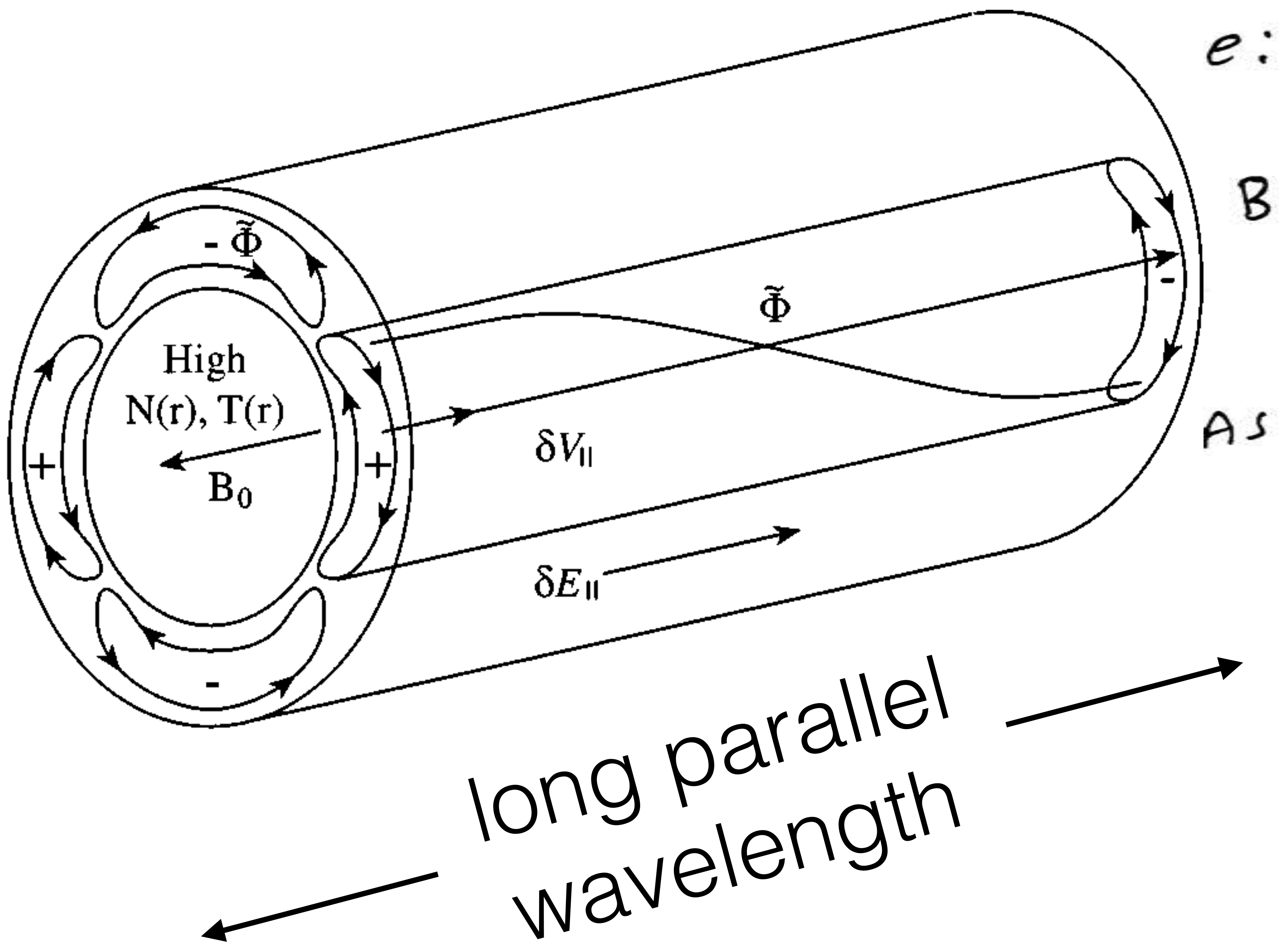
Plasma 2
Lecture 18:
Collisionless Drift Waves

APPH E6102y
Columbia University

Review (1): Adiabatic Electrons



ELECTRON (FAST) PARALLEL MOTION

$$e: \quad \frac{\partial \tilde{v}_{\parallel}}{\partial t} + (\bar{v}_E \cdot \nabla)\tilde{v}_{\parallel} = \frac{e}{m_e}\frac{\partial \tilde{\phi}}{\partial z} - \frac{1}{m_e n_e}\frac{\partial \tilde{p}_e}{\partial z}$$

BUT m_e IS VERY VERY SMALL

ELECTRONS ARE VERY VERY LIGHT AND FAST

AS $m_e \to 0$

$$0 \simeq e\frac{\partial \tilde{\Phi}}{\partial z} - \frac{1}{n_0}\frac{\partial \tilde{p}_e}{\partial z} \approx e\frac{\partial \tilde{\Phi}}{\partial z} - \frac{T}{n_0}\frac{\partial n}{\partial z}$$

$$\frac{1}{n}\frac{\partial n}{\partial z} \simeq \frac{e}{T}\frac{\partial \tilde{\Phi}}{\partial z} \Rightarrow \boxed{\frac{n_e}{n_0} = e^{e\tilde{\Phi}/T}}$$

CALLED "ADIABATIC ELECTRONS"

$$\boxed{\delta\tilde{n} \simeq n_0 \frac{e\tilde{\Phi}}{T}}$$

PROVIDED $k_{\parallel} \neq 0$

Review (2): Continuity (Drifting Ions)

(REMEMBER: $\tilde{n}_e \approx \tilde{n}_i$)

CONTINUITY: $$\frac{\partial \tilde{n}}{\partial t} + \nabla \cdot (n\bar{V}) = 0$$

LET'S USE ION CONTINUITY EQUATION

$$\frac{\partial \tilde{n}}{\partial t} + \bar{\nabla} \cdot (n\bar{V}_E) + \nabla \cdot (n\bar{V}_*) + \nabla \cdot (nV_{pol}) + \bar{\nabla} \cdot (nV_{\parallel}\hat{z}) = 0$$

$E \times B$

DIAMAGNETIC FLUID FLOW

IGNORE AT FIRST (IMPORTANT) "ION INERTIAL"

IONS TOO SLOW ALONG B

$$\nabla \cdot (n\bar{V}_*) = \nabla \cdot (\hat{z} \times \nabla p)\frac{1}{eB} = \frac{1}{eB}\left[\nabla p \cdot (\nabla \times \hat{z}) - \hat{z} \cdot \nabla \times \nabla p\right] = 0$$

$$\nabla \cdot (nV_E) = n\underbrace{\nabla \cdot \bar{V}_E}_{\nabla \cdot (\hat{z} \times \nabla \varphi)/B = 0} + V_E \cdot \nabla n = V_E \cdot \nabla n_0$$

$$\bar{V}_E = \frac{-1}{B}\frac{\partial \Phi}{\partial y}\hat{x}$$

Ion E×B drift across radial density gradient

$$\frac{\partial \tilde{n}}{\partial t} + \hat{V}_E \cdot \frac{\partial n_0}{\partial x} = 0$$

