Plasma 2
Lecture 20:
Interchange, Ballooning, and Kinks

APPH E6102y
Columbia University
Low-Frequency (Electro-)Magnetic Response in a Strongly Magnetized Plasma

- **Alfvén Waves (Review)**
- **Magnetic Induction (Faraday's Law)**
- When $E_{\perp} \equiv 0$, the ideal MHD condition
  
  \[\text{AND WHEN NOT } E_{\perp} \text{ TO}\]

  Drift waves $\widetilde{E}_{\perp} = -i k_{\perp} \widetilde{\phi}$

  MHD waves $\widetilde{E}_{\parallel} = 0$

  \[k_{\perp}^2 \rho^2 < 1 \quad \rho \ll \frac{U_A}{V_A} \quad \text{LOW FREQ}\]

  \[k_{\parallel}^2 > 1 \quad \text{LONG WAVELENGTH} \quad \text{INTERCHANGE FLUTO-LIKE}\]
Parallel Electric Field

\[ E_{\parallel} = -i \hbar \vec{\phi} + \imath \omega \vec{A}_{\parallel} \]

\[ = \hbar \vec{\phi} \left[ \frac{\hbar^2 \vec{\phi} \cdot \vec{A}_{\parallel}}{\omega (\omega - \hbar \omega^2)} - \hbar^2 \vec{\phi} \cdot \vec{A}_{\parallel} \right] \]

**WHEN** \( \hbar^2 \vec{\phi} \cdot \vec{A}_{\parallel} \gg \omega^2 \)

\[ E_{\parallel} = -i \hbar \vec{\phi} \]

**WHEN** \( \hbar \rightarrow 0 \) and/or \( \hbar^2 \vec{\phi}^2 \ll 1 \)

\[ \vec{E}_{\parallel} = 0 \]

**USUAL DRIFT LIMIT**

**USUAL MHD LIMIT**
Density-Potential Relationship

\[
\frac{\varepsilon}{T} = \frac{k_B}{\omega} \frac{\omega - k_F \nu_0^*}{\omega (\omega - k_F \nu_0^* - \frac{\beta^2 \rho^2}{\gamma^2} \omega^2)} \left( \frac{\varepsilon}{T} \right)
\]

Continuity:

\[
\frac{\varepsilon}{n_0} = \frac{b_n S^2}{\omega} \left( \frac{\varepsilon}{T} \right) - \frac{\beta^2 \rho^2}{\gamma^2} \frac{\varepsilon}{T} \left( \frac{\varepsilon}{T} \right)
\]

Eliminating \( \tilde{A}_i \)

\[
\frac{\varepsilon}{n_0} = \left( \frac{\varepsilon}{T} \right) \left( \frac{\varepsilon}{T} \right)
\]

Drift Limit:

\[
\frac{\delta m}{m} = \frac{\varepsilon}{T}
\]

MHD Limit:

\[
\frac{\delta m}{m} = \left( \frac{\varepsilon}{T} \right)
\]

Pure ExB Connection:

When parallel motion is impeded
Low-Frequency MHD Modes

- **Interchange modes** $Q_{11} = 0$ (Pressure driven)
- **Kink modes** $E_{11} = 0$ but electromagnetic $A_{11} \neq 0$ (Current-gradient driven)
- **Ballooning modes** $\lambda_{11} \neq 0$ (Pressure driven)
- **Tearing modes** ($k_{11} = 0$ only or rational surface)
  - Micro-tearing modes (Larmor $k_{11}$)

Comment: Kink modes, ballooning modes, tearing modes are usually developed to leading order by MHD equations but full kinetic and two-fluid dynamics are needed to understand MHD mode dynamics seen in experiments.
F-Layer Spread

- Gravitational Rayleigh-Taylor instabilities are well-studied examples of turbulent interchange dynamics.

- Notes: (i) Large scales, (ii) Rayleigh-Taylor regime & Drift-wave regime

- Refs:

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**Figure 4.1**. Range–time–intensity map displaying the backscatter power at 3-m wavelengths measured at Jicamarca, Peru. The gray scale is decibels above the thermal noise level. [After Kelley et al. (1981). Reproduced with permission of the American Geophysical Union.]

