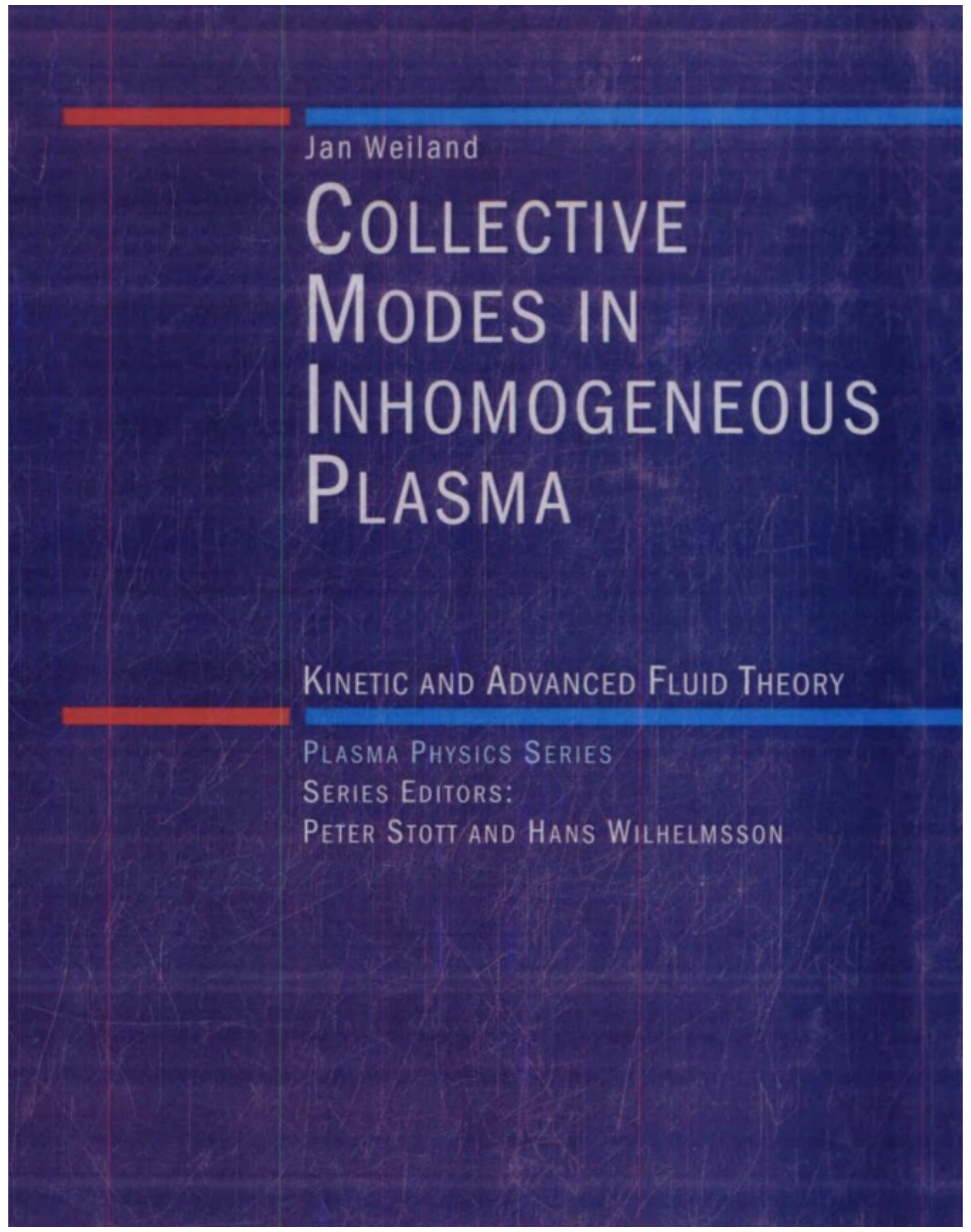
Plasma 2
Lecture 19:
Magnetic Response for Low-Frequency Plasma Dynamics

APPH E6102y
Columbia University

Reference



Collective Modes in Inhomogeneous Plasmas: Kinetic and Advanced Fluid Theory (IOP Publishing 2000)

Jan Weiland

Collective Modes in Inhomogeneous Plasmas: Kinetic and Advanced Fluid Theory presents the collective drift and MHD-type modes in inhomogeneous plasmas from the point of view of two-fluid and kinetic theory. Written by an internationally respected plasma transport theoretician, this introductory monograph emphasizes the description of the plasma rather than the geometry to present a more general approach to a large class of plasma problems. Starting with generalized fluid equations for low frequency phenomena, the author shows how drift waves and MHD-type modes can arise from the effects of inhomogeneities in the plasma. The kinetic description is then presented to reveal a host of phenomena ranging from vortex modes and finite Larmor radius effects to trapped and fast particle instabilities, transport, diffusion, and other advanced fluid effects. Theoretical and computational plasma physicists modeling confined plasmas will find this illustrated book a very valuable addition to their collection.

https://research.chalmers.se/en/person/elfjw

Low-Frequency (Electro-)Magnetic Response in a Strongly Magnetized Plasma

- ALFVEN WAVES (REVIEW) (From *Lecture-9*)
- MAGNETIC INDUCTIONS (FARADAY'S LAW)
- WHEN $E_{\parallel} = 0$ ∴ THE IDEAL MHD CONDITION

  AND WHEN NOT $E_{\parallel} \neq 0$

SHORT WAVELENGTH LOW FREQ

DRIFT WAVES $\tilde{E}_{\parallel} = -i k_{\parallel} \tilde{\phi}$ $k^2 \rho^2 \sim 1$ & $k_{\parallel} v_A \gg \omega$

MHD WAVES $\tilde{E}_{\parallel} = 0$ $k_{\parallel} \to 0$ OR $k^2 \rho^2 \ll 1$

INTERCHANGE FLUTE-LIKE

LONG WAVELENGTH

6.5 Magnetohydrodynamic Waves

(From *Lecture-9*)

$$\frac{\partial \rho_{\mathrm{m1}}}{\partial t} + \rho_{\mathrm{m0}} \boldsymbol{\nabla} \cdot \mathbf{U}_1 = 0, \tag{6.5.1}$$

$$\rho_{\mathrm{m0}} \frac{\partial \mathbf{U}_1}{\partial t} = \frac{1}{\mu_0} (\boldsymbol{\nabla} \times \mathbf{B}_1) \times \mathbf{B}_0 - \boldsymbol{\nabla} P_1, \tag{6.5.2}$$

$$\frac{\partial \mathbf{B}_1}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{U}_1 \times \mathbf{B}_0), \tag{6.5.3}$$

$$P_1 = \gamma \left(\frac{P_0}{\rho_{\mathrm{m0}}} \right) \rho_{\mathrm{m1}}. \tag{6.5.4}$$

$$V_{\mathrm{S}}^2 = \gamma \frac{P_0}{\rho_{\mathrm{m0}}} = \frac{\gamma \kappa T_0}{m}, \tag{6.5.5}$$

