Plasma 2
Lecture 2: Nonlinear Landau Damping (Part 1)

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Collisionless Damping of Nonlinear Plasma Oscillations

THOMAS O'NEIL*

Department of Physics, University of California, San Diego, La Jolla, California

(Received 12 July 1965)

It is well known that the linear theory of collisionless damping breaks down after a time $\tau \equiv (m/e\mathcal{E}k)^{1/2}$, where k is the wavenumber and $\mathcal{E}$ is the amplitude of the electric field. Jacobi elliptic functions are now used to provide an exact solution of the Vlasov equation for the resonant electrons, and the damping coefficient is generalized to be valid for times greater than $t = \tau$. This generalized damping coefficient reduces to Landau's result when $t/\tau \ll 1$; it has an oscillatory behavior when t/τ is of order unity, and it phase mixes to zero as t/τ approaches infinity. The above results are all shown to have simple physical interpretations.

681 Citations to O'Neil, PF, 8 (1965)

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10.1002/2017GL072624

Key Points:
- Nonlinear wave-particle interaction is demonstrated to be nonadiabatic
- The nonlinear evolution timescale is comparable to the particle phase-space trapping timescale
- Subpacket formation is explained using phase-space bounce motion of particles

Identify the nonlinear wave-particle interaction regime in rising tone chorus generation

Xin Tao[1,2], Fulvio Zonca[3,4], and Liu Chen[4,5]

[1]CAS Key Laboratory of Geospace Environment, Department of Geophysics and Planetary SciencesUniversity of Science and Technology of China, Hefei, China, [2]Collaborative Innovation Center of Astronautical Science and Technology, Harbin, China, [3]ENEA C. R. Frascati, Frascati, Italy, [4]Institute of Fusion Theory and Simulation and Department of Physics, Zhejiang University, Hangzhou, China, [5]Department of Physics and Astronomy, University of California, Irvine, California, USA

PHYSICS OF PLASMAS 23, 090702 (2016)

Effects of energetic particles on zonal flow generation by toroidal Alfvén eigenmode

Z. Qiu,[1] L. Chen,[1,2] and F. Zonca[3,1]

[1]*Institute for Fusion Theory and Simulation and Department of Physics, Zhejiang University, Hangzhou, People's Republic of China*
[2]*Department of Physics and Astronomy, University of California, Irvine, California 92697-4575, USA*
[3]*ENEA C. R. Frascati, C. P. 65-00044 Frascati, Italy*

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Generation of zonal flow (ZF) by energetic particle (EP) driven toroidal Alfvén eigenmode (TAE) is investigated using nonlinear gyrokinetic theory. It is found that nonlinear resonant EP contribution dominates over the usual Reynolds and Maxwell stresses due to thermal plasma nonlinear response. ZF can be forced driven in the linear growth stage of TAE, with the growth rate being twice the TAE growth rate. The ZF generation mechanism is shown to be related to polarization induced by resonant EP nonlinearity. The generated ZF has both the usual meso-scale and micro-scale radial structures. Possible consequences of this forced driven ZF on the nonlinear dynamics of TAE are also discussed. *Published by AIP Publishing*. [http://dx.doi.org/10.1063/1.4962997]

PHYSICS OF PLASMAS 23, 053503 (2016)

Theory for the anomalous electron transport in Hall effect thrusters. II. Kinetic model

T. Lafleur,[1,2,a)] S. D. Baalrud,[3] and P. Chabert[1]

[1]*Laboratoire de Physique des Plasmas, CNRS, Sorbonne Universités, UPMC Univ Paris 06, Univ Paris-Sud, Ecole Polytechnique, 91128 Palaiseau, France*
[2]*Centre National d'Etudes Spatiales (CNES), F-31401 Toulouse, France*
[3]*Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52242, USA*

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In Paper I [T. Lafleur *et al.*, Phys. Plasmas **23**, 053502 (2016)], we demonstrated (using particle-in-cell simulations) the definite correlation between an anomalously high cross-field electron transport in Hall effect thrusters (HETs), and the presence of azimuthal electrostatic instabilities leading to enhanced electron scattering. Here, we present a kinetic theory that predicts the enhanced scattering rate and provides an electron cross-field mobility that is in good agreement with experiment. The large azimuthal electron drift velocity in HETs drives a strong instability that quickly saturates due to a combination of ion-wave trapping and wave-convection, leading to an enhanced mobility many orders of magnitude larger than that expected from classical diffusion theory. In addition to the magnetic field strength, B_0, this enhanced mobility is a strong function of the plasma properties (such as the plasma density) and therefore does not, in general, follow simple $1/B_0^2$ or $1/B_0$ scaling laws. *Published by AIP Publishing*. [http://dx.doi.org/10.1063/1.4948496]

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Diagnosing collisionless energy transfer using field–particle correlations: Vlasov–Poisson plasmas

Gregory G. Howes[1,†], Kristopher G. Klein[2,3] and Tak Chu Li[1]

[1]Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52242, USA
[2]Department of Climate and Space Sciences and Engineering, University of Michigan, Ann Arbor, MI 48109, USA
[3]Space Science Center, University of New Hampshire, Durham, NH 03824, USA

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Turbulence plays a key role in the conversion of the energy of large-scale fields and flows to plasma heat, impacting the macroscopic evolution of the heliosphere and other astrophysical plasma systems. Although we have long been able to make

Linearized Vlasov Equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} f + \frac{e}{m} \boldsymbol{\nabla} \Phi \cdot \boldsymbol{\nabla}_{\mathbf{v}} f = 0 \tag{9.1.1}$$

$$\boldsymbol{\nabla}^2 \Phi = -\frac{\rho_q}{\epsilon_0} = -\frac{e}{\epsilon_0} \left[n_0 - \int_{-\infty}^{\infty} f \, \mathrm{d}^3 \upsilon \right]. \tag{9.1.2}$$

$$f(\mathbf{v}) = f_0(\mathbf{v}) + f_1(\mathbf{v}). \tag{9.1.3}$$

$$\frac{\partial f_1}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} f_1 + \frac{e}{m} \boldsymbol{\nabla} \Phi_1 \cdot \boldsymbol{\nabla}_{\mathbf{v}} f_0 = 0 \tag{9.1.4}$$

$$\boldsymbol{\nabla}^2 \Phi_1 = \frac{e}{\epsilon_0} \int_{-\infty}^{\infty} f_1(\mathbf{v}) \, \mathrm{d}^3 \upsilon. \tag{9.1.5}$$

Fourier-Laplace Transform

$$-\mathrm{i}\omega\tilde{f}+\mathrm{i}kv_z\tilde{f}+\mathrm{i}\frac{e}{m}k\tilde{\Phi}\frac{\partial f_0}{\partial v_z}=0 \tag{9.1.6}$$

$$k^2\tilde{\Phi}=-\frac{e}{\epsilon_0}\int_{-\infty}^{\infty}\tilde{f}(\mathbf{v})\,\mathrm{d}^3v. \tag{9.1.7}$$

$$\tilde{f}=\frac{-1}{(kv_z-\omega)}\frac{e}{m}k\tilde{\Phi}\frac{\partial f_0}{\partial v_z}, \tag{9.1.8}$$

$$k^2\tilde{\Phi}=\frac{e^2}{\epsilon_0 m}k\tilde{\Phi}\int_{-\infty}^{\infty}\frac{(\partial f_0/\partial v_z)}{(kv_z-\omega)}\,\mathrm{d}^3v. \tag{9.1.9}$$

$$\left[1-\frac{e^2}{\epsilon_0 mk^2}\int_{-\infty}^{\infty}\frac{\partial f_0/\partial v_z}{(v_z-\omega/k)}\,\mathrm{d}^3v\right]\tilde{\Phi}=0. \tag{9.1.10}$$

$$Đ(k,\omega)=1-\frac{e^2}{\epsilon_0 mk^2}\int_{-\infty}^{\infty}\frac{\partial f_0/\partial v_z}{(v_z-\omega/k)}\,\mathrm{d}^3v=0. \tag{9.1.11}$$

