Plasma 2
Lecture 17:
Collisional Drift Waves

APPH E6102y
Columbia University

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Drift Wave Instability (and Transport)

ADIABATIC ELECTRONS: $\frac{\tilde{n}}{n} \simeq \frac{e\tilde{\Phi}}{T}$ (NO RADIAL TRANSPORT)

NON-ADIABATIC ELECTRONS: $\frac{\tilde{n}}{n} \simeq \frac{e\tilde{\Phi}}{T}(1 - i\delta\psi)$ SMALL PHASE SHIFT

$$\therefore \quad \omega = \frac{k_y v_e^*}{1 - i\delta\psi} \simeq k_y v_e^* (1 + i\delta\psi + \cdots)$$

$\mathrm{Re}(\omega) \sim k_y v_e^* \qquad \mathrm{Im}(\omega) \sim i\,\delta\psi\, k_y v_e^*$

WHAT CAUSES $\delta\psi$?

COLLISIONS ALONG B

LANDAU DAMPING

ELECTROMAGNETIC INDUCTION $\delta B_\perp$

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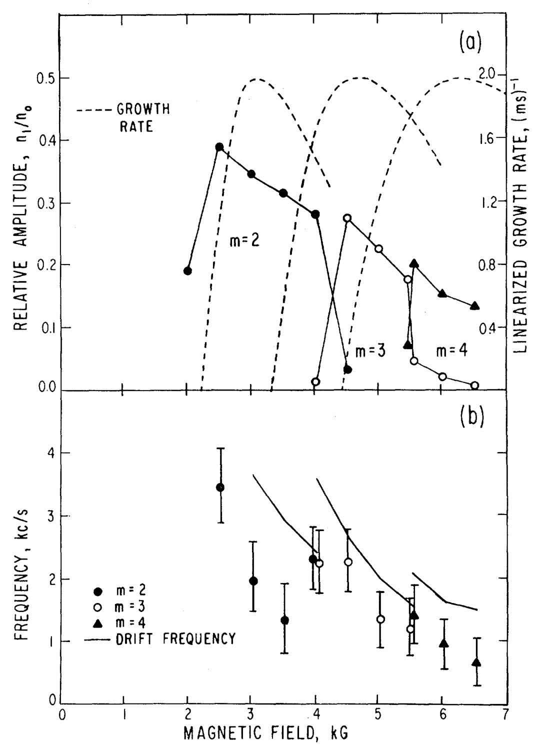
COLLISIONAL EFFECTS IN PLASMAS–DRIFT-WAVE EXPERIMENTS AND INTERPRETATION*

H. W. Hendel,† B. Coppi,‡ F. Perkins, and P. A. Politzer
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey
(Received 25 January 1967)

FIG. 1. (a) Observed oscillation amplitudes are compared with theoretical growth rates as a function of magnetic field strength for various azimuthal mode numbers. The absolute value of the magnetic field strength for the theoretical (slab model) curves has been scaled by a factor of ~1.5 to give a good fit to the data. The relative amplitude is defined as the ratio of the maximum density fluctuation to the central density. (b) The oscillation frequency (after subtraction of the rotational Doppler shift) is compared with the drift frequency $\nu_d = k_y v_d / 2\pi$ as a function of the magnetic field strength. The drift frequency, which has an uncertainty of ±0.5 kc/sec, is computed from the experimental values of k_y, T, and $n^{-1}(dn/dx)$. The data are for a potassium plasma, $n_0 = 3.5 \times 10^{11}$ cm^{-3}, $T = 2800$°K.



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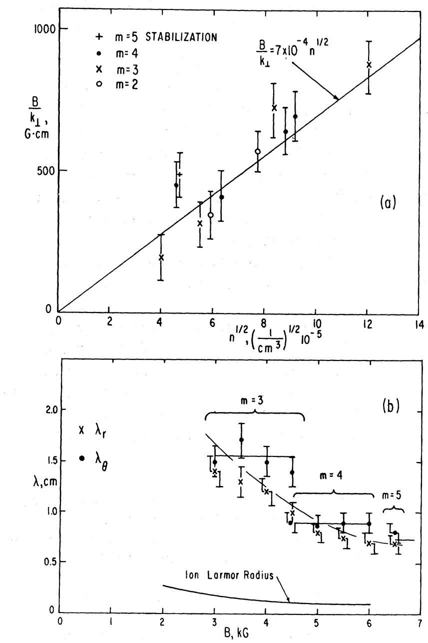

FIG. 2. (a) The ratio of magnetic field strength to perpendicular wave number is plotted versus the square root of the density for the stabilization points of several modes. Theory [Eq. (3)] gives a proportionality factor of 9.7×10^{-4}. (b) The measured radial (λ_r) and azimuthal (λ_θ) wavelengths of the perturbation are displayed as a function of the magnetic field.

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Collisional Drift Waves—Identification, Stabilization, and Enhanced Plasma Transport

H. W. Hendel,* T. K. Chu, and P. A. Politzer
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey
(Received 24 January 1967; final manuscript received 3 July 1968)

Density-gradient-driven collisional drift waves are identified by the dependences of ω and $\mathbf{k}$ on density, temperature, magnetic field, and ion mass, and by comparisons with a linear theory which includes resistivity and viscosity. Abrupt stabilization of azimuthal modes is observed when the stabilizing ion diffusion over the transverse wavelength due to the combined effects of ion Larmor radius and ion–ion collisions (viscosity) balances the destabilizing electron-fluid expansion over the parallel wavelength, determined by electron–ion collisions (resistivity). The finite-amplitude ($\tilde{n}/n_0 \simeq 10\%$) coherent oscillation, involving the entire plasma body, shows a phase difference between density and potential waves (which is predicted by linear theory for growing perturbations). The wave-induced radial transport exceeds classical diffusion, but is below the Bohm value by an order of magnitude. Although observations have been extended to magnetic fields three times those for drift-wave onset, turbulence has not been encountered.

resistivity (due to electron–ion collisions). We consider low-frequency ($\mathrm{Re}\,\omega \ll \Omega_i$) localized waves. Ion motion along lines of force ($k_\parallel v_{i,\mathrm{th}} \ll \mathrm{Re}\,\omega$), electron inertia ($\Omega_e = eB/m_e c \gg \mathrm{Re}\,\omega$), and perpendicular resistivity ($\nu_{ei} \ll \Omega_e$) are neglected. The convective term and the collision-free part of the ion-fluid stress tensor may be omitted because their contributions to the equation of continuity for ions cancel.[8,18] The first-order linearized equations thus become

$$n_0 M \frac{\partial \mathbf{u}_{i\perp}}{\partial t} = -KT\nabla_\perp \tilde{n}_i + n_0 e\left(-\nabla_\perp \phi + \mathbf{u}_{i\perp} \times \frac{\mathbf{B}}{c}\right) - \boxed{\tilde{n}_i e \mathbf{v}_d \times \frac{\mathbf{B}}{c}} + \mu_\perp \nabla_\perp^2 \mathbf{u}_{i\perp}, \quad (2)$$

Diamagnetic Doppler Shift cancels…

$$0 = -KT\nabla_\perp \tilde{n}_e - n_0 e\left(-\nabla_\perp \phi + \mathbf{u}_{e\perp} \times \frac{\mathbf{B}}{c}\right) - \boxed{\tilde{n}_e e \mathbf{v}_d \times \frac{\mathbf{B}}{c}} \quad (3)$$

$$0 = -KT\nabla_\parallel \tilde{n}_e + n_0 e \nabla_\parallel \phi - n_0 m_e \nu_{ei} u_{e\parallel}, \quad (4)$$

$$\frac{\partial \tilde{n}_i}{\partial t} + n_0 \nabla_\perp \cdot \mathbf{u}_{i\perp} + (\mathbf{u}_{i\perp} \cdot \nabla_\perp) n_0 - v_d \frac{\partial \tilde{n}_i}{\partial y} = 0, \quad (5)$$

$$\frac{\partial \tilde{n}_e}{\partial t} + n_0 \nabla_\perp \cdot \mathbf{u}_{e\perp} + (\mathbf{u}_{ei} \cdot \nabla_\perp) n_0 + v_d \frac{\partial \tilde{n}_e}{\partial y} + n_0 \nabla_\parallel u_{e\parallel} = 0, \quad (6)$$