**Figure 2.** Electron density and electric field spectra measured during the downleg of 29.028 on 30 July 1990. The altitude range covers ~20 km near 350 km altitude.
Observation of Ballooning Modes in High-Temperature Tokamak Plasmas


Plasma Physics Laboratory, Princeton University, P.O. Box 451, Princeton, New Jersey 08543

(Received 6 June 1992)

The beta-degradation phase of a high-$\beta$ plasma in the TFTR tokamak is analyzed with x-ray and electron cyclotron emission imaging techniques. Medium-$n$ (toroidal mode number) instabilities with ballooning characteristics are observed near and within the $q=1.5$ surface during a slow degradation in the plasma $\beta$ and precede a sudden partial collapse in the central plasma pressure. This is the first reported observation of a ballooning instability in the interior of a large, collisionless tokamak plasma.

FIG. 2. Contour plots of the time evolution of the ECE profile in shot 54018: (a) Electron temperature profile (the contour step size is 300 eV); (b) perturbation of ECE signal; (c) chord-integrated soft-x-ray emission profile (horizontal view).

3 cm. The soft x rays are detected with a vertical viewing camera (twenty detectors) and a horizontal viewing cam-
Large-Larmor-Radius Interchange Instability


Naval Research Laboratory, Washington, D.C. 20375
(Received 15 June 1987)

We observe linear and nonlinear features of a strong plasma–magnetic-field interchange Rayleigh-Taylor instability in the limit of large ion Larmor radius. The instability undergoes rapid linear growth culminating in free-streaming flute tips.

FIG. 1. Experimental arrangement for instability experiments. A schematic of the equipment is shown; ion detectors are denoted by rectangles and magnetic probes by circles.

FIG. 3. Examples of the instability development. (a) 0.1-T case observed at time 115 ns. (b) Example of density clumps in the early time phase development with $B = 1.0$ T at time 59 ns. (c) Example of curved spike structure with 1.0-T field (field points out of paper) at 115 ns. (d) Same as (c) except field points into paper and $t = 100$ ns; note reversal of curvature sense. $E_1 = 25 - 30$ J and $P < 0.1$ mTorr for these shots.
Plasmapause

Fig. 6. Equatorial drift path of a 20% plasma density enhancement (solid line) and of a plasma hole (dashed line) released at $R = 7R_p$. Otherwise the conditions and notations are the same as in Fig. 5.

Note that the plasma density enhancement now drifts outwards. Under the dominating action of the centrifugal force whose radial component exceeds the gravitational force everywhere in the shaded area beyond the Zero-Radial-Force Surface. The plasma hole spirals in the opposite direction toward the same asymptotic trajectory as the plasma hole in Fig. 5. This asymptotic trajectory of all plasma holes determines the position of the equatorial plasmapause.

Fig. 1 Structures observed by the EUV instrument onboard IMAGE and new morphological nomenclature: examples of shoulders, plumes, fingers, channels, crenulations and notches. The direction to the Sun is shown as a yellow dot for each image. (From http://image.gsfc.nasa.gov/poetry/discoveries/N47big.jpg)
Io Plasma Torus

Magnetospheric Dynamo:
100 TW Auroral Power
Regulates Interchange Motion

plasma! Such a large density differential would make the flux tube extremely buoyant, leading to rapid inward transport.

Changes in the count rate of energetic ions and electrons during the event are illustrated in the center panels of Figure 1. These selected particle channels span a broad energy range from 15 keV to 10 MeV. The energy for each channel, estimates of the typical bounce time, Larmor radius, and gradient drift speed associated with the Ioian magnetic field at 6.03 R$_I$ are listed in Table 1. All particles exhibit a characteristic loss cone distribution with modest depletion along the direction of the ambient magnetic field both before and after the event. During the event, most channels show a pronounced flux enhancement. This increase is most dramatic for the highest energy ions in a direction close to perpendicular to the field. Low energy ions and electrons also exhibit significant flux enhancements and mental effects persist for a brief period following the period of magnetic field enhancement. Notably, higher energy electrons (E > 300 keV) show little change.