Fourier-Laplace Transform

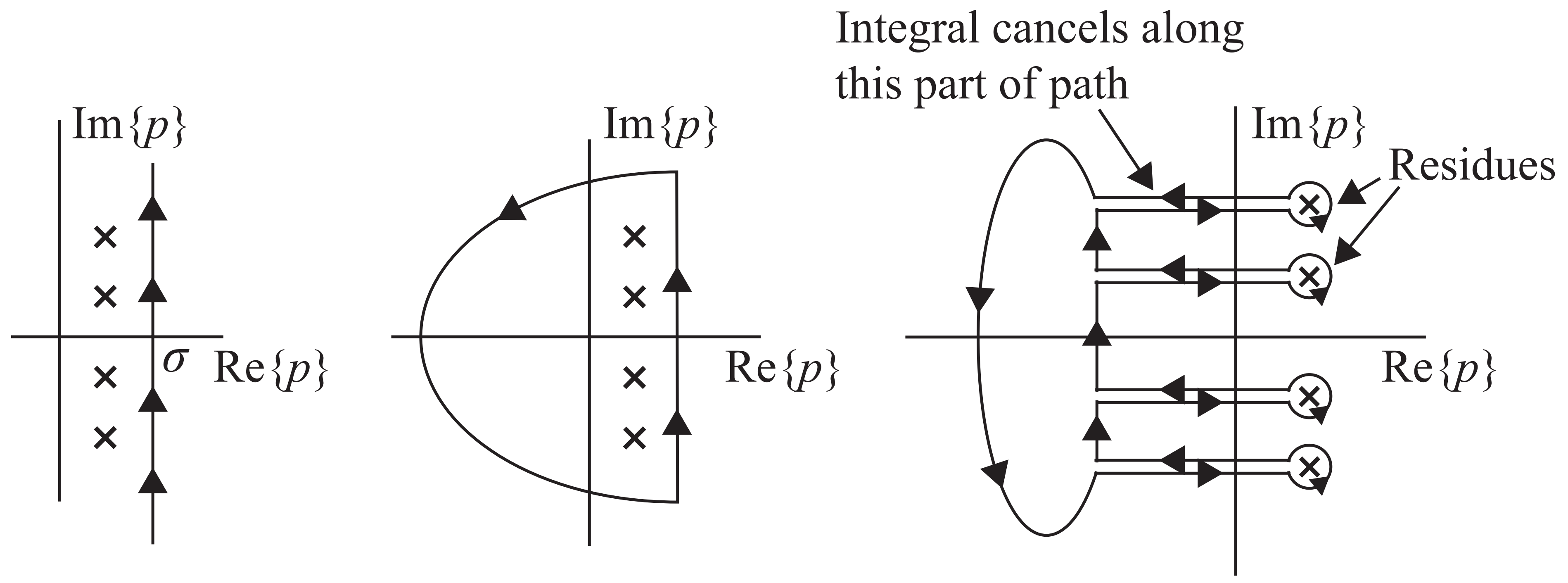


Figure 9.6 The final stage in evaluating the inverse Laplace transform involves distorting the integration contour around the poles and arranging for the integral to cancel around the rest of the contour. The resulting integrals around the poles are called the residues.

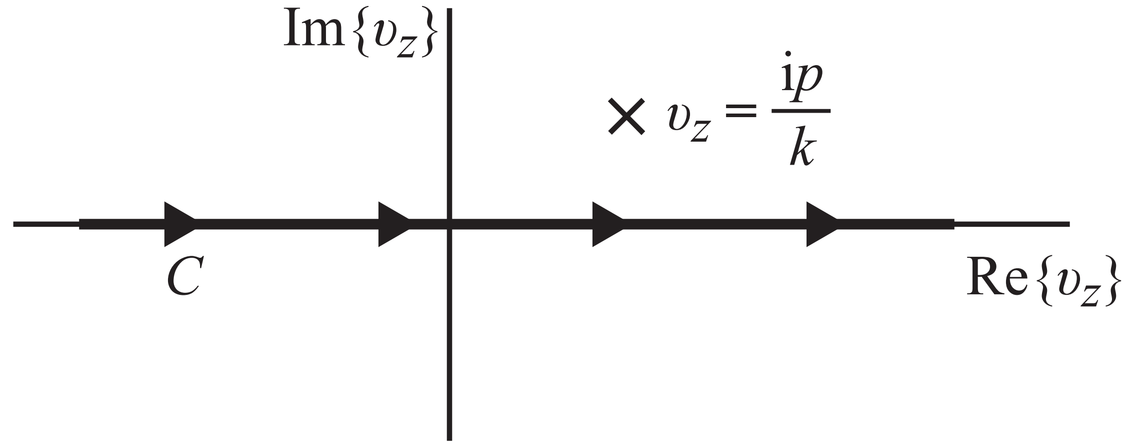


Figure 9.7 For Re$\{p\}$ positive the integration contour in $Đ(k, p)$ is along the Re$\{\upsilon_z\}$ axis.

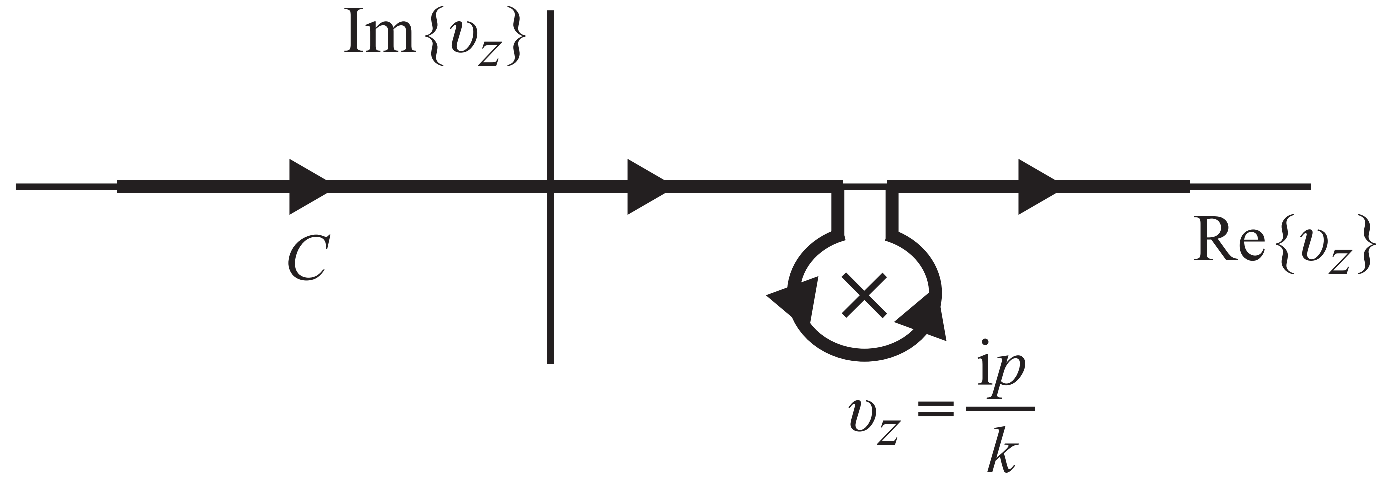


Figure 9.8 To continue $Đ(k, p)$ analytically into the left half of the complex p-plane, the integration contour must be distorted such that it passes below the pole at $\upsilon_z = \mathrm{i}p/k$. This diagram is for $k > 0$.

Electrostatic Dispersion Relation for a Plasma

$$\int_{-\infty}^{\infty} \frac{\partial F_0/\partial v_z}{(v_z-\omega/k)}\,\mathrm{d}v_z = \left[\frac{F_0}{(v_z-\omega/k)}\right]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \frac{F_0}{(v_z-\omega/k)^2}\,\mathrm{d}v_z. \tag{9.1.14}$$

$$Đ(k,\omega) = 1 - \frac{\omega_{\mathrm{p}}^2}{k^2}\int_{-\infty}^{\infty} \frac{\partial F_0/\partial v_z}{(v_z-\omega/k)}\,\mathrm{d}v_z = 0. \tag{9.1.13}$$

$$Đ(k,\omega) = 1 - \frac{\omega_{\mathrm{p}}^2}{k^2}\int_{-\infty}^{\infty} \frac{F_0}{(v_z-\omega/k)^2}\,\mathrm{d}v_z = 0. \tag{9.1.15}$$

Tom O'Neil (1965)

I. REVIEW OF THE LINEAR THEORY OF COLLISIONLESS DAMPING

THE basic equations for the problem are the Vlasov equation for the electron distribution and Poisson's equation. Dawson's model of collisionless damping[1] also uses the equation expressing conservation of energy which is easily derived from the above two equations. The ions cannot participate in the high-frequency plasma oscillations and just form a uniform background charge. Also, the coordinates perpendicular to the propagation vector of the wave may be integrated out of the Vlasov equation at the outset; so, it is only necessary to consider the problem in one dimension.