7 equations; 7 unknowns: $\mathbf{u}_i$, $\mathbf{u}_e$, $u_{e\parallel}$, n_i, n_e

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Max Growth

Damping

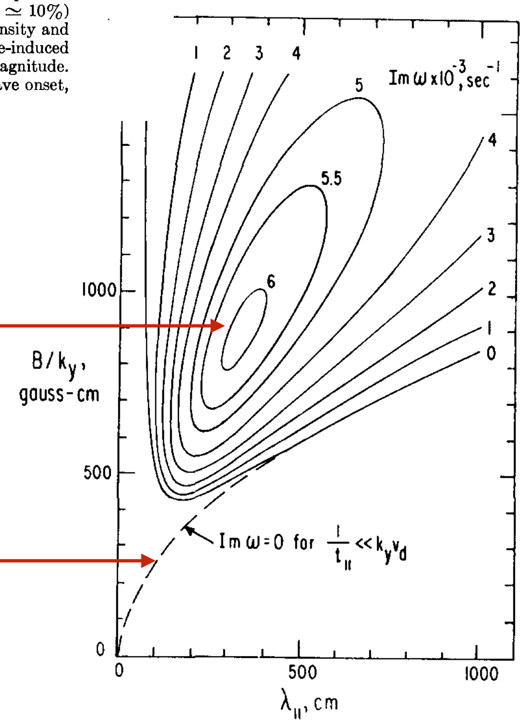

FIG. 1. Stability characteristics of density-gradient-driven collisional drift waves. Contour lines are lines of constant growth rate. Potassium plasma, $T = 2800°\mathrm{K}$, $n_0 = 10^{11}\ \mathrm{cm}^{-3}$, $\nabla n_0/n_0 = -1\ \mathrm{cm}^{-1}$.

Outline

- Review Drift Wave Formalism (last lecture)
- Ion dynamics (⊥) including collisions
- Electron dynamics (||) including collisions
- Characteristics of the collisional drift wave
- Radial ("anomolous") transport
- What's next: Landau damping, (low-frequency) EM: $\delta B_\perp$

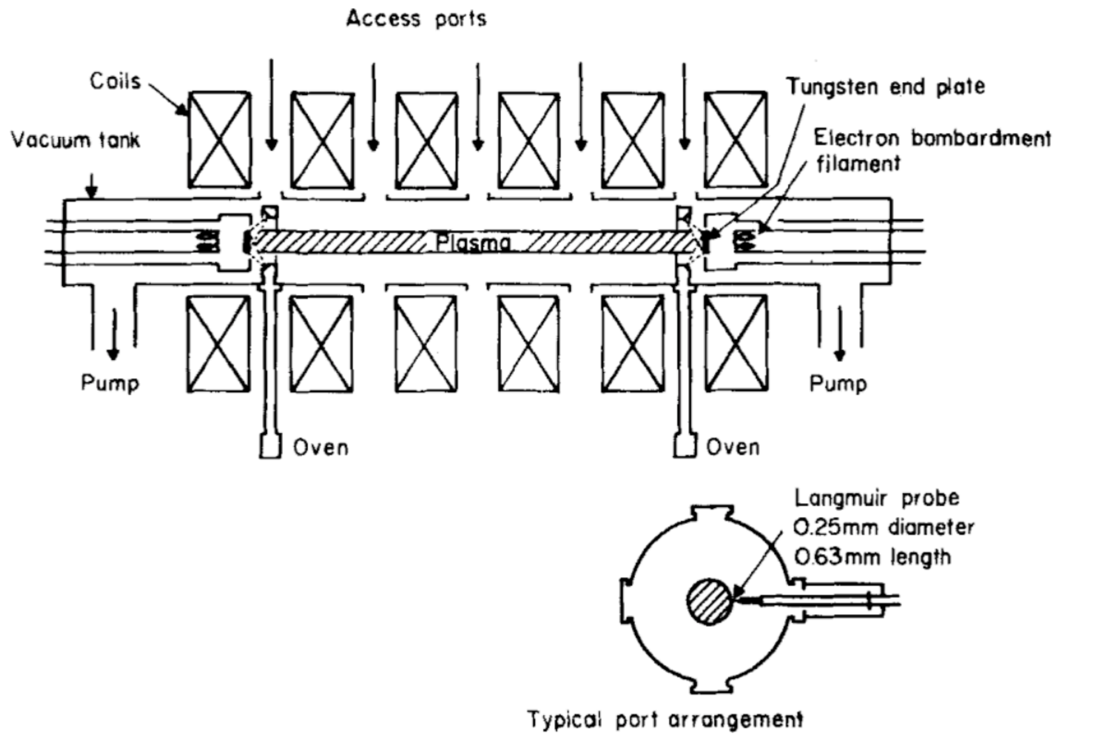

FIG. 5. Schematic of Q-1. Ions are produced by alkali-metal atom beams impinging on hot tungsten ionizer plates. Electrons are emitted thermionically by the same plates. Specifications: vacuum tank, length = 305 cm, diam. = 12.2 cm; plasma column, length = 128 cm, diam. = 3 cm; magnetic field to 7000 G; plasma density to 2×10^{12} cm^{-3}; plasma temperature to 3000° K.