Figure 2 shows the evolution of the pitch angle distribution of energetic ions during the event. Because the event occurred by the magnetometer occurred entirely within one revolution of the orbit, one must carefully separate temporal and angular variations. For the spins which began at 17:33:58 (middle column), only the second passage of the IEP extension through 90 degree of pitch angle (near 17:34:08) occurred during the 10 second interval that the magnetometer measured the

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**Table 1. Energetic Particle Properties near Lo.03**

<table>
<thead>
<tr>
<th>Channel</th>
<th>Species</th>
<th>Energy (keV)</th>
<th>Pitch Angle</th>
<th>Fraction</th>
<th>Recovery</th>
</tr>
</thead>
<tbody>
<tr>
<td>DO</td>
<td>electron</td>
<td>0.067-0.099</td>
<td>5.3</td>
<td>0.08</td>
<td>0.60</td>
</tr>
<tr>
<td>D2</td>
<td>electron</td>
<td>0.062-0.085</td>
<td>0.4</td>
<td>0.17</td>
<td>0.83</td>
</tr>
<tr>
<td>D3</td>
<td>electron</td>
<td>0.074-0.099</td>
<td>1.9</td>
<td>0.35</td>
<td>0.65</td>
</tr>
<tr>
<td>D4</td>
<td>electron</td>
<td>0.083-0.101</td>
<td>4.2</td>
<td>0.71</td>
<td>0.29</td>
</tr>
<tr>
<td>D5</td>
<td>electron</td>
<td>0.088-0.101</td>
<td>9.0</td>
<td>0.61</td>
<td>0.39</td>
</tr>
<tr>
<td>D6</td>
<td>electron</td>
<td>0.093-0.101</td>
<td>22.3</td>
<td>0.55</td>
<td>0.45</td>
</tr>
<tr>
<td>D7</td>
<td>electron</td>
<td>0.098-0.101</td>
<td>33.1</td>
<td>0.68</td>
<td>0.32</td>
</tr>
<tr>
<td>D8</td>
<td>electron</td>
<td>0.103-0.101</td>
<td>44.6</td>
<td>0.56</td>
<td>0.44</td>
</tr>
<tr>
<td>D9</td>
<td>electron</td>
<td>0.107-0.101</td>
<td>55.3</td>
<td>0.65</td>
<td>0.35</td>
</tr>
</tbody>
</table>

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**Figure 5.** Schematic of the transport envisaged to account for the observations. Dashed arcs are placed at the orbit of Io, and 7 R$_I$ Meandering curves are not instantaneous streamlines but some average flow paths that would organize our data. Flux tubes moving out (solid curves) have larger plasma content than do those moving in (dashed curves). The azimuthal meanders indicate that random variations in convection fields may produce fluctuations of average azimuthal velocity. The important point is that well away from Io, the inward and outward flows balance. Along Galileo's orbit in the immediate vicinity of Io (filled circle) where mass loading is concentrated, the flow is predominantly outward. Somewhere else the inward flow dominates. The Galileo trajectory inbound to Jupiter (solid curve), crosses inward and outward moving flux tubes near 7 R$_I$ but principally outward moving flux tubes in the near-Io region.
Kink Instabilities: The Most Dangerous Instability for Current-Carrying Plasma

The Perhapsatron, which was built in 1952-53, was the first Z-pinch device at Los Alamos. The toroidal discharge tube surrounds the central core of an iron transformer.

Toroidal “z-pinch”
Interchange Mode Gravity (& Effective Gravity)

\[ \mathbf{g}_{\text{effective}} = \frac{2 \Omega_c}{m_i R} \]

\[ \mathbf{g}_{\text{effective}} = \mathbf{g}_{\text{cent}} = \left( \frac{\rho \Omega_c^2}{m_i} \right) \mathbf{R} \]

\[ \mathbf{E}_i = 0 \quad \text{because} \quad \mathbf{k}_{ii} = 0 \]

\[ \mathbf{J}_{\text{ion}} = e m \left( \mathbf{V}_{\text{pol}} + \mathbf{V}_{\text{env}} + \mathbf{V}_{\text{gi}} \right) \]

\[ \mathbf{J}_{\text{em}} = -e m \left( \mathbf{V}_{\text{env}} + \mathbf{V}_{\text{ge}} \right) \]

\[ \nabla \cdot (e m \mathbf{V}_{\text{pol}}) + \nabla \cdot (e m (\mathbf{V}_{\text{gi}} - \mathbf{V}_{\text{ge}})) = 0 \]
Dispersion Relation: Gravitational Interchange in Slab Geometry

\[ V_{pol} = \frac{e M_i}{B_0^2} \frac{d \Phi}{d t} = - \frac{e M_i}{B_0^2} \left( \frac{1}{2} \epsilon + n_e V_e \right) \nabla \cdot \Phi \]

\[ V_{g} - V_{pe} = \frac{\gamma}{e^2} \left( \frac{m_i}{m_e} \frac{m_e}{m_i} g \right) \]

