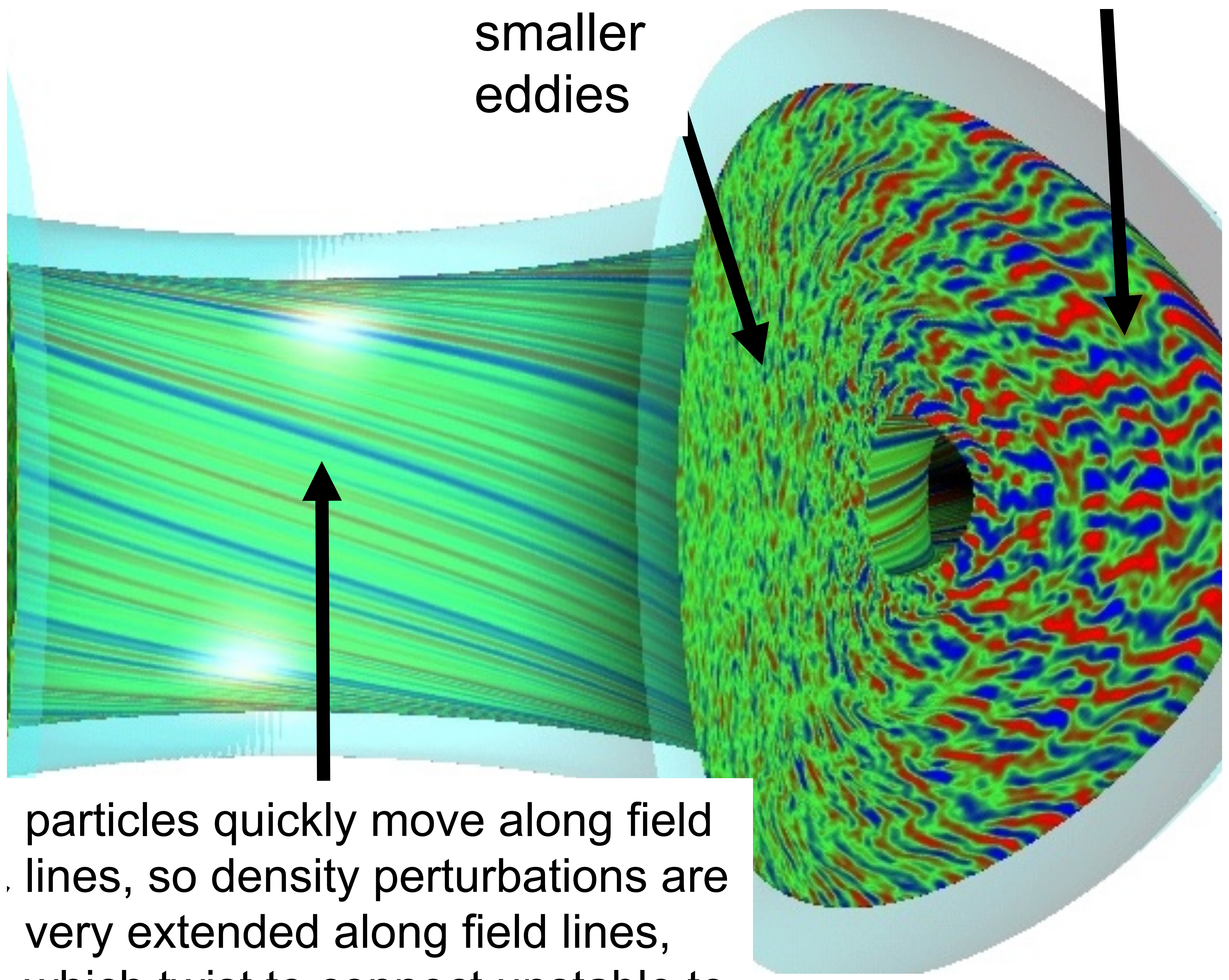
Plasma 2
Lecture 15:
Introduction to Drift Waves

APPH E6102y
Columbia University

Outline

- Goal
- Geometry
- Drifts
- (Fast) electron motion along B ("adiabatic electrons")
- Drift motion across B
- Ion inertial currents (*i.e.* polarization drifts)
- What's next: collisions, Landau damping, (low-frequency) EM: $\delta B_\perp$

Jan Weiland, *Collective Modes in Inhomogeneous Plasma: Kinetic and Advanced Fluid Theory*, IOP Publishing, 2000.

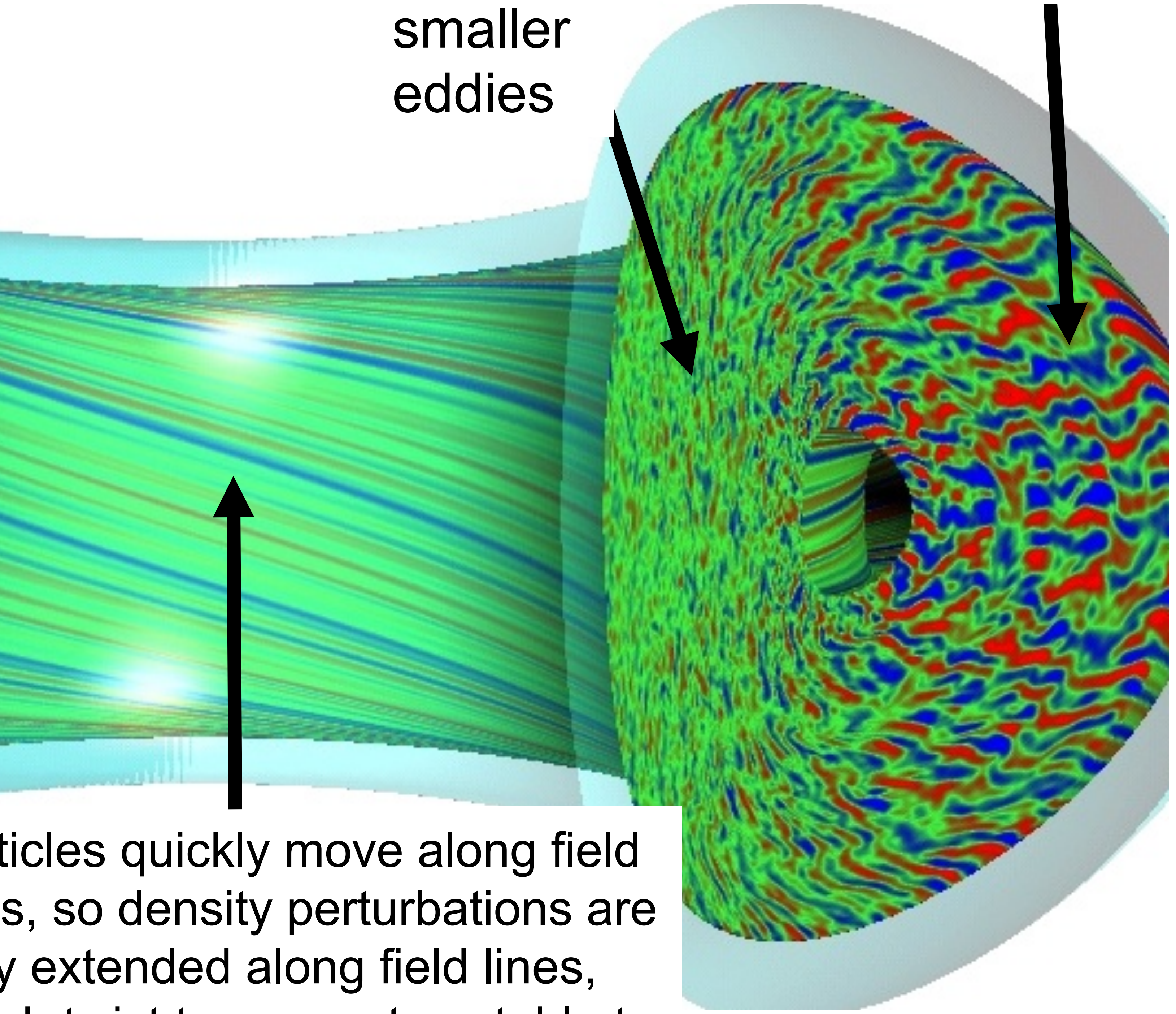


"A gyro-Landau-fluid transport model," R. E. Waltz, G. M. Staebler, W. Dorland, G. W. Hammett, M. Kotschenreuther, and J. A. Konings, Physics of Plasmas 4, 2482 (1997).

Low Frequency Modes in a Magnetized Plasma

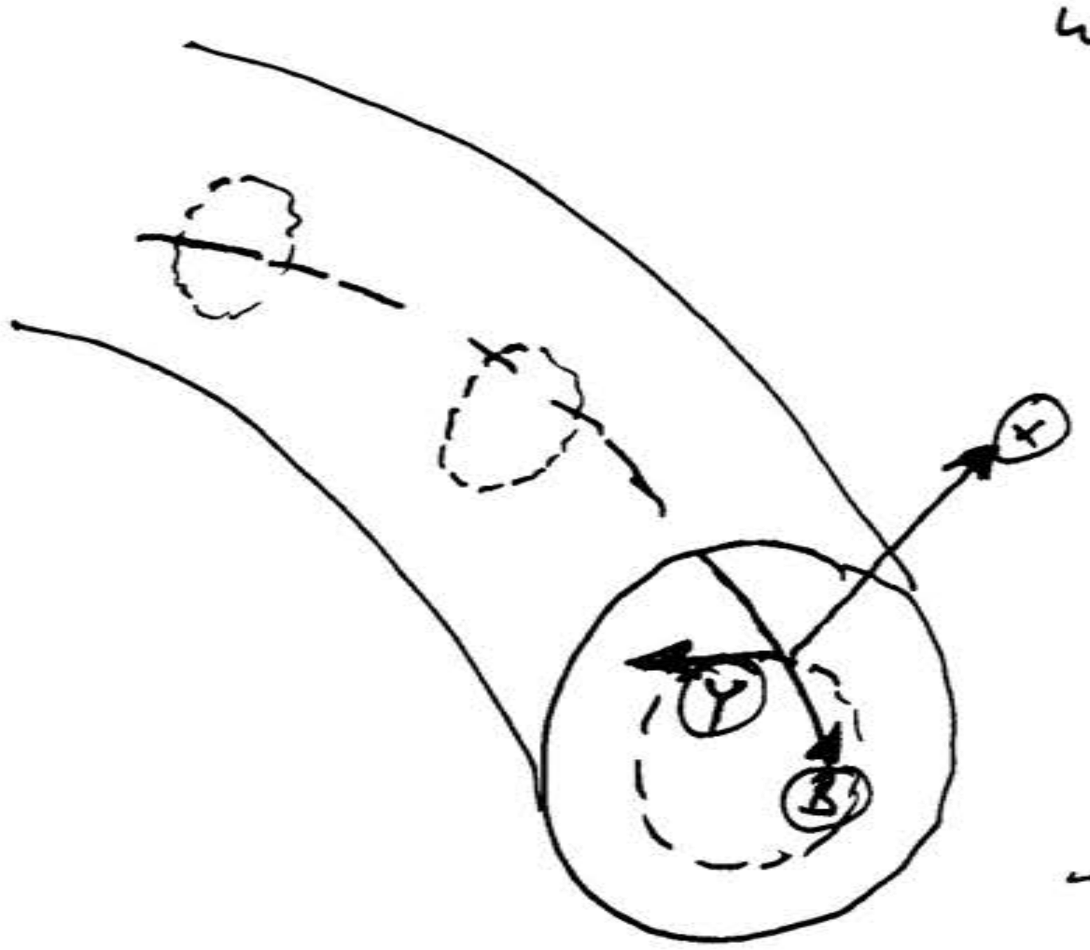
Stable side, smaller eddies

Unstable bad-curvat side, eddies point ou direction of effective gravity



ticles quickly move along field s, so density perturbations are y extended along field lines, ch twist to connect unstable to ble side