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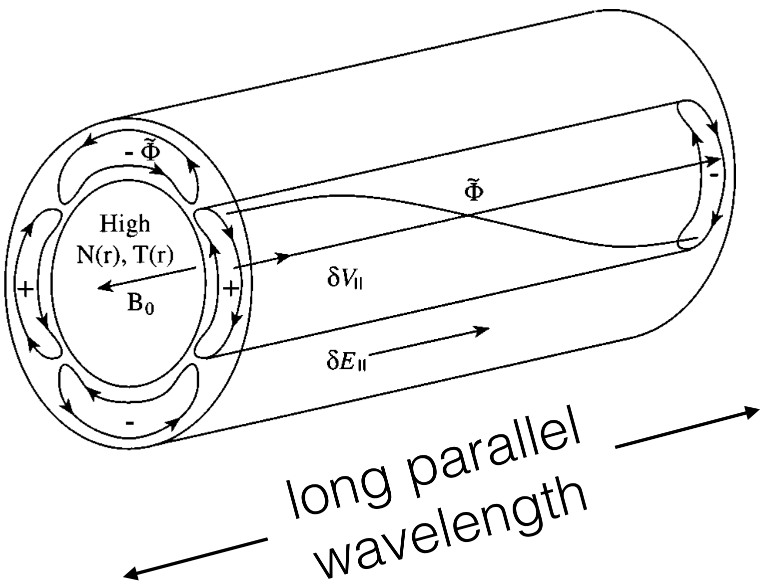
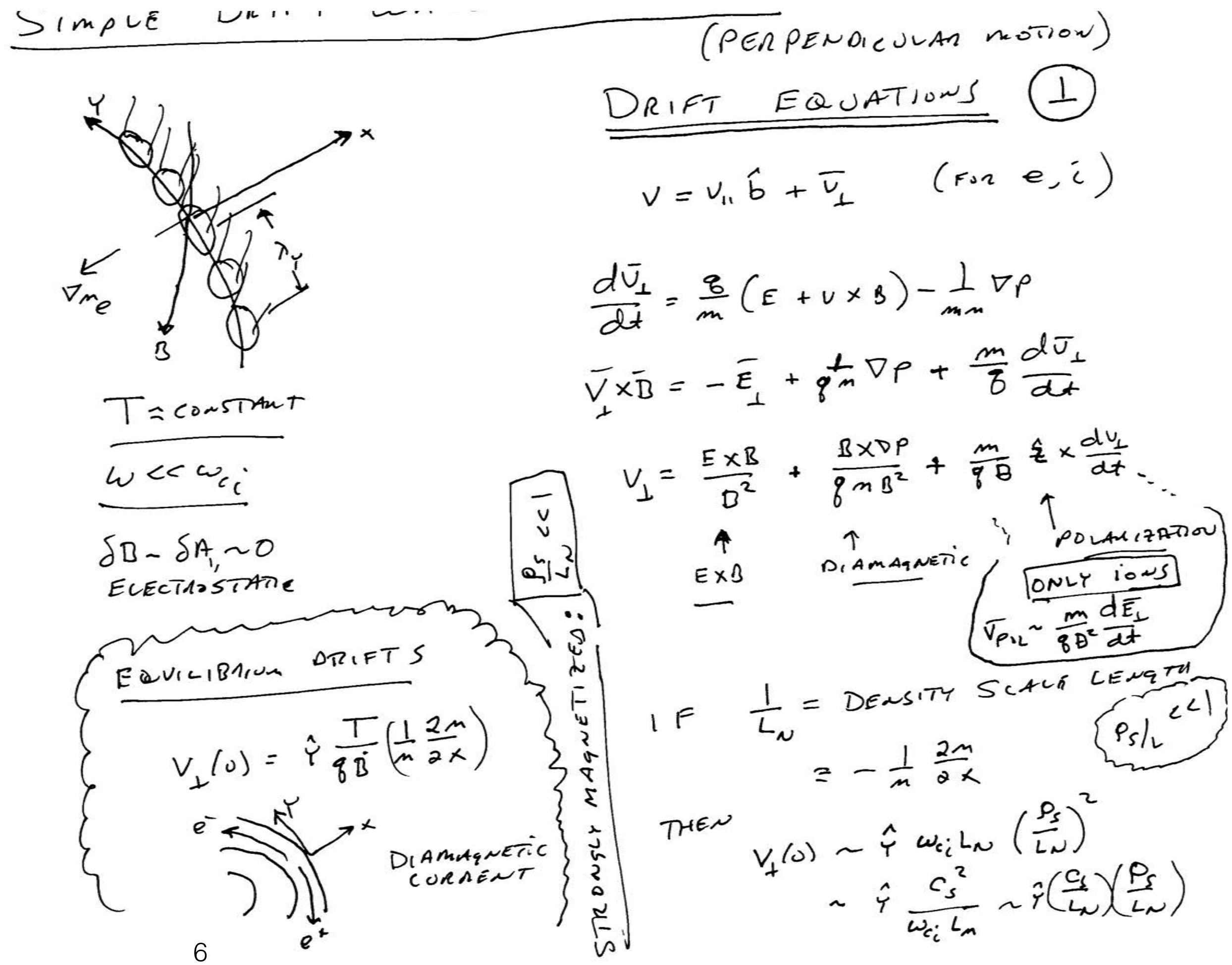
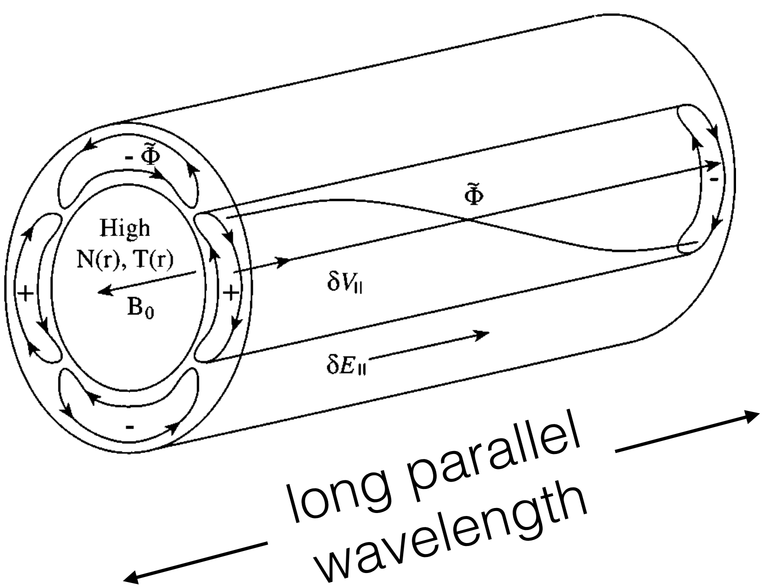
Simple Drift Wave Description

"Collisionless"





Simple Drift Wave Description: Parallel Dynamics



ELECTRON (FAST) PARALLEL MOTION **"Collisionless"**

$$e: \quad \frac{\partial \tilde v_\parallel}{\partial t} + (\bar v_E \cdot \nabla)\tilde v_\parallel = \frac{e}{m_e}\frac{\partial \tilde\phi}{\partial z} - \frac{1}{m_0 n_e}\frac{\partial \tilde p_e}{\partial z}$$

BUT m_e IS VERY VERY SMALL

ELECTRONS ARE VERY VERY LIGHT AND FAST

AS $m_e \to 0$

$$0 \simeq e\frac{\partial \tilde\Phi}{\partial z} - \frac{1}{n_0}\frac{\partial \tilde p_e}{\partial z} \simeq e\frac{\partial \tilde\Phi}{\partial z} - \frac{T}{n_0}\frac{\partial n}{\partial z}$$

$$\frac{1}{n}\frac{\partial n}{\partial z} \simeq \frac{e}{T}\frac{\partial \tilde\Phi}{\partial z} \Rightarrow \frac{n_e}{n_0} = e^{e\tilde\Phi/T}$$

CALLED "ADIABATIC ELECTRONS"

PARALLEL ELECT... FORCE BALA...

FORCE BALANCE

$$\boxed{\delta \tilde n \simeq n_0 \frac{e\tilde\Phi}{T}}$$

PROVIDED $k_\parallel \neq 0$

ELECTRONS MOVE / FLOW QUICKLY ALONG B TO KEEP PARALLEL FORCE ZERO

$$0 \simeq \frac{\partial}{\partial z}\left(e\tilde\Phi - \frac{\tilde p_e}{n}\right) = \frac{q}{m}\tilde E_\parallel - \frac{1}{n}\tilde\nabla_\parallel p = 0$$

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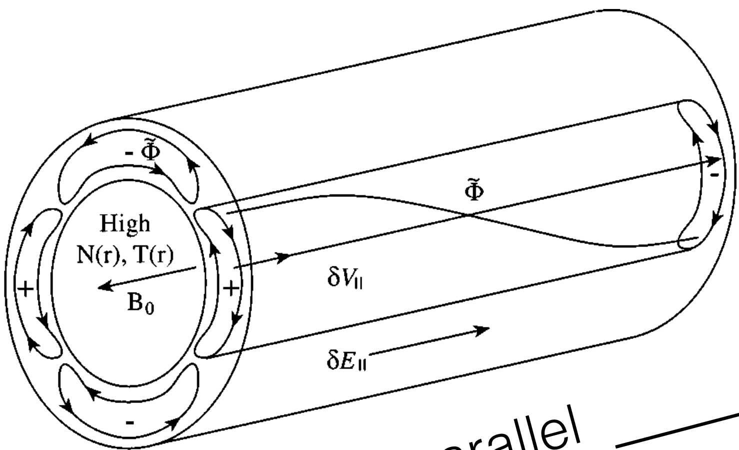
Simple Drift Wave Description: Continuity

“Collisionless”



long parallel wavelength

(only ion E×B drift motion)

CONTINUITY: $\frac{\partial \tilde n}{\partial t} + \nabla\cdot(n\bar V) = 0$ (REMEMBER: $\tilde n_e \approx \tilde n_i$)

LET'S USE ION CONTINUITY EQUATION

$$\frac{\partial \tilde n}{\partial t} + \bar\nabla\cdot(n\bar V_E) + \nabla\cdot(n\bar V_*) + \nabla\cdot(n V_{pol}) + \bar\nabla\cdot(n V_\parallel \hat z) = 0$$