$$-j\omega\tilde{n} + \frac{n_0}{B}\frac{jk_y}{L_N}\tilde{\Phi} = 0$$

$$\boxed{\frac{\tilde{n}}{n_0} = \frac{k_y}{BL_N\omega}\tilde{\Phi}}$$

Review (3): Drift Wave Dispersion Relation

PARALLEL
ADIABATIC
ELECTRONS

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\Phi}}{T}$$

PERPENDICULAR
ION
DRIFT DYNAMICS

$$\frac{\tilde{n}}{n_0} = \frac{k_Y}{B L_N \omega} \tilde{\Phi}$$

DISPERSION RELATION

$$\omega = k_Y \frac{T}{e B L_N} = k_Y V_e^*$$

DRIFT WAVES PROPAGATE IN
ELECTRON DIAMAGNETIC DRIFT
DIRECTION

Ion E×B drift across
radial density gradient

Review (4a): Adding Ion Sound Waves

IONS ARE HEAVY

THEY HAVE POLARIZATION (INERTIAL) DRIFTS
THAT REDUCE $E_\perp$

THEY MOVE SLOWLY (NON ADIABATIC) ALONG B

TAKE DRIFT WAVES LIKE USUAL
ACOUSTIC WAVE

$$V_{THi} \ll \frac{\omega}{k_\parallel} \ll V_{THe}$$

PERPENDICULAR ION

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \bar{v}) = 0$$

$$\frac{\partial \tilde{n}}{\partial t} + \nabla \cdot (n V_{pol} + n V_E) + n \frac{\partial v_{\parallel i}}{\partial z} = 0$$

$$\underbrace{-j\omega\tilde{n} + \nabla \cdot (n V_{pol}) + \tilde{V}_E \cdot \nabla n_0 + i \frac{k_\parallel^2}{\omega} \frac{e\tilde{\Phi}}{m_i} n_0}_{} = 0$$

NEW ION INERTIAL TERM

PARALLEL ION

$$\frac{dv_{\parallel i}}{dt} = -\frac{e}{m_i} \frac{\partial \Phi}{\partial z}$$

$$\hat{V}_{\parallel i} = \frac{e}{m_i} \frac{k_\parallel}{\omega} \tilde{\Phi}$$

SOUND RESPONSE

Review (4b): Adding Ion Inertia

$$\nabla \cdot (n V_{pol}) = \nabla \cdot \left(\frac{n m_i}{q B^2} \frac{d E_\perp}{dt} \right)$$

$$= - \nabla \cdot \left(\frac{n m_i}{q B^2} \nabla_\perp \dot{\tilde{\Phi}} \right)$$

$$= - \nabla \cdot \frac{\epsilon_0}{q} \frac{\omega_{pi}^2}{\omega_{ci}^2} \nabla_\perp \dot{\tilde{\Phi}}$$

$$= k_\perp^2 \frac{n_0 m_i}{q B^2} \dot{\tilde{\Phi}}$$

$$= -j \omega k_\perp^2 n_0 \rho_i^2 \left(\frac{e \tilde{\Phi}}{T} \right)$$

$E_\perp = -\nabla_\perp \tilde{\Phi}$

$\epsilon_0 \frac{\omega_{pi}^2}{\omega_{ci}^2}$ = PLASMA DIELECTRIC

$= \epsilon_0 \frac{n M_i}{B^2} >> \epsilon_0$

VERY LARGE

ADIABATIC ELECTRONS

$\tilde{n} = n_0 \left(\frac{e \tilde{\Phi}}{T} \right)$

$k_\perp^2 \rho^2$ = FINITE LARMOR RADIUS TERM

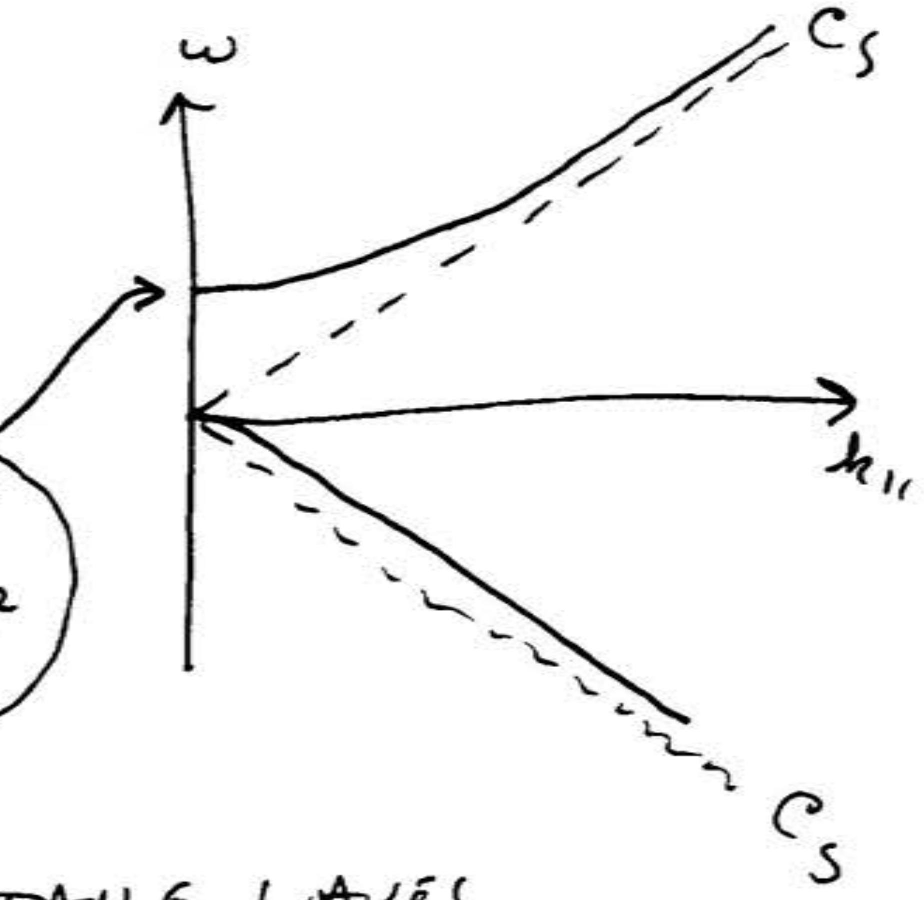
$$\boxed{\omega^2 (1 + k_\perp^2 \rho^2) - \omega k_y v^* - k_\parallel^2 c_s^2 = 0}$$

ION POLARIZATION TERM

DENSITY GRADIENT DRIFT

SOUND WAVE TERM

$\omega = \frac{k_y v^*}{1 + k_\perp^2 \rho^2}$



NOTE: STABLE WAVES

(!! nice)

Next: Collisionless Drift Wave Instability and Transport

Instability

ADIABATIC ELECTRONS: $\frac{\tilde{n}}{n} \approx \frac{e\tilde{\Phi}}{T}$ (NO RADIAL TRANSPORT)

NON-ADIABATIC ELECTRONS: $\frac{\delta n}{n} \approx \frac{e\tilde{\Phi}}{T}\left(1 - i\delta\psi\right)$ ← SMALL PHASE SHIFT

$$\therefore \quad \omega = \frac{k_y V_e^*}{1 - i\delta\psi} \approx k_y V_e^* \left(1 + i\delta\psi + \cdots\right)$$