GOAL: BETTER UNDERSTANDING OF TOROIDALLY CONFINED PLASMA WAVES, INSTABILITIES, TRANSPORT



GRADIENTS: DRIVE FOR WAVES AND TRANSPORT

SIMPLE SLAB GEOMETRY

(x, y, z)

z: ALONG $\bar{B}$

y: WAVE/FLOW DIRECTION

x: GRADIENTS

$\frac{\partial n}{\partial x}$	$\frac{\partial T}{\partial x}$	$\frac{\partial P}{\partial x}$	$\frac{\partial J}{\partial x}$
DENSITY	TEMP	PRESSURE	CURRENT

WHAT SORT OF WAVES?

- ELECTROSTATIC WAVES (LIKE SOUND WAVES)
  "DRIFT WAVES"
- ELECTROMAGNETIC WAVES (LIKE SHEAR ALFVÉN WAVES)

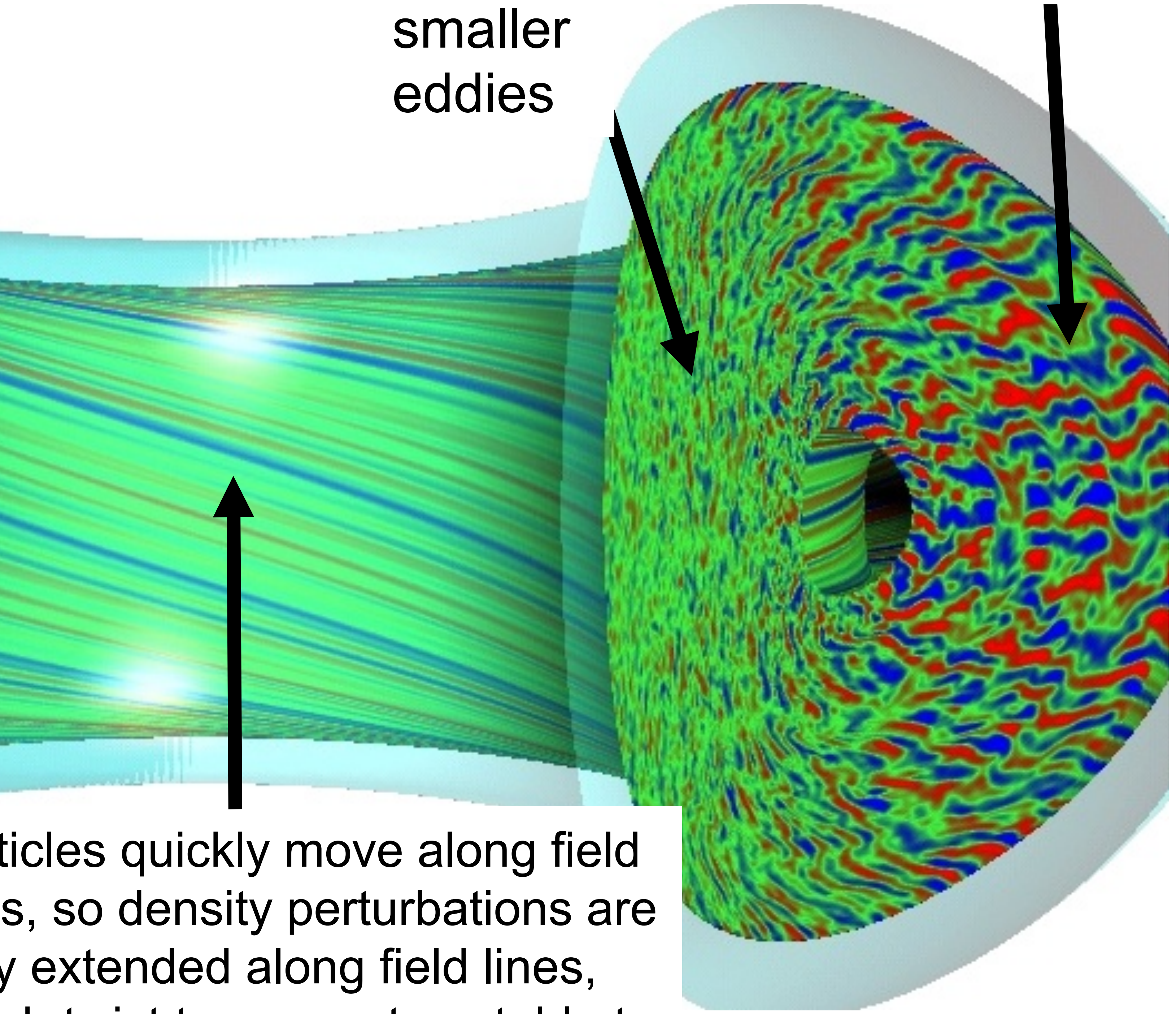
EM: $\delta \bar{B} \approx \nabla \times \tilde{A}$ ($\tilde{A} = \hat{z} A_{\parallel}$)

GO BEYOND MHD

MHD: $E + v \times B \approx 0 \Rightarrow E_{\parallel} \to 0$

ES DRIFT: $E_{\parallel}$ SMALL BUT FINITE

Vorticity and Mixing



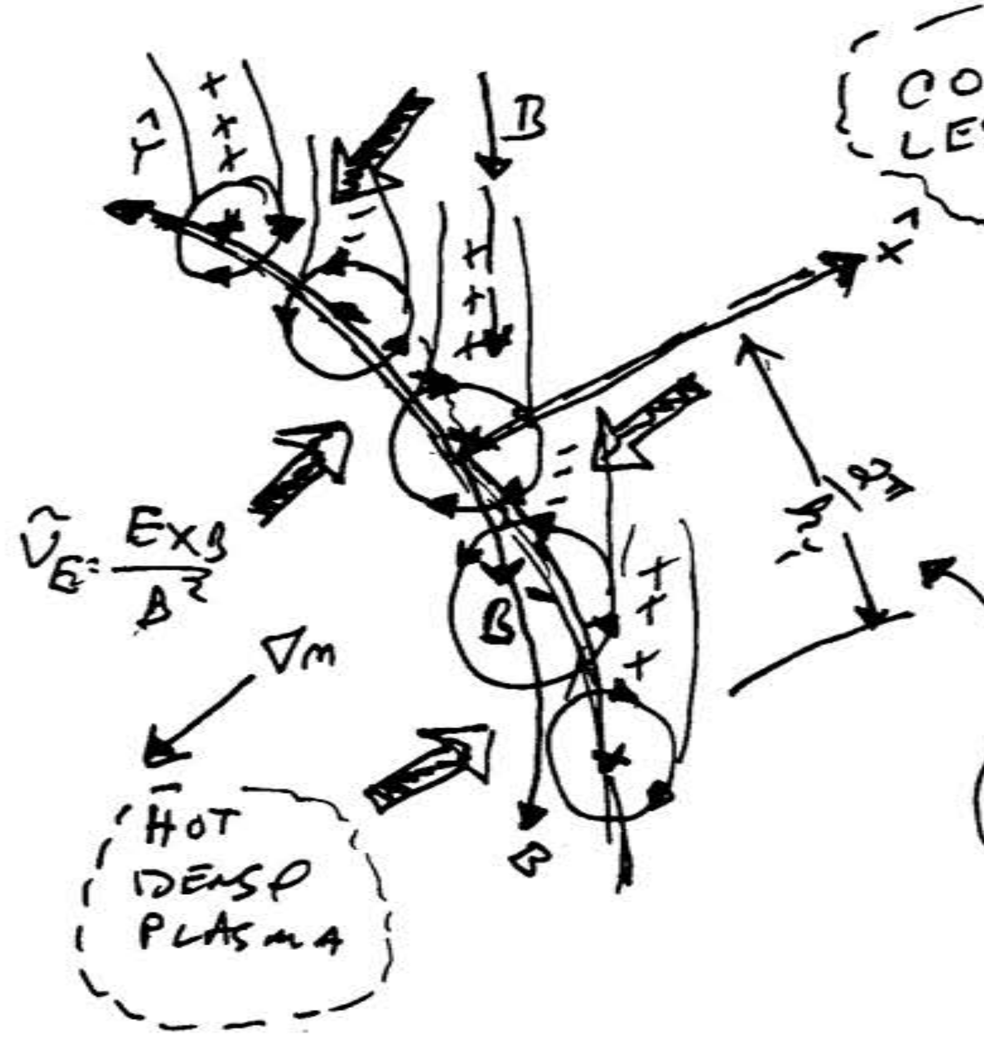
$V_E = \frac{E \times B}{B^2}$

$\Omega = \nabla \times V_E$

$\approx \frac{\hat{z}}{B} \bar{\nabla} \cdot \bar{\nabla}\phi$

$\Omega_z \cong \frac{1}{B} \nabla^2 \phi$

CIRCULATION
MIXING
CONVECTION



COOL LESS DENSE PLASMA

SLAB GEOMETRY

$\delta m, \delta\phi, \delta A_\parallel \sim e^{-j\omega t + j k_y y}$ $\otimes$ $e^{+j k_\parallel z}$