6.5 Magnetohydrodynamic Waves

$$-\mathrm{i}\omega\tilde{\rho}_{\mathrm{m}} + \mathrm{i}\rho_{\mathrm{m}0}\mathbf{k}\cdot\tilde{\mathbf{U}} = 0, \tag{6.5.6}$$

$$-\mathrm{i}\omega\rho_{\mathrm{m}0}\tilde{\mathbf{U}} = \frac{\mathrm{i}}{\mu_0}(\mathbf{k}\times\tilde{\mathbf{B}})\times\mathbf{B}_0 - \mathrm{i}\mathbf{k}\widetilde{P}, \tag{6.5.7}$$

$$-\mathrm{i}\omega\tilde{\mathbf{B}} = \mathrm{i}\mathbf{k}\times(\tilde{\mathbf{U}}\times\mathbf{B}_0), \tag{6.5.8}$$

$$\widetilde{P} = V_{\mathrm{S}}^2\,\tilde{\rho}_{\mathrm{m}}. \tag{6.5.9}$$

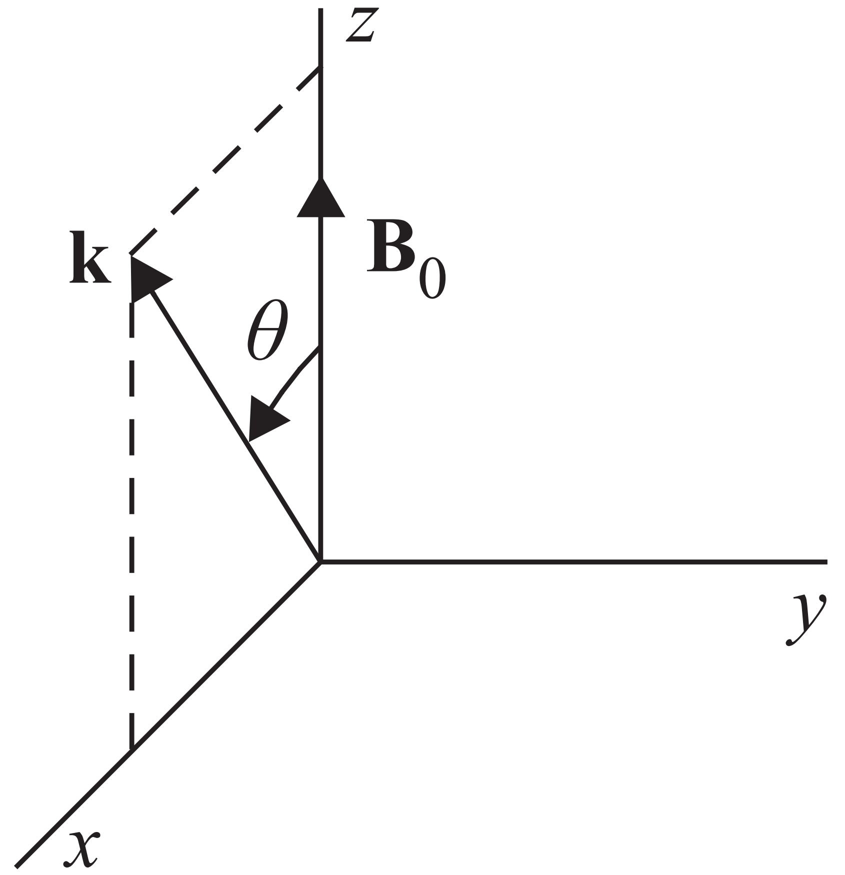
6.5 Magnetohydrodynamic Waves

$$\omega^2 \tilde{\mathbf{U}} = \frac{1}{\mu_0 \rho_{\mathrm{m}0}} \{\mathbf{k} \times (\mathbf{k} \times [\tilde{\mathbf{U}} \times \mathbf{B}_0])\} \times \mathbf{B}_0 + V_{\mathrm{S}}^2 \mathbf{k}(\mathbf{k} \cdot \tilde{\mathbf{U}}). \quad (6.5.12)$$

$$\left(\frac{\omega}{k}\right)^2 \begin{bmatrix} \widetilde{U}_x \\ \widetilde{U}_y \\ \widetilde{U}_z \end{bmatrix} = V_{\mathrm{A}}^2 \begin{bmatrix} \widetilde{U}_x \\ \widetilde{U}_y \cos^2\theta \\ 0 \end{bmatrix} + V_{\mathrm{S}}^2 \begin{bmatrix} \widetilde{U}_x \sin^2\theta + \widetilde{U}_z \sin\theta\cos\theta \\ 0 \\ \widetilde{U}_x \sin\theta\cos\theta + \widetilde{U}_z \cos^2\theta \end{bmatrix}$$