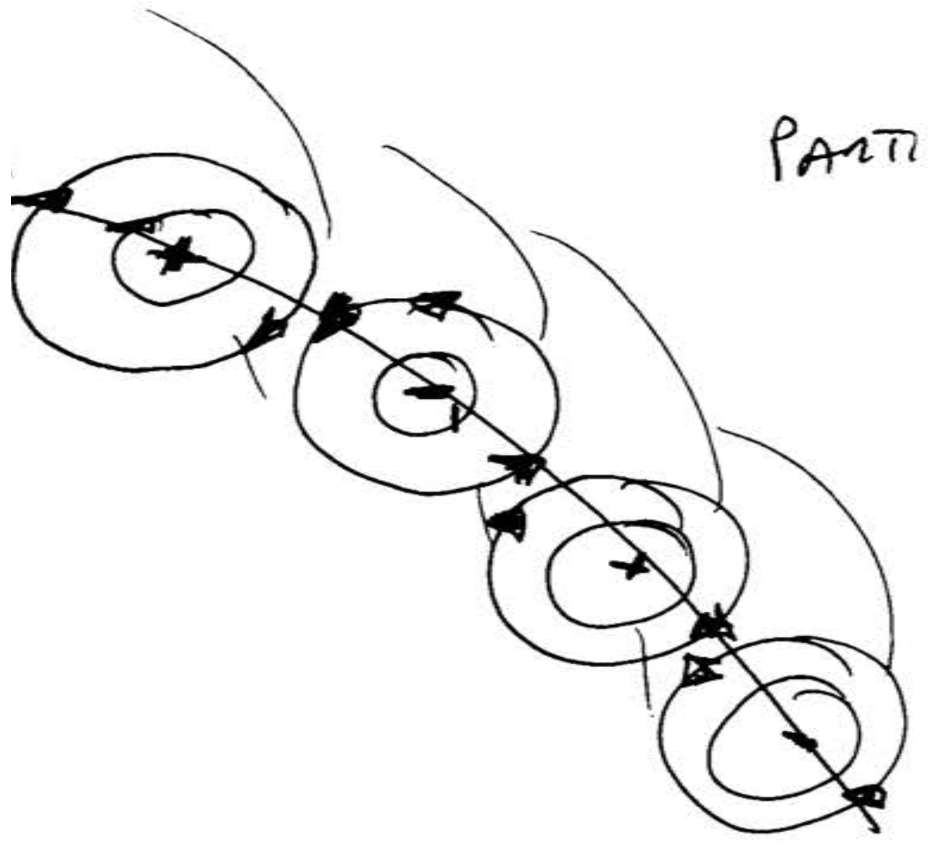
$$\underline{Re(\omega) \sim k_y V_e^* \qquad Im(\omega) \sim i\,\delta\psi\, k_y V_e^*}$$

WHAT CAUSES $\delta\psi$?

COLLISIONS ALONG B	LANDAU DAMPING	ELECTROMAGNETIC INDUCTION $\delta B_\perp$

This lecture

Radial Transport



PARTICLE FLUX $= \frac{1}{2} Re\{\tilde{n}^* \tilde{V}\}$

$$= -\frac{1}{2} Re\left\{ n_0 \frac{e}{T} \tilde{\Phi}^* \frac{i k_y \tilde{\Phi}}{B} \hat{x} \right\}$$

$$= -\frac{1}{2} n_0 \frac{e k_y}{BT} Re\{\underbrace{i \tilde{\Phi}^* \tilde{\Phi}}_{0}\}$$

NO FLUX

WE NEED A PHASE SHIFT BETWEEN δn AND $\delta\phi$ FOR TRANSPORT

Physical Mechanism for the Collisionless Drift Wave Instability

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(Received 26 November 1968)

The growth rate for the collisionless drift instability is calculated in a manner which elucidates the physical mechanism for the instability.

Simple physical models have been useful in understanding fluid instabilities such as the flute instability. However, the collisionless drift wave can be derived only from the Vlasov equation, and its growth rate is determined by resonant particles interacting with the wave. In this note a simple calculation of the growth rate for a drift wave in a nonuniform collisionless plasma immersed in a uniform magnetic field is derived which agrees with the

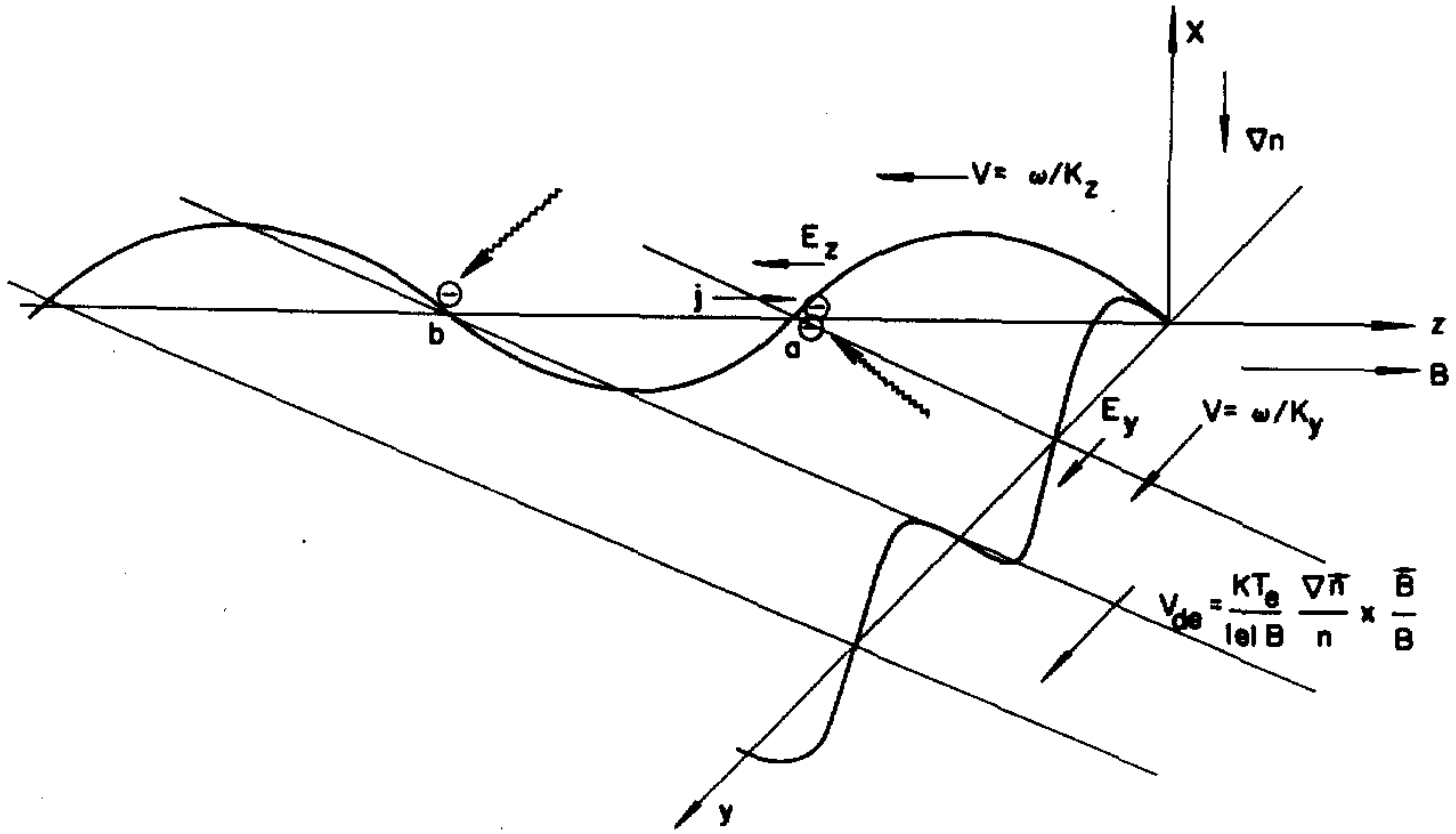

FIG. 1. A schematic view of the equipotential surfaces of a collisionless drift wave showing the motion of the resonant electrons.

electrons will be described here. A resonant electron at position a of Fig. 1 experiences a constant $\mathbf{E} \times \mathbf{B}$ drift upward in the x direction. Similarly, at position b of Fig. 1 a resonant electron $\mathbf{E} \times \mathbf{B}$ drifts downward. However, because of the density gradient there are more resonant electrons at position a than at position b. Since the electrons at position a are pushing against the electric field of the wave, there is a net transfer of parallel energy from the resonant electrons to the drift wave. This is the basic driving mechanism for the collisionless drift wave.