E×B — DIAMAGNETIC FLUID FLOW — IGNORE AT FIRST (IMPORTANT) "ION INERTIAL" — IONS TOO SLOW ALONG B

$$\nabla\cdot(n\bar V_*) = \nabla\cdot(\hat z\times\nabla p)\frac{1}{qB} = \frac{1}{qB}\left[\nabla p\cdot(\nabla\times\hat z) - \hat z\cdot\nabla\times\nabla p\right] = 0$$

$$\nabla\cdot(n V_E) = n\,\underbrace{\nabla\cdot\bar V_E}_{\nabla\cdot(\hat z\times\nabla\varphi)/B = 0} + V_E\cdot\nabla n = V_E\cdot\nabla n_0$$

$$\bar V_E = -\frac{1}{B}\frac{\partial\Phi}{\partial y}\hat x$$

$$\frac{\partial\tilde n}{\partial t} + \tilde V_E\cdot\frac{\partial n_0}{\partial x} = 0$$

$$-j\omega\tilde n + \frac{n_0}{B}\frac{jk_y}{L_N}\tilde\Phi = 0$$

$$\frac{\tilde n}{n_0} = \frac{k_y}{BL_N\omega}\tilde\Phi$$

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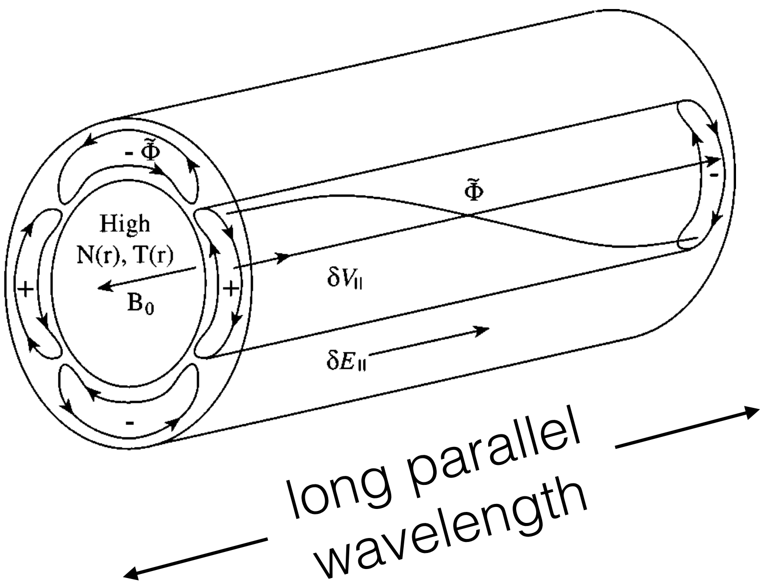
Basic “Drift Wave”

“Collisionless”

(only ion E×B drift motion)



PARALLEL ADIABATIC ELECTRONS

$$\frac{\tilde n}{n_0} = \frac{e\tilde\Phi}{T}$$

PERPENDICULAR ION DRIFT DYNAMICS

$$\frac{\tilde n}{n_0} = \frac{k_y}{B L_n \omega}\tilde\Phi$$

DISPERSION RELATION

$$\omega = k_y \frac{T}{eBL_n} = k_y V_e^*$$

DRIFT WAVES PROPAGATE IN ELECTRON DIAMAGNETIC DRIFT DIRECTION

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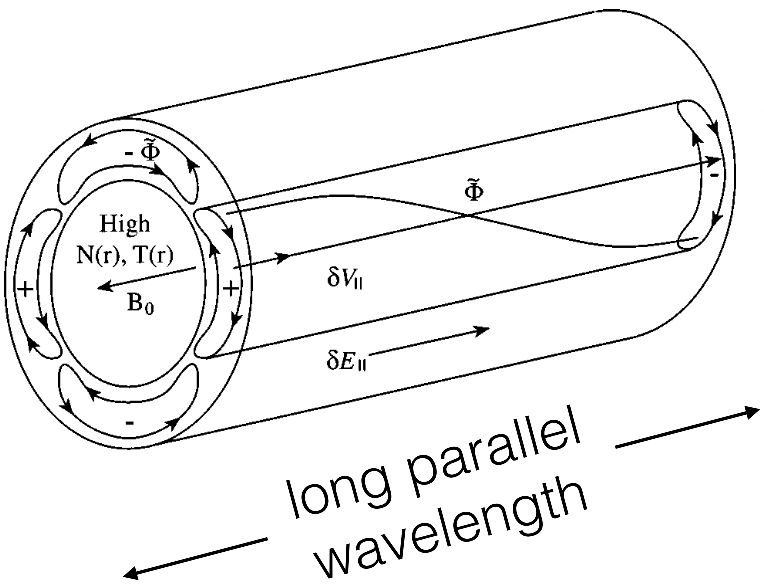
Ion Inertial Currents (Acoustic & Polarization Drift)



IONS ARE HEAVY

THEY HAVE POLARIZATION (INERTIAL) DRIFTS THAT REDUCE $E_\perp$

THEY MOVE SLOWLY (NON ADIABATIC) ALONG B

TAKE DRIFT WAVES LIKE USUAL ACOUSTIC WAVE

$$V_{THi} << \frac{\omega}{k_\parallel} << V_{The}$$

PERPENDICULAR ION

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\bar{v}) = 0$$

$$\frac{\partial \tilde{n}}{\partial t} + \nabla\cdot(n v_{pol} + n v_E) + n\frac{\partial v_{\parallel i}}{\partial z} = 0$$

$$\underbrace{-i\omega\tilde{n} + \nabla\cdot(n v_{pol}) + \tilde{v}_E\cdot\nabla n_0 + i\frac{k_\parallel^2}{\omega}\frac{e\tilde{\Phi}}{m_i}n_0 \approx 0}_{\text{NEW ION INERTIAL TERM}}$$

PARALLEL ION

$$\frac{dv_{\parallel i}}{dt} = -\frac{e}{m_i}\frac{\partial\Phi}{\partial z}$$

$$\hat{v}_{\parallel i} = \frac{e}{m_i}\frac{k_\parallel}{\omega}\tilde{\Phi}$$

SOUND RESPONSE

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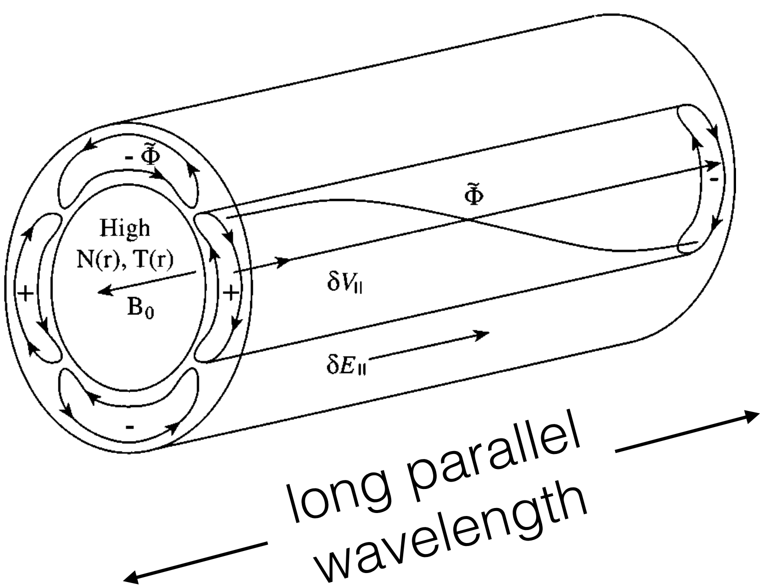
Ion Inertial Currents (Polarization Drift)



$$\nabla \cdot (n V_{pol}) = \nabla \cdot \left(\frac{n m_i}{q B^2} \frac{d E_\perp}{dt} \right)$$

$$= -\nabla \cdot \left(\frac{n m_i}{q B^2} \nabla_\perp \dot{\tilde{\Phi}} \right)$$

$$= -\nabla \cdot \frac{\epsilon_0}{q} \frac{\omega_{pi}^2}{\omega_{ci}^2} \nabla_\perp \dot{\tilde{\Phi}}$$

$$= k_\perp^2 \frac{n_0 m_i}{q B^2} \dot{\tilde{\Phi}}$$

$$= -j\omega k_\perp^2 n_0 \rho_i^2 \left(\frac{e\tilde{\Phi}}{T} \right)$$

$E_\perp = -\nabla_\perp \tilde{\Phi}$

$\epsilon_0 \frac{\omega_{pi}^2}{\omega_{ci}^2}$ = PLASMA DIELECTRIC $= \epsilon_0 \frac{n m_i}{B^2} >> \epsilon_0$