AND k_x TOO

WAVELENGTH

LOW FREQUENCY: $\omega \ll \omega_{ci}$

LONG WAVELENGTH: $k \ll \frac{1}{\lambda_D}$

$\tilde{m}_i - \tilde{m}_e \cong 0$

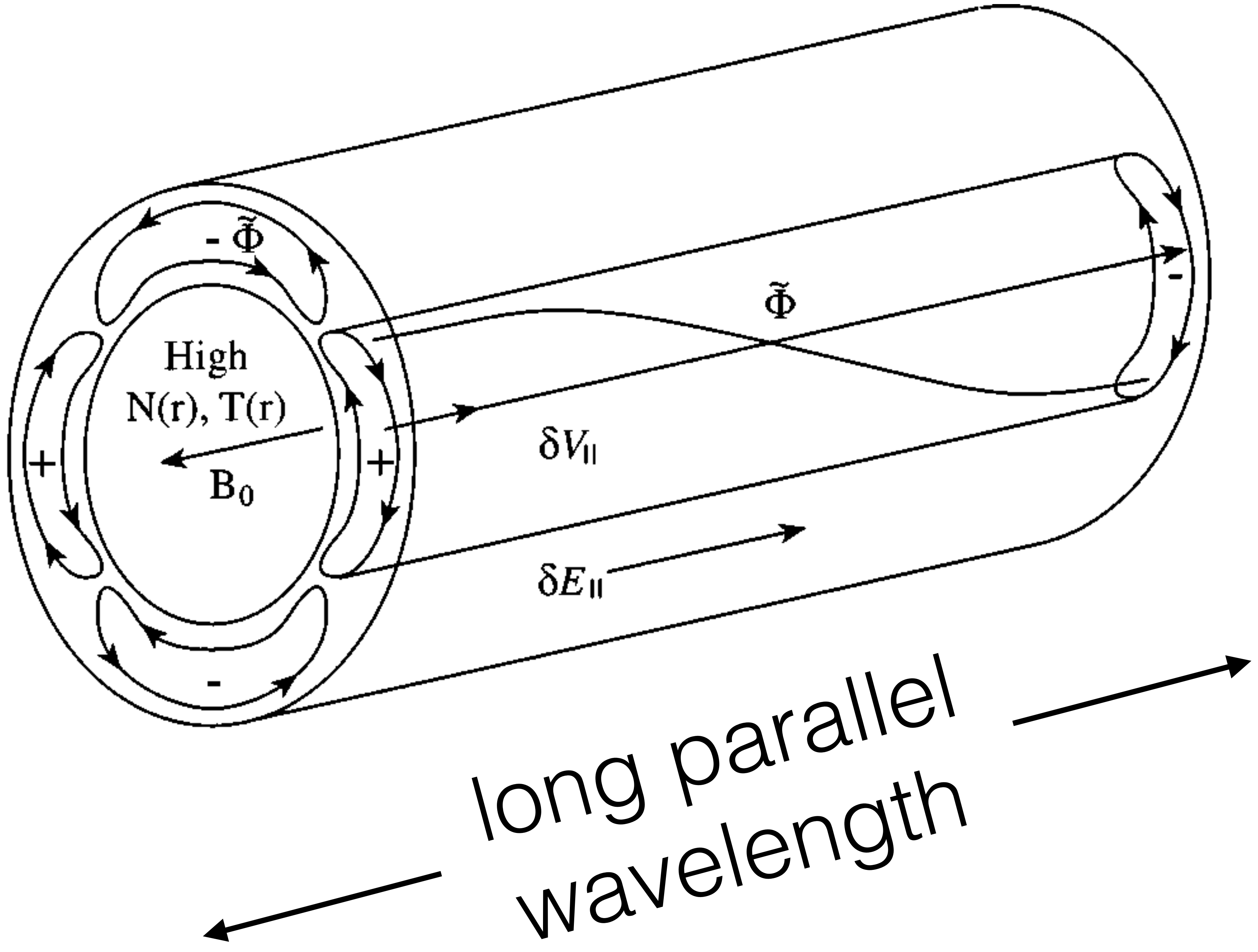
$\nabla \cdot \bar{J} = 0$

DRIFT APPROXIMATION FOR PERPENDICULAR MOTION/CURRENT

MOTION ALONG B:

- COLLISIONLESS: $\frac{\omega}{k_\parallel} \gg v_{TH}$ (LANDAU DAMPING)
- COLLISIONAL $\nu_{COL} \gg \omega$, $k_\parallel \lambda_{MFP} \ll 1$
- MHD-LIKE ALFVENIC OR $\frac{\omega}{k_\parallel} \approx 0 \ll v_{TH}$

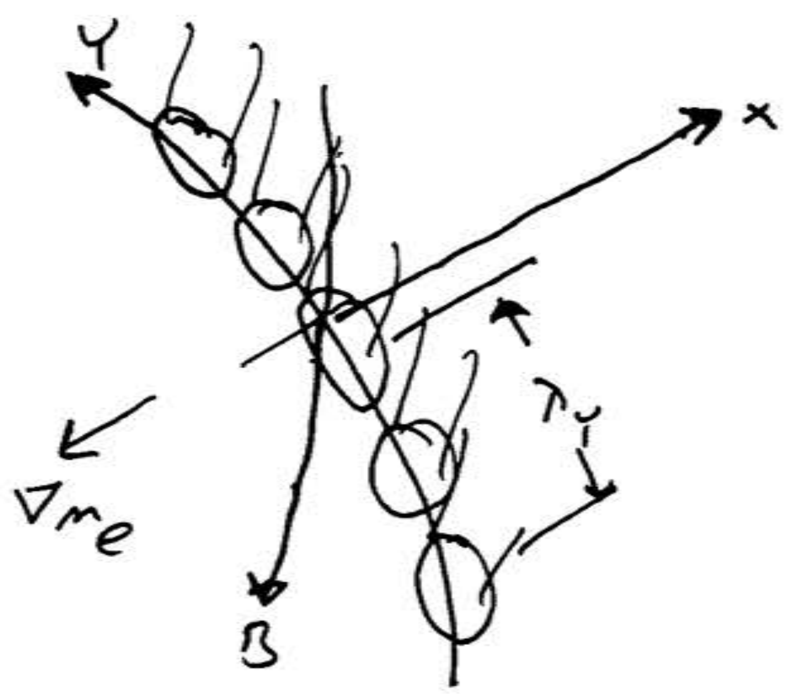
Simple Drift Wave Description



SIMPLE

(PERPENDICULAR MOTION)

DRIFT EQUATIONS ⊥



$T \simeq$ CONSTANT

$\omega << \omega_{ci}$

$\delta B \sim \delta A_{\parallel} \sim 0$
ELECTROSTATIC

$\rho_s / L_N << 1$

STRONGLY MAGNETIZED:

$$v = v_{\parallel} \hat{b} + \bar{v}_{\perp} \quad \text{(FOR } e, i\text{)}$$

$$\frac{d\bar{v}_{\perp}}{dt} = \frac{q}{m}(E + v \times B) - \frac{1}{mn}\nabla p$$

$$\bar{V}_{\perp} \times \bar{B} = -\bar{E}_{\perp} + \frac{1}{qn}\nabla p + \frac{m}{q}\frac{d\bar{v}_{\perp}}{dt}$$

$$V_{\perp} = \frac{E \times B}{B^2} + \frac{B \times \nabla p}{q n B^2} + \frac{m}{qB}\hat{b} \times \frac{dv_{\perp}}{dt} \ldots$$

E×B

DIAMAGNETIC

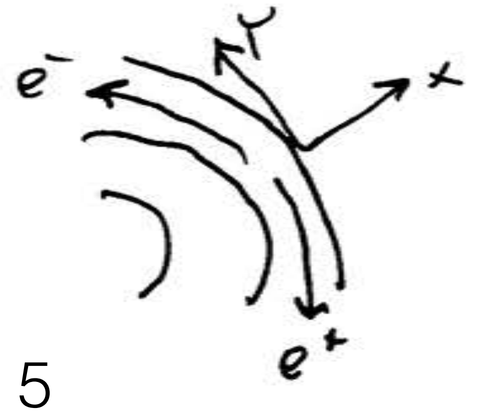
POLARIZATION

ONLY IONS

$$V_{POL} \sim \frac{m}{qB^2}\frac{dE_{\perp}}{dt}$$

EQUILIBRIUM DRIFTS

$$V_{\perp}(0) = \hat{y}\frac{T}{qB}\left(\frac{1}{n}\frac{\partial n}{\partial x}\right)$$