$$V_{\mathrm{A}} = B_0 / \sqrt{\mu_0 \rho_{\mathrm{m}0}}$$

$$\cos\theta = k_{||}/k$$
$$\sin\theta = k_{\perp}/k$$

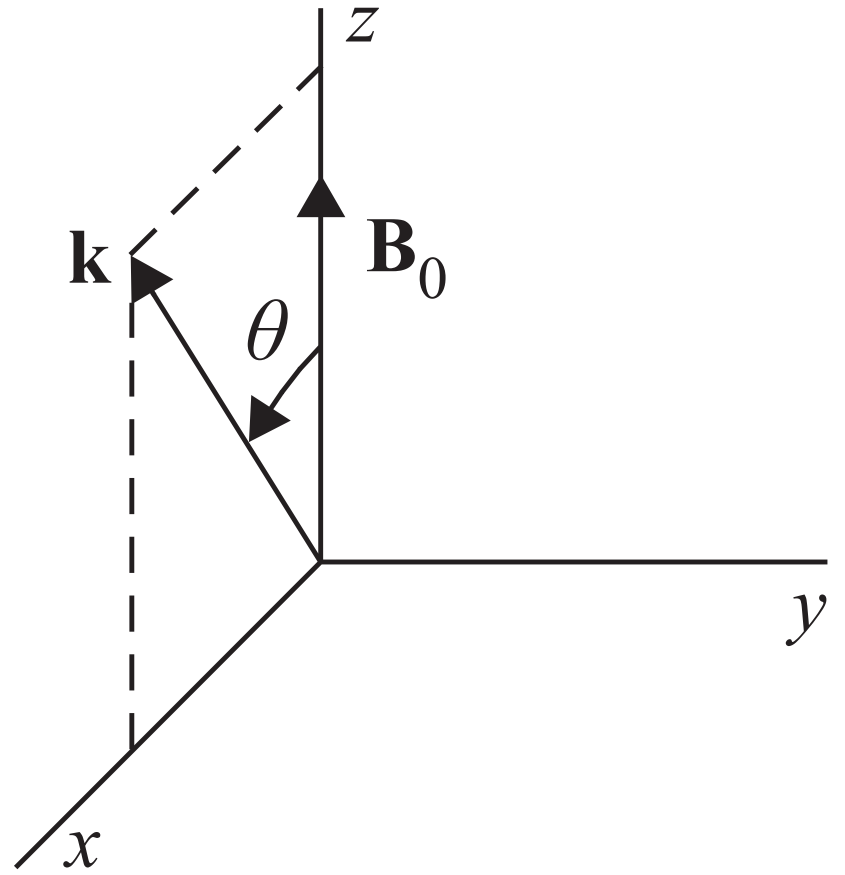


6.5 Magnetohydrodynamic Waves

$$\begin{bmatrix} \upsilon_{\mathrm{p}}^2 - V_{\mathrm{S}}^2 \sin^2\theta - V_{\mathrm{A}}^2 & 0 & -V_{\mathrm{S}}^2 \sin\theta\cos\theta \\ 0 & \upsilon_{\mathrm{p}}^2 - V_{\mathrm{A}}^2 \cos^2\theta & 0 \\ -V_{\mathrm{S}}^2 \sin\theta\cos\theta & 0 & \upsilon_{\mathrm{p}}^2 - V_{\mathrm{S}}^2 \cos^2\theta \end{bmatrix} \begin{bmatrix} \widetilde{U}_x \\ \widetilde{U}_y \\ \widetilde{U}_z \end{bmatrix} = 0. \qquad (6.5.15)$$

$$\cos\theta = k_{||}/k$$

$$\sin\theta = k_{\perp}/k$$



$\upsilon_{\mathrm{p}} = \omega/k$

6.5 Magnetohydrodynamic Waves

This equation has non-trivial solutions for $\tilde{\mathbf{U}}$ if and only if the determinant of the matrix is zero, which gives the dispersion relation

$$\mathcal{D}(k,\omega) = \left(v_{\mathrm{p}}^2 - V_{\mathrm{A}}^2\cos^2\theta\right)\left[v_{\mathrm{p}}^4 - v_{\mathrm{p}}^2\left(V_{\mathrm{A}}^2 + V_{\mathrm{S}}^2\right) + V_{\mathrm{A}}^2 V_{\mathrm{S}}^2\cos^2\theta\right] = 0. \qquad (6.5.16)$$

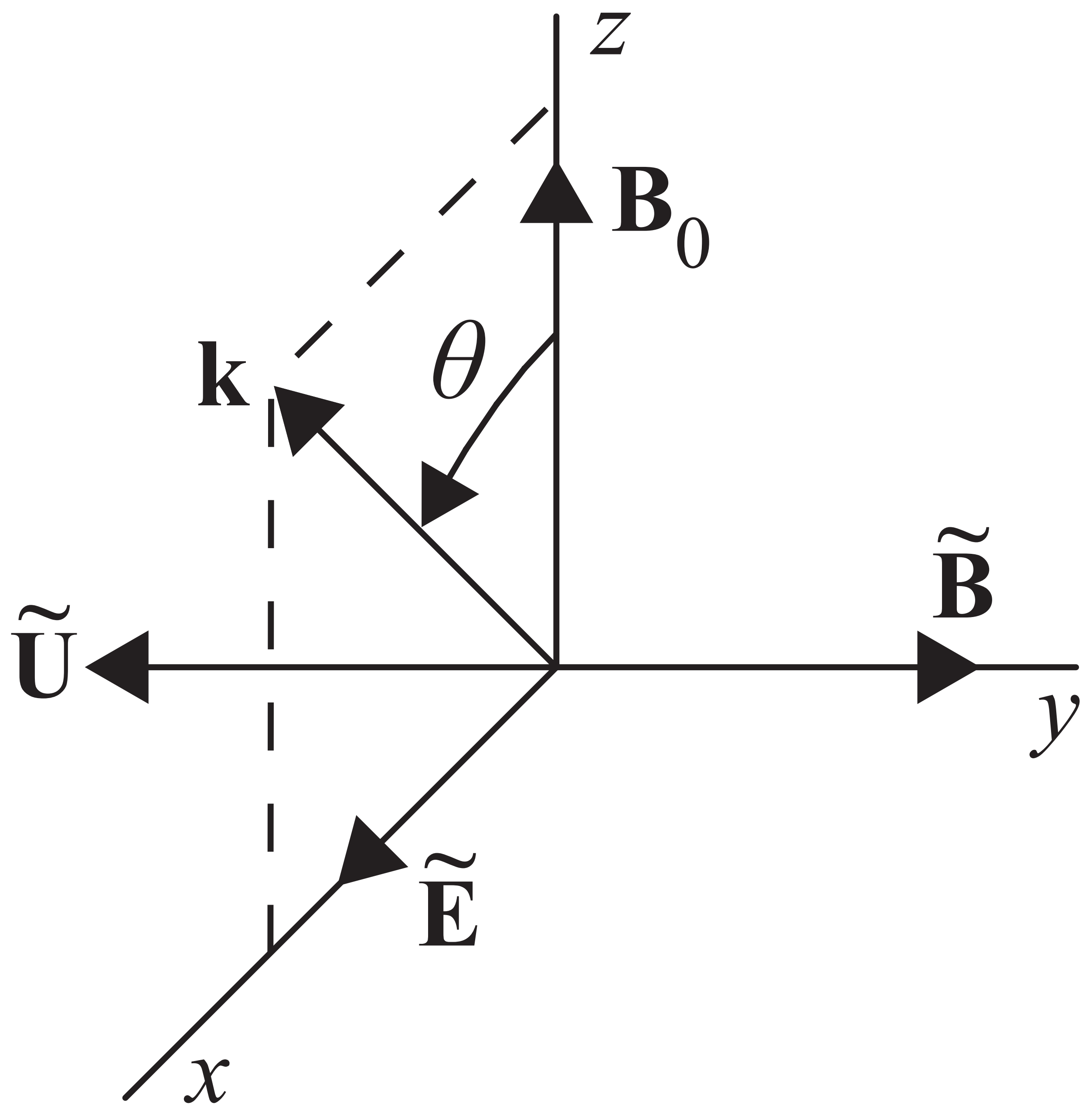
It can be shown that the dispersion relation has three roots:

$$v_{\mathrm{p}}^2 = \frac{1}{2}\left(V_{\mathrm{A}}^2 + V_{\mathrm{S}}^2\right) - \frac{1}{2}\left[\left(V_{\mathrm{A}}^2 - V_{\mathrm{S}}^2\right)^2 + 4V_{\mathrm{A}}^2 V_{\mathrm{S}}^2\sin^2\theta\right]^{1/2}, \qquad (6.5.17)$$

