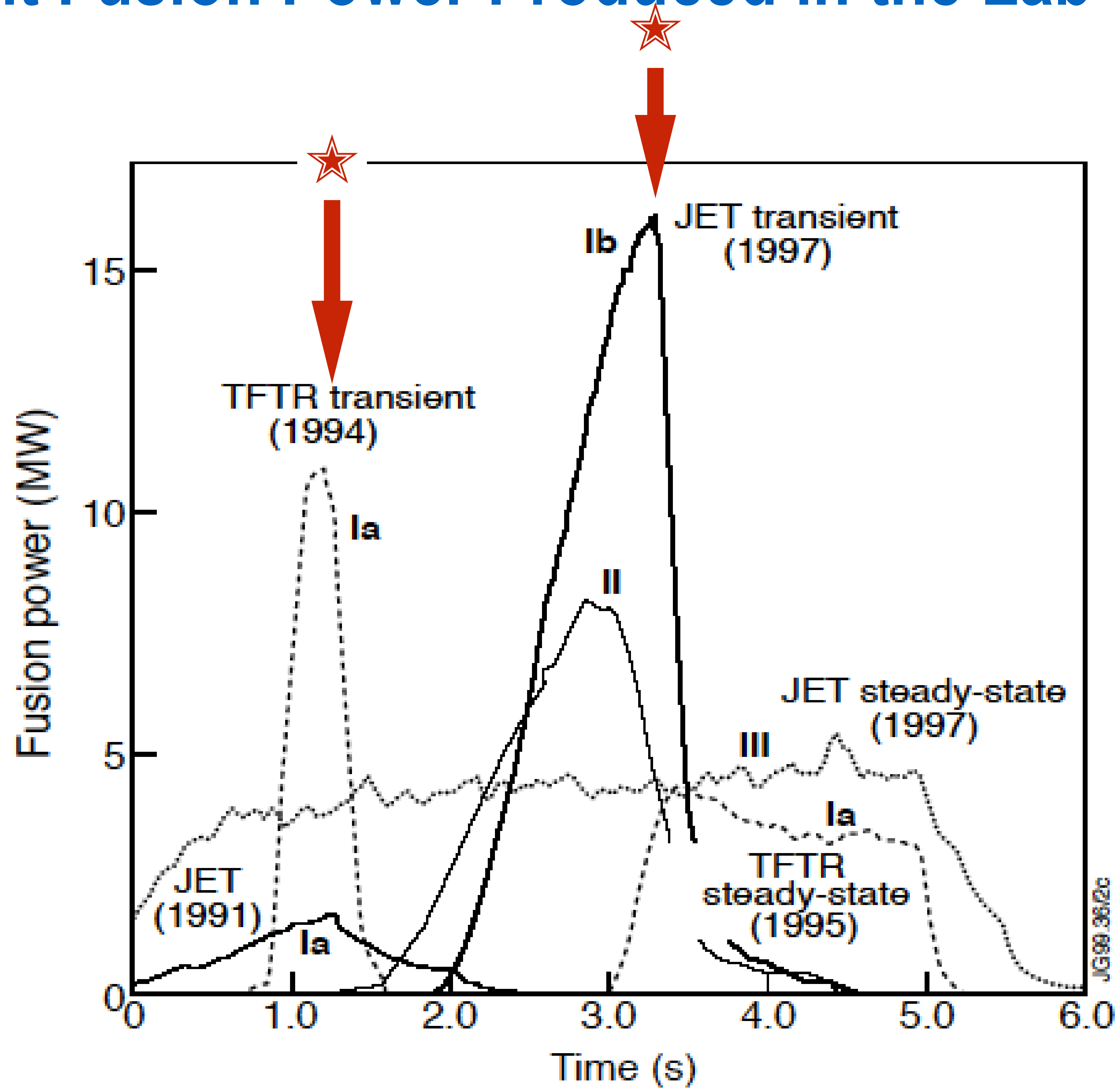
Plasma 2
Lecture 23-RWM:
Kink Mode Modeling with a Resistive Wall

APPH E6102y
Columbia University

20 Years Ago: Significant Fusion Power Produced in the Lab

- ✓ 2.5 MW/m^3 achieved in TFTR!
- ✓ Establishes basic "scientific feasibility", but power out ~ power in.
- ➡ Fusion self-heating, characteristic of a "burning plasma", to be explored in ITER.
- ★ Control instabilities, disruptions & transients still T.B.D.
- ◉ Steady state, maintainability, high-availability still T.B.D.
- ◉ The technologies needed for net power still T.B.D.



Fusion power development in the D-T campaigns of JET (full and dotted lines) and TFTR (dashed lines), in different regimes:

(Ia) Hot-Ion Mode in limiter plasma; (Ib) Hot-ion H-Mode;

(II) Optimized shear; and (III) Steady-state ELMY-H Modes.

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IDENTIFICATION OF EXTERNAL KINK MODES IN JET

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ABSTRACT. The 'outer mode' is one of the MHD modes that limits the fusion performance of the hot ion H mode discharges in JET. It has previously been proposed that the outer mode is a non-linearly saturated external kink mode. This was based on the localization of the perturbation close to the edge as observed in soft X ray (SXR), electron temperature and electron density measurements. In addition, MHD stability calculations showed that the plasma edge is close to the ideal external kink stability boundary at the time when the outer mode is observed. The SXR data of the outer mode are compared with predictions based on the mode structure of the ideal $n = 1$ external kink mode. Excellent agreement is found, confirming the identification of the outer mode as an external kink mode.

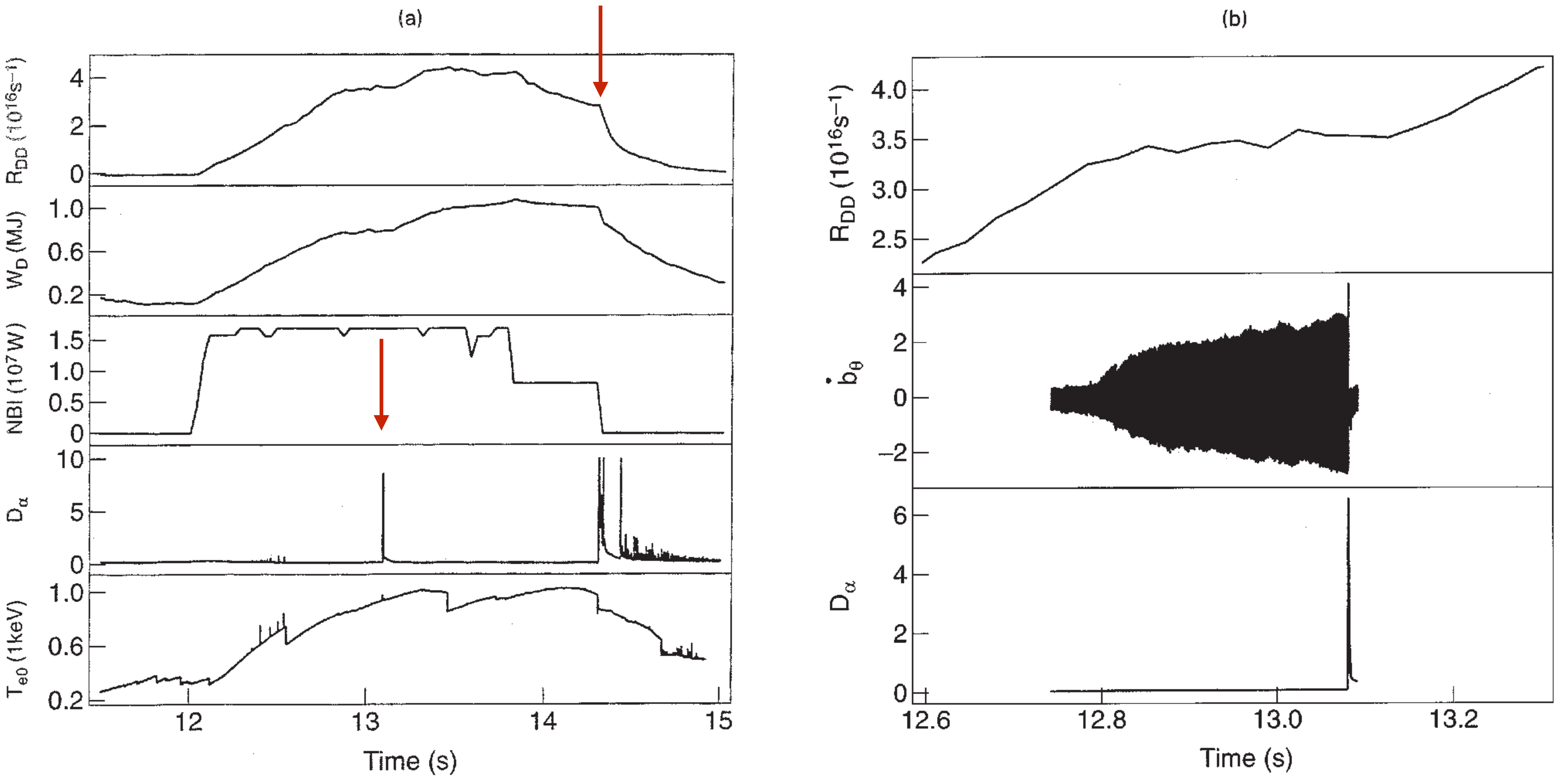


FIG. 1. (a) DD reaction rate R_{DD}, stored energy W, neutral beam injection (NBI) power, D_α signal and central electron temperature T_{e0}, showing the effect of the outer mode between 12.8 and 13.1 s. (b) Expanded time trace, showing the outer mode growth on an outboard midplane Mirnov coil.