DIAMAGNETIC CURRENT

IF $\frac{1}{L_N}$ = DENSITY SCALE LENGTH

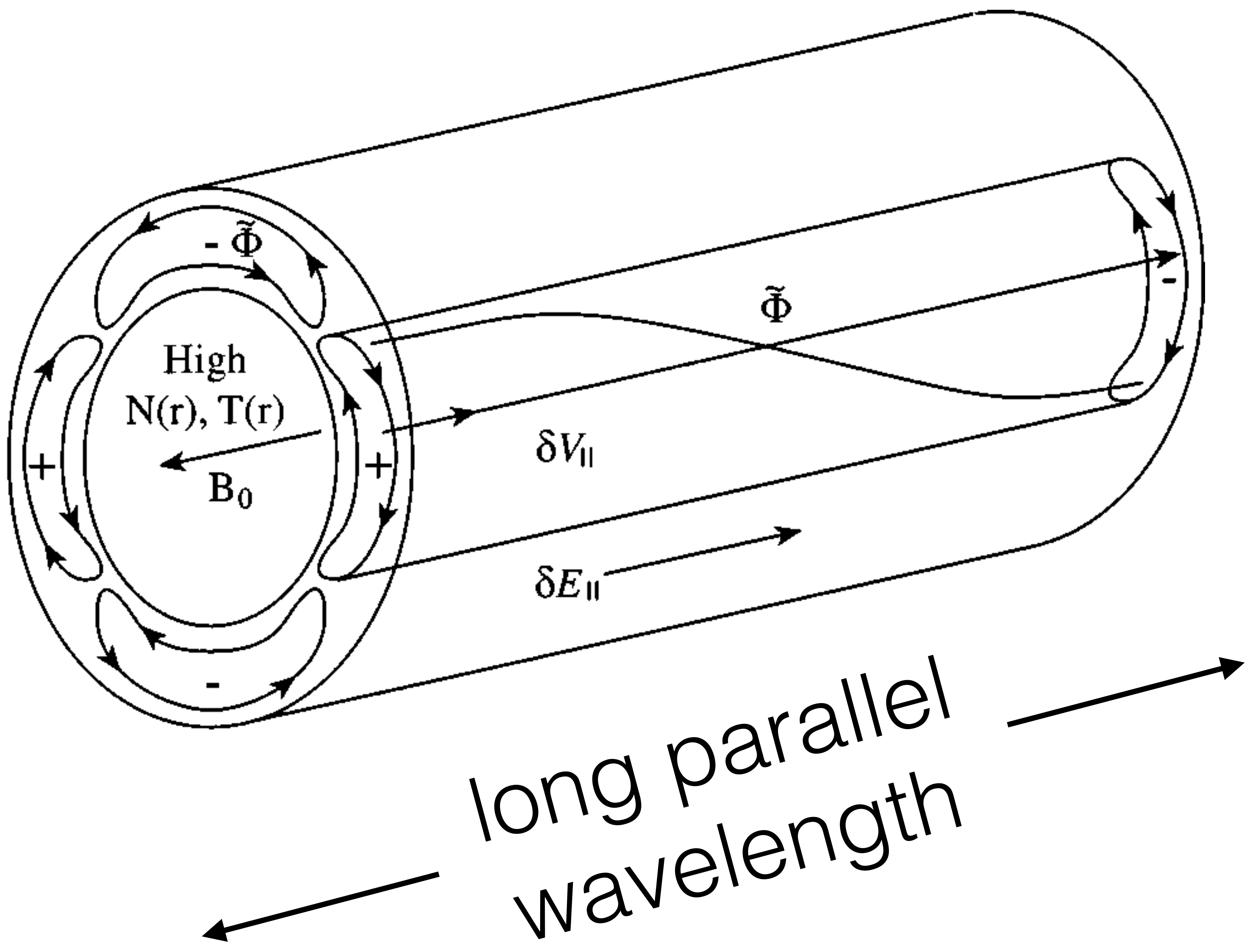
$$= -\frac{1}{n}\frac{\partial n}{\partial x}$$

$\rho_s / L << 1$

THEN

$$V_{\perp}(0) \sim \hat{y}\,\omega_{ci} L_N \left(\frac{\rho_s}{L_N}\right)^2 \sim \hat{y}\frac{c_s^2}{\omega_{ci} L_n} \sim \hat{y}\left(\frac{c_s}{L_N}\right)\left(\frac{\rho_s}{L_N}\right)$$

How Fast is the Diamagnetic Drift?



HBT-EP:

$$\boxed{V_e^* = \hat{y}\,\frac{T}{eBL_N}}$$

$B \sim 3.2\,\mathrm{kg}$ $\qquad L_n \sim 7\,\mathrm{cm}$

$T \sim 25\,\mathrm{eV}$ $\qquad n_0 \sim 10^{13}\,\mathrm{cm}^{-3}$

$V_{TH_i} = 3.5\times 10^6\ \frac{\mathrm{cm}}{\mathrm{sec}} \simeq C_s$

$V_{ALFVEN} = 150\times 10^6\ \frac{\mathrm{cm}}{\mathrm{sec}}$

$\rho_i \sim 0.23\,\mathrm{cm}$ $\qquad V_A \gg V_{TH_i} \gg V_{DIA}$

$V_{DIA} \sim (B\cdot a)\left(\frac{\rho}{a}\right)^2$

$B\cdot a \propto$ PLASMA CURRENT (I_p) $\qquad$ MAGNETIZATION

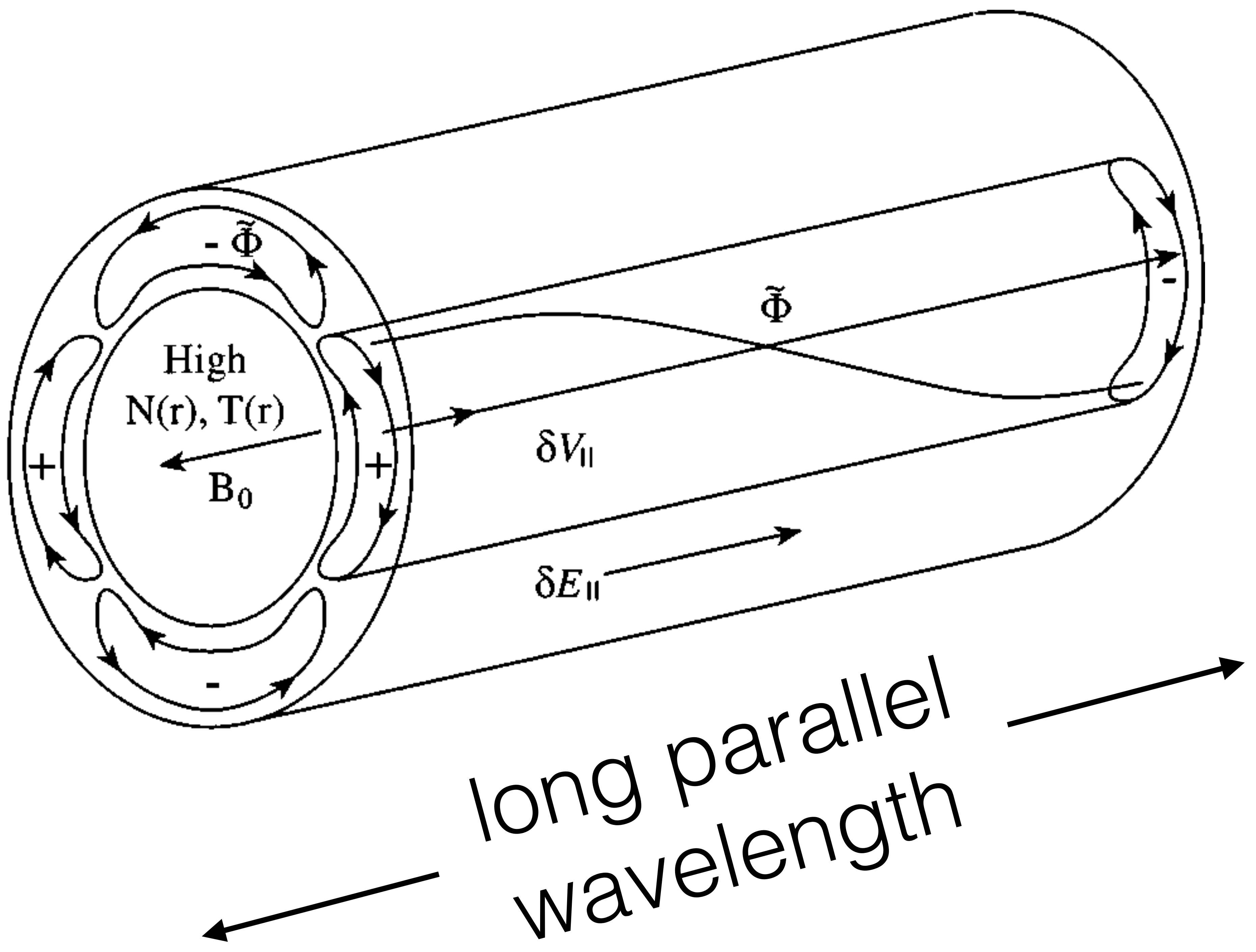
$V_{DIA} \sim 0.1\times 10^6\ \frac{\mathrm{cm}}{\mathrm{sec}}$

$\frac{V_{DIA}}{2\pi\, 14\,\mathrm{cm}} \simeq \underline{1.2\,\mathrm{kHz}}$

SO DIAMAGNETIC DRIFT FREQUENCY IS COMPARABLE TO OBSERVED MODES

$(\rho_i/a)_{HBTEP} \sim 0.015 \sim 1/60$

Simple Drift Wave Description: Parallel Dynamics



ELECTRON (FAST) PARALLEL MOTION

$$e: \quad \frac{\partial \tilde{v}_{\parallel}}{\partial t} + (\bar{v}_E \cdot \nabla)\tilde{v}_{\parallel} = \frac{e}{m_e}\frac{\partial \tilde{\phi}}{\partial z} - \frac{1}{m_0 m_e}\frac{\partial \tilde{P}_e}{\partial z}$$

BUT m_e IS VERY VERY SMALL

ELECTRONS ARE VERY VERY LIGHT AND FAST

AS $m_e \to 0$

$$0 \simeq e\frac{\partial \tilde{\Phi}}{\partial z} - \frac{1}{n_0}\frac{\partial \tilde{P}_e}{\partial z} \simeq e\frac{\partial \tilde{\Phi}}{\partial z} - \frac{T}{n_0}\frac{\partial n}{\partial z}$$

$$\frac{1}{n}\frac{\partial n}{\partial z} \simeq \frac{e}{T}\frac{\partial \tilde{\Phi}}{\partial z} \Rightarrow \frac{n_e}{n_0} = e^{e\tilde{\Phi}/T}$$

CALLED "ADIABATIC ELECTRONS"

