Plasma 2
Lecture 11: Midterm Assignment Notes

APPH E6102y
Columbia University

NAS Burning Plasma Strategic Planning

Outline

- General Advice: you must complete a calculation/derivation
- Echos (Alex S.)
- Quasilinear plasma wave heating
- Kinetic Alfvén Wave Heating (Jessica): **Electron/Ion Landau Damping**
- Cyclotron heating (Alex B., Tyler)
- Radial Diffusion / Alpha Channeling (Juan)

Unit Seven

Constructing a Research Paper I

TASK ONE

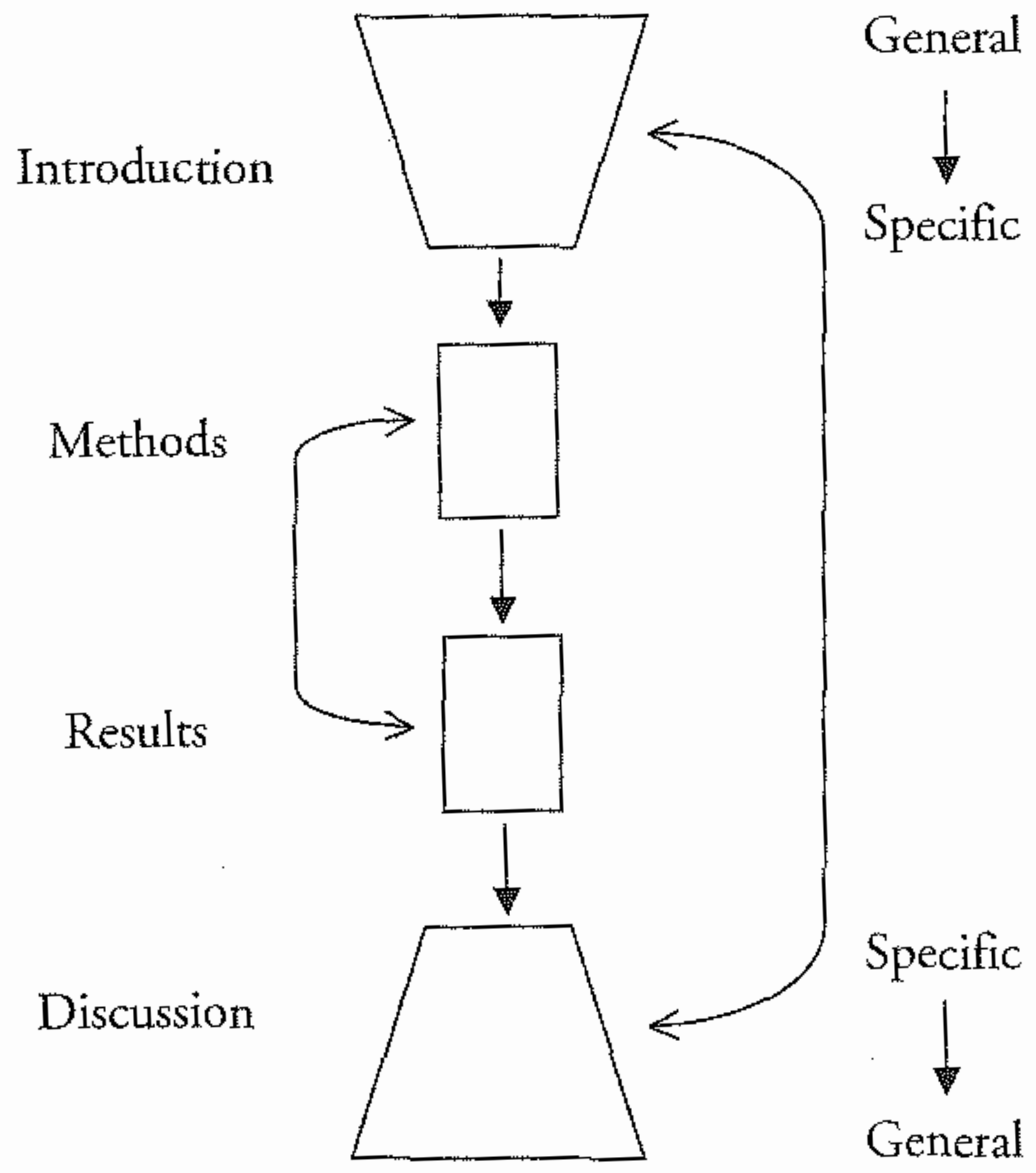
If you have not done so already, find 5–10 well-written published research papers that are typical of papers in your area of study. It does not matter whether these are seminal papers or where the research was conducted. We simply want you to have a small data set (a corpus) that you can analyze to gain some insights into the important characteristics of published work in your discipline.