* This work was submitted in partial fulfillment of the requirements for the Ph.D. degree, University of California, San Diego.
[1] J. Dawson, Phys. Fluids **4**, 869 (1961).
[2] The formalism used in this section is different from that used by Dawson; however, it is quite similar to the quasi-linear formalism of W. E. Drummond and D. Pines.

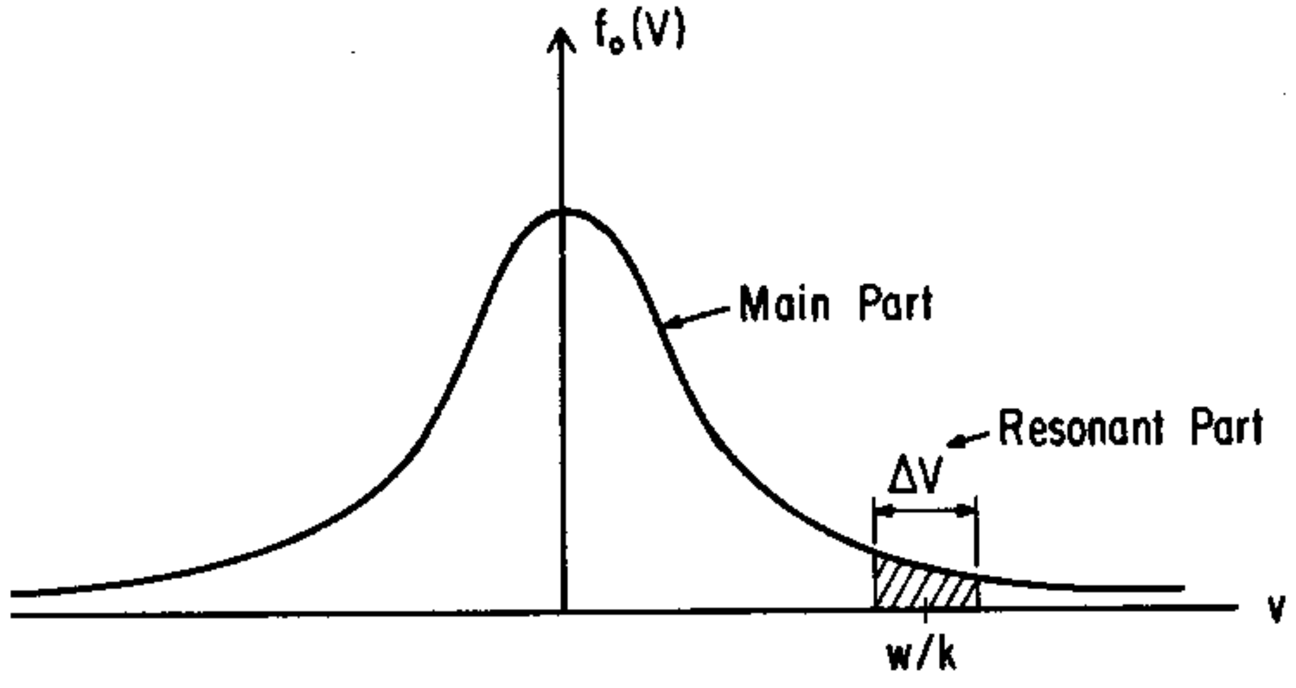


FIG. 1. The division of the electron distribution into a main part and a resonant part.

To explain the mechanism of collisionless damping, Dawson divides the electron distribution into a main part and a resonant part (see Fig. 1). He shows that the main part of the distribution supports the oscillatory motion of the plasma wave and that the resonant part of the distribution damps the wave. To get the damping coefficient, he first calculates the rate of increase of the kinetic energy of the resonant electrons. By invoking the conservation of energy, he sets this rate of increase of kinetic energy equal to the rate of decrease of wave energy. The latter quantity immediately gives the damping coefficient of the wave.

Electrostatic Dispersion Relation for a Plasma

$$Ð(k,\omega) = 1 - \frac{\omega_{\mathrm{p}}^2}{k^2} \int_{-\infty}^{\infty} \frac{\partial F_0/\partial v_z}{(v_z - \omega/k)}\, \mathrm{d}v_z = 0. \tag{9.1.13}$$

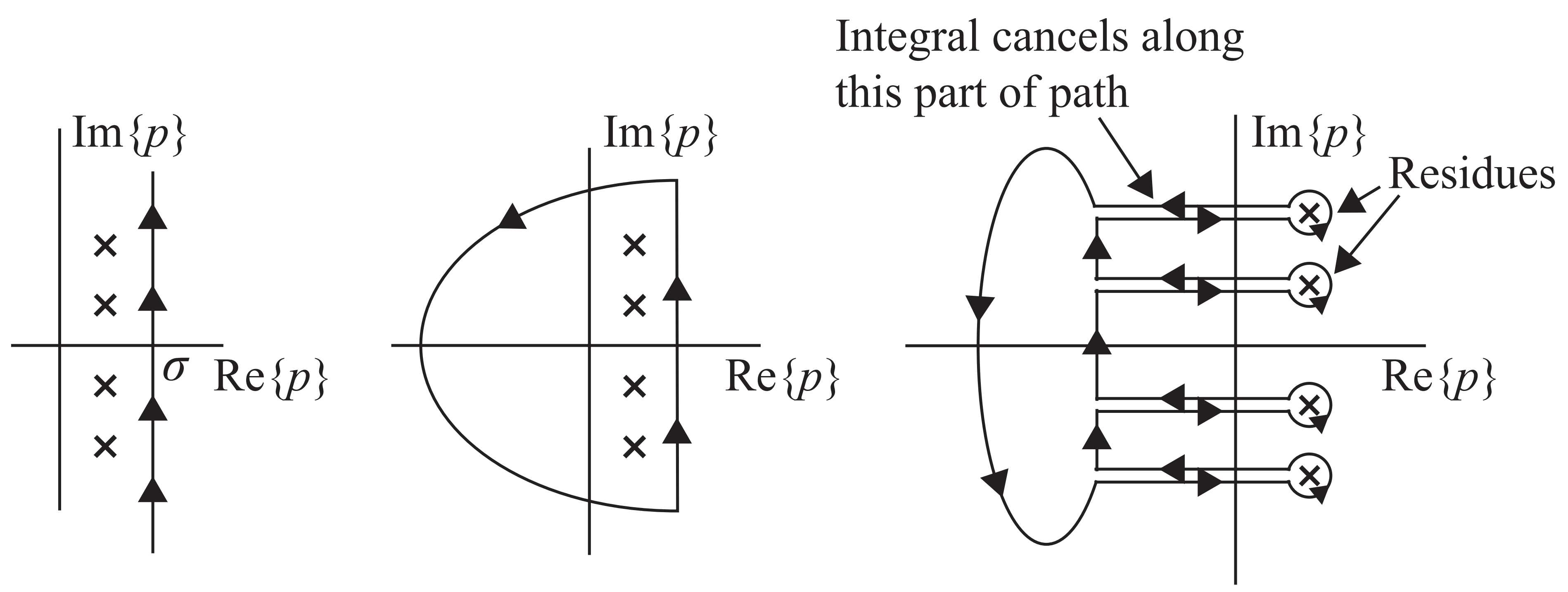
$$\int_{-\infty}^{\infty} \frac{\partial F_0/\partial v_z}{(v_z - \omega/k)}\, \mathrm{d}v_z = \left[\frac{F_0}{(v_z - \omega/k)}\right]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \frac{F_0}{(v_z - \omega/k)^2}\, \mathrm{d}v_z. \tag{9.1.14}$$

$$Ð(k,\omega) = 1 - \frac{\omega_{\mathrm{p}}^2}{k^2} \int_{-\infty}^{\infty} \frac{F_0}{(v_z - \omega/k)^2}\, \mathrm{d}v_z = 0. \tag{9.1.15}$$