**Charge Neutrality**

\[ \nabla \cdot J = 0 \]

\[ \nabla \cdot (e m \vec{V}_{pol}) + e (V_{g} - V_{pe}) \nabla \cdot \vec{m} = 0 \]

**Continuity**

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \]

**Electrons**

\[ \frac{\partial \rho_e}{\partial t} + \vec{V}_e \cdot \nabla \rho_e = 0 \]

\[ -j \omega \vec{m} - \frac{1}{m_0} j \vec{V}_e \cdot \nabla \Phi \left( m'/m \right) = 0 \]

**Magnetic Field**

\[ \vec{B} = \frac{\mu_0}{c} \frac{\partial \vec{A}}{\partial t} + \frac{e}{c} \vec{V} \times \vec{m} \]

\[ \omega (\omega - \lambda_\gamma V_{g}) = \left( \frac{e^2 c}{m_0} \right) \gamma \omega \frac{B^2}{m_0} \hat{l} = 0 \]

\[ (\omega_0, \gamma) < 0 \text{ unstable} \]
Gravitational Interchange in Slab Geometry

\[ \omega^2 - \omega k_y v_g^2 - \left( \frac{m'}{\rho_0} \right) \frac{k_x^2}{k_y^2} = 0 \]

\[ \omega = \frac{k_x v_g^2}{k_y} \pm \sqrt{\left( \frac{k_x v_g^2}{k_y} \right)^2 + \left( \frac{m'}{\rho_0} \right) \frac{k_x^2}{k_y^2}} \]

Mode propagates in \( \omega \approx \beta \) direction

\[ \frac{m'}{\rho_0} \gg k_x^2 \]

\( \mathbf{\Pi} = \mathbf{B}_0 \hat{z} \)

\( v_{g_i} = -\frac{m'}{eQ} (\mathbf{B}_0 \times \mathbf{G}) \)

\( \mathbf{G} = -2 \mathbf{g} \)

(Check signs)

\[ \text{This is } \omega < 0 \text{ for instability} \]
Estimate: Curvature-Driven Interchange ("Cylindrical" Geometry)

\[ \nabla \cdot \left( \varepsilon m (v_{gi} - v_{ge}) \right) \rightarrow \nabla \cdot (\varepsilon m v_{\text{drive}}) \]

\[ \text{Gravity} \]

\[ \text{Curvature} \]

\[ v_{\text{drive}} \leftrightarrow v_{\text{drive}} \]

\[ \nabla \cdot (\varepsilon m v_{\text{drive}}) \]

\[ T' = \frac{2(T_e + T_i)}{m_i R_C} \frac{A_y^2}{\lambda^2} \]

\[ g = \frac{2T_e}{m_i R_C} \]

\[ v_y(\text{eff}) = -\frac{m}{e_B} g \]

\[ \frac{\rho'}{\rho} = \frac{m^2 (1 + \eta)}{m} \]

\[ \frac{d\lambda}{dA} \]

\[ \eta \]

\[ \text{This actually discusses ion FEL stabilization.} \]

\[ \gamma \text{ ion FELs} \]

\[ \frac{\lambda_y}{R_C} \rightarrow \frac{\lambda_y}{R_C} \]

\[ \alpha \]
Ballooning Modes

Example

\[ \nabla \cdot \mathbf{J}_{\perp} = - \nabla_{11} \cdot \mathbf{J}_{11} \]

\[ \nabla \cdot \mathbf{J}_{\text{pol}} + \nabla_{11} \cdot \mathbf{J}_{11} + \nabla \cdot \mathbf{J}_{\text{curv}} = 0 \]

\[ \hat{A}_n = \frac{k_n^2}{\omega} \hat{\Phi} \]

\[ \sin \theta \hat{e} = 0 \]

\[ -j \frac{M_i^2 m_0}{B^2} \omega \hat{a}_n \hat{\Phi} (1 - \frac{\hat{b}_n^2 V_A^2}{\omega^2}) \]

\[ V_A = \frac{B}{\mu_0 m_i} \]
Ballooning Stability

\[ \omega^2 - \frac{1}{\text{te}} \frac{V_A^2}{\text{te}} = -\text{(Drive)} \frac{k_r^2}{\text{te}} \]