Physics Fluids **12**, 947 (1969); [https://doi.org/10.1063/1.1692583]

Observation of radially propagating collisionless-drift-wave instability

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(Received 19 September 1972)

The radially traveling drift-wave instability is observed in a fully ionized collisionless plasma. The observed radial wave number is larger than the azimuthal wave number. The observed frequency, in the frame of $E_r B_z$ plasma rotation, is in good agreement with the calculated frequency of the drift instability after the correction of finite ion inertia including the radial wave number, where E_r is a radial electric field and B_z is an axial magnetic field.

The experiment has been performed in the magnetic channel of a QP-Machine[7] which has an 8-m-long uniform magnetic field, variable from 0.6 to 3.5 kG. The vacuum chamber is made of glass, and its inner diameter is 15 cm. The plasma, produced by a PIG discharge, diffused along the magnetic field into the experimental region. The residual gas pressure was $(1 \sim 2) \times 10^{-6}$ Torr. The helium plasma density was about 5×10^9 cm^{-3}, the electron temperature 2.5 ~ 3.5 eV, and the ion temperature below 1 eV. A set of two probes, separated 90° each in the azimuthal direction θ, is inserted into the plasma column at the same axial position. One of the probes can be moved radially across the plasma column, and the other is fixed at the desired position.

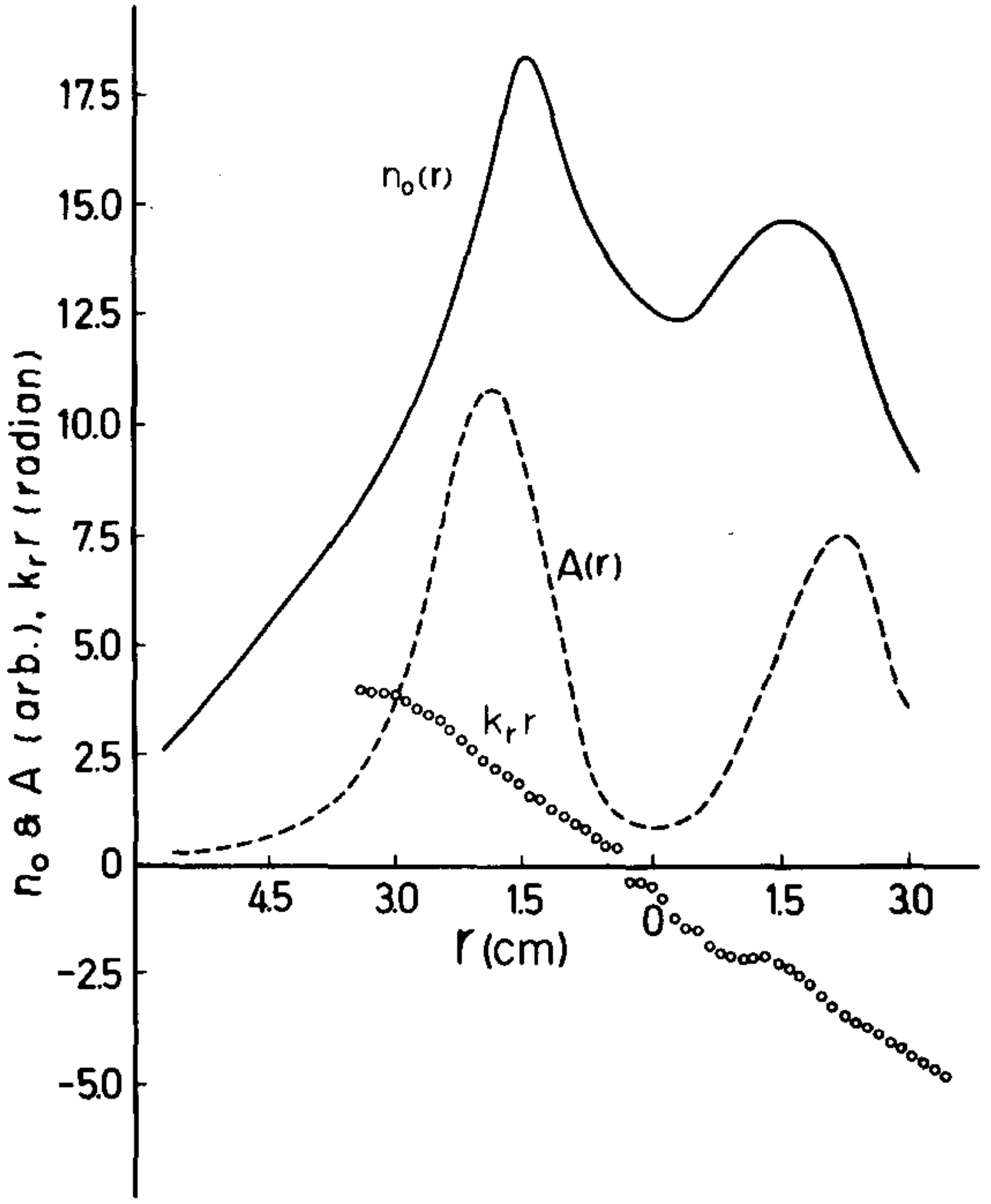

FIG. 1. The calculated amplitude (---) and phase (○) of the fluctuation as a function of the radial position. The plasma density profile is also shown by solid line. Vertical scales are a radian for the phase and an arbitrary linear for the amplitude and the density, respectively.

Journal Applied Physics **44**, 1541 (1973); [https://doi.org/10.1063/1.1662408]

Anomalous Transport and Stabilization of Collisionless Drift-Wave Instabilities*

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(Received 10 November 1975)

The nonlinear evolution of collisionless drift-wave instabilities and the associated plasma transport have been studied extensively using particle-code simulations. It is found that the quasilinear decay of the density profile gives rise to the nonlinear saturation. The results also indicate that a new mechanism of wave absorption is responsible for the observed anomalous energy transport, which, in general, is larger than the corresponding particle diffusion and is also less sensitive to shear.

To simulate a collisionless plasma,[13] the standard dipole-expansion technique with finite-size particles is used.[14] The guiding centers of the particles are loaded initially according to a prescribed density profile and with spatially uniform temperatures on a 64×32 $(L_x\times L_y)$ spatial grid. Maxwellian velocity distributions are also used. The parameters of the simulations are $m_i/m_e = 25$, $T_e/T_i = 4$, $\lambda_{De}/\Delta = 2$, $\omega_{ce}/\omega_{pe} = 2$, the average number density $\langle n\rangle = 8/\Delta^2$, and a(rms of a Gaussian particle) $= 1.5\Delta$. The mesh size Δ is taken as the unit length. All the frequencies are measured in terms of ω_{pe}. These parameters give $k_x\rho_i = 0.12m$ and $k_y\rho_i = 0.49n$ where $m, n = 1, 2, 3, \ldots$. Exact dynamics for the particle pushing have been used with $\Delta t = 0.5$.