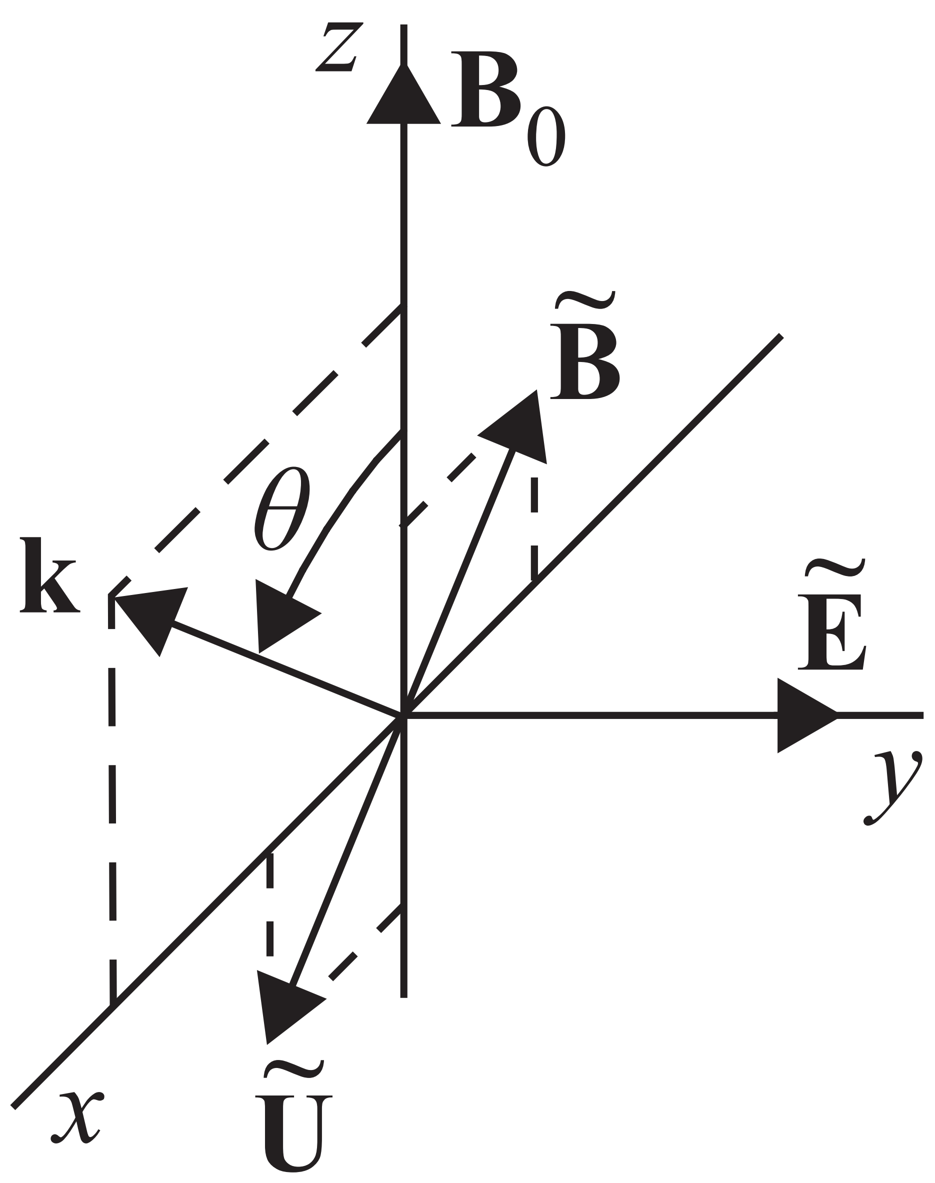
$$v_{\mathrm{p}}^2 = V_{\mathrm{A}}^2\cos^2\theta, \quad \leftarrow \text{Shear, or “transverse” Alfvén wave} \qquad (6.5.18)$$

$$v_{\mathrm{p}}^2 = \frac{1}{2}\left(V_{\mathrm{A}}^2 + V_{\mathrm{S}}^2\right) + \frac{1}{2}\left[\left(V_{\mathrm{A}}^2 - V_{\mathrm{S}}^2\right)^2 + 4V_{\mathrm{A}}^2 V_{\mathrm{S}}^2\sin^2\theta\right]^{1/2}. \qquad (6.5.19)$$