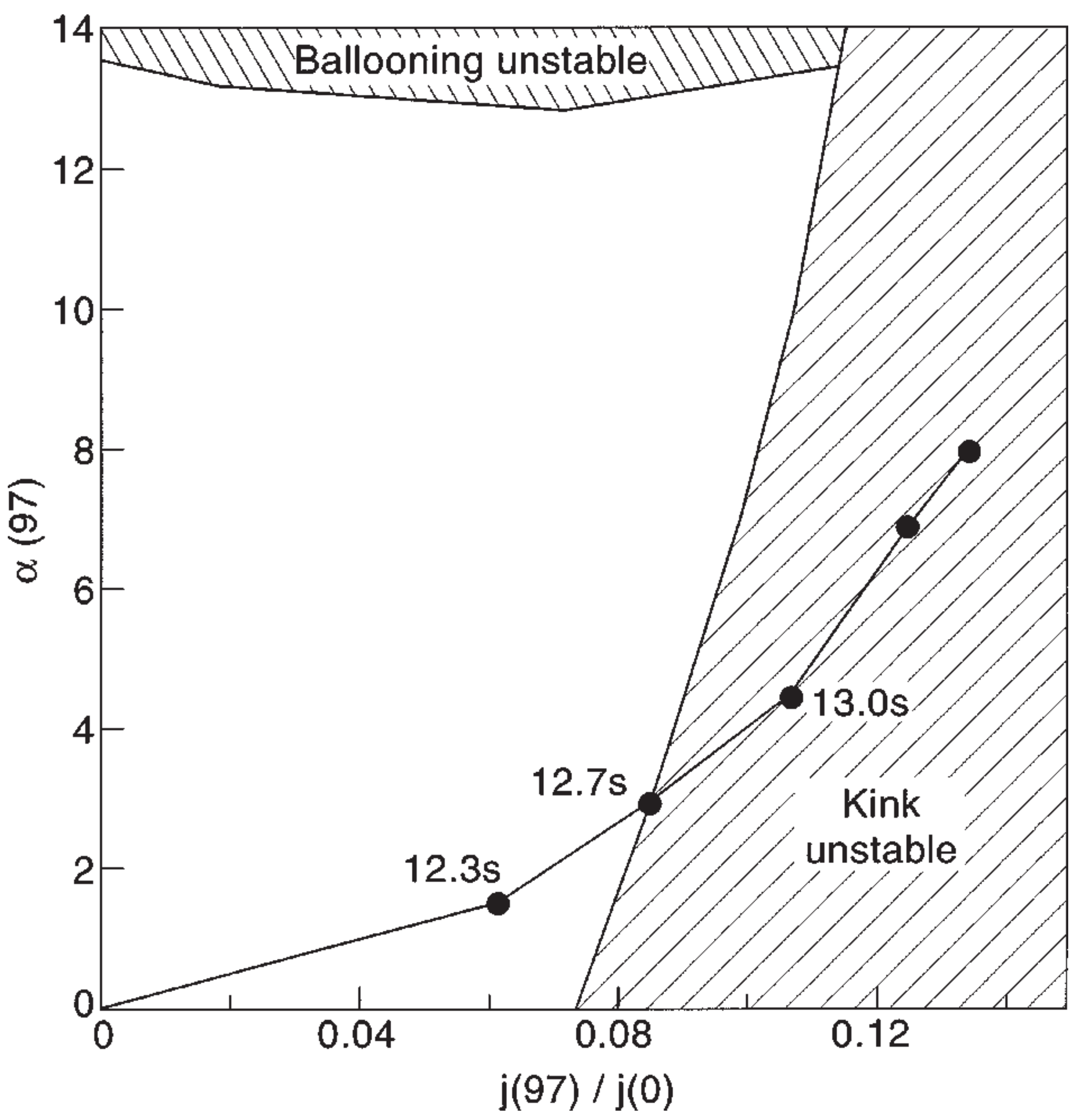


FIG. 3. Edge stability diagram for discharge 38 675, showing the kink and ballooning stability limits in the edge pressure gradient, edge current density plane. The pressure gradient and the current density are taken at the $\psi = 0.97$ flux surface. The time trace of the edge plasma parameters of discharge 38 675 is included.

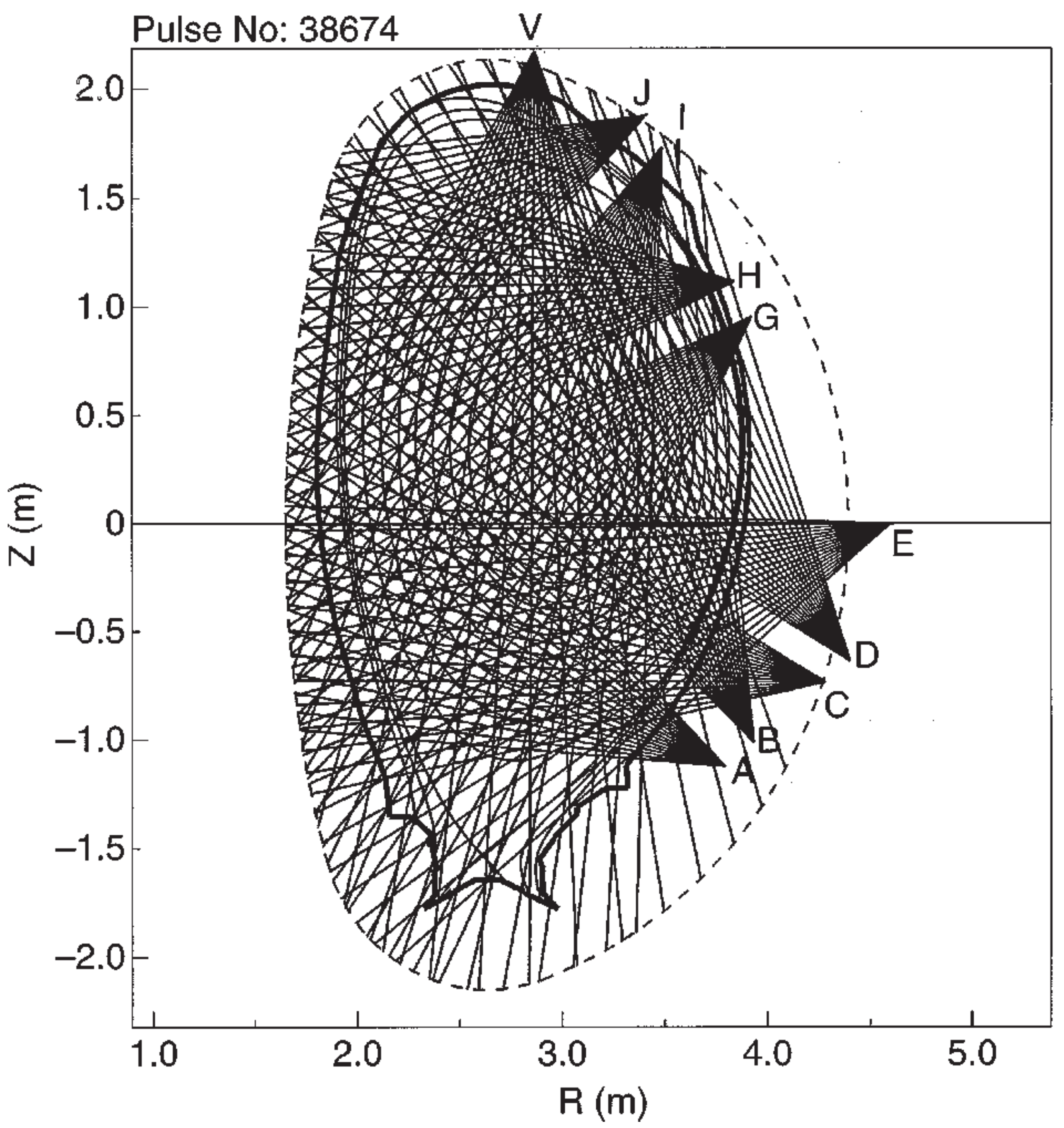
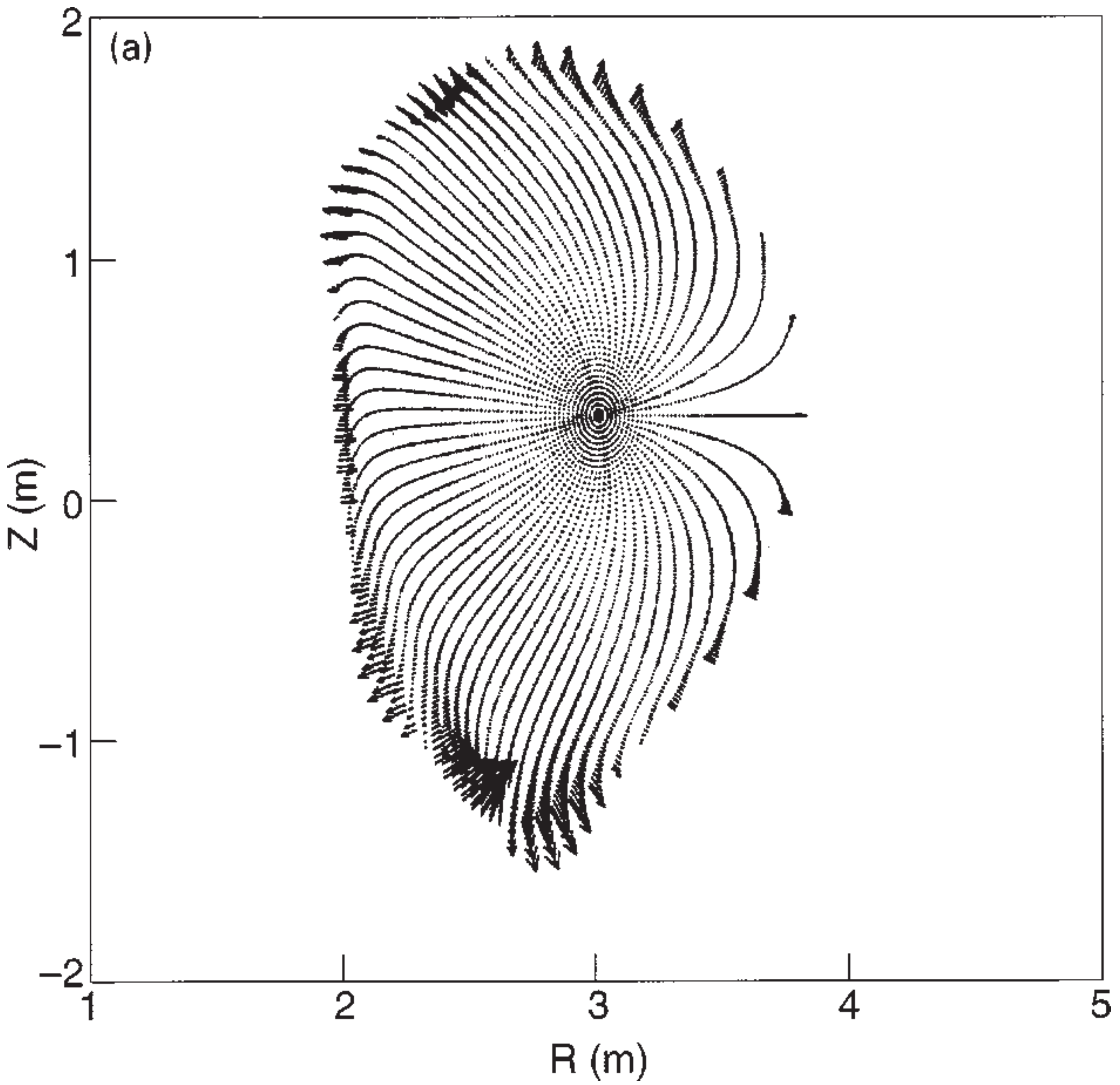