PARALLEL ELECT FORCE BALA

FORCE BALANCE

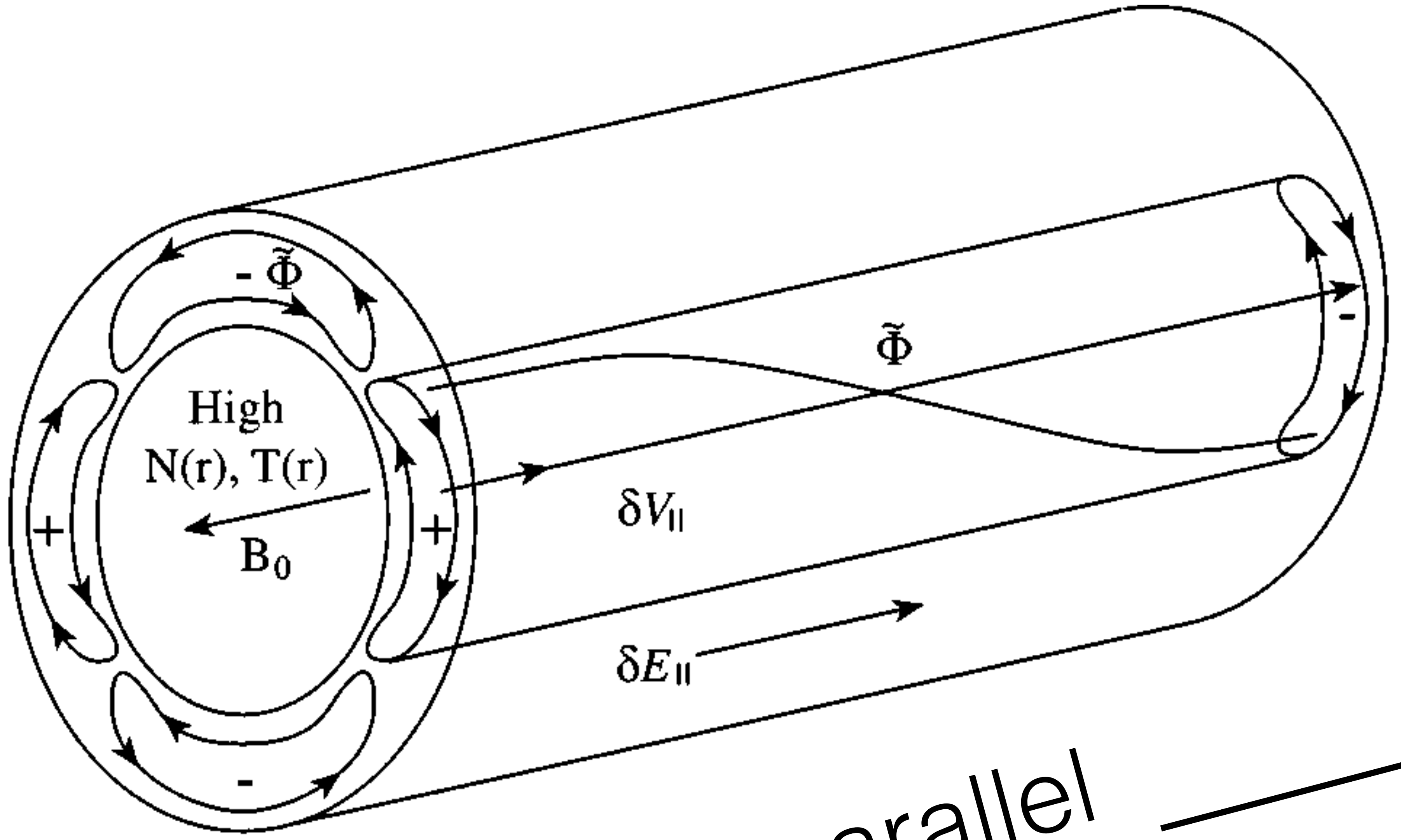
$$\boxed{\delta \tilde{n} \simeq n_0 \frac{e\tilde{\Phi}}{T}}$$

PROVIDED $k_{\parallel} \neq 0$

ELECTRONS ~~FLOW~~ MOVE QUICKLY ALONG B TO KEEP PARALLEL FORCE ZERO

$$0 \simeq \frac{\partial}{\partial z}\left(e\tilde{\Phi}\right) - \frac{\partial}{\partial z}\left(\frac{P_e}{n}\right) = \frac{q}{m}\tilde{E}_{\parallel} - \frac{1}{n}\tilde{\nabla}_{\parallel} p = 0$$

Simple Drift Wave Description: Continuity



long parallel wavelength

CONTINUITY: $\dfrac{\partial \tilde{n}}{\partial t} + \nabla \cdot (n \bar{v}) = 0$ (REMEMBER: $\tilde{n}_e \approx \tilde{n}_i$)

LET'S USE ION CONTINUITY EQUATION

$$\frac{\partial \tilde{n}}{\partial t} + \underbrace{\bar{\nabla} \cdot (n \bar{v}_E)}_{E\times B} + \underbrace{\nabla \cdot (n \bar{v}_*)}_{\text{DIAMAGNETIC FLUID FLOW}} + \underbrace{\nabla \cdot (n v_{pol})}_{\text{IGNORE AT FIRST (IMPORTANT) "ION INERTIAL"}} + \underbrace{\bar{\nabla} \cdot (n v_{\parallel} \hat{z})}_{\text{IONS TOO SLOW ALONG } B} = 0$$

$$\nabla \cdot (n \bar{v}_*) = \nabla \cdot (\hat{z} \times \nabla p) \frac{1}{qB} = \frac{1}{qB} \left[\nabla p \cdot (\nabla \times \hat{z}) - \hat{z} \cdot \nabla \times \nabla p \right] = 0$$

$$\nabla \cdot (n v_E) = n \nabla \cdot \bar{v}_E + v_E \cdot \nabla n = v_E \cdot \nabla n_0$$

$$\nabla \cdot \bar{v}_E \to \nabla \cdot (\hat{z} \times \nabla \varphi)/B = 0$$

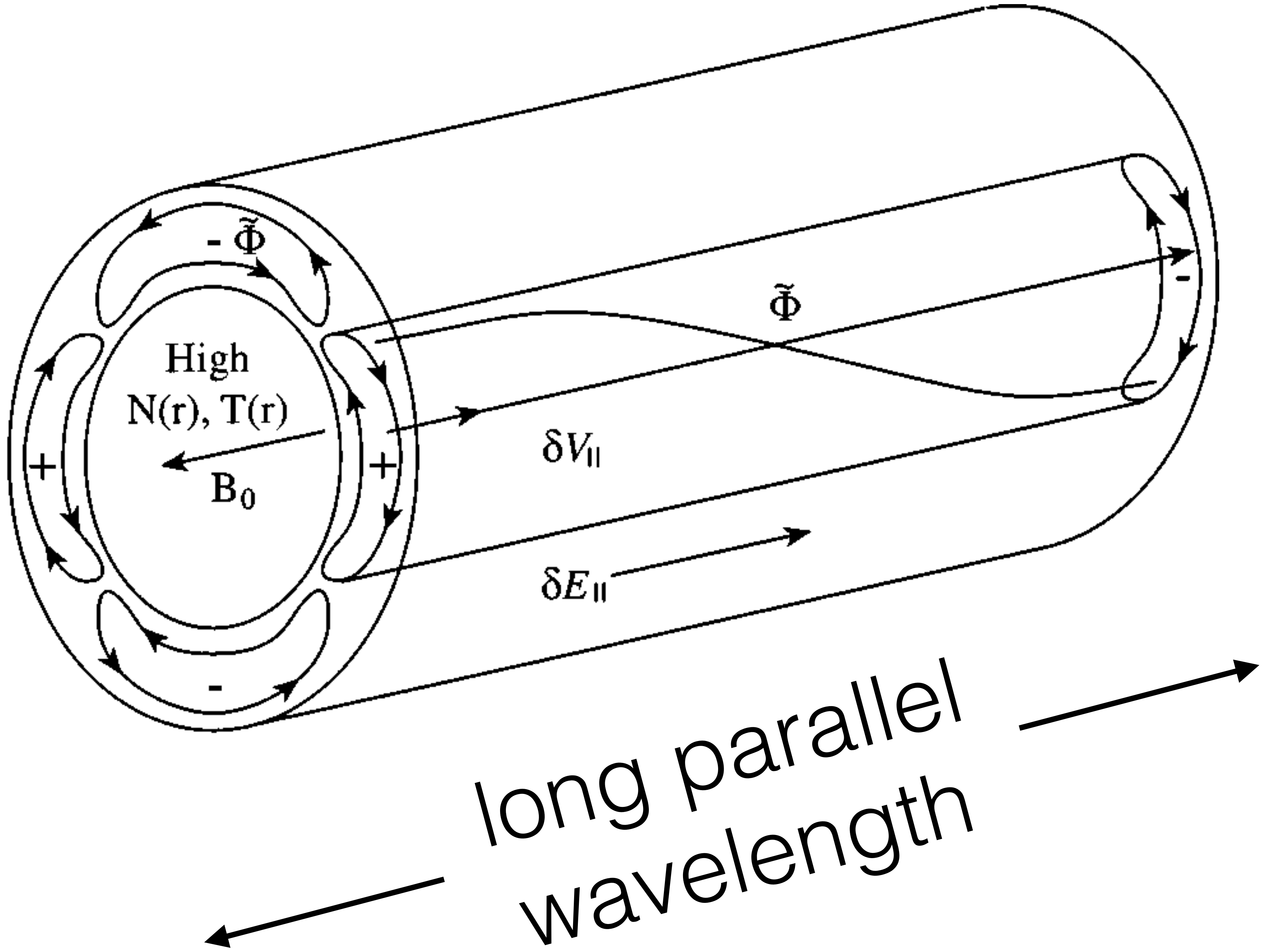
$$\bar{V}_E = \frac{-1}{B} \frac{\partial \Phi}{\partial y} \hat{x}$$

$$\frac{\partial \tilde{n}}{\partial t} + \tilde{v}_E \cdot \frac{\partial n_0}{\partial x} = 0$$

$$-j\omega \tilde{n} + \frac{n_0}{B} \frac{j k_y}{L_N} \tilde{\Phi} = 0$$

$$\frac{\tilde{n}}{n_0} = \frac{k_y}{B L_N \omega} \tilde{\Phi}$$

Basic “Drift Wave”