$\partial(\bar{n}_0, \bar{T}_{e\parallel})/\partial t = (D_\perp, K_{e\perp})\partial^2(\bar{n}_0, \bar{T}_{e\parallel})/\partial x^2$, is used. The average coefficients versus shear for the hyperbolic tangent profile are shown in Fig. 2. For the shear-free case, the measured $D_\perp$ is about 25% of the values given by the Bohm diffusion, $D_\perp \sim cT_e/16eB_0$, and the turbulent diffusion, $D_\perp \sim \gamma/k_\perp^2$, by Kadomtsev,[1] while $K_{e\perp}$ is about four times bigger. These are also true for the shear-

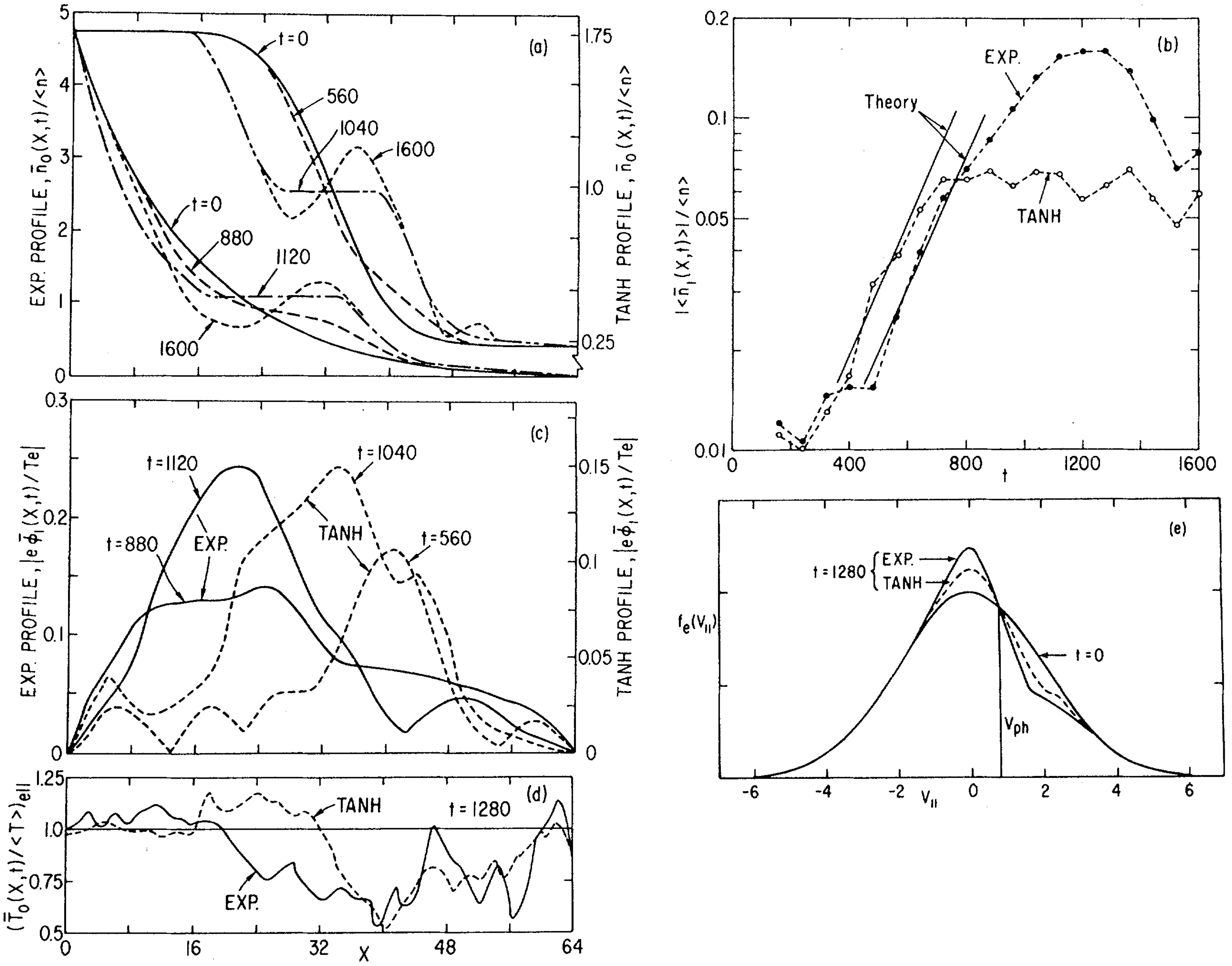

FIG. 1. (a) Time evolution of the density profiles. (b) Growth of the density modulation for $n=1$ mode. (c) Mode structures for $n=1$ mode. (d) Parallel electron heat transfer patterns. (e) Average velocity distributions.

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Numerical computation of collisionless drift wave turbulence

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(Received 28 December 1998; accepted 17 March 1999)

Collisionless drift waves are studied by means of nonlinear numerical simulations in a three-dimensional sheared slab geometry. The electron dynamics is described by a drift-kinetic equation, and the ions are treated as a cold fluid. The energy spectra of the turbulent fluctuations and the dependence of the resulting anomalous transport on various dimensionless plasma parameters are investigated. It is shown that this model resolves fundamental contradictions between experimental results and linear drift wave theory, especially the dependence of turbulent transport on radial position but also the scaling with ion mass.
[S1070-664X(99)03606-X]

E. Particle and energy transport

The fluxes

$$\Gamma = \langle n v_x \rangle , \quad Q = \tfrac{3}{2}\langle p v_x \rangle \tag{30}$$

with

$$\tfrac{3}{2}p = \tfrac{1}{2}p_\parallel + p_\perp = \tfrac{3}{2}n + \tfrac{1}{2}T_\parallel + T_\perp \tag{31}$$

and $v_x = -\partial_y \phi$ characterize the turbulent particle and heat transport in the radial direction. Here $\langle \cdots \rangle$ denotes spatial averaging over the simulation domain. As a consequence of the above normalizations, Γ and Q are normalized to $D_{GB}(n_{e0}/L_\perp)$ and $D_{GB}(p_{e0}/L_\perp)$, respectively, with the gyro-Bohm transport coefficient