FIG. 4. JET soft X ray system, showing the 197 lines of sight for cameras A to J and camera V.

ECE and reflectometer signals, are collected and processed through the JET Central Acquisition and Trig-



Identification of a Low Plasma-Rotation Threshold for Stabilization of the Resistive-Wall Mode

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The plasma rotation necessary for stabilization of resistive-wall modes (RWMs) is investigated by controlling the toroidal plasma rotation with external momentum input by injection of tangential neutral beams. The observed threshold is 0.3% of the Alfvén velocity and much smaller than the previous experimental results obtained with magnetic braking. This low critical rotation has a very weak β dependence as the ideal wall limit is approached. These results indicate that for large plasmas such as in future fusion reactors with low rotation, the requirement of the additional feedback control system for stabilizing RWM is much reduced.

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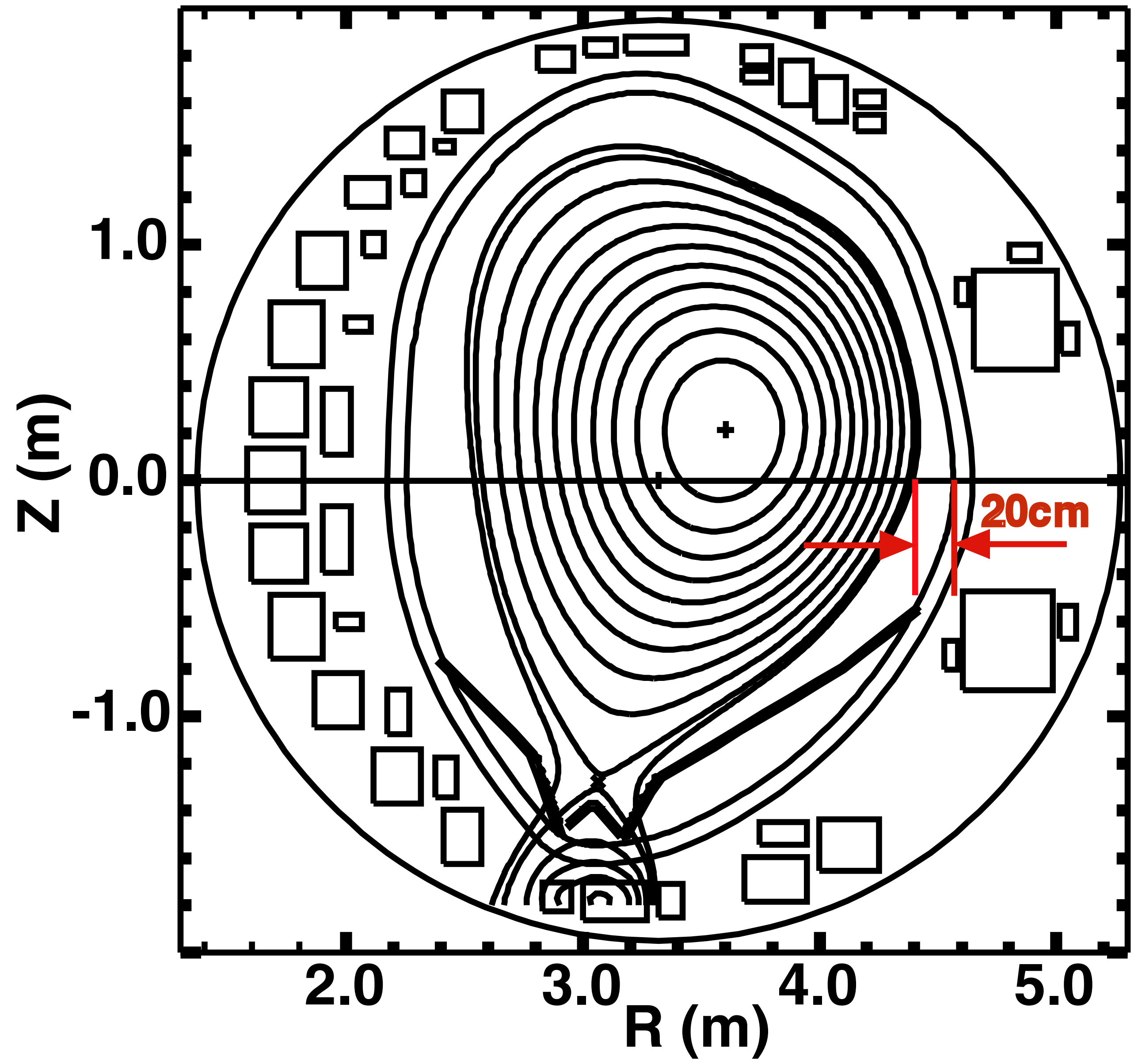

FIG. 1 (color online). Poloidal cross section of the plasma for the RWM experiment.

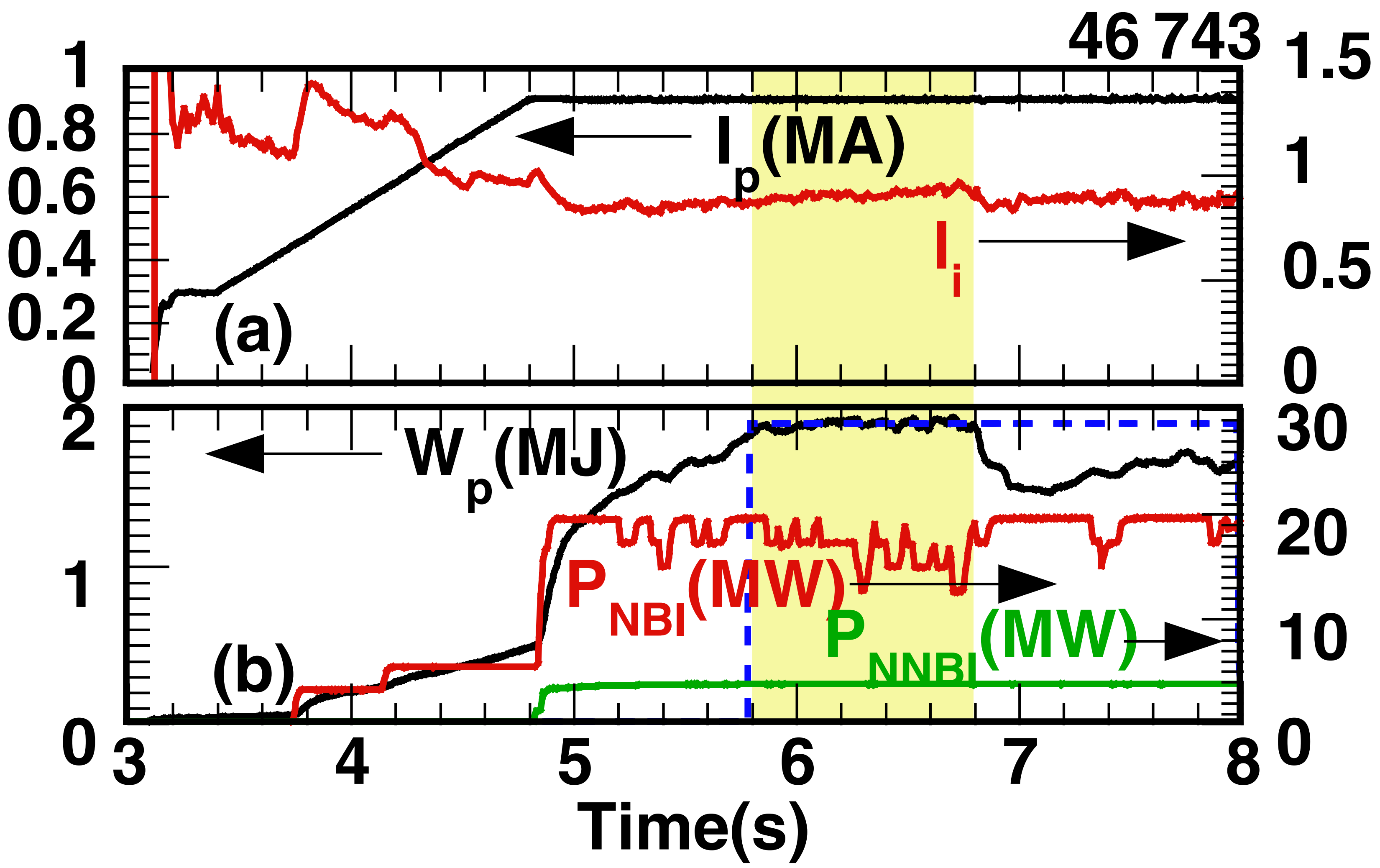

FIG. 2 (color online). The temporal evolution of plasma current and internal inductance (a), and stored energy and injection power of NBs (b) of the plasma for the critical rotation experiment of RWM.