VERY LARGE

ADIABATIC ELECTRONS

$\hat{n} = n_0 \left(\frac{e\tilde{\Phi}}{T} \right)$

$k_\perp^2 \rho^2$ = FINITE LARMOR RADIUS TERM

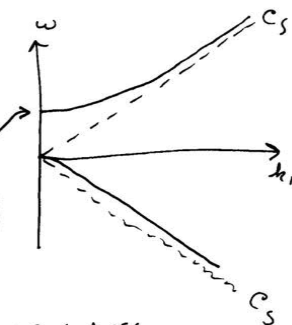
$$\boxed{\omega^2 (1 + k_\perp^2 \rho^2) - \omega k_y v^* - k_\parallel^2 c_s^2 = 0}$$

ION POLARIZATION TERM

DENSITY GRADIENT DRIFT

SOUND WAVE TERM

$\omega = \frac{k_y v^*}{1 + k_\perp^2 \rho^2}$



NOTE: STABLE WAVES

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Ch. 12: Collisional Processes

Based on the above analysis, we can now estimate the mean-free path, $\lambda_{\rm m}$, required for multiple small-angle collisions to produce a deflection of the order of 90°. Multiple small-angle collisions produce a change $\langle(\Delta v_\perp)^2\rangle/\Delta\ell$, given by Eq. (12.2.7). To produce a deflection of the order of 90° in a path length, $\lambda_{\rm m}$, we require that

$$\frac{\langle(\Delta v_\perp)^2\rangle}{\Delta\ell}\lambda_{\rm m} = V^2, \tag{12.2.10}$$

which, after substituting Eq. (12.2.7) for $\langle(\Delta v_\perp)^2\rangle/\Delta\ell$, gives

$$\lambda_{\rm m} = \frac{1}{8\pi n_0 b_0^2 \ln\Lambda}. \tag{12.2.11}$$

Next, we calculate the mean-free path for a single 90° large-angle collision, $\lambda_{\rm S}$, which, using Eq. (12.1.6), is given by

$$\lambda_{\rm S} = \frac{1}{n_0\sigma_{\rm S}} = \frac{1}{n_0\pi b_0^2}. \tag{12.2.12}$$

$$b_0 = \frac{e_s e_{s'}}{4\pi\epsilon_0\mu_{ss'}V^2}$$

$$\mu_{ss'} = \frac{m_s m_{s'}}{m_s + m_{s'}}$$

Plasma Resistivity

The plasma resistivity η is the inverse of the conductivity σ. A more detailed calculation of the resistivity that takes into account the effect of electron–electron collisions, first carried out by Spitzer and Harm (1953), yields the following formula for the resistivity:

$$\eta \simeq 5.2 \times 10^{-5} \frac{\ln \Lambda}{(\kappa T_e)^{3/2}} \text{ ohm m}, \tag{12.4.25}$$

where κT_e is expressed in electron volts. To be precise, the above equation gives the Spitzer–Harm resistivity parallel to an equilibrium magnetic field. The resistivity

Table 12.1 *Comparison of the resistivity of various types of plasmas with some common materials*

Material	Resistivity, η (ohm m)
Copper	2×10^{-8}
Stainless steel	7×10^{-7}
100 eV plasma	5×10^{-7}
5 keV plasma	1×10^{-9}
Interstellar gas (1 eV)	5×10^{-7}
Solar corona (10 eV)	5×10^{-5}
Earth's ionosphere (0.1 eV)	2×10^{-2}

$$\nu_{ei} \simeq \frac{n_0 e^2}{m_e \sigma} \simeq \frac{n_0 e^4 \ln \Lambda}{32\pi^{1/2} \epsilon_0^2 m_e^{1/2} (2\kappa T_e)^{3/2}}.$$

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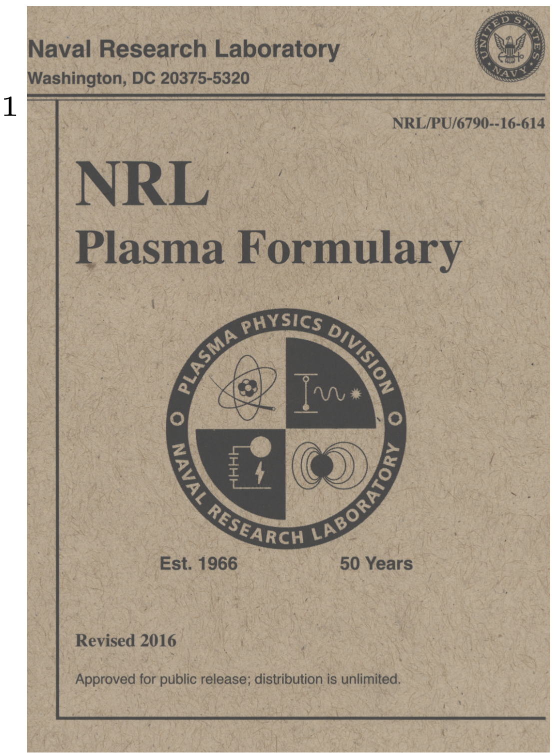
NRL Plasma Formulary

https://www.nrl.navy.mil/ppd/content/nrl-plasma-formulary

electron collision rate $\quad \nu_e = 2.91 \times 10^{-6} n_e \ln \Lambda T_e{}^{-3/2} \,\text{sec}^{-1}$

ion collision rate $\quad \nu_i = 4.80 \times 10^{-8} Z^4 \mu^{-1/2} n_i \ln \Lambda T_i{}^{-3/2} \,\text{sec}^{-1}$



e^+ - e^- energy exchange:

$$\frac{dT_\alpha}{dt} = \sum_\beta \bar{\nu}_\epsilon^{\alpha\backslash\beta}(T_\beta - T_\alpha),$$

where

$$\bar{\nu}_\epsilon^{\alpha\backslash\beta} = 1.8 \times 10^{-19} \frac{(m_\alpha m_\beta)^{1/2} Z_\alpha{}^2 Z_\beta{}^2 n_\beta \lambda_{\alpha\beta}}{(m_\alpha T_\beta + m_\beta T_\alpha)^{3/2}} \,\text{sec}^{-1}.$$

For electrons and ions with $T_e \approx T_i \equiv T$, this implies

$$\bar{\nu}_\epsilon^{e|i}/n_i = \bar{\nu}_\epsilon^{i|e}/n_e = 3.2 \times 10^{-9} Z^2 \lambda/\mu T^{3/2} \text{cm}^3 \,\text{sec}^{-1}.$$

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Braginskii Equations

21. S. I. Braginskii, "Transport Processes in a Plasma," *Reviews of Plasma Physics*, Vol. 1 (Consultants Bureau, New York, 1965), p. 205.