Electrostatic Dispersion Relation for a Plasma

$$Đ(k, \omega) \cong 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}\left(1 + 3\frac{k^2}{\omega^2}\langle v_z^2 \rangle\right) = 0, \tag{9.1.20}$$

Fourier-Laplace Transform



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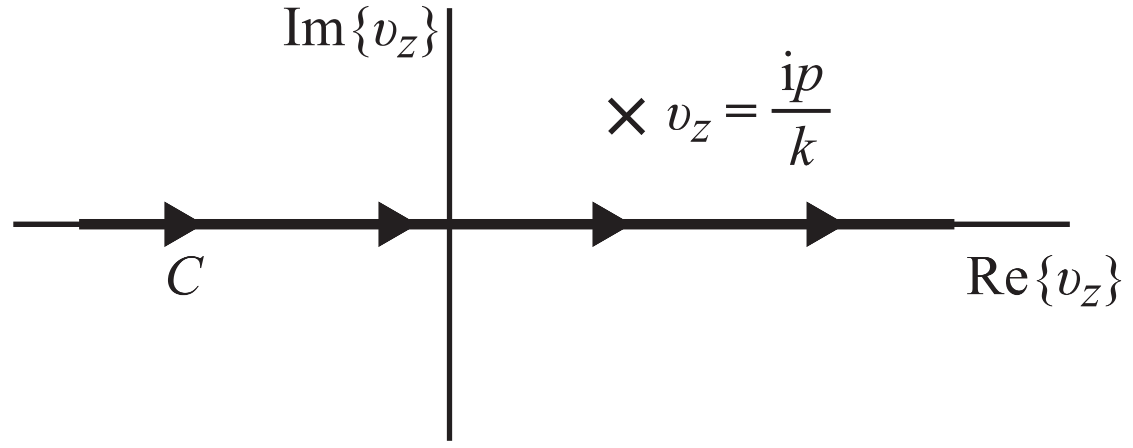

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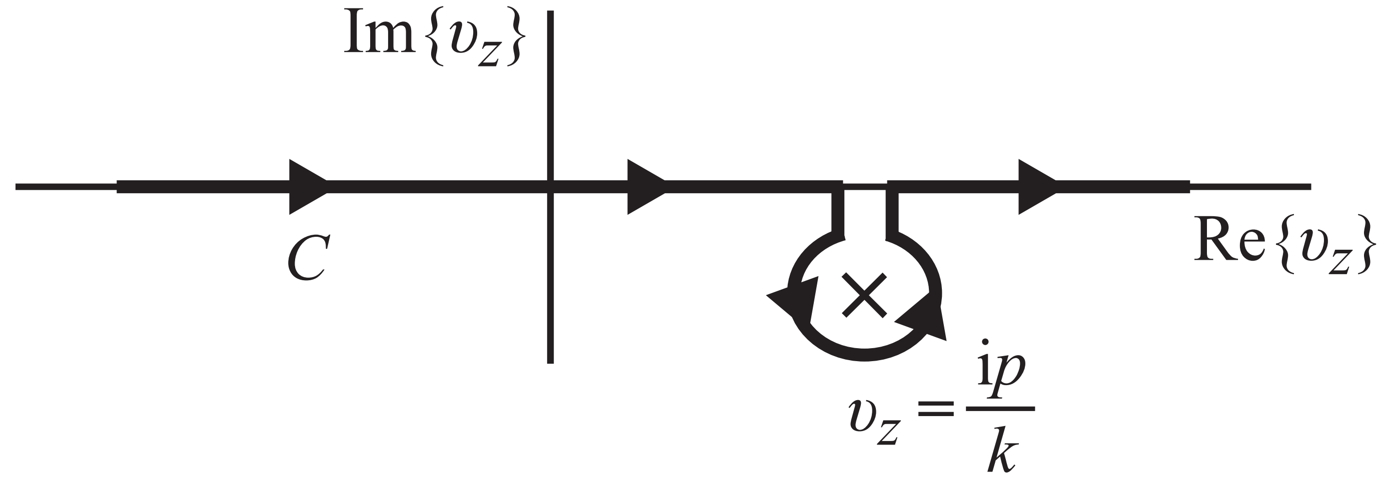

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Fourier-Laplace Transform

$$\frac{\partial f_1}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} f_1 + \frac{e}{m} \boldsymbol{\nabla} \Phi_1 \cdot \boldsymbol{\nabla}_{\mathbf{v}} f_0 = 0 \qquad (9.1.4)$$

$$p\tilde{f} - f(0) + \mathrm{i}k\upsilon_z \tilde{f} + \mathrm{i}\frac{e}{m}\, k\tilde{\Phi}\, \frac{\partial f_0}{\partial \upsilon_z} = 0 \qquad (9.2.15)$$

$$k^2\tilde{\Phi} = -\frac{e}{\epsilon_0} \int_{-\infty}^{\infty} \tilde{f}\, \mathrm{d}^3\upsilon, \qquad (9.2.16)$$

$$\boxed{\begin{gathered} \tilde{f} = \frac{-\mathrm{i}(e/m)\, k\tilde{\Phi}\, (\partial f_0/\partial \upsilon_z) + f(0)}{p + \mathrm{i}k\upsilon_z}. \qquad (9.2.17) \\[2em] k^2\tilde{\Phi} = -\frac{en_0}{\epsilon_0} \int_{-\infty}^{\infty} \frac{F(0)}{p + \mathrm{i}k\upsilon_z}\, \mathrm{d}\upsilon_z + \mathrm{i}\omega_{\mathrm{p}}^2\, k\tilde{\Phi} \int_{-\infty}^{\infty} \frac{\partial F_0/\partial \upsilon_z}{p + \mathrm{i}k\upsilon_z}\, \mathrm{d}\upsilon_z, \qquad (9.2.18) \end{gathered}}$$