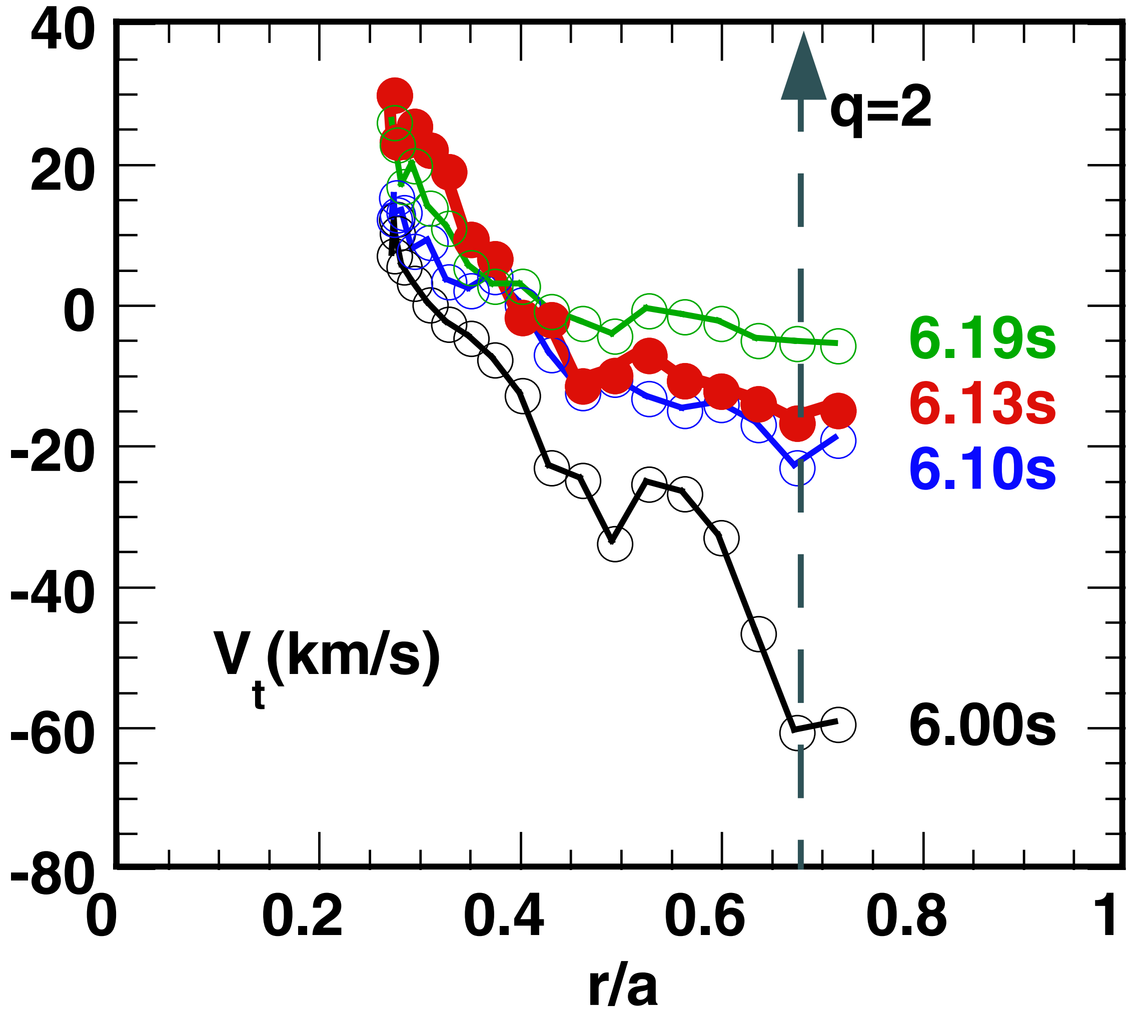

FIG. 4 (color online). Temporal evolution of rotation profile for E46 710 before disruption. The error in rotation velocity is ± 1.5 km/s at $q = 2$ and is smaller than the open circle.

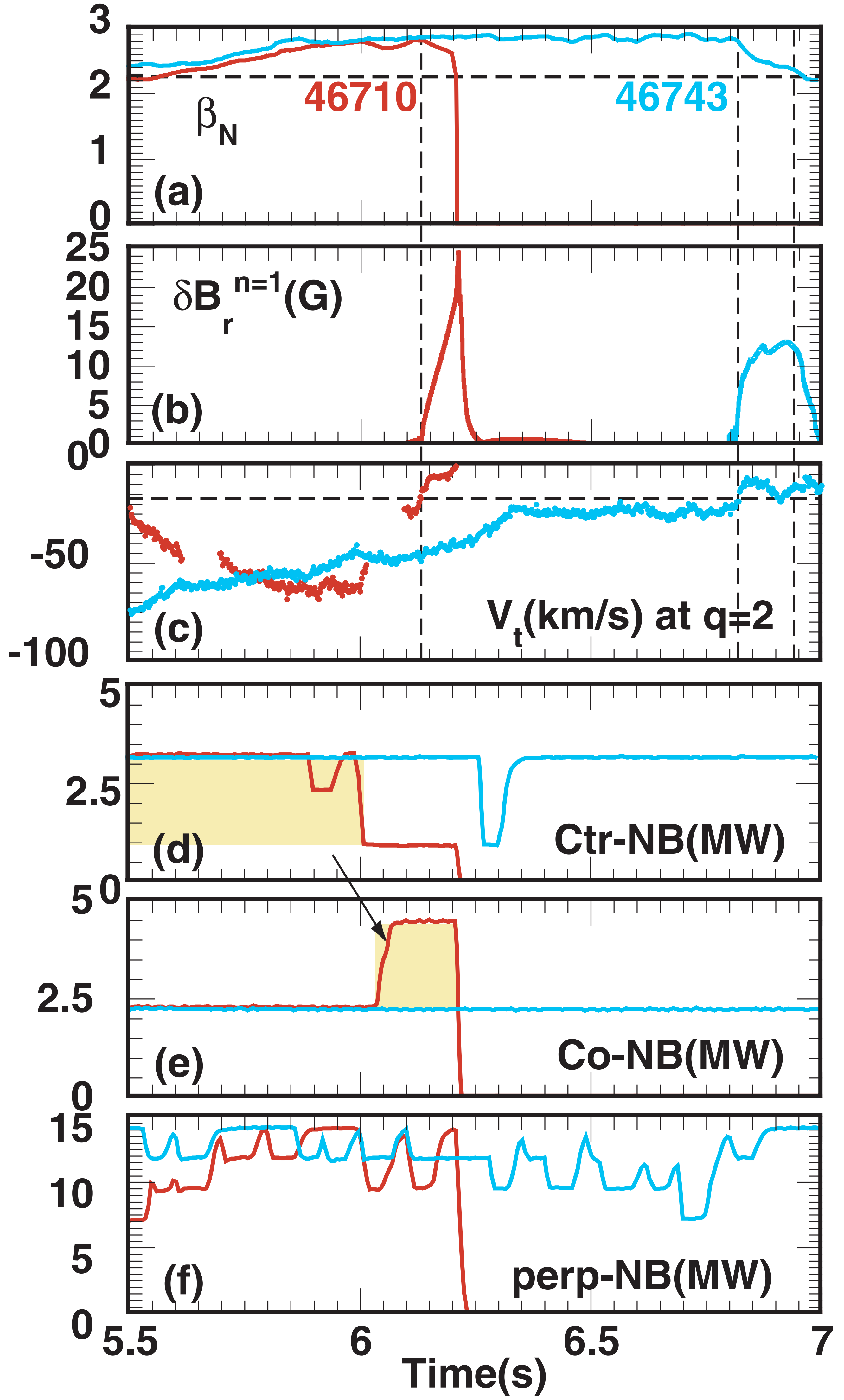

FIG. 3 (color online). Waveforms of E46 710(red) and E46 743(blue). (a) β_N, (b) $n = 1$ radial magnetic fluctuation and (c) toroidal rotation at $q = 2$ measured by CXRS. The power of counter NBs (d), co NBs (e), and perpendicular NBs (f). The horizontal dotted lines are calculated critical β (a) and critical rotation (c), respectively.

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Overview of JT-60U results towards the establishment of advanced tokamak operation