Transport Coefficients

Transport equations for a multispecies plasma:

$$\frac{d^\alpha n_\alpha}{dt} + n_\alpha \nabla \cdot \mathbf{v}_\alpha = 0;$$

$$m_\alpha n_\alpha \frac{d^\alpha \mathbf{v}_\alpha}{dt} = -\nabla p_\alpha - \nabla \cdot P_\alpha + Z_\alpha e n_\alpha \left[\mathbf{E} + \frac{1}{c} \mathbf{v}_\alpha \times \mathbf{B} \right] + \mathbf{R}_\alpha;$$

$$\frac{3}{2} n_\alpha \frac{d^\alpha k T_\alpha}{dt} + p_\alpha \nabla \cdot \mathbf{v}_\alpha = -\nabla \cdot \mathbf{q}_\alpha - P_\alpha : \nabla \mathbf{v}_\alpha + Q_\alpha.$$

Here $d^\alpha/dt \equiv \partial/\partial t + \mathbf{v}_\alpha \cdot \nabla$; $p_\alpha = n_\alpha k T_\alpha$, where k is Boltzmann's constant; $\mathbf{R}_\alpha = \sum_\beta \mathbf{R}_{\alpha\beta}$ and $Q_\alpha = \sum_\beta Q_{\alpha\beta}$, where $\mathbf{R}_{\alpha\beta}$ and $Q_{\alpha\beta}$ are respectively the momentum and energy gained by the αth species through collisions with the βth; P_α is the stress tensor; and $\mathbf{q}_\alpha$ is the heat flow.

stress tensor (either species)

$$P_{xx} = -\frac{\eta_0}{2}(W_{xx} + W_{yy}) - \frac{\eta_1}{2}(W_{xx} - W_{yy}) - \eta_3 W_{xy};$$
$$P_{yy} = -\frac{\eta_0}{2}(W_{xx} + W_{yy}) + \frac{\eta_1}{2}(W_{xx} - W_{yy}) + \eta_3 W_{xy};$$
$$P_{xy} = P_{yx} = -\eta_1 W_{xy} + \frac{\eta_3}{2}(W_{xx} - W_{yy});$$
$$P_{xz} = P_{zx} = -\eta_2 W_{xz} - \eta_4 W_{yz};$$
$$P_{yz} = P_{zy} = -\eta_2 W_{yz} + \eta_4 W_{xz};$$
$$P_{zz} = -\eta_0 W_{zz}$$

(here the z axis is defined parallel to $\mathbf{B}$);

ion viscosity

$$\eta_0^i = 0.96 nkT_i\tau_i; \quad \eta_1^i = \frac{3nkT_i}{10\omega_{ci}{}^2\tau_i}; \quad \eta_2^i = \frac{6nkT_i}{5\omega_{ci}{}^2\tau_i};$$
$$\eta_3^i = \frac{nkT_i}{2\omega_{ci}}; \quad \eta_4^i = \frac{nkT_i}{\omega_{ci}};$$

electron viscosity

$$\eta_0^e = 0.73 nkT_e\tau_e; \quad \eta_1^e = 0.51\frac{nkT_e}{\omega_{ce}{}^2\tau_e}; \quad \eta_2^e = 2.0\frac{nkT_e}{\omega_{ce}{}^2\tau_e};$$
$$\eta_3^e = -\frac{nkT_e}{2\omega_{ce}}; \quad \eta_4^e = -\frac{nkT_e}{\omega_{ce}}.$$

For both species the rate-of-strain tensor is defined as

$$W_{jk} = \frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} - \frac{2}{3}\delta_{jk}\nabla \cdot \mathbf{v}.$$

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Ion Collisional ($\perp$) Dynamics

$$\frac{d\bar{v}_\perp}{dt} = \frac{e}{m_i}\left(\bar{E} + \bar{v}\times\bar{B}\right) - \frac{1}{n m_i}\bar{\nabla} p_i + \frac{\mu_\perp}{n m_i}\nabla_\perp^2 \bar{v}_\perp$$

$$\bar{v}_\perp \simeq \frac{E\times B}{B^2} + \frac{B\times\nabla p}{q n B^2} + \underbrace{\frac{m_i}{qB}\hat{z}\times\frac{d v_\perp}{dt}}_{\frac{m}{qB^2}\frac{dE_\perp}{dt}} - \underbrace{\frac{m_i}{qB}\nu_{ii}\rho_i^2\nabla_\perp^2(\hat{z}\times\bar{v}_\perp)}_{-\underbrace{\frac{m}{qB^2}\nu_{ii}\rho_i^2}\nabla_\perp^2\bar{E}_\perp}$$

$\hookrightarrow$ NOTE: VERY STRONG $|B|$ SCALING $\propto \frac{1}{B^4}$!!

$\boxed{\text{NEXT:}}$ CONTINUITY

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\bar{v}) = 0$$

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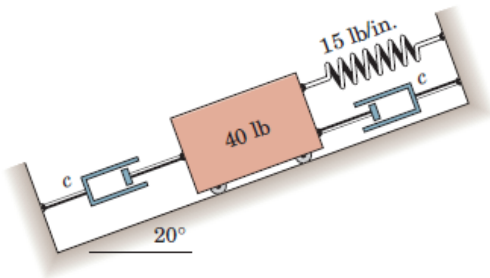
Ion Collisional Dynamics (w Viscous Damping)



ACCOUSTIC PARALLEL DYNAMICS

$$\frac{dV_{\parallel i}}{dt} = -\frac{e}{m_i}\frac{\partial \Phi}{\partial z}$$

CONTINUITY

$$\frac{\partial \tilde{n}}{\partial t} + \nabla \cdot (n \bar{V}) = 0$$

WHERE...

$$\nabla \cdot (n V_{DIA}) = 0$$

$$\nabla \cdot V_E = 0$$

PERPENDICULAR DYNAMICS

$$\underbrace{\frac{d\bar{V}_\perp}{dt}}_{\text{POLARIZATION DRIFT}} = \frac{e}{m_i}\underbrace{(\bar{E} + \bar{V}\times\bar{B})}_{E\times B \text{ DRIFT}} \underbrace{- \frac{1}{n m_i}\nabla p_i}_{\text{DIAMAGNETIC DRIFT}} + \underbrace{\frac{\mu_\perp}{n m_i}\nabla_\perp^2 \bar{V}_\perp}_{\text{VISCOUS DAMPING}}$$

$$\frac{\mu_\perp}{n m_i} = \underbrace{\nu_{ii}}_{\text{ION-ION COLLISIONS}} \underbrace{\rho_i^2}_{\text{ION GYRORADIUS}}$$

"PERPENDICULAR ION MOMENTUM DIFFUSION" !

NOTE: VISCOSITY ALSO RESULTS FROM ION-NEUTRAL COLLISIONS AT EDGE

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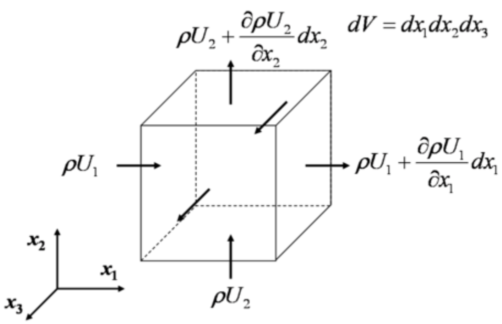
Ion Continuity



$$\frac{\partial \tilde{n}}{\partial t} + \bar{V}_E \cdot \bar{\nabla} n_0 + \nabla \cdot (n V_{pol}) + n_0 \frac{\partial v_{\parallel}}{\partial z} + \nabla \cdot (n V_{\mu}) = 0$$

ION POLARIZATION

ION VISCOSITY

E×B CROSS-FIELD CONVECTION

PARALLEL ACOUSTIC

$$-j\omega \tilde{n}$$

$$i k_y V_D^* n_0 \left(\frac{e\tilde{\Phi}}{T}\right)$$

$$\frac{\partial v_{\parallel}}{\partial z} = i \frac{e}{M_i} \frac{k_{\parallel}^2}{\omega} \tilde{\Phi}$$

$$\nabla \cdot (n V_{\mu}) = -\nu_{ii} (k_\perp \rho_i)^4 n_0 \left(\frac{e\tilde{\Phi}}{T}\right)$$

$$\nabla \cdot (n V_{pol}) = k_\perp^2 \frac{n_0 M_i}{q B^2} \dot{\tilde{\Phi}} = -j\omega k_\perp^2 \rho_i^2 n_0 \left(\frac{e\tilde{\Phi}}{T}\right)$$

NEXT

$$\tilde{n}, \tilde{\Phi} \propto e^{-j\omega t} e^{j k_y y} e^{j k_x x} e^{j k_{\parallel} z}$$