$$D_{GB} = \frac{c T_{e0} \rho_s}{e B L_\perp}$$

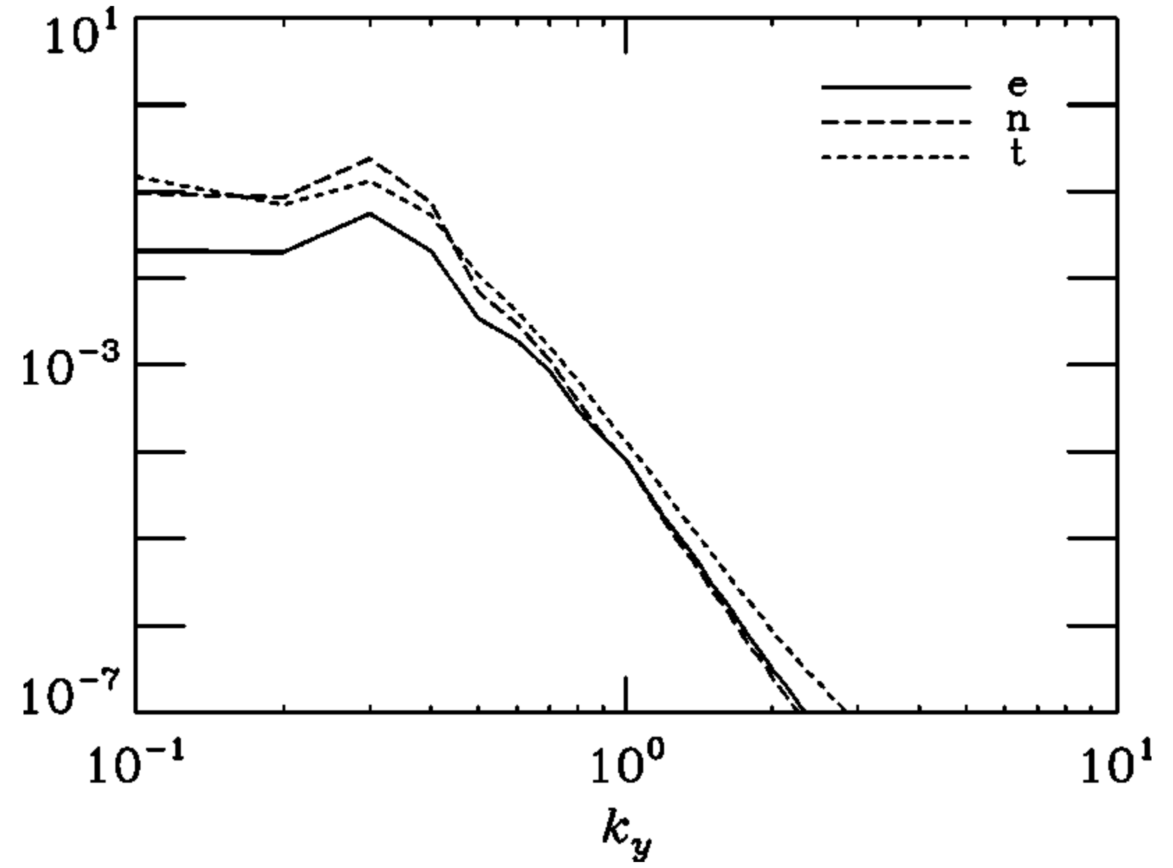


FIG. 2. k_y spectra for the fluctuation energies of the $\mathbf{E}\times\mathbf{B}$ drift (E_e), electron density (E_n), and electron temperature (E_t). The system is driven primarily at intermediate length scales, $2\pi/k_y \sim 20\rho_s$, corresponding to 1.3 cm for the nominal plasma edge parameters given in Sec. II.

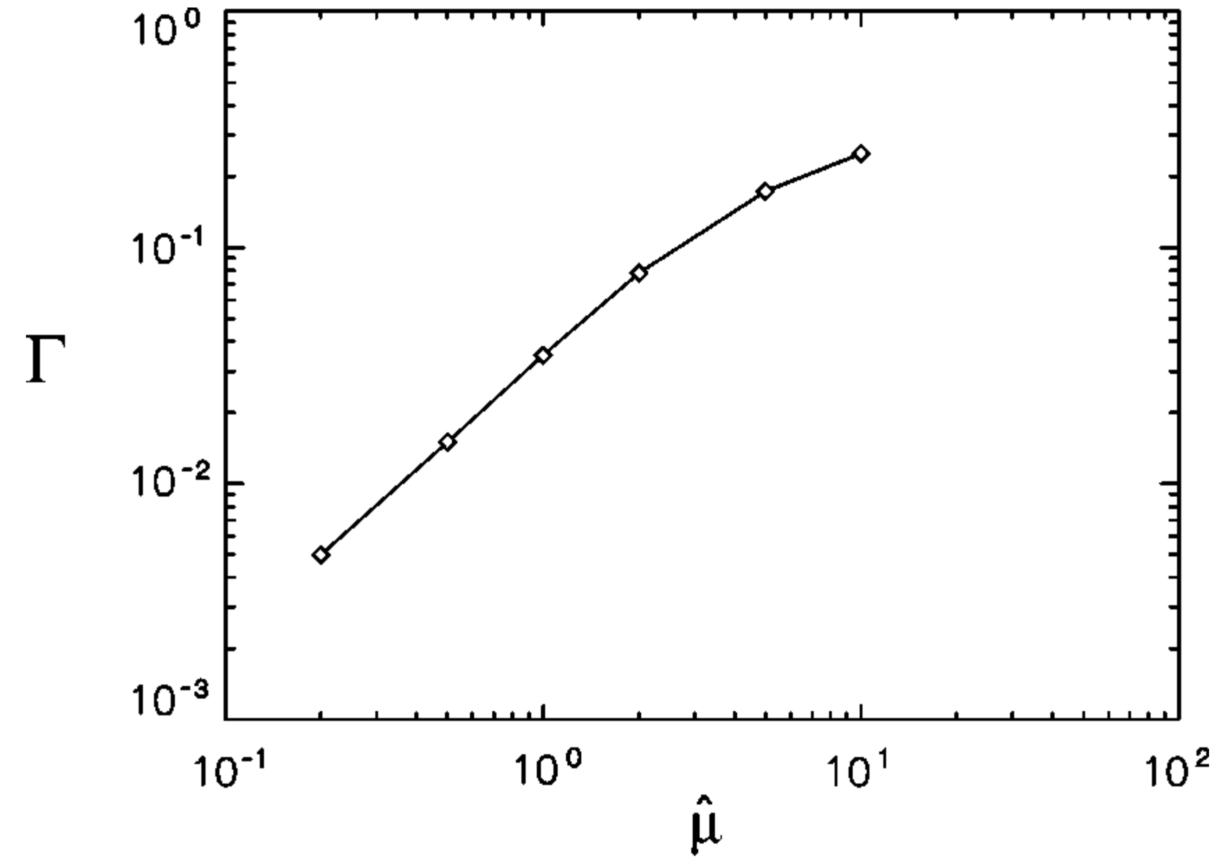


FIG. 3. Dependence of the turbulent particle transport Γ on the parameter $\hat{\mu} = 2/\alpha_e^2$ which controls the adiabaticity of the collisionless drift wave system $(\hat{s} = 2/\pi,\ \omega_t = 0)$.

portant than electron Landau damping. The most important parameter in the present electrostatic collisionless model is $\alpha_e = (L_\perp/qR)(2M_p/m_e)^{1/2}$, which is given by the ratio of the parallel to the perpendicular dynamical frequencies, i.e., thermal electron transit frequency $k_\parallel v_e \sim v_T/qR$ to electron

Physics Plasmas 6, 2418 (1999); [https://doi.org/10.1063/1.873513]

Review of Sound Waves

IONS: $-j\omega\tilde{n} + jk\tilde{v}n_0 = 0$ $\quad -j\omega\tilde{v} = -\frac{e}{m_i} jk\tilde{\Phi}$

$$\frac{\tilde{n}}{n_0} = \frac{k}{\omega}\tilde{v} = \frac{k^2}{\omega^2}\underbrace{\left(T/m_i\right)}_{C_s^2}\left(\frac{e\tilde{\Phi}}{T}\right)$$

$$\Downarrow$$

ADIABATIC ELECTRONS: $\frac{\tilde{n}}{n_0} \approx \frac{e\tilde{\Phi}}{T} \Rightarrow \omega^2 = k^2 C_s^2$

WHAT OTHER PHYSICS IS IMPORTANT TO PLASMA SOUND WAVES?