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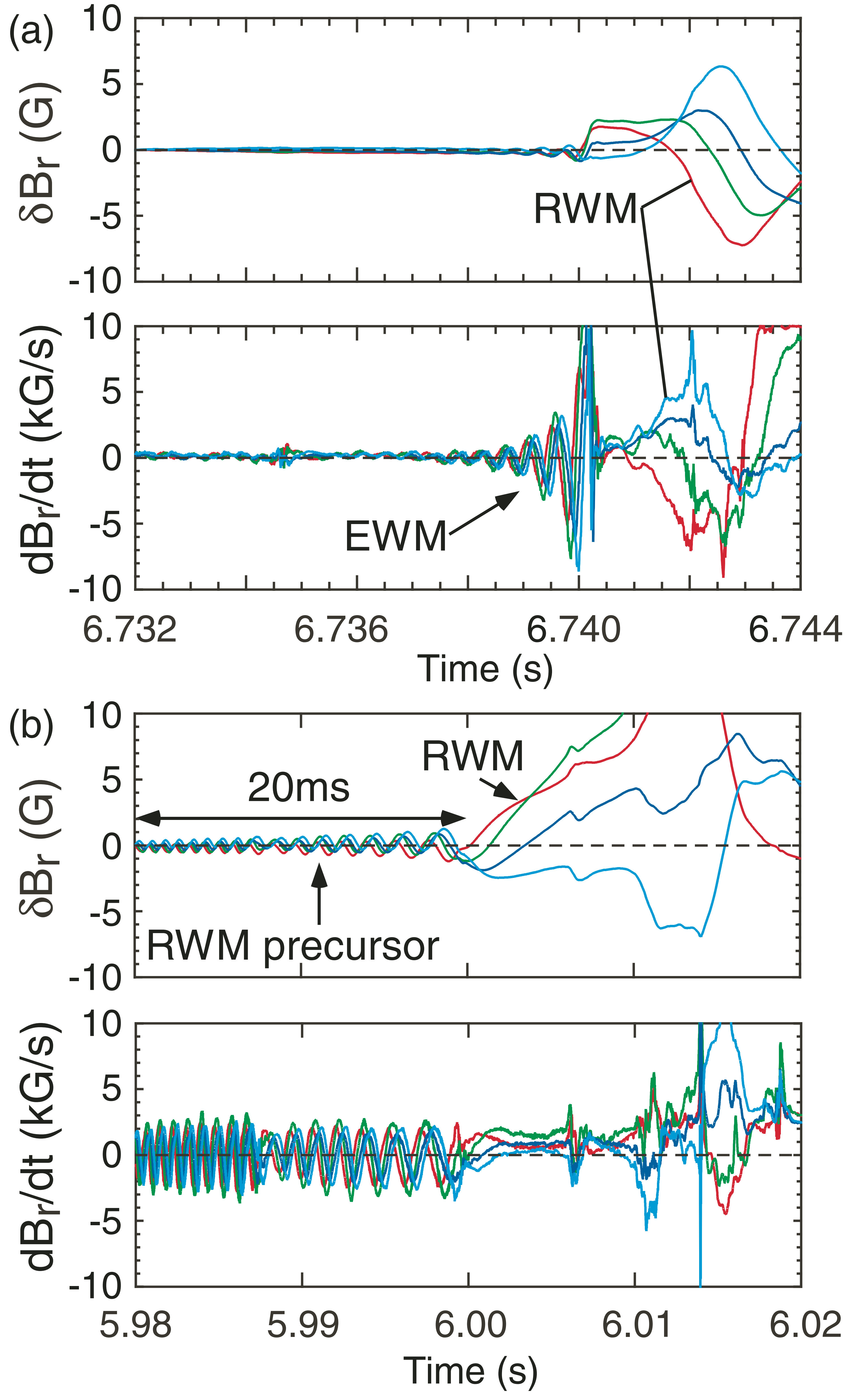
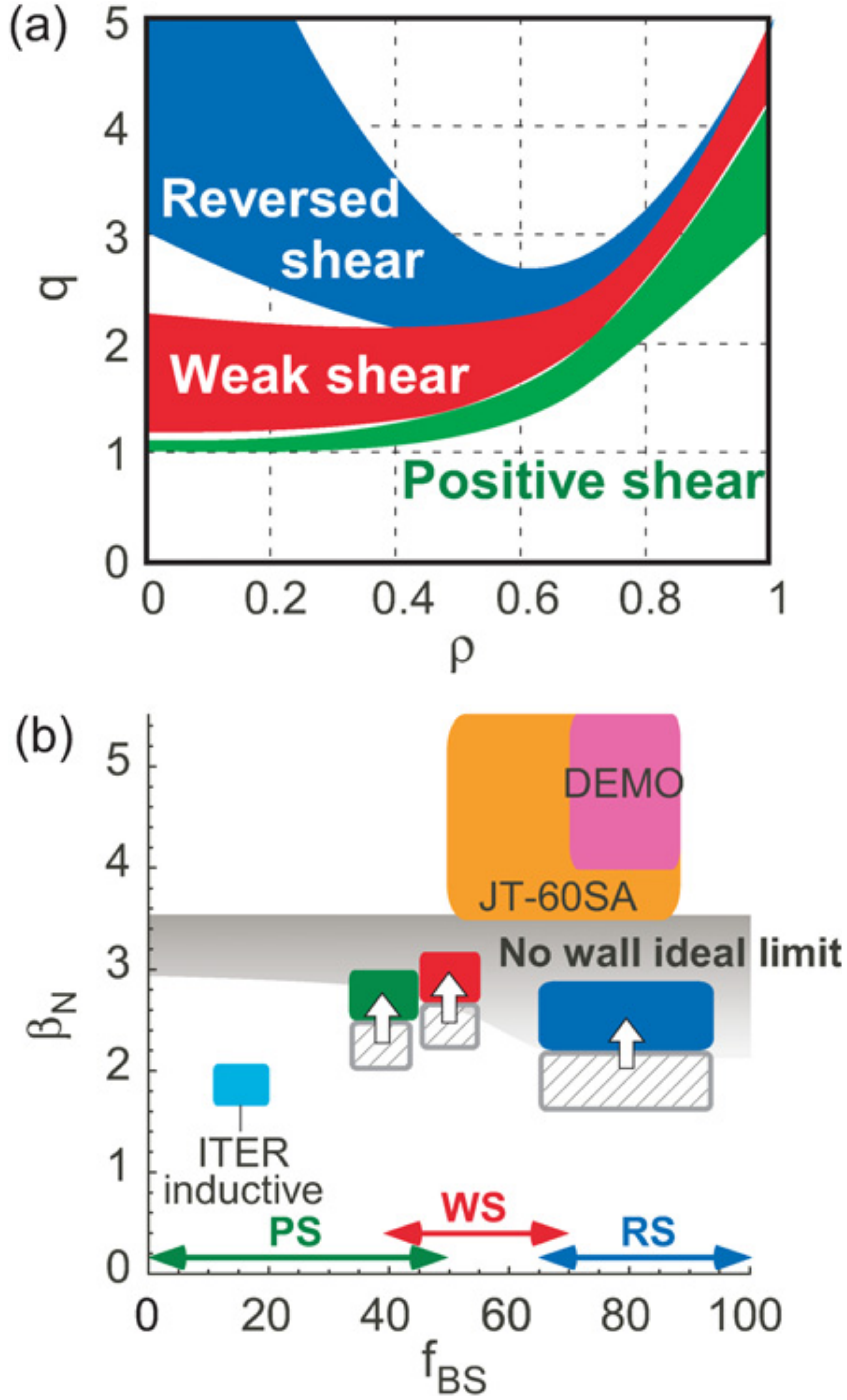


Figure 6. Time evolution of magnetic fluctuations for (*a*) EWM and (*b*) the RWM precursor. Upper figures in (*a*) and (*b*) show integrated mode amplitude of $n = 1$ fluctuation.



Global Kink Eigenmodes

WHAT ARE $(\psi(r), \chi(r))$ INSIDE AND OUTSIDE PLASMA?

Boundary conditions

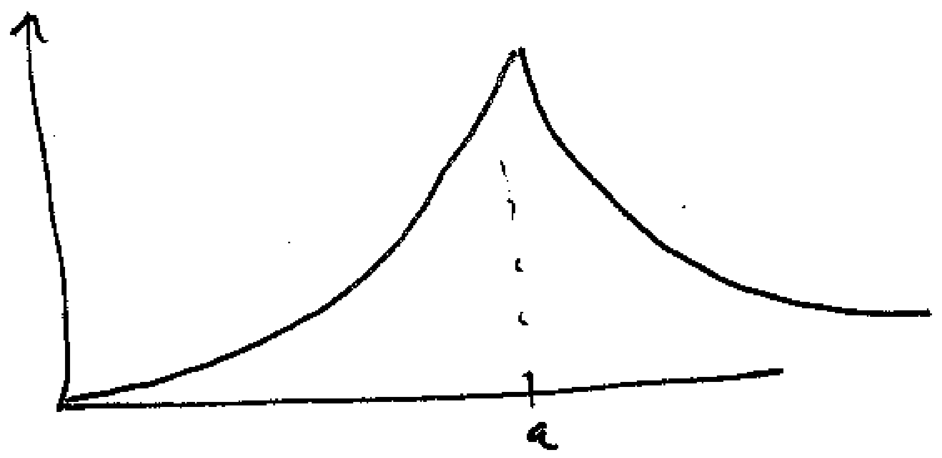
$\nabla^2 \psi = 0$ (NO CURRENT) OUTSIDE PLASMA

$\nabla^2 \chi = 0$ (NO FLOW, VORTICITY, NO SLIP)

$\nabla^2 \psi = 0$ (NO CURRENTS INSIDE PLASMA TOO)

$\nabla^2 \chi = 0$ (NO VORTICITY WITHIN PLASMA)

PERTURBED FIELDS + PLASMA MOTION
BUT NO CURRENTS OR VORTICITY



NO WALL		WITH WALL	
$\psi(r) \sim \left(\frac{r}{a}\right)^m$	$r<a$	$\sim \left(\frac{r}{a}\right)^m$	$r<a$
$\sim \left(\frac{a}{r}\right)^m$	$r>a$	$\sim \dfrac{\left(\frac{b}{r}\right)^m - \left(\frac{r}{b}\right)^m}{\left(\frac{b}{a}\right)^m - \left(\frac{a}{b}\right)^m}$	$(a<r<b)$

$$V_\perp = \hat{z} \times \nabla \chi = \hat{\theta} \frac{m}{r}\left(\frac{r}{a}\right)^m - \hat{r}\,\frac{im}{r}\left(\frac{r}{a}\right)^m \quad \text{INSIDE}$$

$$= -\hat{\theta} \frac{m}{r}\left(\frac{a}{r}\right)^m - \hat{r}\,\frac{im}{r}\left(\frac{a}{r}\right)^m \quad \text{OUTSIDE}$$