Drift Wave Dynamics (Adiabatic Electrons)

IONS

$$-i\omega \frac{\tilde{n}}{n_0} + i k_y v_D^* \left(\frac{e\tilde{\Phi}}{T}\right) - i\omega (k_\perp \rho)^2 \left(\frac{e\tilde{\Phi}}{T}\right) + i \frac{k_\parallel^2 c_s^2}{\omega} \left(\frac{e\tilde{\Phi}}{T}\right) - \gamma_{ii} (k_\perp \rho_i)^4 \left(\frac{e\tilde{\Phi}}{T}\right) = 0$$

ELECTRONS

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\Phi}}{T}$$ (ADIABATIC PARALLEL DYNAMICS)

$$\omega^2 (1 + (k_\perp \rho)^2) - \omega \left(k_y v_D^* + i \gamma_{ii} (k_\perp \rho)^4\right) - k_\parallel^2 c_s^2 = 0$$

FOR LONG PARALLEL WAVELENGTHS $k_\parallel \to 0$

$$\omega \simeq k_y v_D^* - i \gamma_{ii} (k_\perp \rho)^4$$

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Electron (||) Dynamics w Collisions

$$\underbrace{\frac{\partial v_{\parallel e}}{\partial t} + (V_E \cdot \nabla) V_{\parallel e}}_{\substack{\text{ELECTRON} \\ \text{PARALLEL} \\ \text{INERTIA} \\ \approx 0}} = \frac{e}{m_e}\frac{\partial \Phi}{\partial z} - \frac{1}{n m_e}\frac{\partial p_0}{\partial z} - \nu_{ei} V_\parallel^e$$

ION-ELECTRON COLLISIONS !!

$$\therefore \quad \tilde{V}_{\parallel e} = i \frac{k_\parallel T}{\nu_{ei} m_e}\left(\frac{e\tilde{\Phi}}{T} - \frac{\tilde{n}}{n_0}\right)$$

<u>ELECTRON CONTINUITY</u>:

$$\frac{\partial \tilde{n}_e}{\partial t} + \nabla \cdot (V_E n_0) + \frac{\partial}{\partial z}(n_0 V_{\parallel e}) = 0$$

$$-j\omega\tilde{n} - j k_y V_D^* n_0 + j k_\parallel V_{\parallel e} n_0 = 0$$

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Electron (||) Dynamics w Collisions

$$\omega \frac{\tilde{n}}{n_0} - k_y V_0^* \left(\frac{e\tilde{\Phi}}{T}\right) - i \underbrace{\frac{k_{\parallel}^2 T}{\nu_{ei} m_e}}_{k_{\parallel}^2 D_e} \left(\frac{e\tilde{\Phi}}{T} - \frac{\tilde{n}}{n_0}\right) = 0$$

$$D_e \equiv \frac{T}{\nu_{ei} m_e}$$

PARALLEL COLLISIONAL DIFFUSION

$$\left(\omega + i k_{\parallel}^2 D_e\right) \frac{\tilde{n}}{n_0} - \left(k_y V_0^* + i k_{\parallel}^2 D_e\right)\left(\frac{e\tilde{\Phi}}{T}\right) \approx 0$$

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\Phi}}{T}\left[\frac{k_y V_0^* + i k_{\parallel}^2 D_e}{\omega + i k_{\parallel}^2 D_e}\right]$$

NICE LIMITS TO EXAMINE

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Strong Collisions (yields unstable drift wave)

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\Phi}}{T}\left[\frac{k_y v_0^* + i k_\parallel^2 D_e}{\omega + i k_\parallel^2 D_e}\right]$$

ASSUME $|k_\parallel^2 D_e| \gg |\omega|$, THEN

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\Phi}}{T}\left[\left(1 - \frac{i k_y v_0^*}{k_\parallel^2 D}\right)\left(1 + i\frac{\omega}{k_\parallel^2 D} + \cdots\right)\right] = \frac{e\tilde{\Phi}}{T}\left[1 - i\frac{(k_y v_0^* - \omega)}{k_\parallel^2 D} + \cdots\right]$$

$\boxed{\text{LOOK: PHASE SHIFT !}}$

DRIFT WAVES (WITH ION POLARIZATION) $\quad \omega \approx k_y v_0^* / (1 + (k_\perp \rho)^2) \simeq k_y v_0^* (1 - (k_\perp \rho)^2)$

SO

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\Phi}}{T}\left[1 - i\underbrace{\frac{(k_y v_0^*)(k_\perp \rho)^2}{k_\parallel^2 D_e}}_{\left(\frac{m_e}{m_i}\right)\frac{k_y v_0^* \nu_{ei} (k_\perp \rho)^2}{(k_\parallel c_s)^2}}\right]$$

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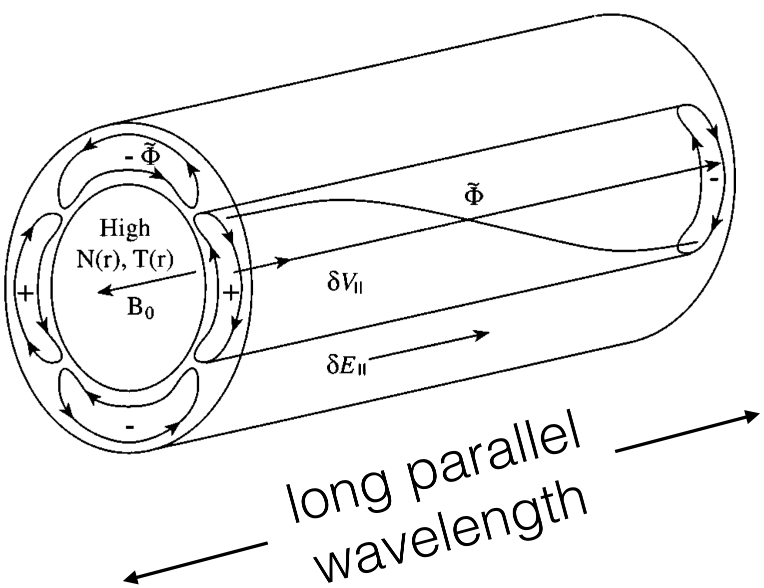
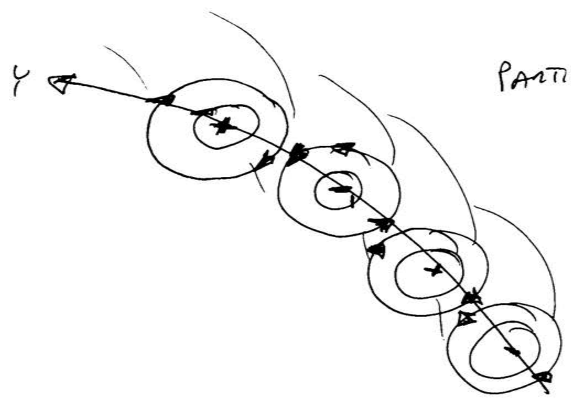
How Much Transport from Drift Waves?



$$\omega^2(1+k_\perp^2\rho_i^2) - \omega k_y v^* - k_\parallel^2 c_s^2 = 0$$

$$\frac{\delta n}{n_0} = \frac{e\tilde{\Phi}}{T} \implies$$ DENSITY AND POTENTIAL FLUCTUATIONS ARE <u>IN</u> <u>PHASE</u>



PARTICLE FLUX $= \frac{1}{2}\mathrm{Re}\{\tilde{n}^*\tilde{v}\}$

$$= -\frac{1}{2}\mathrm{Re}\left\{n_0\frac{e}{T}\tilde{\Phi}^*\frac{ik_y\tilde{\Phi}}{B}\hat{x}\right\}$$

$$= -\frac{1}{2}n_0\frac{ek_y}{BT}\,\mathrm{Re}\underbrace{\{i\tilde{\Phi}^*\tilde{\Phi}\}}_{0}$$