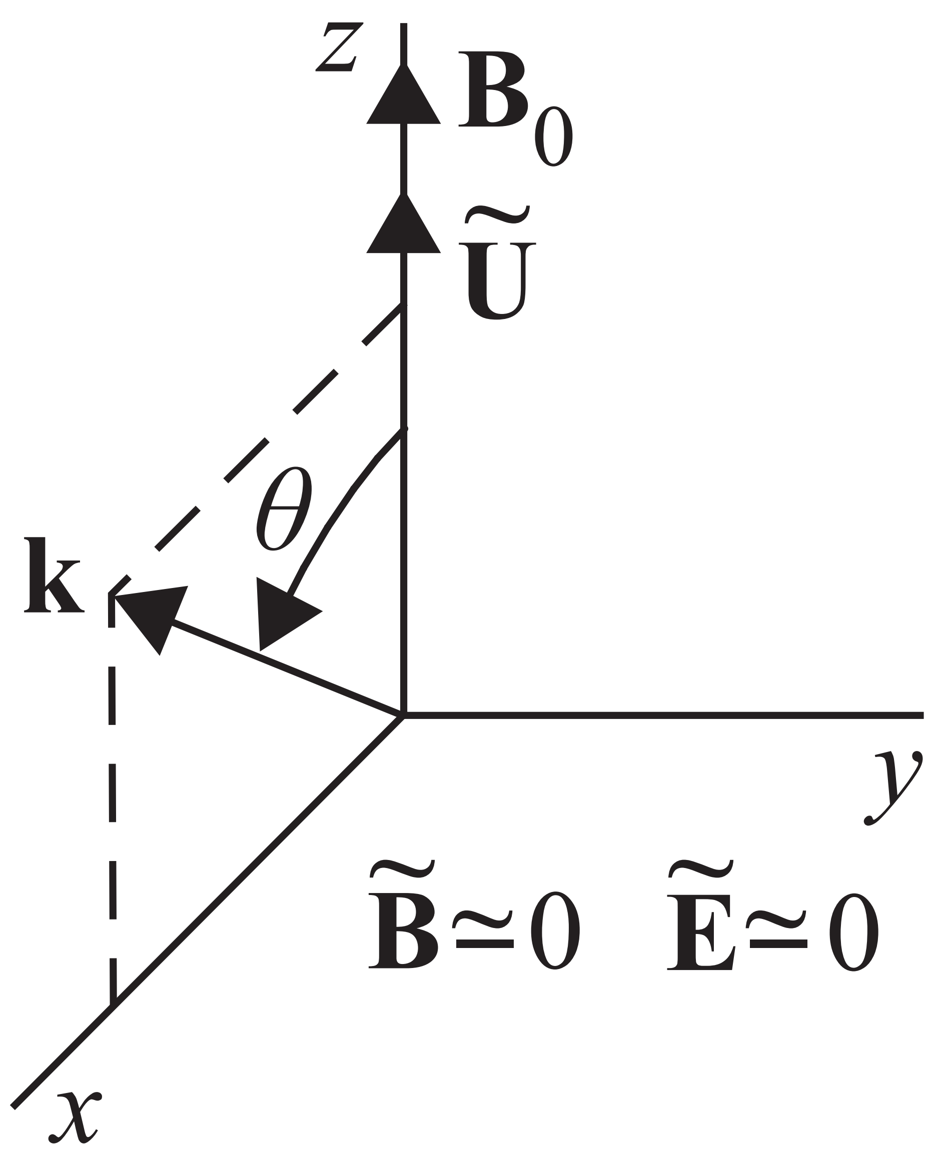
6.5 Magnetohydrodynamic Waves



Shear Alfvén
Wave



Fast magnetosonic
wave



Slow magnetosonic
wave

6.5 Magnetohydrodynamic Waves

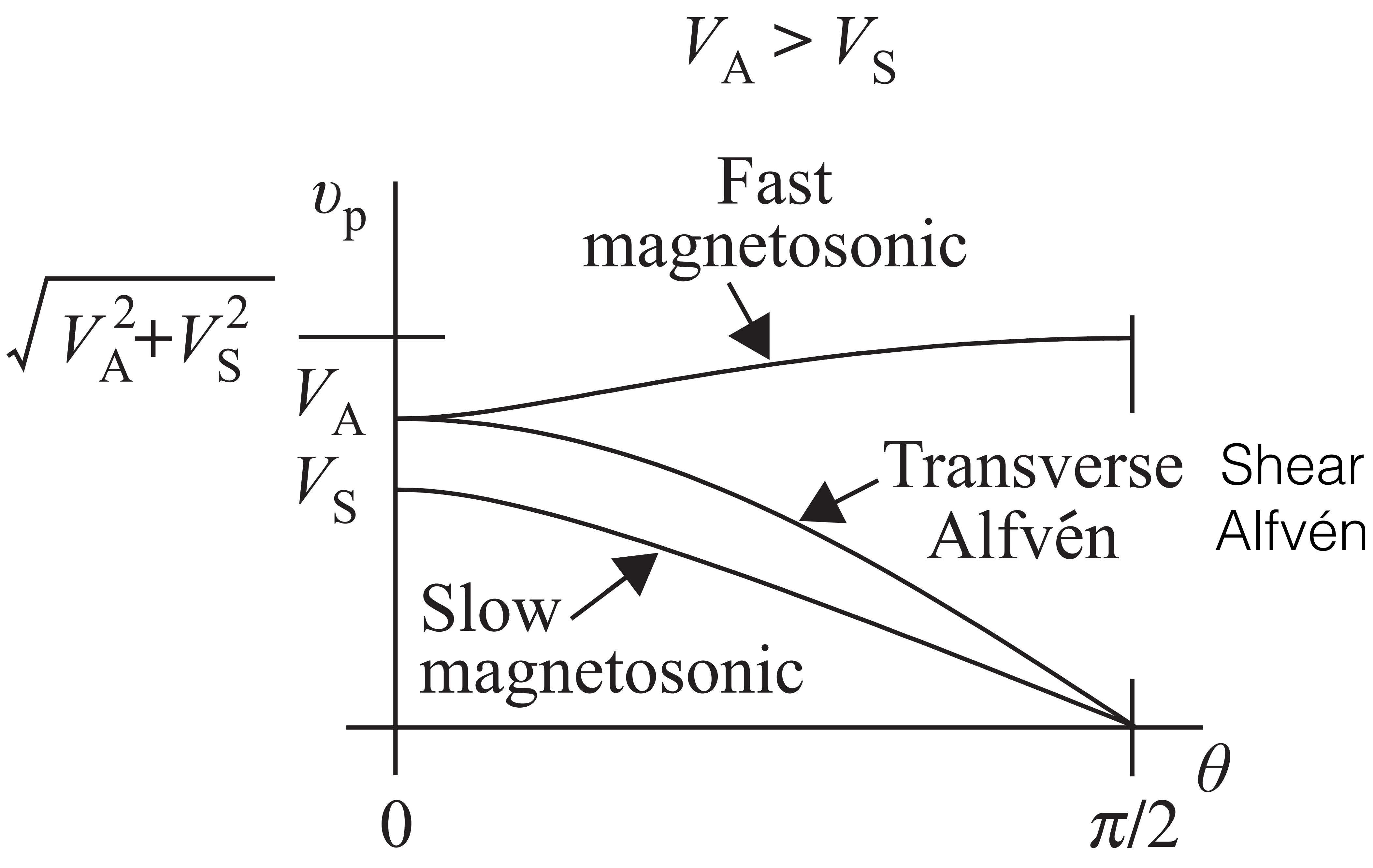


Figure 6.7 Plots of the phase velocities for the three MHD modes as a function of the wave normal angle for two cases, $V_A > V_S$ and $V_A < V_S$.

$$\beta \propto V_s^2 / V_A^2$$

$$v_p = \omega / k$$

Review: Shear Alfvén Wave (Basic Equations)

QUASI-NEUTRAL $\tilde{n}_i = \tilde{n}_e$ $\boxed{\nabla \cdot \bar{J} = 0}$

FARADAY'S LAW: $\nabla \times E = -\frac{\partial B}{\partial t} = -\nabla \times \frac{\partial \bar{A}}{\partial t}$ $(B = \nabla \times A)$

$$\nabla \times \left(E + \frac{\partial A}{\partial t}\right) = 0$$

$$\bar{E} = -\nabla \phi - \frac{\partial A}{\partial t}$$

$$\boxed{\tilde{E}_{\parallel} = -j k_{\parallel} \tilde{\Phi} + j \omega \tilde{A}_{\parallel}}$$