Kink Mode

$$-\omega\tilde{\varphi}_a = \frac{B_p}{r}(m-nq)\tilde{\chi}_a$$

$$\omega\rho\frac{\partial\chi}{\partial r}\Big|_{a^-} = \frac{2mB_p}{\mu_s a^2}\tilde{\varphi}_a\left[(m-nq)\left(\frac{-\Delta'(a)}{2m/a}\right)-1\right]$$

$$\Delta'(a) = -\frac{2m}{a}\frac{(b/a)^m}{(b/a)^m-\left(\frac{a}{b}\right)^m}$$

FOR A WALL AT $r=b$

$$\frac{\tilde{\chi}_a}{\tilde{\varphi}_a} = -\frac{\omega q_a R}{B_0(m-nq_a)}$$

EIGENVECTOR

$$\frac{\partial\chi}{\partial r}\Big|_{a^-} = -\frac{m}{a}\tilde{\chi}_a$$

EIGENVALUE

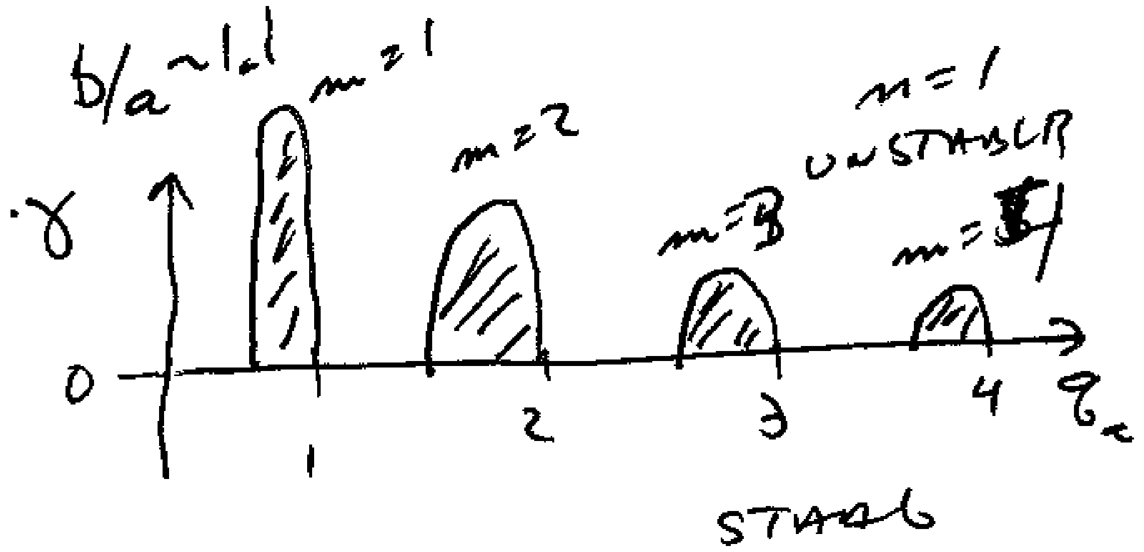
$$\omega^2 = 2\omega_A^2(m-nq) \times\left[(m-nq)\frac{(\Lambda+1)}{2}-1\right]$$

GLOBAL KINK MODES

$$\begin{pmatrix} \omega & \frac{B_p}{a}(m-nq) \\ \frac{2mB_p}{\mu_0 a}\left[(m-nq)\frac{(\Lambda+1)}{2}-1\right] & \omega\rho \end{pmatrix}\begin{pmatrix}\tilde{\varphi}_a \\ \tilde{\chi}_a\end{pmatrix} = 0$$

$$\boxed{\Delta'(a) = -\frac{m}{a}(\Lambda+1)}$$

SHAFRANOV'S FORMULA



Wesson's Kink Modes

LINEARIZED EQUATIONS FOR PERTURBED STREAM FUNCTION (χ) AND PERTURBED POLOIDAL FLUX (ψ)

$$-\rho\,\omega\,\nabla_\perp^2\tilde{\chi} = -\frac{m}{r}\frac{\partial J_z}{\partial r}\tilde{\psi} + \frac{B_\theta}{\mu_0 r}(n - nq)\nabla_\perp^2\tilde{\psi} \qquad \text{(MHD)}$$

$$-\omega\tilde{\psi} = \frac{B_\theta}{r}(m - nq)\tilde{\chi} \qquad \text{(INDUCTION)}$$

LET'S TAKE $\rho \simeq$ UNIFORM, WITH A SHARP JUMP AT THE PLASMA'S EDGE:

$$\omega\,\rho\,\frac{\partial\chi}{\partial r}\bigg|_{a^-} = \frac{B_\theta(a)}{\mu_0 a}(m - nq_a)\,\tilde{\psi}_a\,\boxed{\Delta'(a)}$$

PERTURBED SURFACE CURRENT AT PLASMA'S EDGE

WITH INDUCTION EQUATION:

$$\omega^2 = -\omega_A^2(m - nq_a)^2\,\Delta'(a)\,\frac{\chi_a}{(\partial\chi/\partial r)_{a^-}}$$

INSTABILITY REQUIRES $\Delta'(a) > 0$

BUT, HOW TO FIGURE OUT $\tilde{\psi}(r)$?

Wesson's Kink Modes

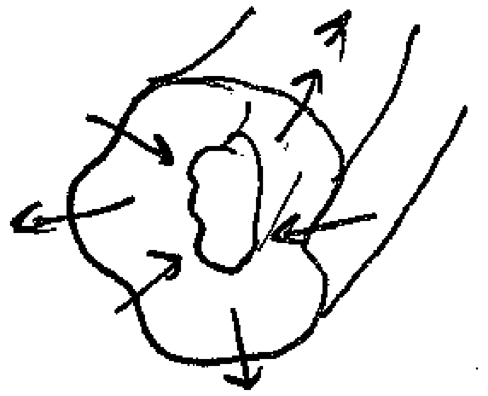
Since $|\omega| < \omega_A$, the kink mode causes the "internal" plasma to respond "quickly", so quickly that we can ignore the time it takes to form a distorted, 3D, quasi-equilibrium.

Inside, the plasma is a "forced" equilibrium

$$0 \approx -\frac{m}{r}\frac{\partial J_z}{\partial r}\tilde{\Psi} + \frac{B_\theta}{\mu_0 r}(n - mq)\nabla_\perp^2 \tilde{\Psi}$$

Outside, the response is the "vacuum" response.

With $J_z(r)$, we have to solve for $\tilde{\Psi}$ using a computer. (This is <u>very</u> <u>easy</u> for the cylindrical "tokamak")



← The surface current "pushes"/"pulls" plasma, and the "distorted" plasma is measured by <u>$\tilde{\Psi}(r, \theta, z)$</u>

With a "vacuum" and "perfectly conducting" exterior boundary.

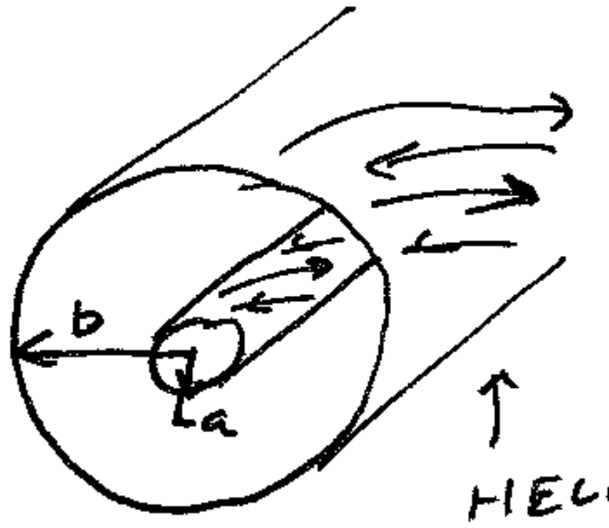
Resistive Wall Modes

- EXTERNAL KINKS WITH A PERFECTLY CONDUCTING (IDEAL) WALL

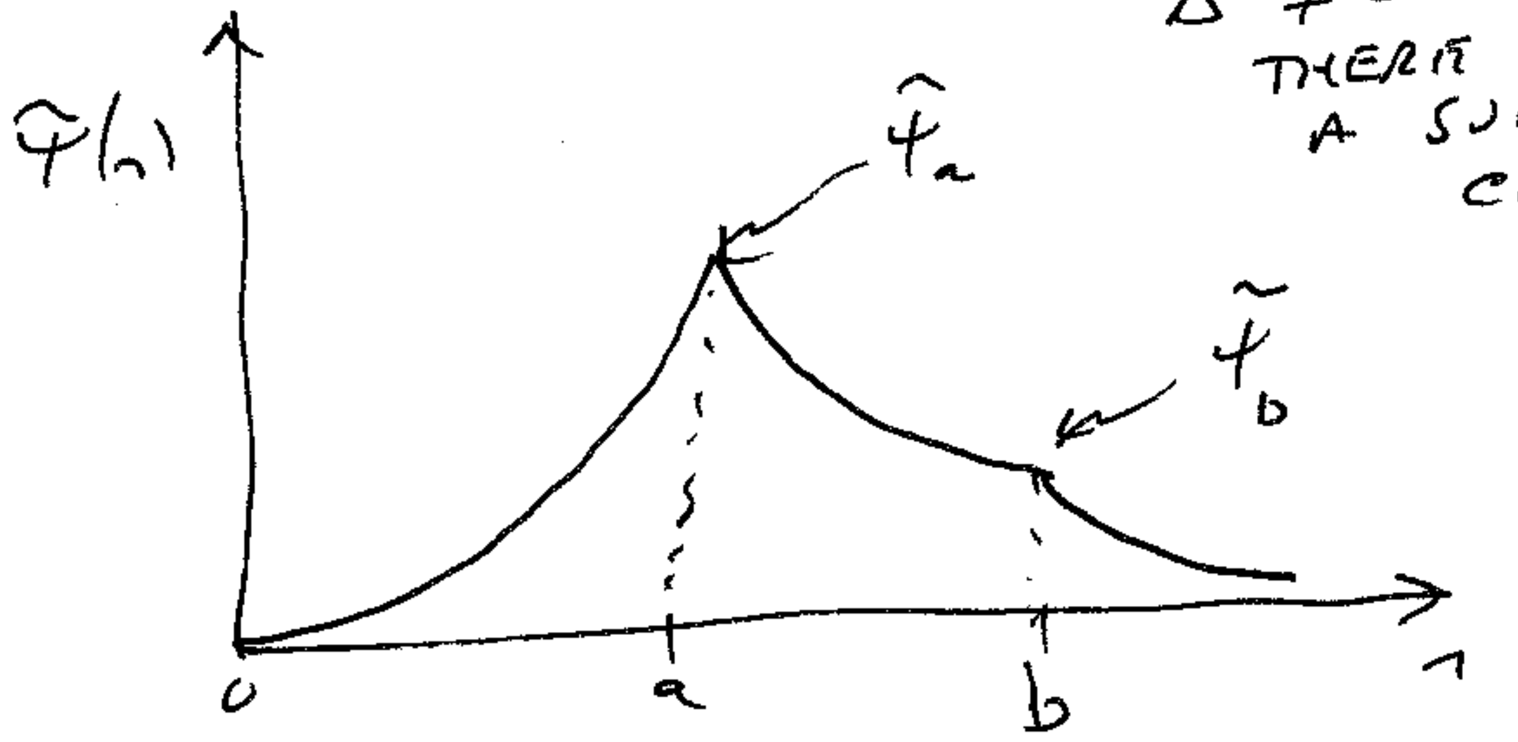
  HAVE $\hat{\psi}(r=b)=0$ i.e. $\hat{B}\cdot\hat{n}=0$