<u>NO</u> <u>FLUX</u> $\underline{\underline{0}}$

WE NEED A <u>PHASE</u> SHIFT BETWEEN δn AND $\delta\phi$ FOR <u>TRANSPORT</u>

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Observing Drift-Wave Transport

VI. MEASUREMENTS OF ENHANCED PLASMA TRANSPORT CAUSED BY COLLISIONAL DRIFT WAVES

Drift (universal) instabilities are potentially perilous to low-β plasma confinement as they may cause enhanced loss in such plasmas. However, the mechanism responsible for enhanced plasma transport due to a finite-amplitude instability cannot, in general, be deduced from linearized calculations, and a rigorous nonlinear theory of collisional drift-wave induced plasma losses, predicting experimental amplitudes and phase difference, has not been reported.[32] Experimentally, the relation of enhanced plasma loss to specific instabilities is difficult to establish due to problems in the identification of instabilities and to the simultaneous presence in many plasma devices of different unstable modes. Furthermore, direct measurements of classical and enhanced radial fluxes are complicated because of the small velocities involved and because of changes in local plasma density and its gradient in the presence of probes. Indirect measurements of wave-induced enhanced fluxes are complicated by problems in the separation of these fluxes from other losses such as charge exchange, dc drifts, and volume or end-plate recombination; and in the measurement of local wave parameters from which enhanced plasma transport is determined. In addition, for a conclusive measurement it must be shown that the presence of the instability does not cause changes in the boundary conditions and concomitant losses.

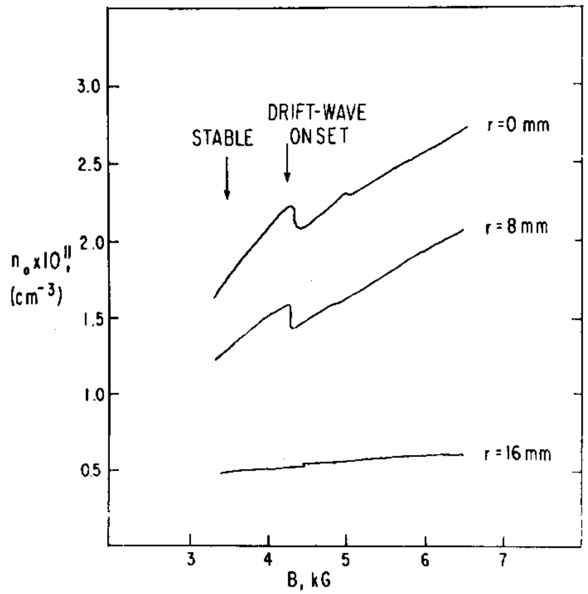

FIG. 7. Plasma density at different radial positions in relation to drift-wave onset. Cesium, T = 2800°K.

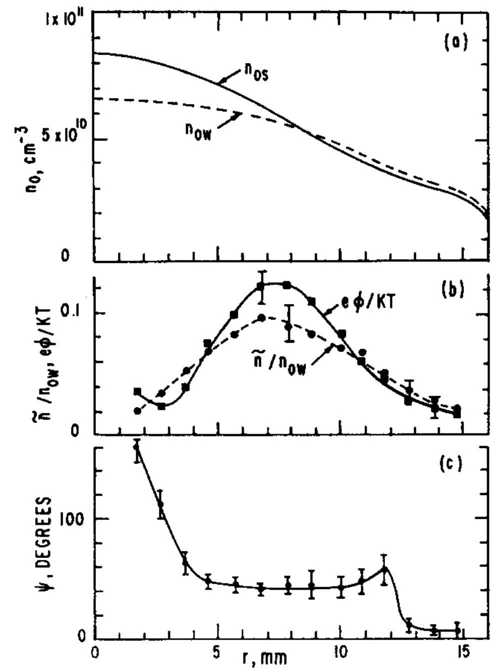

FIG. 9. Measured density profiles and wave parameters versus radius for a potassium plasma at T = 2760°K. (a) Density profiles in the stable (n_{0s}, B = 1964 G) and drift-wave regimes (n_{0w}, m = 1, B = 2050 G). (b) Relative amplitude of density ($\tilde{n}/n_0$) and potential ($e\phi/kT$) oscillations. (c) Phase angle ψ by which the density wave leads the potential wave.

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Conclusions

VII. DISCUSSION AND CONCLUSION

The present experiment is conducted in a region of the plasma where the free-energy reservoirs available for self-sustained instabilities are limited; i.e., the plasma is close to thermal equilibrium, no current is applied, and regions dominated by density and temperature gradients[36] are separated. In this plasma region, where the only known excitation mechanism for instability is the density gradient, we observe a self-sustained, coherent oscillation whose behavior agrees with results from the linear theory of density-gradient-driven collisional drift waves.

The identification of this wave is based on its measured dependence of frequency and wavenumber on all experimentally accessible parameters. The wave is observed to show abrupt stabilization of azimuthal modes occurring when the stabilizing ion diffusion over the transverse wavelength due to the combined effects of ion–ion collisions and finite ion Larmor radius (viscosity) balances the destabilizing effect of electron motion over the parallel wavelength due to, but also limited by, resistivity.

The present experimental results show that the observed pattern of mode amplitudes is similar to that of the calculated linear growth rate, and that the observed frequencies are those predicted for highest linear growth rates. If we take these results as an indication that at the saturation stage the amplitude is proportional to a power of the linear growth rate, in agreement with the theory considering higher-order terms discussed above, we can associate the observed finite-amplitude modes with those of highest growth rate predicted from linear theory.

The crucial plasma-confinement experiment is the measurement of anomalous losses, i.e., losses above classical diffusion. Drift waves are of particular interest since the only required sustaining mechanism is a generally present density (or temperature) gradient. The growth rates predicted by linear theory for collisional drift waves are large, $2\gamma \sim \mathrm{Re}\,\omega \sim k_y v_d/2$, as is the transverse wavelength, $\lambda_\perp \gg r_L$, making the collisional drift wave suitable to cause enhanced plasma transport. Such causal relation between collisional drift waves and enhanced plasma transport is observed in the present experiment. The plasma transport induced by the wave electric field is larger than that due to classical binary-collision diffusion by one order of magnitude for a 10% relative wave amplitude. This transport is a direct consequence of the phase difference between the coherent density and potential waves, which is predicted for a growing wave by linear theory. The coherent wave, which has

To conclude, the present results on the parametric dependence of frequency ω and wavenumber $\mathbf{k}$ constitute a comprehensive identification of the density-gradient-driven collisional drift wave at the oscillatory finite-amplitude saturation stage, based on, and in agreement with, results from linear theory. The coherent wave, involving the entire plasma body, is shown to produce enhanced plasma transport. This finding of plasma transport induced by the single-mode coherent drift wave represents a departure from the prevalent concept of plasma loss from magnetic confinement, which associates plasma loss with plasma turbulence. Furthermore, this work indicates the presence of an extensive regime of coherent oscillations beyond wave onset ($B \gg B_c$); although azimuthal modes have been observed up to $m = 7$, turbulence, potentially important and desirable for further study, has yet to be encountered.

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Next: Collisionless Drift Wave Instability and Transport

ADIABATIC ELECTRONS: $\frac{\tilde{n}}{n} \approx \frac{e\tilde{\Phi}}{T}$ (NO RADIAL TRANSPORT)

NON-ADIABATIC ELECTRONS: $\frac{\tilde{n}}{n} \approx \frac{e\tilde{\Phi}}{T}(1 - i\delta\psi)$ SMALL PHASE SHIFT

$$\therefore \quad \omega = \frac{k_y v_e^*}{1 - i\delta\psi} \approx k_y v_e^*(1 + i\delta\psi + \cdots)$$

$\mathrm{Re}(\omega) - k_y v_e^*$ $\quad \mathrm{Im}(\omega) \sim i\delta\psi k_y v_e^*$

WHAT CAUSES $\delta\psi$?

COLLISIONS ALONG B

LANDAU DAMPING

ELECTROMAGNETIC INDUCTION $\delta B_\perp$

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