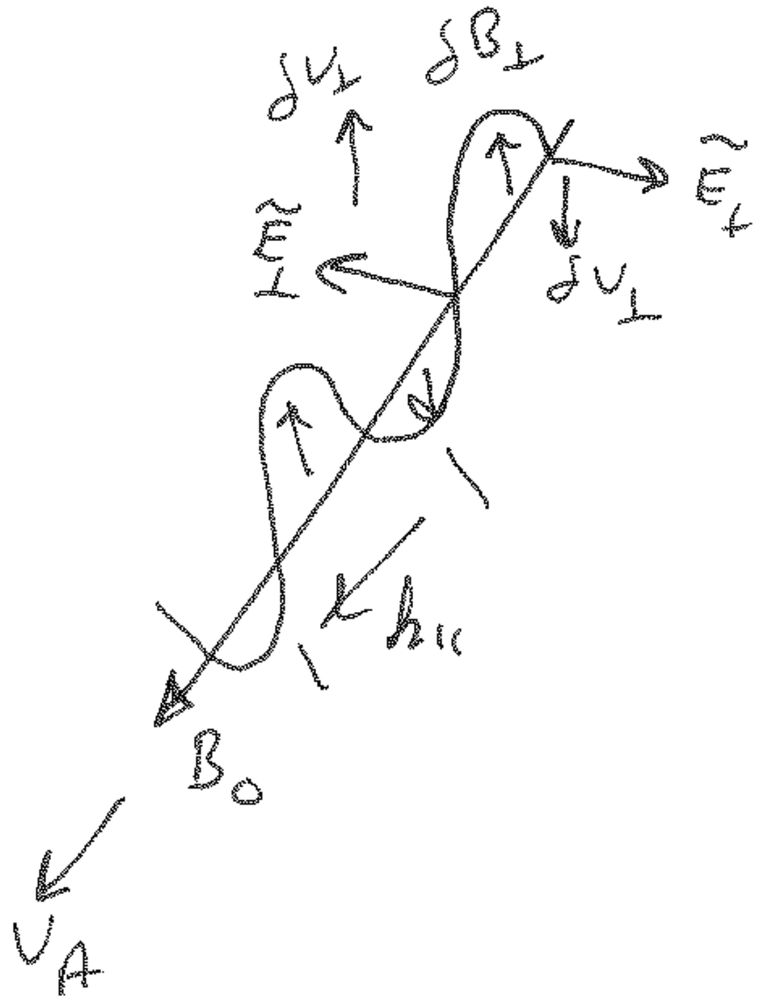
INDUCED ELECTRIC FIELD

AMPERE'S LAW: $J = \frac{1}{\mu_0} \nabla \times B$

$$= \frac{1}{\mu_0} \nabla \times (\nabla \times A) = -\frac{1}{\mu_0} \nabla^2 \bar{A} \quad \left(\begin{array}{l}\text{gauge} \\ \nabla \cdot A = 0\end{array}\right)$$

$$\boxed{J_{\parallel} = -\frac{1}{\mu_0} \nabla^2 A_{\parallel}}$$

Review: Shear Alfvén Wave



ONLY BENDING OF MAGNETIC FIELD (RELATIVELY EASY) VS. COMPRESSION

- FARADAY'S LAW AND $\tilde{E}_\parallel = 0$

$$\tilde{E}_\parallel = 0 = -i k_\parallel \tilde{\Phi} + j\omega \tilde{A}_\parallel$$

$$\tilde{A}_\parallel = \frac{k_\parallel}{\omega} \tilde{\Phi}$$

- AMPERE'S LAW AND $\nabla \cdot J = 0$

WHY ONLY $V_{ION\,POL}$?

$$\nabla \cdot J = 0 = \frac{\partial \tilde{J}_\parallel}{\partial z} + \nabla_\perp \cdot \tilde{J}_\perp$$

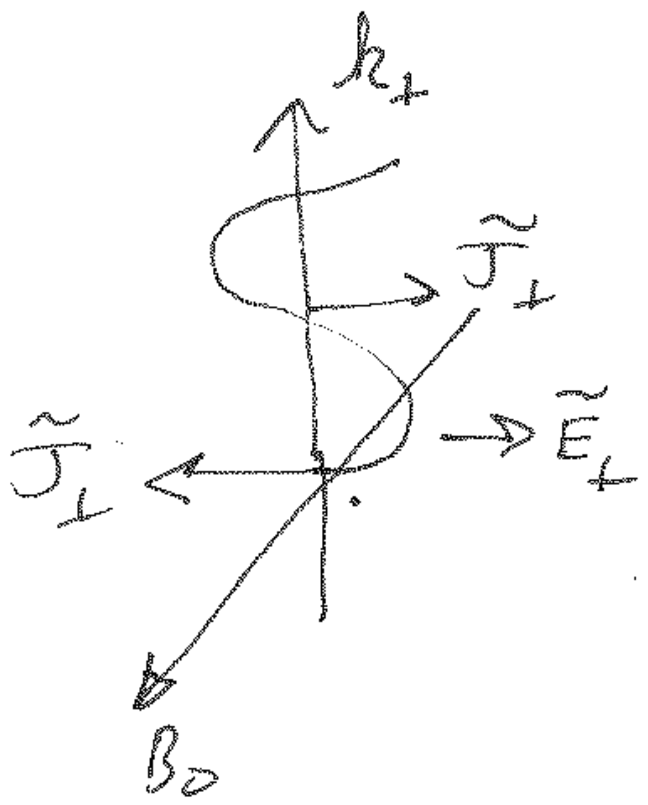
$$\tilde{J}_\parallel = -\frac{1}{\mu_0} \nabla_\perp^2 \tilde{A}_\parallel \qquad \tilde{J}_\perp = en V_{ION\ POLARIZATION} = \frac{n m_i}{B_0^2} \frac{\partial E_\perp}{\partial t}$$

$$\nabla \cdot J = 0 = -\frac{1}{\mu_0} \frac{\partial}{\partial z} \nabla_\perp^2 \tilde{A}_\parallel + \nabla_\perp \cdot \frac{n m_i}{B_0^2} \left(-\nabla_\perp \frac{\partial \tilde{\Phi}}{\partial t} \right)$$

$$0 = -\frac{k_\parallel}{\mu_0} \nabla_\perp^2 \tilde{A}_\parallel + \frac{n m_i}{B_0^2} \omega \nabla_\perp^2 \tilde{\Phi} \quad \leftarrow \tilde{\Phi} = \frac{\omega}{k_\parallel} \tilde{A}_\parallel$$

$$\omega^2 = k_\parallel^2 V_A^2 \leftarrow \boxed{0 = -k_\parallel^2 V_A^2 \nabla_\perp^2 \tilde{A}_\parallel + \omega^2 \nabla_\perp^2 \tilde{A}_\parallel}$$

Compressional (Magnetosonic) Alfvén Wave



FOR COMPRESSIONAL WAVES

$$\nabla \cdot nV \neq 0 \quad \leftarrow \text{COMPRESS PLASMA}$$

$$\frac{\partial B_\parallel}{\partial t} \neq 0 \quad \leftarrow \text{COMPRESS MAGNETIC FIELD}$$