PARALLEL ADIABATIC ELECTRONS

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\Phi}}{T}$$

PERPENDICULAR ION DRIFT DYNAMICS

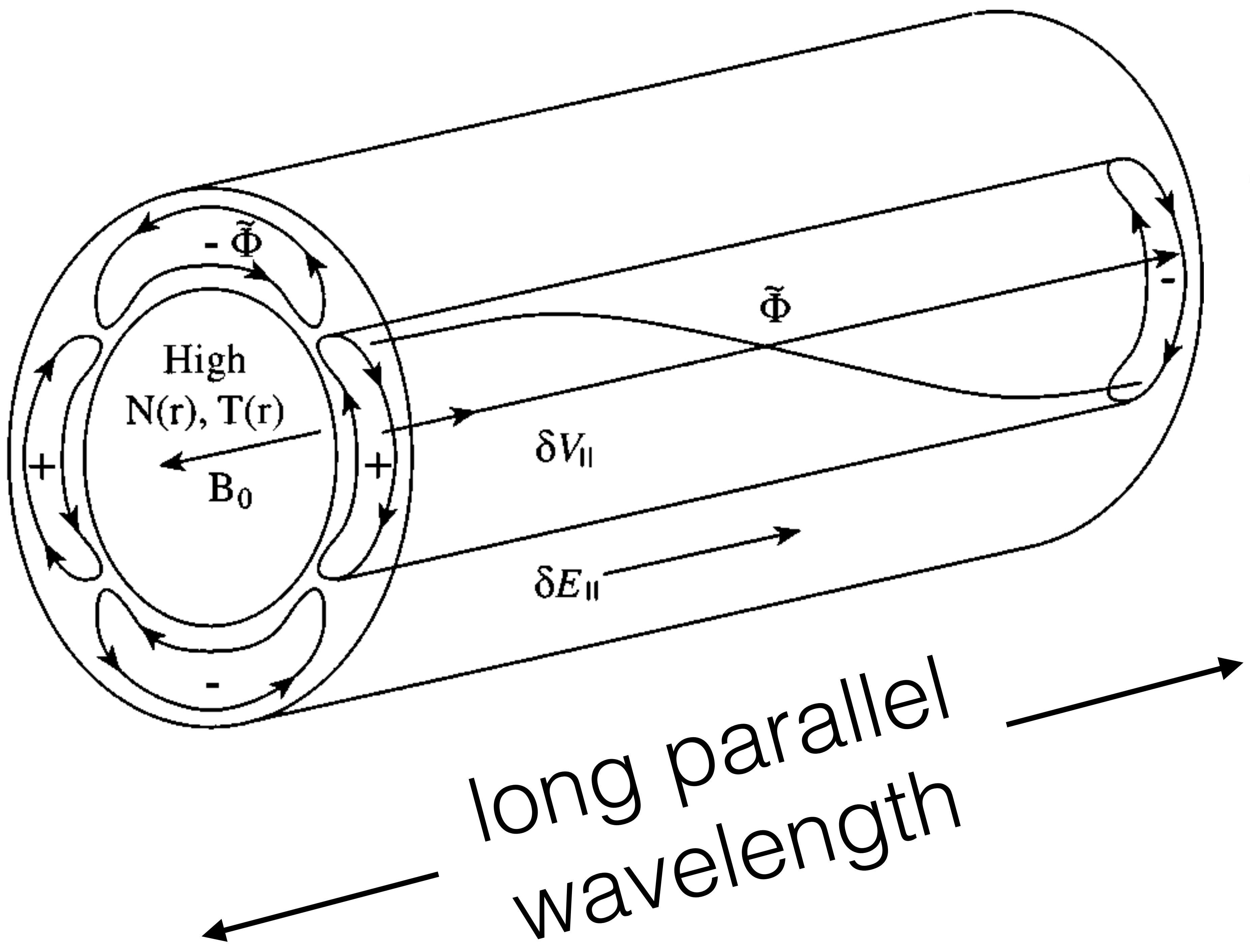
$$\frac{\tilde{n}}{n_0} = \frac{k_y}{B L_n \omega}\tilde{\Phi}$$

DISPERSION RELATION

$$\omega = k_y \frac{T}{e B L_n} = k_y V_e^*$$

DRIFT WAVES PROPAGATE IN ELECTRON DIAMAGNETIC DRIFT DIRECTION

Ion Inertial Currents (Polarization Drift)



IONS ARE HEAVY

THEY HAVE POLARIZATION (INERTIAL) DRIFTS
THAT REDUCE $E_\perp$

THEY MOVE SLOWLY (NON ADIABATIC) ALONG B

TAKE DRIFT WAVES LIKE USUAL
ACOUSTIC WAVE

$$V_{THi} << \frac{\omega}{k_\parallel} << V_{THe}$$

PERPENDICULAR ION

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\bar{v}) = 0$$

$$\frac{\partial \tilde{n}}{\partial t} + \nabla\cdot(nV_{pol} + nV_E) + n\frac{\partial v_{\parallel i}}{\partial z} = 0$$

$$\underbrace{-i\omega\tilde{n} + \nabla\cdot(nV_{pol}) + \tilde{V}_E\cdot\nabla n_0 + i\frac{k_\parallel^2}{\omega}\frac{e\tilde{\Phi}}{m_i}n_0}\simeq 0$$

NEW ION INERTIAL TERM

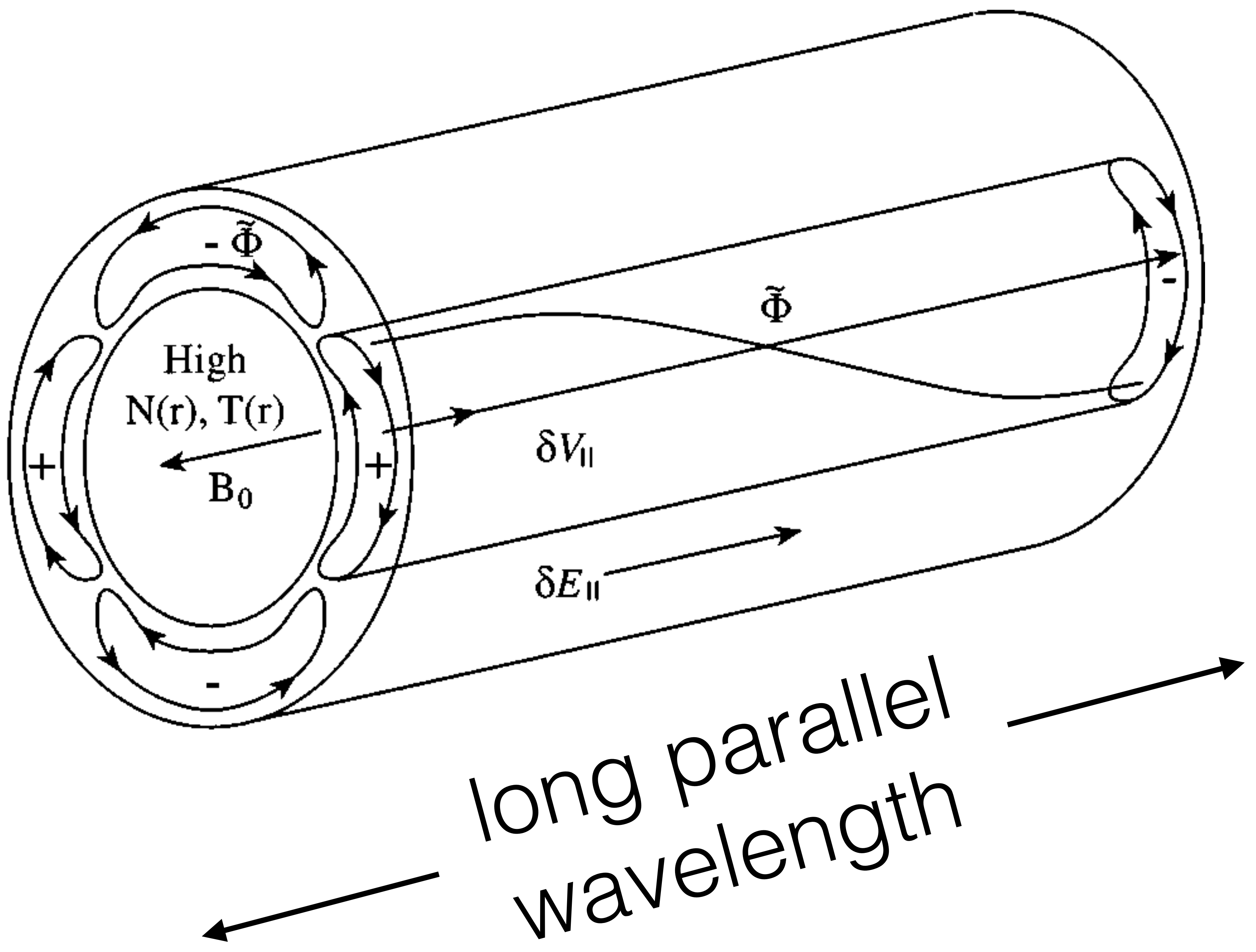
PARALLEL ION

$$\frac{dV_{\parallel i}}{dt} = -\frac{e}{m_i}\frac{\partial\Phi}{\partial z}$$

$$\hat{V}_{\parallel i} = \frac{e}{m_i}\frac{k_\parallel}{\omega}\tilde{\Phi}$$

SOUND RESPONSE

Ion Inertial Currents (Polarization Drift)



$$\nabla\cdot(n V_{POL}) = \nabla\cdot\left(\frac{n m_i}{B^2}\frac{d E_\perp}{dt}\right)$$

$$E_\perp = -\nabla_\perp \tilde{\Phi}$$

$$= -\nabla\cdot\left(\frac{n m_i}{B^2}\nabla_\perp \dot{\tilde{\Phi}}\right)$$

$$= -\nabla\cdot\frac{\epsilon_0\,\omega_{pi}^2}{\omega_{ci}^2}\nabla_\perp \dot{\tilde{\Phi}}$$

$$= k_\perp^2 \frac{n_0 m_i}{B^2}\dot{\tilde{\Phi}}$$

$$= -j\omega k_\perp^2 n_0 \rho_i^2 \left(\frac{e\tilde{\Phi}}{T}\right)$$

$k_\perp^2\rho^2$ = FINITE LARMOR RADIUS TERM

$\epsilon_0 \frac{\omega_{pi}^2}{\omega_{ci}^2}$ = PLASMA DIELECTRIC $= \epsilon_0 \frac{n M_i}{B^2} \gg \epsilon_0$