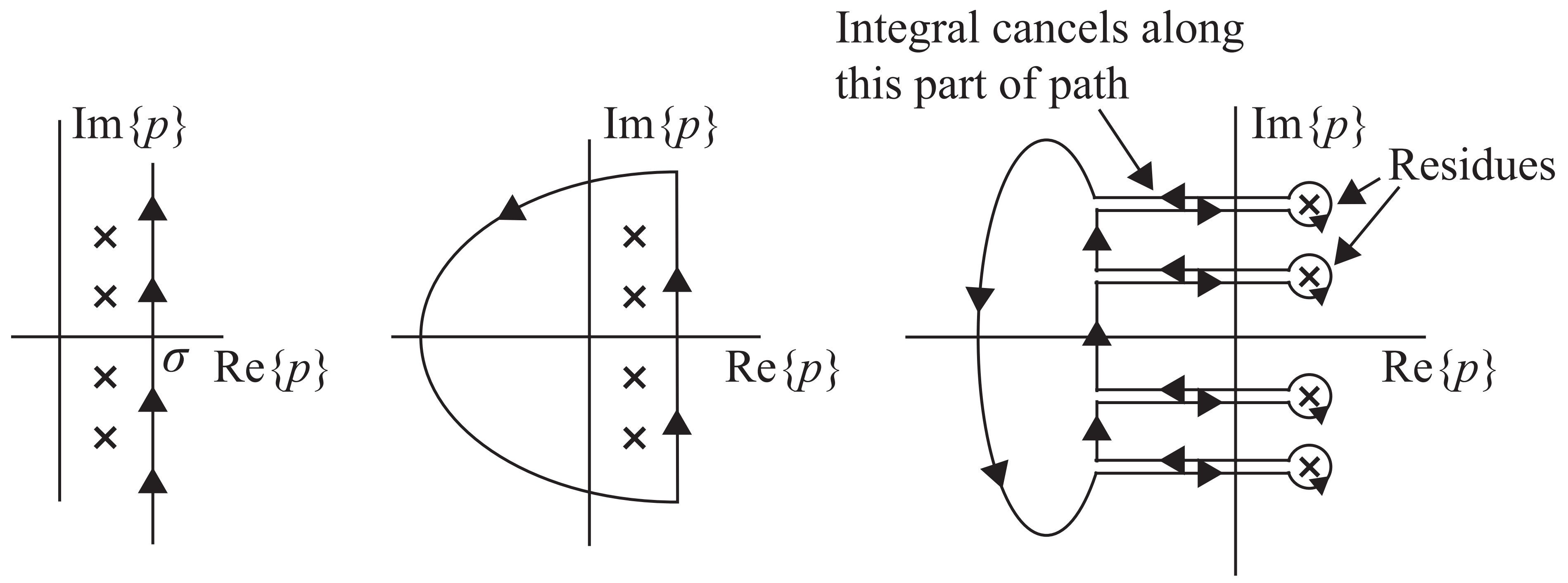
Fourier-Laplace Transform

$$\tilde{\Phi}(k,p) = \frac{N(k,p)}{Đ(k,p)}, \tag{9.2.19}$$

$$N(k,p) = \mathrm{i}\,\frac{en_0}{\epsilon_0 k^3} \int_{-\infty}^{\infty} \frac{F(0)}{\upsilon_z - \mathrm{i}p/k}\, \mathrm{d}\upsilon_z \tag{9.2.20}$$

$$Đ(k,p) = 1 - \frac{\omega_{\mathrm{p}}^2}{k^2} \int_{-\infty}^{\infty} \frac{\partial F_0/\partial \upsilon_z}{\upsilon_z - \mathrm{i}p/k}\, \mathrm{d}\upsilon_z. \tag{9.2.21}$$

Fourier-Laplace Transform



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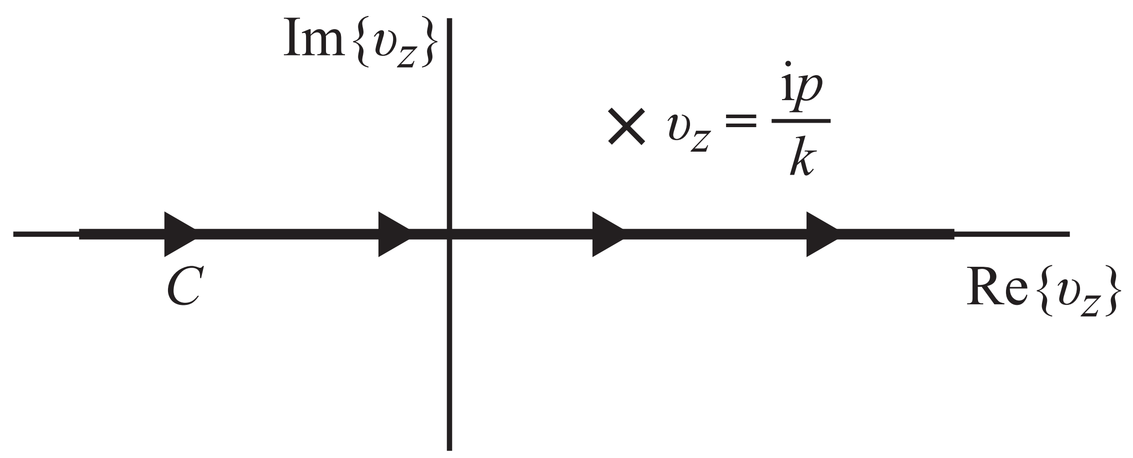

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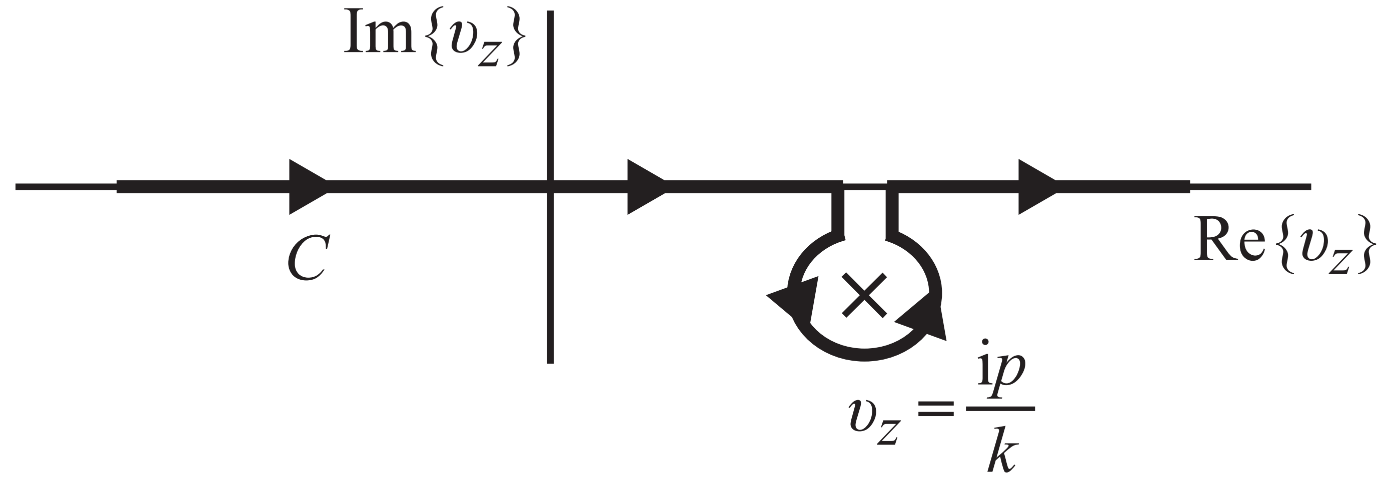

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Fourier-Laplace Transform

$$F_0(v_z) = \frac{C}{\pi}\frac{1}{C^2 + v_z^2} \tag{9.2.28}$$

$p = -\,i\,\omega = -\,i\,\omega_r + \omega_i$

$$Đ(k,p) = 1 - \frac{\omega_p^2}{k^2}\int_C \frac{F_0(v_z)}{(v_z - \mathrm{i}p/k)^2}\,\mathrm{d}v_z = 0. \tag{9.2.29}$$

$$Đ(k,p) = 1 - \frac{\omega_p^2}{k^2}\frac{C}{\pi}\int_C \frac{\mathrm{d}v_z}{(v_z - \mathrm{i}C)(v_z + \mathrm{i}C)(v_z - \mathrm{i}p/k)^2} = 0. \tag{9.2.30}$$