PLASMA COMPRESSION

$$\frac{\partial \tilde{n}}{\partial t} + \nabla \cdot (n_0 \tilde{V}) = 0$$

$$\hookrightarrow \tilde{V} = \frac{\tilde{E} \times B_0}{B_0^2} = -\frac{\hat{b}}{B_0} \times \nabla \tilde{\Phi} - \frac{\hat{b}}{B_0} \times \frac{\partial \tilde{A}}{\partial t}$$

$$\frac{\partial \tilde{n}}{\partial t} + \frac{n_0}{B_0} \nabla \cdot \left[-\hat{b} \times \nabla \tilde{\Phi} - \hat{b} \times \frac{\partial \tilde{A}}{\partial t} \right] = 0$$

BUT $\nabla \cdot (\bar{A} \times \bar{B}) = \bar{B} \cdot \nabla \times \bar{A} - \bar{A} \cdot \nabla \times \bar{B}$

$$\nabla \cdot \tilde{V}_E = -\frac{1}{B_0} \frac{\partial \tilde{B}_\parallel}{\partial t}$$

$$\frac{\partial \tilde{n}}{\partial t} - \frac{n_0}{B_0} \hat{b} \cdot \underbrace{\nabla \times \dot{\tilde{A}}}_{\tilde{B}} = 0$$

$$\boxed{\frac{\partial \tilde{n}}{\partial t} - \frac{n_0}{B_0} \frac{\partial \tilde{B}_\parallel}{\partial t} = 0} \Rightarrow \boxed{\frac{\tilde{n}}{n_0} = \frac{\tilde{B}_\parallel}{B_0}}$$

COMPRESSION

Compressional Alfvén Wave: Perpendicular Current

AMPERE'S LAW: $\tilde{J}_\perp = -\frac{1}{\mu_0}\nabla^2 \tilde{A}_\perp$

PLASMA CURRENTS: $\tilde{J}_\perp = e n_0 \tilde{U}_{\substack{ion \\ pol}} - e n_0 \tilde{V}_{e\,\mathrm{DIAMAG}}$ $\quad (T_e \gg T_i)$

$$\frac{n_0 M_i}{B^2}\frac{\partial \tilde{E}_\perp}{\partial t} \qquad\qquad -\frac{1}{e n_0 B_0}\,\hat{b}\times\nabla \tilde{p}_e$$

$\nabla \tilde{p}_e = T\nabla \tilde{n}$

PERTURBED COMPRESSED DENSITY

$$\tilde{J}_\perp = \frac{n_0 M_i}{B^2}\frac{\partial \tilde{E}_\perp}{\partial t} + \frac{T_e}{B_0}\,\hat{b}\times\nabla \tilde{n}$$

$$\tilde{E}_\perp = -\frac{\partial \tilde{A}_\perp}{\partial t}$$

$$\tilde{n} = \frac{n_0}{B_0}\tilde{B}_\parallel$$

$$\tilde{B}_\parallel = \hat{b}\cdot\nabla\times\tilde{A}_\perp = i\,\hat{b}\cdot\bar{k}\times\tilde{A}_\perp$$

$$= i\,(\hat{b}\times\bar{k})\,\tilde{A}_\perp$$

$$\therefore\quad J_\perp = -\frac{n_0 M_i}{B^2}\frac{\partial^2 A_\perp}{\partial t^2} - \frac{n_0 T_e}{B_0^2}(\hat{b}\times k)^2 A_\perp$$

Compressional Alfvén Wave

$$\underbrace{\tilde{J}_\perp = -\frac{1}{\mu_0}\nabla^2 \tilde{A}_\perp}_{\text{AMPERE'S LAW}} = -\underbrace{\frac{n_0 m_i}{B^2}}_{\text{ION POLARIZATION}}\frac{\partial^2 A_\perp}{\partial t^2} - \underbrace{\frac{n_0 T_e}{B_0^2}(\hat{b}\times k)^2 A_\perp}_{\text{PERTURBED ELECTRON DIAMAGNETIC DRIFT}}$$

$$\nabla^2 \sim k_\perp^2 \qquad (\hat{b}\times k)^2 = k_\perp^2$$

$$\boxed{T_e/m_i = c_s^2}$$

$$\boxed{\frac{B_0^2}{\mu_0 n_0 m_i} = v_A^2}$$

$$k_\perp^2 V_A^2 = \omega^2 - c_s^2 k_\perp^2$$

$$\boxed{\omega^2 = (v_A^2 + c_s^2)\, k_\perp^2}$$

What is $E_{||}$?

DRIFT WAVES: $\frac{\delta n}{n} \sim \frac{e\delta\tilde{\Phi}}{T}$ $\tilde{E}_{||} = -ik_{||}\tilde{\Phi}$

SHEAR ALFVEN WAVES $\tilde{E}_{||} = -ik_{||}\tilde{\Phi} + i\omega\tilde{A}_{||}$

$= 0$

COMPRESSIONAL ALFVEN WAVES $\tilde{E}_{\perp} = -\frac{\partial\tilde{A}_{\perp}}{\partial t}$

$\tilde{E}_{||} = 0$

MHD $E + U \times B = 0 \Rightarrow E_{||} = 0$

DRIFT $\tilde{E}_{||} \neq 0$

Two Fluid Electron Electromagnetics

INDUCTIVE

PERTURBED DIAMAGNETIC FORCE

- PARALLEL ELECTRON DYNAMICS :

$$m_e \frac{\partial v_{\parallel}}{\partial t} = e\left(\frac{\partial \tilde{\Phi}}{\partial z} + \frac{\partial A_{\parallel}}{\partial t}\right) + e\,(\bar{v} \times \tilde{B})_{\parallel} - \frac{\partial P_e}{\partial z}\frac{1}{n_e}$$

ADIABATIC RESPONSE

- PARALLEL ELECTRON CONTINUITY :

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \bar{v}) = 0$$

$$\bar{v} = \bar{v}_E + \hat{b}\, v_{\parallel}$$

CAN WE USE THESE TO PLACE BOUNDS OF $\tilde{E}_{\parallel}$?