Kinetic Parallel Electron Dynamics for (Slow) Sound Waves

KINETIC ELECTRONS:

$$\frac{\partial \tilde{f}}{\partial t} + v\frac{\partial \tilde{f}}{\partial x} + i\frac{e}{m_0} k\tilde{\Phi}\frac{\partial F_0}{\partial v} \simeq 0$$

$$-i(\omega - kv)\tilde{f} = -i\frac{e}{m_0} k\tilde{\Phi}\frac{\partial F_0}{\partial v}$$

$$F_0(v) \sim \frac{n_0}{\sqrt{\pi v_{TH}^2}} e^{-v^2/2v_{TH}^2} \qquad v_{TH}^2 = T/m_e$$

$$n_0 = \int_{-\infty}^{\infty} dv\, F_0(v)$$

$$\tilde{f} = \frac{e}{m_0} k\tilde{\Phi}\frac{\frac{\partial F_0}{\partial v}}{\omega - kv}$$

EQUILIBRIUM DISTRIBUTION IS FUNCTION OF UNPERTURBED KINETIC CONSTANTS OF MOTION

$$\frac{dF_0}{dt} = 0$$

$$\frac{\tilde{n}}{n_0} = \int_{-\infty}^{\infty} dv\, \tilde{f}$$

$$\tilde{f} = \left(\frac{e\tilde{\Phi}}{T}\right)\left(\frac{m_i}{m_0}\right)\frac{kc_s^2\frac{\partial F_0}{\partial v}}{\omega - kv} \Rightarrow \frac{\tilde{n}}{n_0} = \left(\frac{e\tilde{\Phi}}{T}\right)\left(\frac{m_i}{m_e}\right)\int_{-\infty}^{\infty} dv\, \frac{kc_s^2\frac{\partial F_0}{\partial v}}{\omega - kv}$$

Electron Landau Damping for Acoustic Waves

$$\frac{\tilde{n}}{n_0} = \left(\frac{e\tilde{\Phi}}{T}\right)\left(\frac{m_i}{m_e}\right)\int_{-\infty}^{\infty} dv \frac{k c_s^2 \frac{\partial F_0}{\partial v}}{\omega - k v}$$

[WAVE-PARTICLE RESONANCE
$\omega \simeq k v$
$v \simeq c_s \ll v_{The}$]

LEADING ORDER ELECTRON RESPONSE ...

$$\frac{\partial F_0}{\partial v} = -\frac{v}{v_{Th}^2} F_0 \qquad \frac{1}{\omega - kv} \simeq -\frac{1}{kv\left(1-\frac{\omega}{kv}\right)}$$

$$\simeq -\frac{1}{kv}\left(1 + \frac{\omega}{kv} + \dots\right)$$

$$\frac{\tilde{n}}{n_0} = \left(\frac{e\tilde{\Phi}}{T}\right)\left(\frac{m_i}{m_e}\right)\frac{c_s^2}{v_{The}^2} P\int_{-\infty}^{\infty} F_0 \left(1 + \frac{\omega}{kv} + \dots\right)$$

$$\simeq \left(\frac{e\tilde{\Phi}}{T}\right) \qquad \text{SINCE } \frac{m_i}{m_e}\frac{c_s^2}{v_{The}^2} = 1$$

BUT WHAT ABOUT LANDAU DAMPING ...

Electron Landau Damping

$$\frac{\tilde{n}}{n_0} = \left(\frac{e\tilde{\Phi}}{T}\right)(-1)\int_{-\infty}^{\infty} dv\, \frac{v_{THe}^2 \frac{\partial F_0}{\partial v}}{\left(v - \frac{\omega}{k}\right)}$$

$$= \left(\frac{e\tilde{\Phi}}{T}\right)(-1)\left[\underbrace{P\!\!\int_{-\infty}^{\infty} dv\, \frac{v_{THe}^2 \frac{\partial F_0}{\partial v}}{v - \frac{\omega}{k}}}_{\text{PRINCIPLE PART}} \underbrace{- i\pi\, v_{THe}^2 \frac{\partial F_0}{\partial v}\bigg|_{v=\frac{\omega}{k}}}_{\text{RESIDUE ASSUMING } |\omega_R| \gg |\omega_I|} \right]$$

$$= \left(\frac{e\tilde{\Phi}}{T}\right)\left[1 + i\pi\, v_{THe}^2 \frac{\partial F_0}{\partial v}\bigg|_{v=\frac{\omega}{k}=c_s} \right]$$

$$\frac{\partial F_0}{\partial v} = \frac{-1}{\sqrt{\pi}\, v_{TH}} \frac{v}{v_{TH}^2}\, e^{-v^2/2v_{THe}^2}$$

$$\simeq \left(\frac{e\tilde{\Phi}}{T}\right)\left[1 + \underbrace{i\sqrt{\pi}\,\frac{(\omega/k)}{v_{THe}^2}}_{\longrightarrow\ i\sqrt{\pi}\sqrt{\frac{m_e}{M_i}}} \right] \qquad !!$$

(What about ion Landau damping?)

Are Plasma Sound Waves Damped?

(ELECTRONS)

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\Phi}}{T}\left[1 + i\sqrt{\pi}\sqrt{\frac{m_e}{m_i}}\right]$$

(IONS)

$$\frac{\tilde{n}}{n_0} = \frac{k^2}{\omega^2} C_s^2 \left(\frac{e\tilde{\Phi}}{T}\right)$$

$$D(\omega, k) = 1 - \frac{k^2 C_s^2}{\omega^2} + i\sqrt{\pi}\sqrt{\frac{m_e}{m_i}}$$

WEAK DAMPING APPROXIMATION...

$$D = D_R + i D_I \qquad |\omega_R| \gg |\omega_I|$$

$$\left.\begin{aligned} &D_R(\omega_R) = 0 \\ &D_I + \omega_I \frac{\partial D_R}{\partial \omega} \simeq 0 \end{aligned}\right\} \qquad \begin{aligned} &\omega^2 = k^2 C_s^2 \\ &\omega_I = -\frac{D_I}{\frac{\partial D_R}{\partial \omega}} = \frac{-\sqrt{\pi}\sqrt{\frac{m_e}{m_i}}}{2/\omega_R} \end{aligned}$$

$$\therefore \quad \frac{\omega_I}{\omega_R} \simeq -\frac{1}{2}\sqrt{\pi}\sqrt{\frac{m_e}{m_i}}$$

Electron Kinetics in the "Drift" Regime

$$\frac{\partial \tilde{f}}{\partial t} + v\cdot\nabla\tilde{f} + \frac{e}{m_0}\nabla\tilde{\Phi}\cdot\frac{\partial F_0}{\partial v} = 0$$

BUT IN DRIFT REGIME $F_0(x_{gc}, v_\parallel)$

WHERE $(x_{gc}, v_\parallel)$ MUST BE UNPERTURBED CONSTANTS OF MOTION !!

$v_\parallel$ = C.O.M. IF $\frac{\partial\tilde{\Phi}}{\partial z} = E_\parallel \to 0$ CHECK ✓

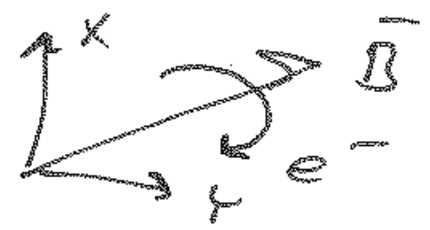
$$\bar{\rho} = -\frac{1}{\omega_c}\hat{b}\times\bar{v}_\perp$$

BUT $x = x_{gc} + \rho_x(t)$ ← GYROMOTION

$$\rho_x(t) = v_y(t)/\omega_c$$

x PARTICLE POSITION

GYRO CENTER C.O.M.