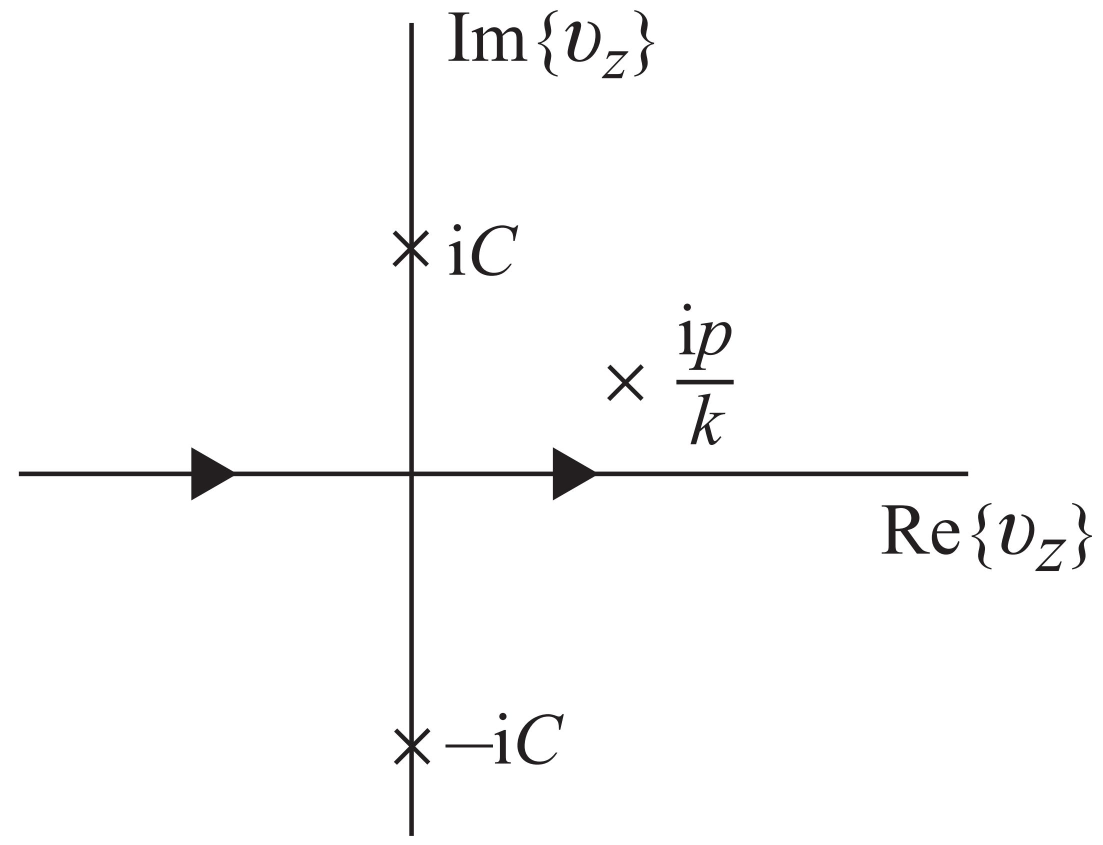


Figure 9.9 The location of the poles in the integral for $Đ(k,p)$ for the Cauchy distribution function. This diagram assumes that Re$\{p\}$ is positive and that $k > 0$.

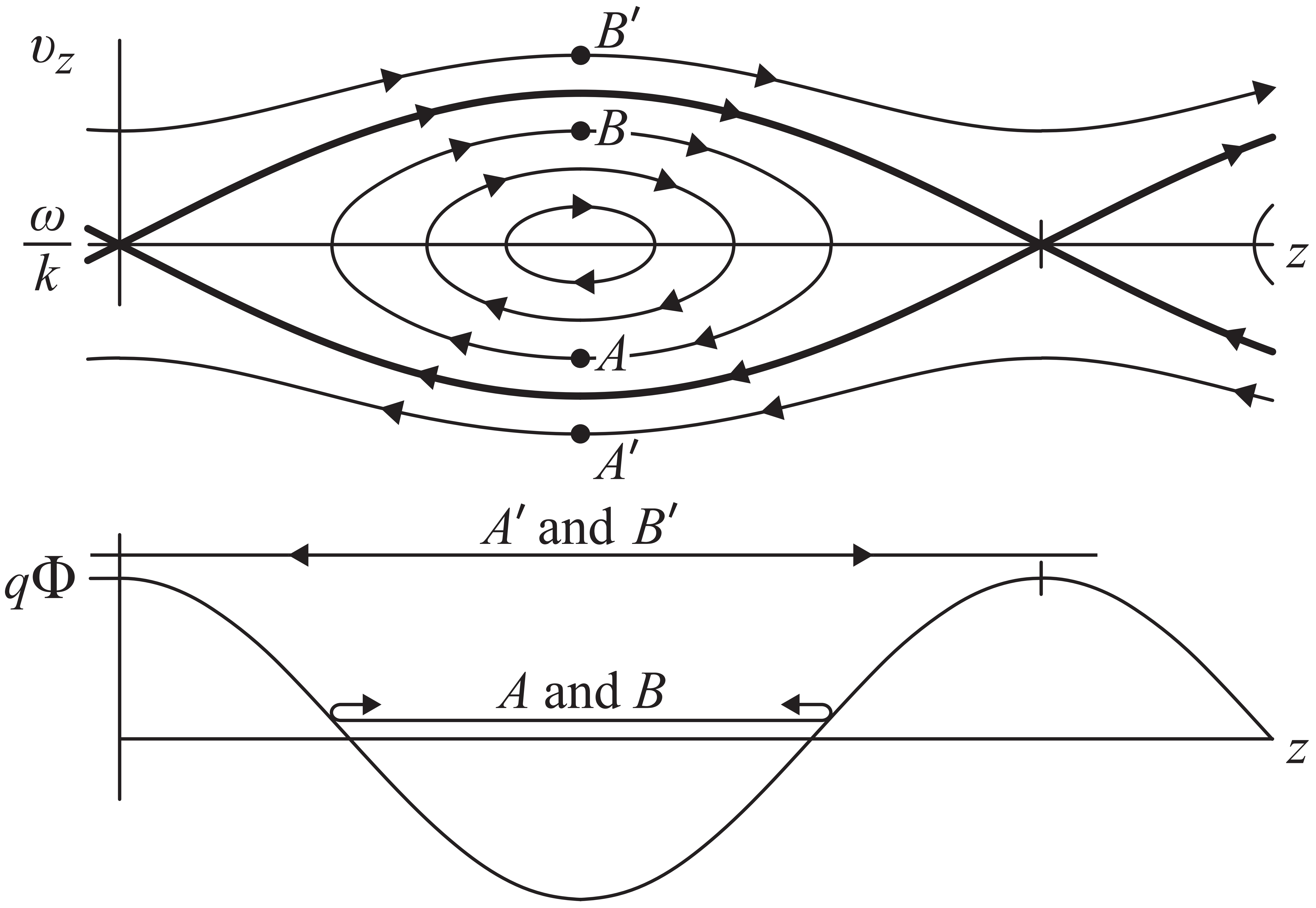


Figure 9.13 Phase-space trajectories in a frame of reference (z, v_z) moving at the phase velocity for a sinusoidal electrostatic potential $\Phi(z) = \Phi_0 \cos kz$.