VERY LARGE

ADIABATIC ELECTRONS

$$\tilde{n} = n_0\left(\frac{e\tilde{\Phi}}{T}\right)$$

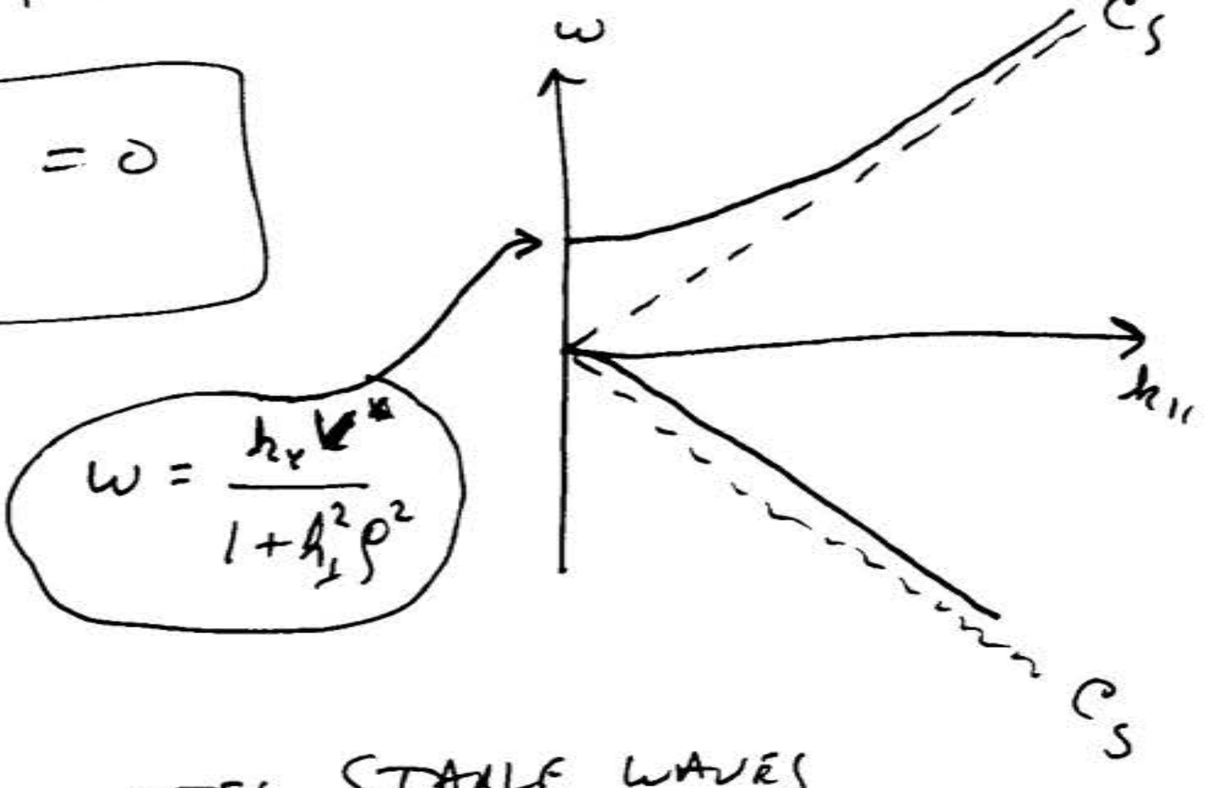
$$\boxed{\omega^2(1+k_\perp^2\rho^2) - \omega k_y v^* - k_\parallel^2 c_s^2 = 0}$$

ION POLARIZATION TERM

DENSITY GRADIENT DRIFT

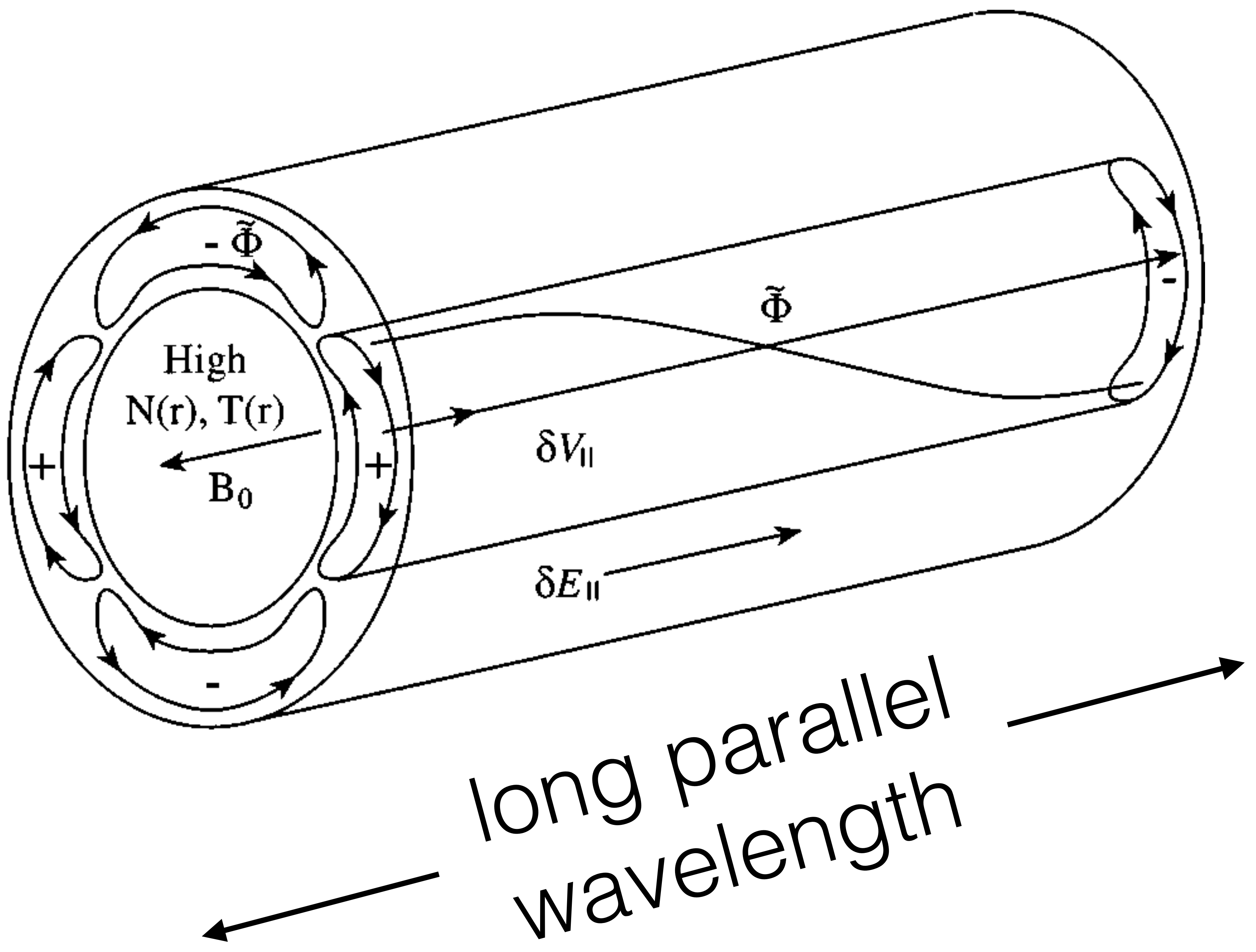
SOUND WAVE TERM

$$\omega = \frac{k_y v^*}{1+k_\perp^2\rho^2}$$



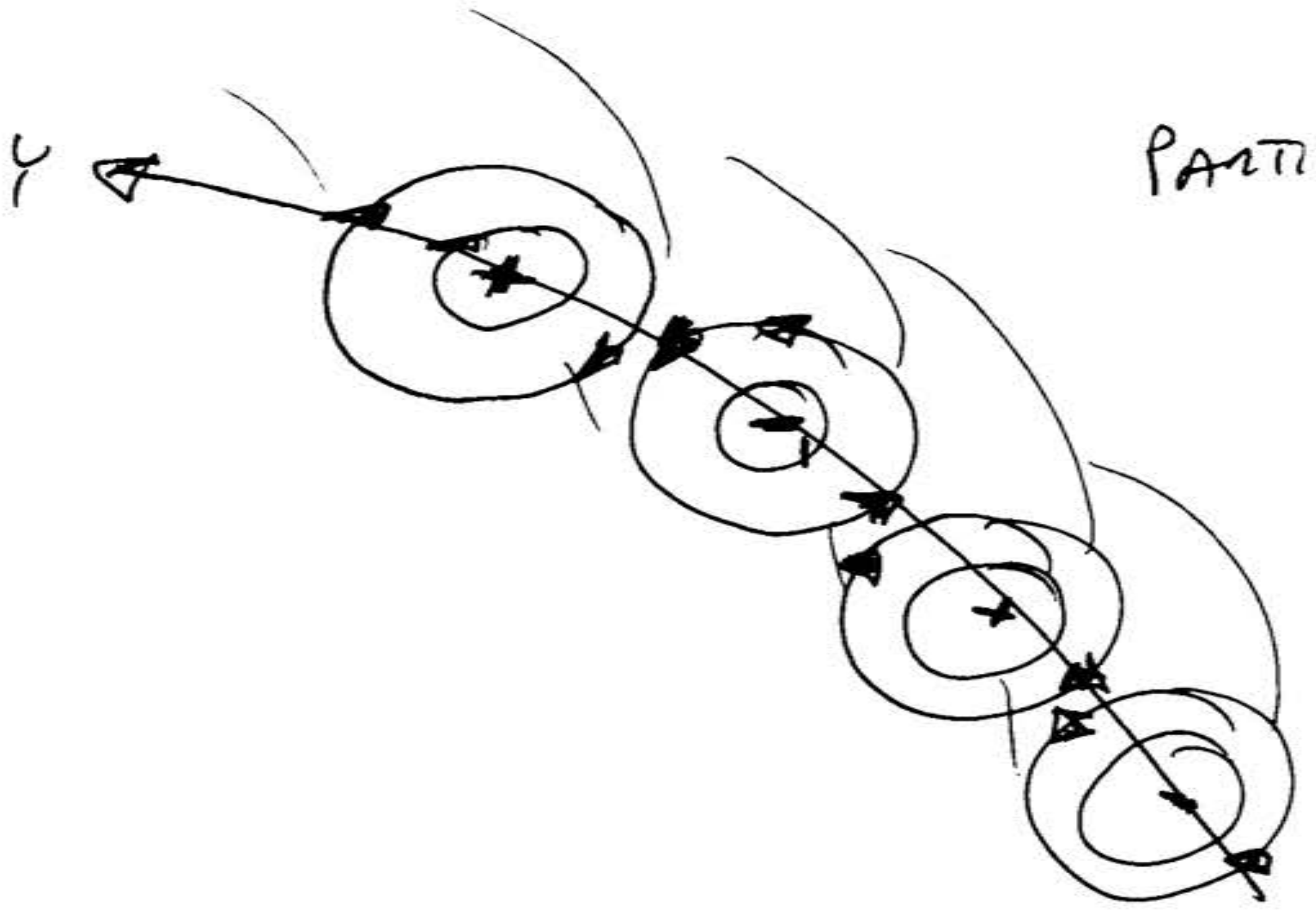
NOTE: STABLE WAVES

How Much Transport from Drift Waves?



$$\omega^2(1+k_\perp^2\rho_i^2) - \omega k_y V^* - k_{||}^2 C_s^2 = 0$$

$$\frac{\delta n}{n_0} = \frac{e\tilde{\Phi}}{T}$$ $\Rightarrow$ DENSITY AND POTENTIAL FLUCTUATIONS ARE <u>IN</u> <u>PHASE</u>



PARTICLE FLUX $= \frac{1}{2}\,\mathrm{Re}\{\tilde{n}^*\tilde{v}\}$

$$= -\frac{1}{2}\,\mathrm{Re}\left\{n_0\frac{e}{T}\tilde{\Phi}^*\frac{ik_y\tilde{\Phi}}{B}\hat{x}\right\}$$

$$= -\frac{1}{2}\,n_0\frac{e k_y}{BT}\underbrace{\mathrm{Re}\{i\tilde{\Phi}^*\tilde{\Phi}\}}_{\phi}$$

<u>NO</u> <u>FLUX</u>

WE NEED A <u>PHASE</u> SHIFT BETWEEN δn AND $\delta\phi$ FOR <u>TRANSPORT</u>

Next: Drift Wave Instability and Transport

ADIABATIC ELECTRONS: $\dfrac{\tilde{n}}{n} \approx \dfrac{e\tilde{\Phi}}{T}$ (NO RADIAL TRANSPORT)

NON-ADIABATIC ELECTRONS: $\dfrac{\tilde{n}}{n} \approx \dfrac{e\tilde{\Phi}}{T}(1 - i\delta\varphi)$ SMALL PHASE SHIFT

$$\therefore \quad \omega = \frac{k_y V_e^*}{1 - i\delta\varphi} \approx k_y V_e^*(1 + i\delta\varphi + \cdots)$$

$$\mathrm{Re}(\omega) \sim k_y V_e^* \qquad \mathrm{Im}(\omega) \sim i\,\delta\varphi\, k_y V_e^*$$

WHAT CAUSES $\delta\varphi$?