Parallel Electron Dynamics: Force Balance

$$m_e \frac{\partial V_\parallel}{\partial t} \simeq 0 \simeq e\left(\frac{\partial \Phi}{\partial z} + \frac{\partial A_\parallel}{\partial t}\right) - e(V \times B)_\parallel - \frac{T}{n_0}\nabla_\parallel \tilde{n}$$

IGNORE INERTIA

$$\vec{A} = \vec{b}\,\tilde{A}_\parallel$$

$$(V \times B)_\parallel \sim \vec{b}\cdot\left(V^*_{DIA} \times (\nabla \times A)\right)$$

$$= \vec{b}\cdot\left(V^*_{DIA} \times \underbrace{(\nabla \times \vec{b}\,A_\parallel)}_{-\vec{b}\times\nabla\tilde{A}_\parallel}\right)$$

$$= -\vec{b}\cdot\left(V^*_{DIA} \times (\vec{b} \times \nabla\tilde{A}_\parallel)\right)$$

$$= -V^*_{DIA}\cdot\nabla\tilde{A}_\parallel$$

$$= -j\,k_y V^*_{DIA}\,\tilde{A}_\parallel$$

$$0 \simeq e(jk_\parallel\tilde{\Phi} - j\omega\tilde{A}_\parallel) + j\,k_y V^*_D\,\tilde{A}_\parallel - jk_\parallel\frac{T}{n_0}\tilde{n}$$

$$\frac{\tilde{n}}{n_0} = \left(\frac{e\tilde{\Phi}}{T}\right) + \frac{k_y V^*_D - \omega}{k_\parallel}\left(\frac{e\tilde{A}_\parallel}{T}\right)$$

"ADIABATIC" "INDUCED/MAGNETIC"

Parallel Electron Dynamics: Continuity

$$\frac{\partial n_0}{\partial t} + \nabla \cdot (\tilde{n}_0 \vec{V}) = 0$$

$$\vec{V} = \vec{V}_{E\times B} + \hat{b}\, V_{\parallel}$$

$$\nabla \cdot (n V_{E\times B}) = \nabla \cdot \left(\hat{b} \times \nabla\phi \frac{n_0}{B}\right) = \hat{b} \times \nabla\phi \cdot \frac{\nabla n_0}{B}$$

$$= -j k_y \tilde{\Phi} \frac{n_0}{B} \left(\frac{\partial n_0/\partial x}{n_0}\right)$$

$$= +j k_y V_{DIA}^{*} \left(\frac{e\tilde{\Phi}}{T}\right) n_0$$

$$\nabla \cdot (n v_{\parallel} \hat{b}) = -\frac{1}{e} \nabla_{\parallel} J_{\parallel}$$

$$J_{\parallel} = -e n_e V_{\parallel e}$$

$$J_{\parallel} = -\frac{1}{\mu_0} \nabla^2 A_{\parallel}$$

AMPERE'S LAW

$$-j\omega\tilde{n} + j k_y V_D^{*} n_0 \left(\frac{e\tilde{\Phi}}{T}\right) + \frac{1}{e\mu_0} j k_{\parallel} (-k^2) \tilde{A}_{\parallel} = 0$$

$$\frac{\tilde{n}}{n_0} = \frac{k_y V_D^{*}}{\omega} \left(\frac{e\tilde{\Phi}}{T}\right) - \frac{k_{\parallel} k^2 \tilde{A}_{\parallel}}{\mu_0 e n_0 \omega}$$

$$\frac{1}{\mu_0 e n_0} = \left(\frac{B^2}{\mu_0 n_0 M_i}\right)\left(\frac{M_i}{e B^2}\right) = V_A^2 \rho^2 \left(\frac{e}{T}\right)$$

Parallel Electron Dynamics

PARALLEL FORCE BALANCE:
$$\frac{\tilde{n}}{n_0} = \left(\frac{e\tilde{\Phi}}{T}\right) + \frac{k_y V_0^* - \omega}{k_\parallel}\left(\frac{e\tilde{A}_\parallel}{T}\right)$$

CONTINUITY:
$$\frac{\tilde{n}}{n_0} = \frac{k_y V_0^*}{\omega}\left(\frac{e\tilde{\Phi}}{T}\right) - k^2\rho^2 V_A^2 \frac{k_\parallel}{\omega}\left(\frac{e\tilde{A}_\parallel}{T}\right)$$

ELIMINATING $\frac{\tilde{n}}{n_0}$ ...

$$\left(\frac{e\tilde{A}_\parallel}{T}\right) = \frac{k_\parallel\left(\omega - k_y V_0^*\right)}{\omega\left(\omega - k_y V_0^*\right) - k^2\rho^2 V_A^2 k_\parallel^2}\left(\frac{e\tilde{\Phi}}{T}\right)$$

Parallel Electric Field

$$E_\parallel = -i k_\parallel \tilde{\Phi} + i\omega \tilde{A}_\parallel$$

$$= i k_\parallel \tilde{\Phi} \left[\frac{k^2 \rho^2 k_\parallel^2 V_A^2}{\omega(\omega - k v_D^*) - k^2 \rho^2 k_\parallel^2 V_A^2} \right]$$

(wow!!)

WHEN $k^2 \rho^2 k_\parallel^2 V_A^2 \gg \omega^2$

$$E_\parallel \cong -i k_\parallel \tilde{\Phi}$$

USUAL DRIFT WAVE LIMIT

WHEN $k_\parallel \to 0$ AND/OR $k^2 \rho^2 \ll 1$

$$\tilde{E}_\parallel = 0$$

USUAL MHD LIMIT

Density-Potential Relationship

MAGNETIC FLUCTUATION POTENTIAL :

$$\left(\frac{e\tilde{A}}{T}\right) = \frac{k_\parallel (\omega - k_y v_D^*)}{\omega(\omega - k_y v_D^*) - k^2\rho^2 k_\parallel^2 v_A^2}\left(\frac{e\tilde{\Phi}}{T}\right)$$

CONTINUITY :

$$\frac{\tilde{n}}{n_0} = \frac{k_y v_D^*}{\omega}\left(\frac{e\tilde{\Phi}}{T}\right) - k^2\rho^2 v_A^2 \frac{k_\parallel}{\omega}\left(\frac{e\tilde{A}_\parallel}{T}\right)$$

ELIMINATING $\tilde{A}_\parallel$ ...

$$\frac{\tilde{n}}{n_0} = \left(\frac{e\tilde{\Phi}}{T}\right)\left[1 - \frac{(\omega - k_y v_D^*)^2}{\omega(\omega - k_y v_D^*) - k^2\rho^2 k_\parallel^2 v_A^2}\right]$$

DRIFT LIMIT

$k_\parallel v_A \gg \omega \quad k_y\rho \sim 1$

$$\frac{\delta n}{n} = \frac{e\tilde{\Phi}}{T}$$

MHD LIMIT

$k^2\rho^2 \ll 1 \quad (\text{or } k_\parallel \to 0)$

$$\frac{\delta n}{n} = \left(\frac{k_y v_D^*}{\omega}\right)\left(\frac{e\tilde{\Phi}}{T}\right)$$

PURE E×B CONVECTION WHEN PARALLEL MOTION IS IGNORED

Next: Interchange and Kink Modes

- Read Ch. 7 in textbook
- Equilibrium: review
- Sec. 7.3.2: Stability of Ideal Magnetostatic Equilibrium (interchange and sausage)
- Sec. 7.3.3: Linear force operator