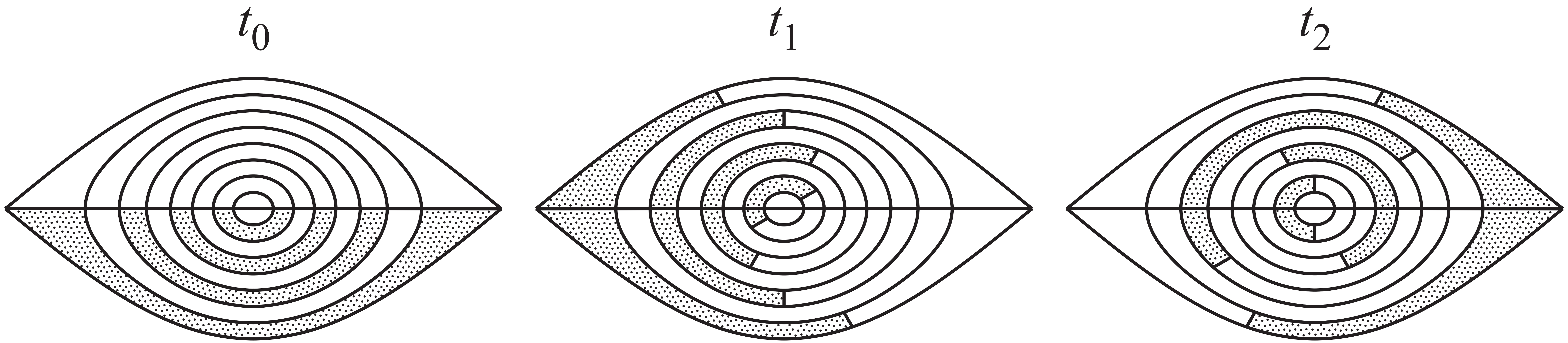
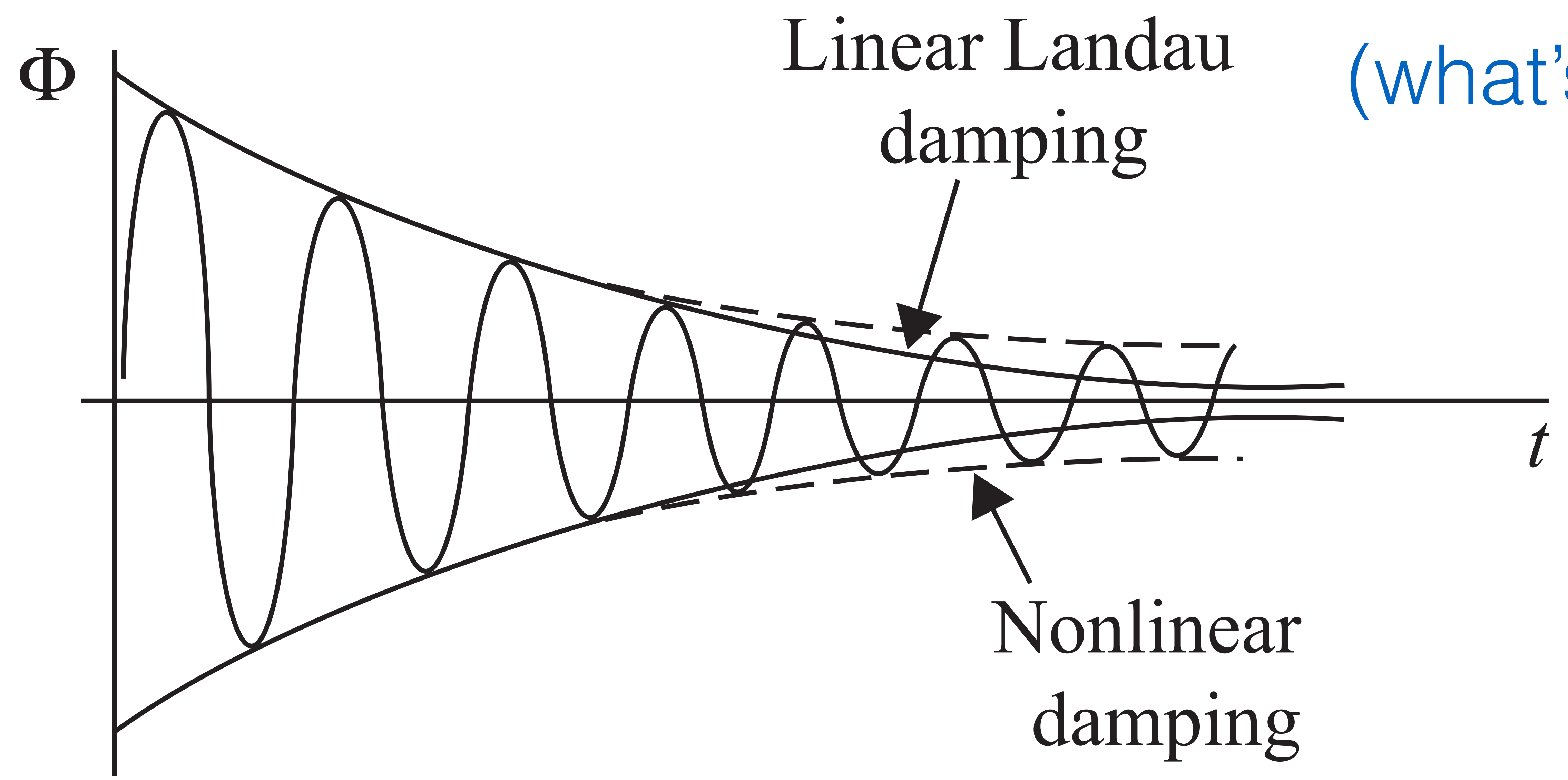


Figure 9.14 The relative phase-space locations of trapped particles at three successive phases of the bounce cycle.

Bounce (Trapping) Frequency

$$0 < t \ll \omega_{\mathrm{b}}^{-1} = \frac{1}{k}\sqrt{\frac{m}{q\Phi_0}}. \tag{9.2.53}$$



(what’s wrong with this picture?)

Figure 9.15 The nonlinear effects of particle trapping tend to increase the wave amplitude relative to the predictions of linear Landau damping.

https://doi.org/10.1103/PhysRevLett.13.184

COLLISIONLESS DAMPING OF ELECTROSTATIC PLASMA WAVES*

J. H. Malmberg and C. B. Wharton
John Jay Hopkins Laboratory for Pure and Applied Science,
General Atomic Division of General Dynamics Corporation, San Diego, California
(Received 6 July 1964)

It has been predicted by Landau[1] that electrostatic electron waves in a plasma of finite temperature will be damped, even in the absence of collisions. Landau's theory has been challenged on various grounds[2] and a number of experiments designed to detect the effect for electrostatic electron waves or ion acoustic waves have been reported.[3] The existence of the damping is of interest not only for its own sake, but because the method of calculation has been widely used for related problems. We report here preliminary results of an experiment designed to measure the Landau damping of electrostatic electron waves. We observe heavy damping which exhibits the expected dependence on phase velocity.

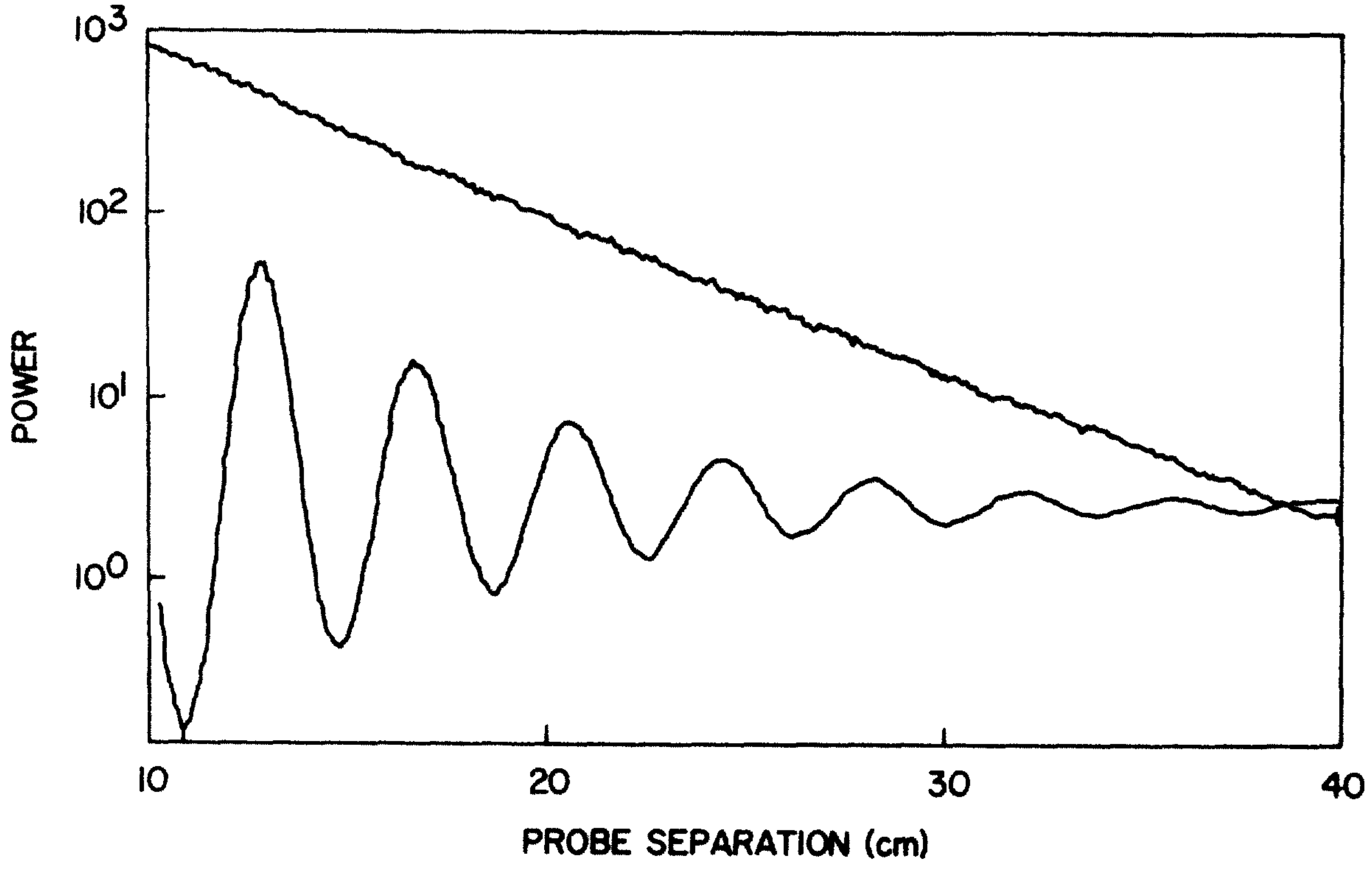


FIG. 1. Raw data. Upper curve is the logarithm of received power. Lower curve is interferometer outpu Abscissa is probe separation.