SO $x_{gc} = x - v_y/\omega_c$

$$\boxed{F_0(x - v_y/\omega_c,\ v_\parallel)}$$

← ALSO CONSERVATION OF TOTAL CANONICAL MOMENTUM

$$\bar{P} = m\bar{v} + e\bar{A}$$

VECTOR POTENTIAL

$A_y = -xB$

Electron Slow ($\omega \ll \omega_{ci}$) Drift Response

$$\frac{\partial \tilde{f}}{\partial t} + v_{\parallel}\frac{\partial \tilde{f}}{\partial z} + \frac{e}{m_0}\frac{\partial \tilde{\Phi}}{\partial z}\frac{\partial F_0}{\partial v_{\parallel}} + \frac{e}{m}\frac{\partial \tilde{\Phi}}{\partial y}\frac{\partial F_0}{\partial v_y} \cong 0$$

$$\frac{\partial F_0}{\partial v_{\parallel}} \downarrow \; -\frac{v_{\parallel}}{v_{TH0}^2}F_0 \qquad \frac{\partial F_0}{\partial v_y} \downarrow \; -\frac{1}{\omega_{c0}}\frac{\partial F_0}{\partial x} = +\frac{1}{\omega_{c0}L_N}F_0$$

(IGNORE FAST $v_y(t)$ CYCLOTRON TERMS)

L_N = DENSITY GRAD SCALE LENGTH

$$-j(\omega - k_{\parallel}v_{\parallel})\tilde{f} = -i\frac{e}{m_0}\tilde{\Phi}\left(k_{\parallel}v_{\parallel} - \frac{k_y T/m_e}{\omega_{c0}L_N}\right)\left(-\frac{F_0}{v_{TH0}^2}\right)$$

$$V_D = \frac{T/m_0}{\omega_{c0}L_N}$$

$$\tilde{f} = -\left(\frac{e\tilde{\Phi}}{T}\right)\left[\frac{k_{\parallel}v_{\parallel} - k_y v_D}{\omega - k_{\parallel}v_{\parallel}}F_0\right]$$

LIKE SOUND WAVE: ADIABATIC RESPONSE (COLLECTIVE)
LANDAU DAMPING (WAVE-PARTICLE RESONANCE)

Evaluating Electron Landau Damping Term…

$$\hat{f} = -\left(\frac{e\tilde{\Phi}}{T}\right)\left[\frac{k_\parallel v_\parallel - k_y v_D}{\omega - k_\parallel v_\parallel} F_0\right]$$

$$\hat{f} = \left(\frac{e\tilde{\Phi}}{T}\right)\left[\frac{(\omega - k_\parallel v_\parallel) - (\omega - k_y v_D)}{\omega - k_\parallel v_\parallel} F_0\right]$$

$$= \left(\frac{e\tilde{\Phi}}{T}\right)\left[F_0 - \left(\frac{\omega - k_y v_D}{\omega - k_\parallel v_\parallel}\right) F_0\right]$$

$$\frac{\tilde{n}}{n_0} = \left(\frac{e\tilde{\Phi}}{T}\right)\left[1 - \underbrace{\int_{-\infty}^{\infty} dv\, F_0 \frac{\omega - k_y v_D}{\omega - k_\parallel v_\parallel}}_{\text{PRINCIPAL PART PLUS RESONANT RESIDUAL}}\right]$$

$$= \left(\frac{e\tilde{\Phi}}{T}\right)\left[1 - i\frac{\sqrt{\pi}}{k\, v_{the}}\left(k_y v_D - \omega\right)\right]$$

Collisionless Drift Wave

ION RESPONSE

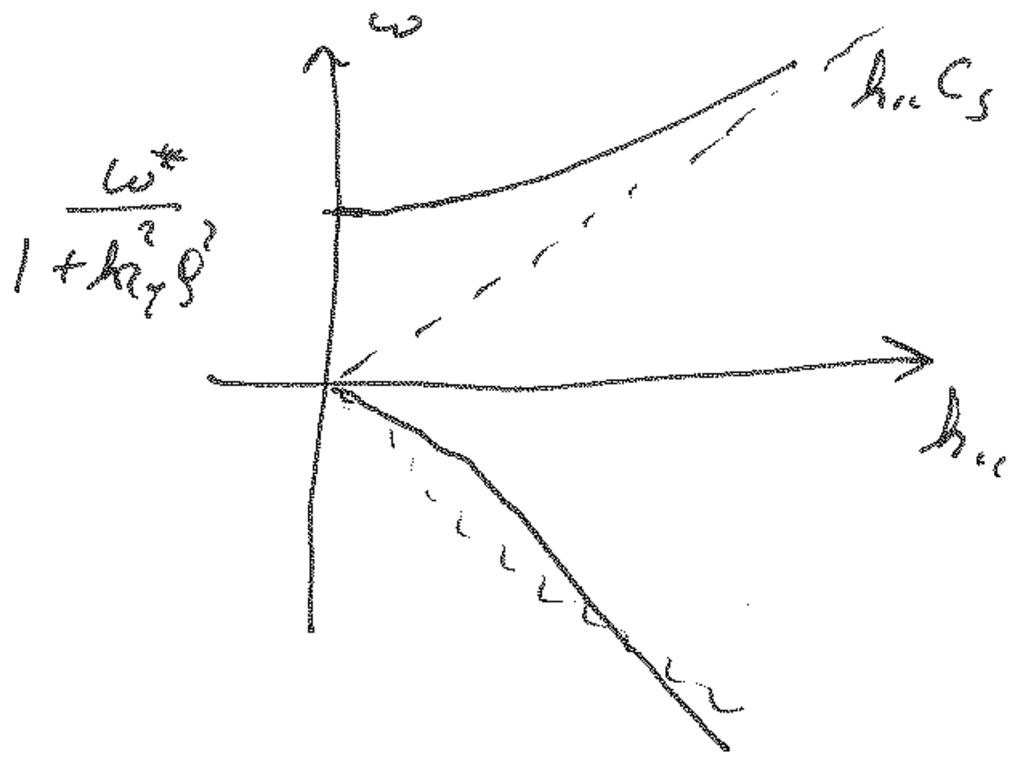
$$\frac{\tilde{n}}{n_0} = \left(\frac{e\tilde{\Phi}}{T}\right)\left[\frac{k_y v_D}{\omega} - k_\perp^2 \rho_i^2 + \frac{k_\parallel^2 c_s^2}{\omega^2}\right]$$

ION DRIFT RESPONSE $E\times B\cdot\nabla$

ION POLARIZATION

ION PARALLEL

$$D(\omega, k_y, k_\parallel) = 1 - \frac{k_y v_D}{\omega} - \frac{k_\parallel^2 c_s^2}{\omega^2} + k_\perp^2 \rho_i^2 - i\frac{\sqrt{\pi}}{k\, v_{The}^2}(k_y v_D - \omega)$$



FOR $k_\parallel$ SMALL

$$D \sim 1 - \frac{k_y v_D}{\omega} + k_\perp^2 \rho_i^2 + iD_I$$

$$\omega = \frac{k_y v_D}{1 + k_\perp^2 \rho_i^2}$$

$$\omega_I = \frac{-D_I}{\partial D_R/\partial\omega}$$

$$= \frac{i\sqrt{\pi}}{k_\parallel v_{The}}(k_y v_D - \omega)\left(\frac{\omega^2}{k_y v_D}\right)$$

SO $\omega < k_y v_D \rightarrow$ INSTABILITY

Next: Low-Frequency Magnetic Terms

ADIABATIC ELECTRONS: $\frac{\tilde{n}}{n} \approx \frac{e\tilde{\Phi}}{T}$ (NO RADIAL TRANSPORT)

NON-ADIABATIC ELECTRONS: $\frac{\tilde{n}}{n} \approx \frac{e\tilde{\Phi}}{T}(1 - i\delta\psi)$ SMALL PHASE SHIFT

$$\therefore \quad \omega = \frac{k_y V_e^*}{1 - i\delta\psi} \approx k_y V_e^* (1 + i\delta\psi + \cdots)$$

$\mathrm{Re}(\omega) \sim k_y V_e^*$ $\quad \mathrm{Im}(\omega) \sim i\delta\psi\, k_y V_e^*$

WHAT CAUSES $\delta\psi$?

COLLISIONS ALONG B

LANDAU DAMPING

ELECTROMAGNETIC INDUCTION $\delta B_\perp$