- WITH A RESISTIVE WALL, THE NORMAL FIELD DIFFUSES THROUGH THE WALL ON A "LONG" TIME SCALE

- IMAGINE TWO CONCENTRIC CONDUCTING SHELLS WITH $a/R \ll 1$



HELICAL CURRENTS WITH (m,n)



$\Delta' \neq 0$ WHEN THERE IS A SURFACE CURRENT

CURRENTS FLOW ON THIN SURFACES

$$\Delta'(a)\,\hat{\psi}(a) = \mu_0 K_z^a(\theta,\varphi) \leftarrow \text{SURFACE CURRENT AT } r=a$$

$$\Delta'(b)\,\hat{\psi}(b) = \mu_0 K_z^w(\theta,\varphi) \leftarrow \text{SURFACE CURRENT AT } r=b$$

Cylindrical Form of Perturbed Magnetic Flux

$$\bar{B} = \hat{z} \times \nabla\psi = \hat{\theta}\frac{\partial\psi}{\partial r} - \hat{r}/_r\left(\frac{\partial\psi}{\partial\theta}\right) = \hat{\theta}\frac{\partial\psi}{\partial r} - \frac{i\hat{r}m}{r}\tilde{\psi} \qquad \left(\frac{a}{R} \ll 1\right)$$

$\hat{\psi}$ = CONSTANT ACROSS CURRENT LAYER $\Rightarrow$ $\tilde{B}\cdot\hat{n} = 0$

$$\Delta' = \frac{1}{\hat{\psi}}\left(\frac{\partial\psi}{\partial r}\Big|_+ - \frac{\partial\psi}{\partial r}\Big|_-\right) \neq 0$$ WHEN THERE EXISTS A PERTURBED CURRENT

** NOTE: THIS CHANGES INSIDE A PLASMA

$$\psi(r) = \begin{cases} \psi_a \left(\frac{r}{a}\right)^m & r < a \\ \psi_a \left[\frac{\left(\frac{r}{b}\right)^m - \left(\frac{b}{r}\right)^m}{\left(\frac{a}{b}\right)^m - \left(\frac{b}{a}\right)^m}\right] + \psi_b \left[\frac{\left(\frac{a}{r}\right)^m - \left(\frac{r}{a}\right)^m}{\left(\frac{a}{b}\right)^m - \left(\frac{b}{a}\right)^m}\right] & a < r < b \\ \psi_b \left(\frac{b}{r}\right)^m & r > b \end{cases}$$

$$c \equiv \left(\frac{a}{b}\right)^{2m}$$ APPROX COUPLING COEFFICIENT

$$\therefore \quad \Delta'(a) = \frac{1}{\hat{\psi}_a}\left(\frac{\partial\psi}{\partial r}\Big|_{a+} - \frac{\partial\psi}{\partial r}\Big|_{a-}\right) = -\frac{2m}{a}\frac{1}{1-c} + \frac{\sqrt{c}}{1-c}\frac{2m}{a}\frac{\hat{\psi}_b}{\hat{\psi}_a}$$

$$\Delta'(b) = \frac{1}{\hat{\psi}_b}\left(\frac{\partial\psi}{\partial r}\Big|_{b+} - \frac{\partial\psi}{\partial r}\Big|_{b-}\right) = -\frac{2m}{b}\frac{1}{1-c} + \frac{\sqrt{c}}{1-c}\frac{2m}{b}\frac{\hat{\psi}_a}{\hat{\psi}_b}$$

Cylindrical Form of Perturbed Magnetic Flux

"EFFECT OF CONDUCTING WALL"

ALWAYS STABILIZING

$$\Delta'(a) = \frac{1}{\tilde{\psi}_a}\left(\frac{\partial \psi}{\partial r}\bigg|_{a+} - \frac{\partial \psi}{\partial r}\bigg|_{a-} \right)$$

VACUUM REGION

PLASMA REGION

$$\frac{1}{\tilde{\psi}_a}\frac{\partial \tilde{\psi}}{\partial r}\bigg|_{a+} = \begin{cases} -\dfrac{m}{a} & \text{WITH NO WALL} \\[2ex] -\dfrac{m}{a} - \dfrac{2m}{a}\dfrac{c}{1-c} + \dfrac{\tilde{\psi}_b}{\tilde{\psi}_a}\dfrac{2m}{a}\dfrac{\sqrt{c}}{1-c} \end{cases}$$

$$c = \left(\frac{a}{b}\right)^{2m}$$

THIS IS ALWAYS STABILIZING SINCE $\Delta'(a) > 0$ FOR INSTABILITY.

Resistive Relaxation of Wall Eddy Currents

INDUCTION: $\frac{\partial \bar{B}}{\partial t} = -\nabla \times E$ $\bar{B} = \hat{z} \times \nabla \psi$ $\bar{E} = \eta \bar{J}$

RESISTIVITY OF WALL

$$\hat{z} \times \nabla \frac{\partial \psi}{\partial t} = -\nabla \times \eta \bar{J}$$

$$\nabla \times \left(\hat{z} \frac{\partial \psi}{\partial t}\right) = \nabla \times \eta \bar{J}$$

SURFACE WALL CURRENT

δ_w = THICKNESS OF WALL ($\sim 1\,mm$)

$$\frac{\partial \tilde{\psi}_b}{\partial t} = \eta J_z = \frac{\eta}{\delta_w} K_z$$

$$= \frac{\eta}{\mu_0 \delta_w} \Delta'(b)\, \tilde{\psi}(b)$$

$$= \frac{\eta}{\mu_0 \delta_w}\left[-\frac{2m}{b}\frac{1}{1-c}\tilde{\psi}_b + \frac{\sqrt{c}}{1-c}\frac{2m}{b}\tilde{\psi}_a \right]$$

OR

$$\frac{\partial \tilde{\psi}_b}{\partial t} + \frac{\gamma_w}{1-c}\tilde{\psi}_b = \frac{\gamma_w \sqrt{c}}{1-c}\tilde{\psi}_a$$

$$\gamma_w \equiv \frac{\eta\, 2m}{\mu_0\, b\, \delta_w}$$

WALL TIME $= 1/\gamma_w$

EXAMPLES:

IF $\tilde{\psi}_a = 0$, THEN

$$\tilde{\psi}_b \sim \psi_b(0)\, e^{-\gamma_w t/1-c}$$

IF $\tilde{\psi}_a = \sqrt{c}\,\tilde{\psi}_b$, THEN $\tilde{\psi}_b \sim \tilde{\psi}_b(0)\, e^{-\gamma_w t}$

External Kink and Resistive Wall Mode

SHAFRANOV EQUILIBRIUM

PERTURBED SURFACE CURRENT ON PLASMA

$$\frac{d\tilde{\psi}_a}{dt} = \frac{B_p}{a}\, i\,(m - nq)\cdot \tilde{\chi}_a$$

$$m\rho\, \frac{d\tilde{\chi}_a}{dt} = i\, \frac{2m B_p}{\mu_0 a}\left[(m-nq)\frac{\Delta'(a)}{(2m/a)} + 1\right]\tilde{\psi}_a$$

$$\frac{d^2\tilde{\psi}_a}{dt^2} = -2\omega_A^2\,(m-nq)\left[(m-nq)\,\frac{\Delta'(a)}{(2m/a)} + 1\right]\tilde{\psi}_a$$

$$\frac{\Delta'(a)}{2m/a}\,\tilde{\psi}_a = -\frac{1}{1-c}\,\tilde{\psi}_a + \frac{\sqrt{c}}{1-c}\,\tilde{\psi}_b$$

PLASMA DYNAMICS

$$\frac{d^2\tilde{\psi}_a}{dt^2} = -2\,\omega_A^2\,(m-nq)^2\,\frac{c}{1-c}\left[\frac{\tilde{\psi}_b}{\sqrt{c}} + \left\{\frac{1-c}{c(m-nq)} - \frac{1}{c}\right\}\tilde{\psi}_a\right]$$

WALL COUPLING

External Kink and Resistive Wall Mode

"WESSON" EQUILIBRIUM

$$\omega^2 = -\,\omega_A^2\,(m - nq_a)^2\,\Delta'(a)\,\frac{X_a}{\partial X/\partial r)_{a^-}}$$