Physics of Fluids 13, 2422 (1970); [https://doi.org/10.1063/1.1693255]

Nonlinear Development of the Beam–Plasma Instability

W. E. DRUMMOND
University of Texas at Austin, Austin, Texas 78712
J. H. MALMBERG
Gulf General Atomic Incorporated, San Diego, California and *University of California, San Diego, La Jolla, California 92037*
T. M. O'NEIL
University of California, San Diego, La Jolla, California 92037
AND
J. R. THOMPSON
University of Texas at Austin, Austin, Texas 78712
(Received 12 January 1970; final manuscript received 27 April 1970)

The nonlinear limit of wave growth induced by a low density cold electron beam in a collisionless plasma is calculated from a simple physical model. The bandwidth of the growing "noise" is so small that the beam interacts with a nearly sinusoidal electric field.

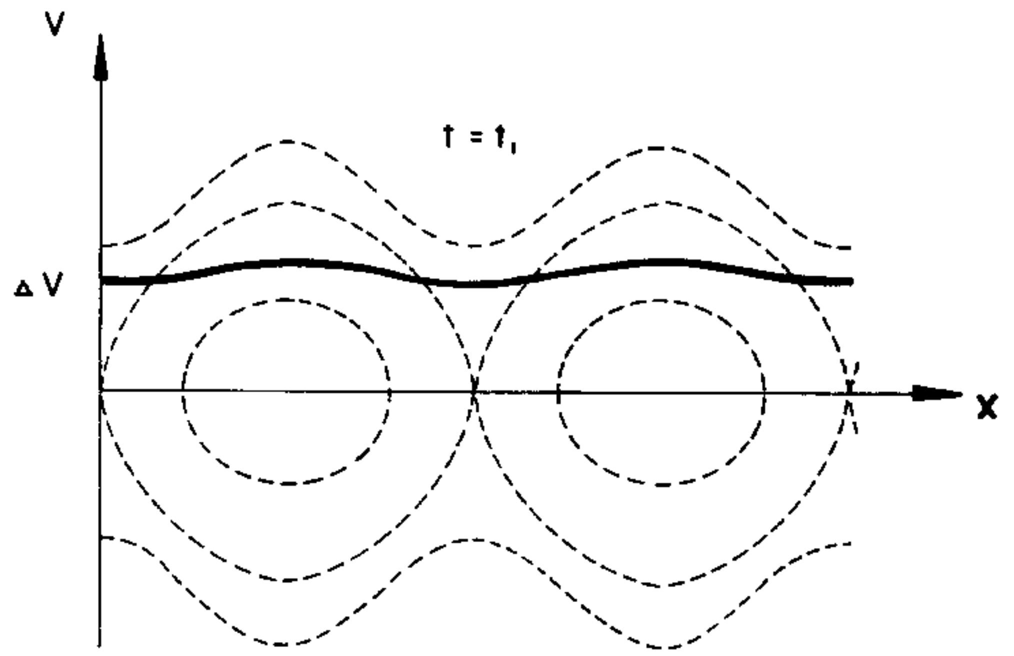


FIG. 1. Phase space.

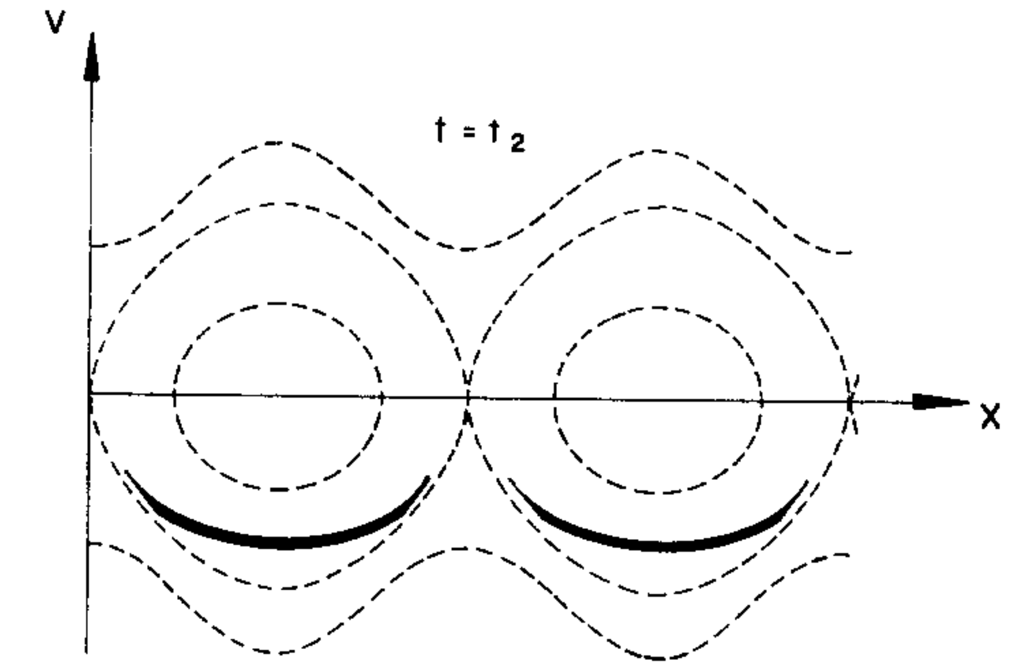


FIG. 2. Phase space.

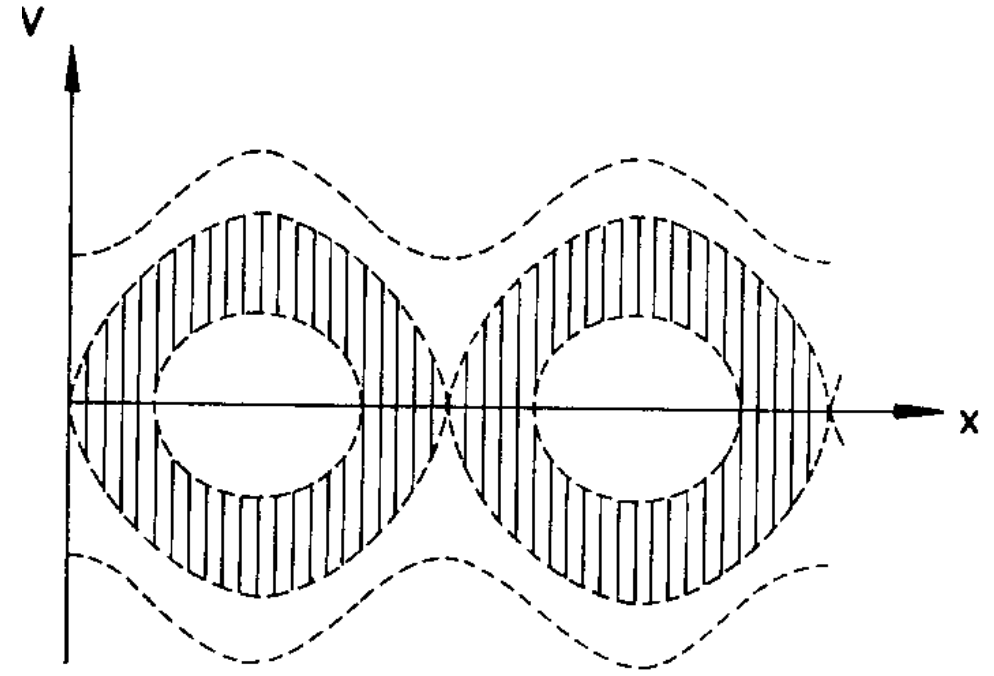


FIG. 3. Phase space.