COLLISIONS ALONG B

LANDAU DAMPING

ELECTROMAGNETIC INDUCTION $\delta B_\perp$

COLLISIONAL EFFECTS IN PLASMAS—DRIFT-WAVE EXPERIMENTS AND INTERPRETATION*

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(Received 25 January 1967)

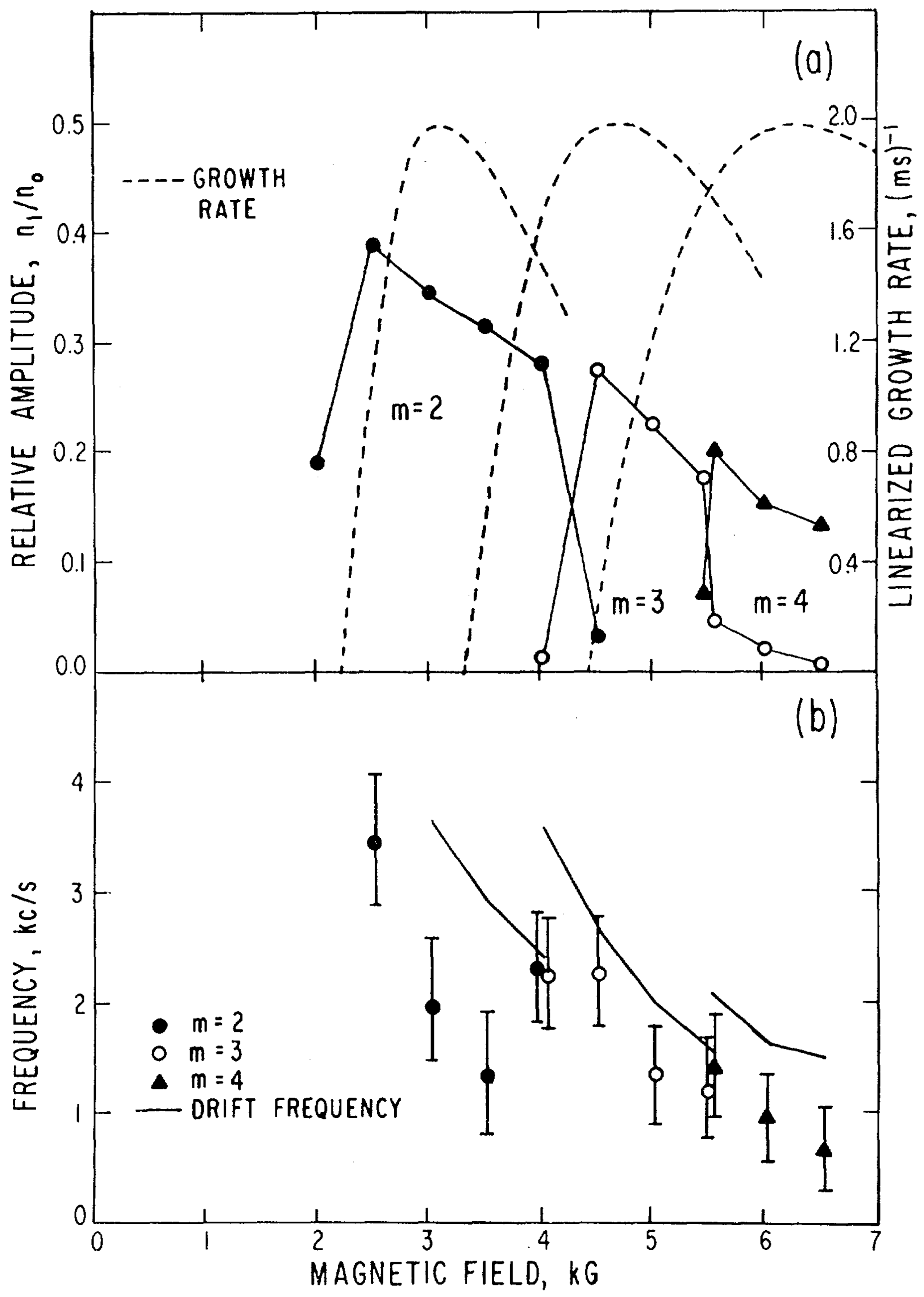

FIG. 1. (a) Observed oscillation amplitudes are compared with theoretical growth rates as a function of magnetic field strength for various azimuthal mode numbers. The absolute value of the magnetic field strength for the theoretical (slab model) curves has been scaled by a factor of ~1.5 to give a good fit to the data. The relative amplitude is defined as the ratio of the maximum density fluctuation to the central density. (b) The oscillation frequency (after subtraction of the rotational Doppler shift) is compared with the drift frequency $\nu_d = k_y v_d / 2\pi$ as a function of the magnetic field strength. The drift frequency, which has an uncertainty of ±0.5 kc/sec, is computed from the experimental values of k_y, T, and $n^{-1}(dn/dx)$. The data are for a potassium plasma, $n_0 = 3.5 \times 10^{11}$ cm^{-3}, $T = 2800$°K.

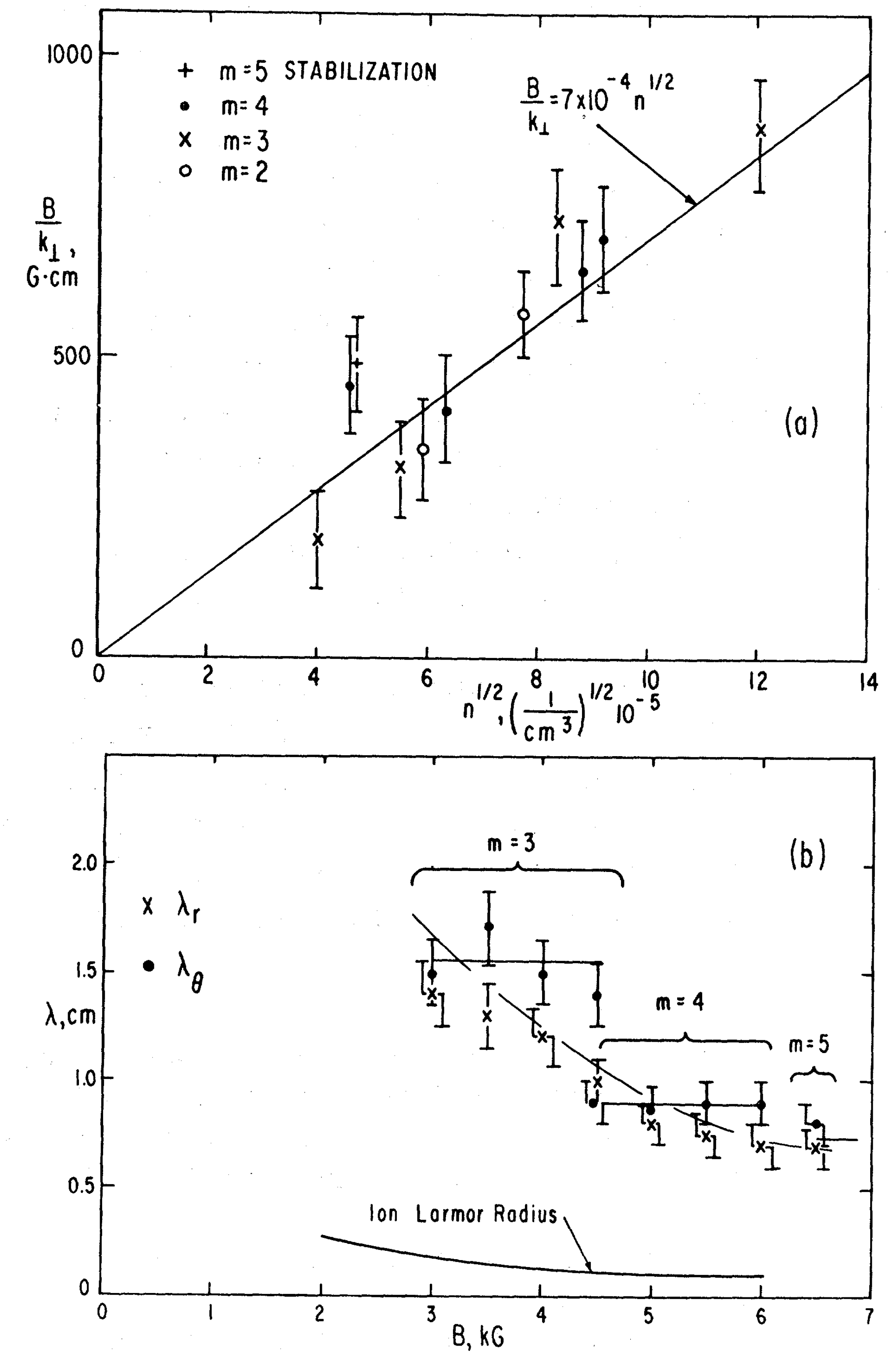

FIG. 2. (a) The ratio of magnetic field strength to perpendicular wave number is plotted versus the square root of the density for the stabilization points of several modes. Theory [Eq. (3)] gives a proportionality factor of 9.7×10^{-4}. (b) The measured radial (λ_r) and azimuthal (λ_θ) wavelengths of the perturbation are displayed as a function of the magnetic field.