INSTABILITY REQUIRES $\Delta'(a) > 0$

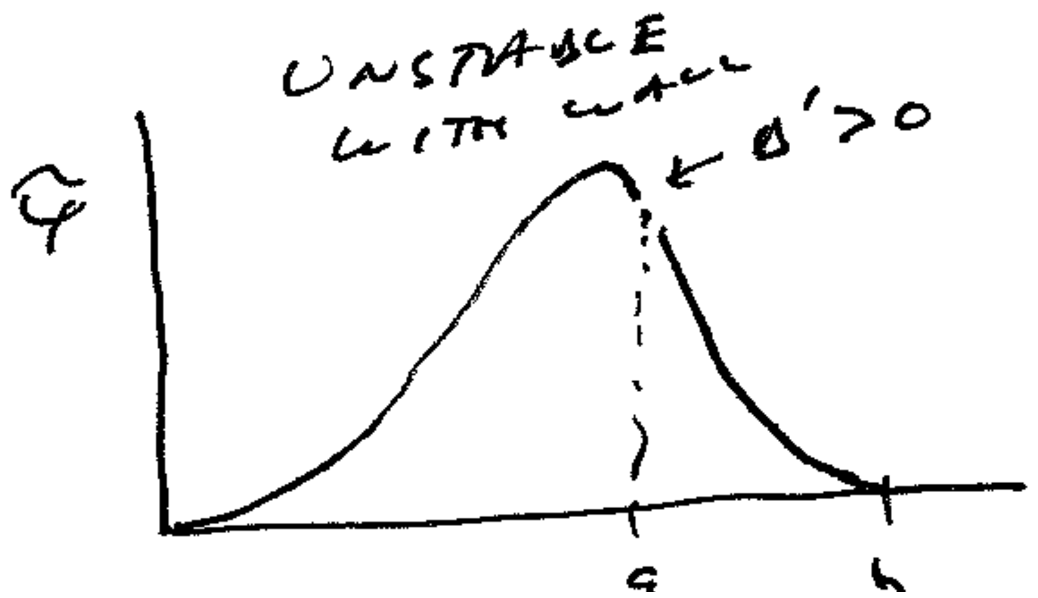
DEPENDS UPON PLASMA RESPONSE

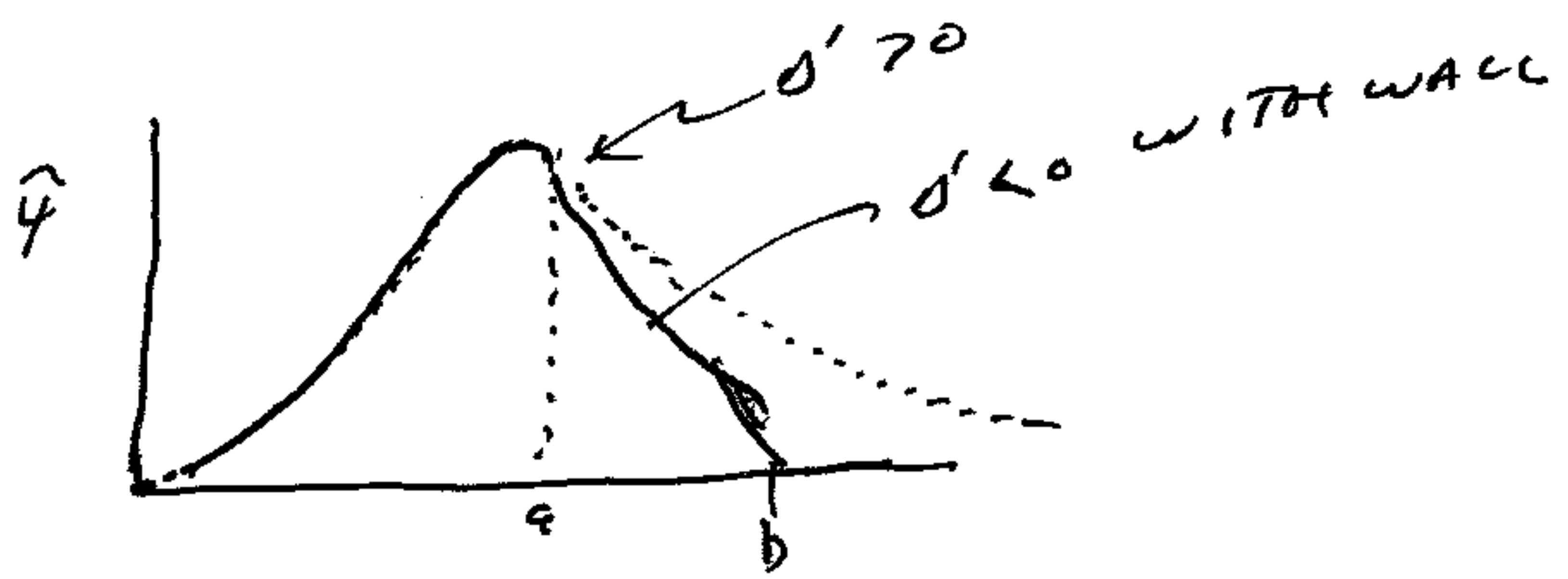
RECALL

$$\Delta'(a) = \frac{1}{\tilde{\psi}_a}\left(\frac{\partial\psi}{\partial r}\Big|_{a^+} - \frac{\partial\psi}{\partial r}\Big|_{a^-}\right)$$

DEPENDS ON VACUUM/WALL RESPONSE

$$\frac{1}{\tilde{\psi}_a}\frac{\partial\psi}{\partial r}\Big|_{a^+} \sim -\frac{m}{a} - \frac{2m}{a}\frac{c}{1-c} + \frac{\tilde{\psi}_b}{\tilde{\psi}_a}\frac{2m}{a}\frac{\sqrt{c}}{1-c}$$





General RWM Dynamics w No Flow/No Dissipation

$$\frac{d^2\tilde{\psi}_a}{dt^2} + \gamma_{MHD}^2 (1-\bar{s})\,\tilde{\psi}_a = \gamma_{MHD}^2 \frac{\tilde{\psi}_b}{v_c}$$

$$\frac{d\tilde{\psi}_b}{dt} + \frac{\gamma_w}{1-c}\,\tilde{\psi}_b = \gamma_w \frac{v_c}{1-c}\,\tilde{\psi}_a$$

$$\gamma_{MHD}^2 \equiv 2\,\omega_A^2 (n-mq)^2 \frac{c}{1-c}$$

$$\bar{s} = s/s_{CRIT}$$

$$s_{CRIT} = \frac{c}{1-c}$$

<u>THREE ROOTS</u> !

ROOT 1 = STABLE ALFVEN WAVE

ROOT 2 = EXTERN KINK MODE – COUPLES TO WALL

ROOT 3 = WALL MODE

<u>3 EIGEN VECTORS</u>

Usual "RWM Limit"...

$$\frac{d}{dt} \sim \gamma_w \ll \gamma_{MHD}$$

THEN: "PLASMA INERTIA" IS IGNORABLE

$$(1-\bar{s})\hat{\psi}_a \hat{=} \hat{\psi}_b / \sqrt{c}$$

$$\frac{d\hat{\psi}_b}{dt} + \frac{\gamma_w}{1-c}\hat{\psi}_b = \gamma_w \frac{\sqrt{c}}{1-c}\hat{\psi}_a \approx \gamma_w \frac{\hat{\psi}_b}{(1-c)(1-\bar{s})}$$

OR

$$\frac{d\hat{\psi}}{dt} + \underbrace{\frac{\gamma_w}{1-c}\left(1 - \frac{1}{(1-\bar{s})}\right)}\hat{\psi} = 0$$

$$\text{GROWTH RATE} = \gamma_w \frac{\bar{s}}{(1-\bar{s})}\frac{1}{(1-c)}$$ (VALID FOR SMALL $\bar{s}$)

Fitzpatrick-Aydemir Model

(Nuc Fusion **36** (1996) pp. 11)

STABILIZATION OF THE RESISTIVE SHELL MODE IN TOKAMAKS

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ABSTRACT. The stability of current-driven external-kink modes is investigated in a tokamak plasma surrounded by an external shell of finite electrical conductivity. According to conventional theory, the ideal mode can be stabilized by placing the shell sufficiently close to the plasma, but the non-rotating 'resistive shell mode', which grows as the characteristic L/R time of the shell, always persists. It is demonstrated, using both analytic and numerical techniques, that a combination of strong edge plasma rotation and dissipation somewhere inside the plasma is capable of stabilizing the resistive shell mode. This stabilization mechanism is similar to that found recently by Bondeson and Ward, except that it does not necessarily depend on toroidicity, plasma compressibility or the presence of resonant surfaces inside the plasma. The general requirements for the stabilization of the resistive shell mode are elucidated.

Further Reading…

Simplified forms of the dynamical equations for the perturbed flux at the plasma edge, ψ_a, and the conducting wall, ψ_w, were summarized by Fitzpatrick [3] who also included the perturbed flux at the wall driven by control coils, ψ_c. The perturbed flux is related to perturbed radial magnetic field as

$$b_r(r,\theta,\phi,t) = \Re\left\{ i\frac{m}{r}\psi(r)e^{i(m\theta-\phi)+\gamma t}\right\}. \tag{1}$$

The three model equations describe an external kink mode coupled to a surrounding wall, the resistive dissipation of wall eddy currents, and toroidal torque balance. These equations are:

$$\frac{d^2\psi_a}{dt^2} + (\nu^* - 2i\Omega_\phi)\frac{d\psi_a}{dt} + \left[\gamma_{MHD}^2(1-\bar{s}) - \Omega_\phi^2 - i\nu^*\Omega_\phi\right]\psi_a = \gamma_{MHD}^2\frac{\psi_w}{\sqrt{c}} \tag{2}$$

$$\frac{d\psi_w}{dt} + \frac{\gamma_w}{1-c}\left(\psi_w - \sqrt{c}\psi_a\right) = \gamma_w\psi_c \tag{3}$$

$$\frac{d\Omega_\phi}{dt} + \nu^*(\Omega_\phi - \Omega_\phi^{(0)}) = -\frac{1}{2}\nu^*\Omega_\phi\frac{1+c}{1-c}\frac{|\psi_a|^2}{\delta M_a L_p} \tag{4}$$

where c is a coupling coefficient between the wall and the plasma, $\bar{s} \equiv s/s_{crit} = s(1-c)/c$ is the normalized Boozer stability parameter, Ω_ϕ is the plasma rotation at the edge, ν^* is the (anomalous) viscosity, δM_a is the effective mass of the inertial layer, and L_p is the effective inductance of the perturbed plasma skin current. For a complete cylindrical wall, $c \approx (a/b)^{2m}$. `VALEN` [9] can be used to calculate c and γ_w for an incomplete wall, since c is related to the ideal wall stability limit through the definition $s_{crit} = c/(1-c)$ and since the resistive wall mode growth rate is proportional to γ_w when $0 < \bar{s} \ll 1$.