If \( \frac{1}{\text{te}} \) could go to zero (\( \text{z-pinch} \))
then \( \omega^2 = -\text{(Drive)} \frac{k_r^2}{\text{te}} \) unstable with \( \text{(Drive)} > 0 \)

But with finite \( \frac{1}{\text{te}} \), field-line bending is stabilizing.

Example: Tokamak: \( k_r \sim \frac{1}{\text{GR}} \), connection length from outside to inside

Unstable when \( \text{Drive} > \frac{k_r^2 V_A^2}{\text{te} V_A^2 / (\text{GR})^2} \)

\( \text{Drive} \sim \frac{2 (\text{te} + \text{te}^*)}{\text{m} \cdot \text{ac}} > \frac{V_A^2}{\text{te}^* \text{RC}} \Rightarrow \frac{2 \text{m} (\text{te} + \text{te}^*)}{\text{Baco} \cdot R} > \frac{\text{ac}^2}{\text{Baco} \cdot R} \)
Kink Modes
(The most dangerous instabilities for current-carrying plasma.)

\[ \nabla \cdot \mathbf{J} = 0 = \nabla \cdot \mathbf{J}_{\text{pol}} + \nabla \cdot \mathbf{J}_{\text{te}} + \nabla \cdot \mathbf{J}_{\text{cur}} + \nabla \cdot \mathbf{J}_{\text{grad}} \]

\[ \nabla \cdot \left( \mathbf{J} \frac{\partial \mathbf{B}}{\partial \mathbf{B}} \right) = \frac{\delta \mathbf{B}}{\delta \mathbf{B}} \cdot \nabla \mathbf{J}_{\text{grad}} \]

\[ \mathbf{J}_{\text{Lagr}} \sim \mathbf{J}_{\text{grad}} \frac{\delta \mathbf{B}}{\mathbf{B}} \]

\[ \mathbf{J}_{\text{Lagr}} \sim \mathbf{J}_{\text{grad}} \frac{\delta \mathbf{B}}{\mathbf{B}} \]

\[ \omega^2 = \frac{\kappa^2}{m_i} \frac{V_A^2}{A} = -(\text{coulomb}) \frac{\kappa^2}{m_i} \frac{V_A^2}{A} \]

\[ \mathbf{A}_{\text{damp}} = \frac{1}{Q_0} (m - mg) \]

So if \( mg > m \), STAB!!
Next: Reduced MHD

- Cylindrical Reduced MHD
- Ideal instabilities
- Tearing instabilities
- RWMs and FWMs
Cylindrical Reduced MHD

the order of $\epsilon^2 B_0$. To lowest order in $\epsilon$ this unknown vari-
ation of the toroidal field can be eliminated from the problem by taking the curl of the momentum equation. The resulting equations are the standard low-$\beta$ tokamak reduced equations that describe free-boundary kink modes:

$$ R_0^2 \frac{d\nabla^2 u}{dt} = \mathbf{B} \cdot \nabla (\nabla^2 \psi), $$

$$ \frac{\partial \psi}{\partial t} = R_0^2 \mathbf{B} \cdot \nabla u, $$

$$ \mathbf{B} = \nabla \psi \times \nabla \zeta + I_0 \nabla \zeta, $$

$$ \mathbf{V} = R_0^2 \nabla u \times \nabla \zeta, $$

$$ \nabla^2_1 = \frac{\partial^2}{\partial R^2} + \frac{\partial^2}{\partial z^2}. $$

$$ A_\phi = I_0 (r/2) $$

$$ A_\parallel \approx A_z = \psi_0 (r) + \tilde{\psi} (r, \phi) $$

Important

Here $I_0 = B_0 R_0$ and $\nabla \zeta = \frac{\zeta}{R_0}.$