TASK TWO

Read a review article of relevance to you. Does it include one of the aspects proposed by Noguchi? Or is the approach different? What kind of section headings does it have? How long is it? How many references does it have?

TASK THREE

FIGURE 14. Overall Shape of a Research Paper



See also Lecture 10

THE PHYSICS OF FLUIDS VOLUME 11, NUMBER 6 JUNE 1968

Observation of Plasma Wave Echoes

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(Received 22 September 1967; final manuscript received 29 January 1968)

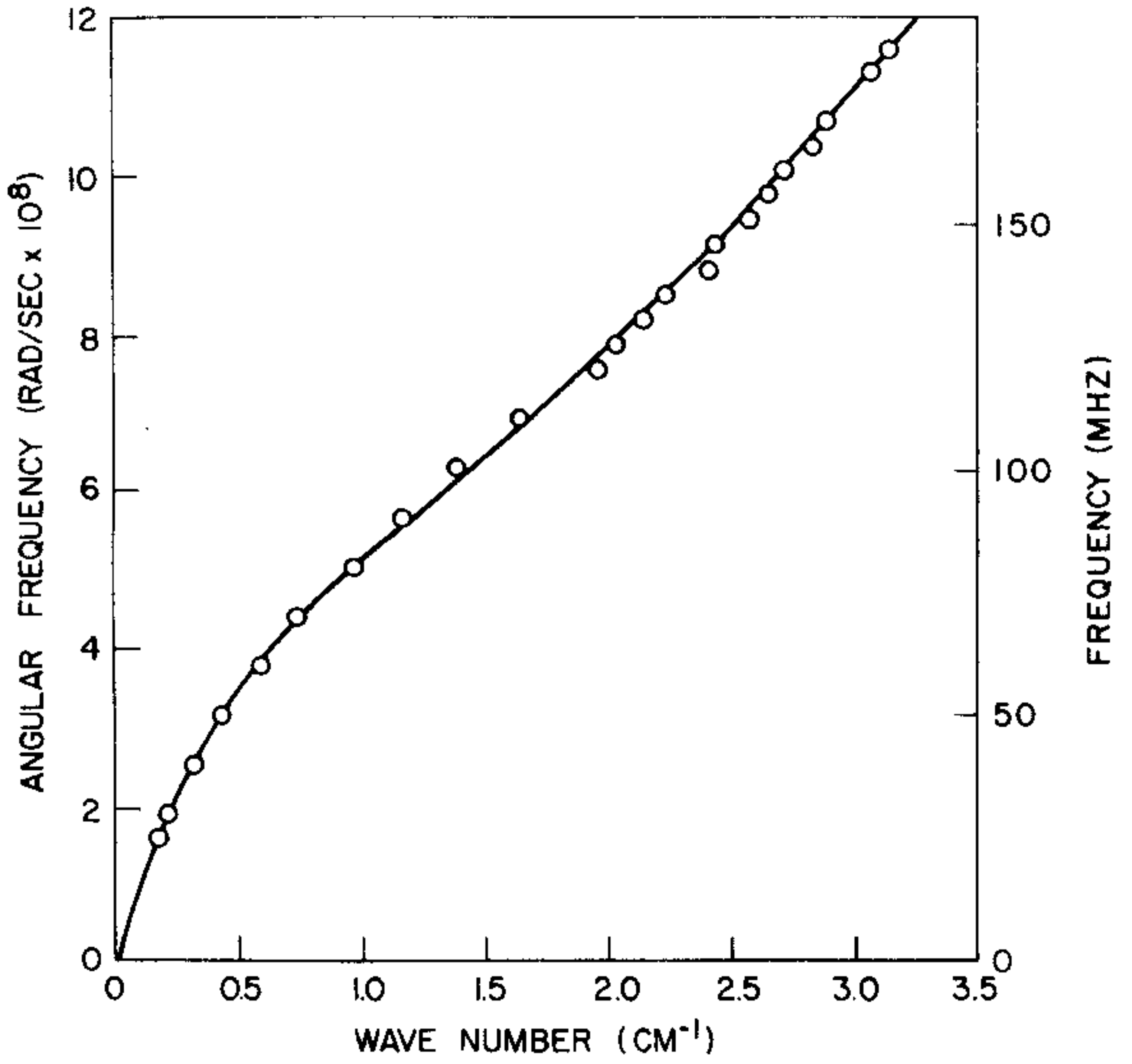

FIG. 1. Plasma wave dispersion curve. Circles are experimental result. The curve is theoretical.

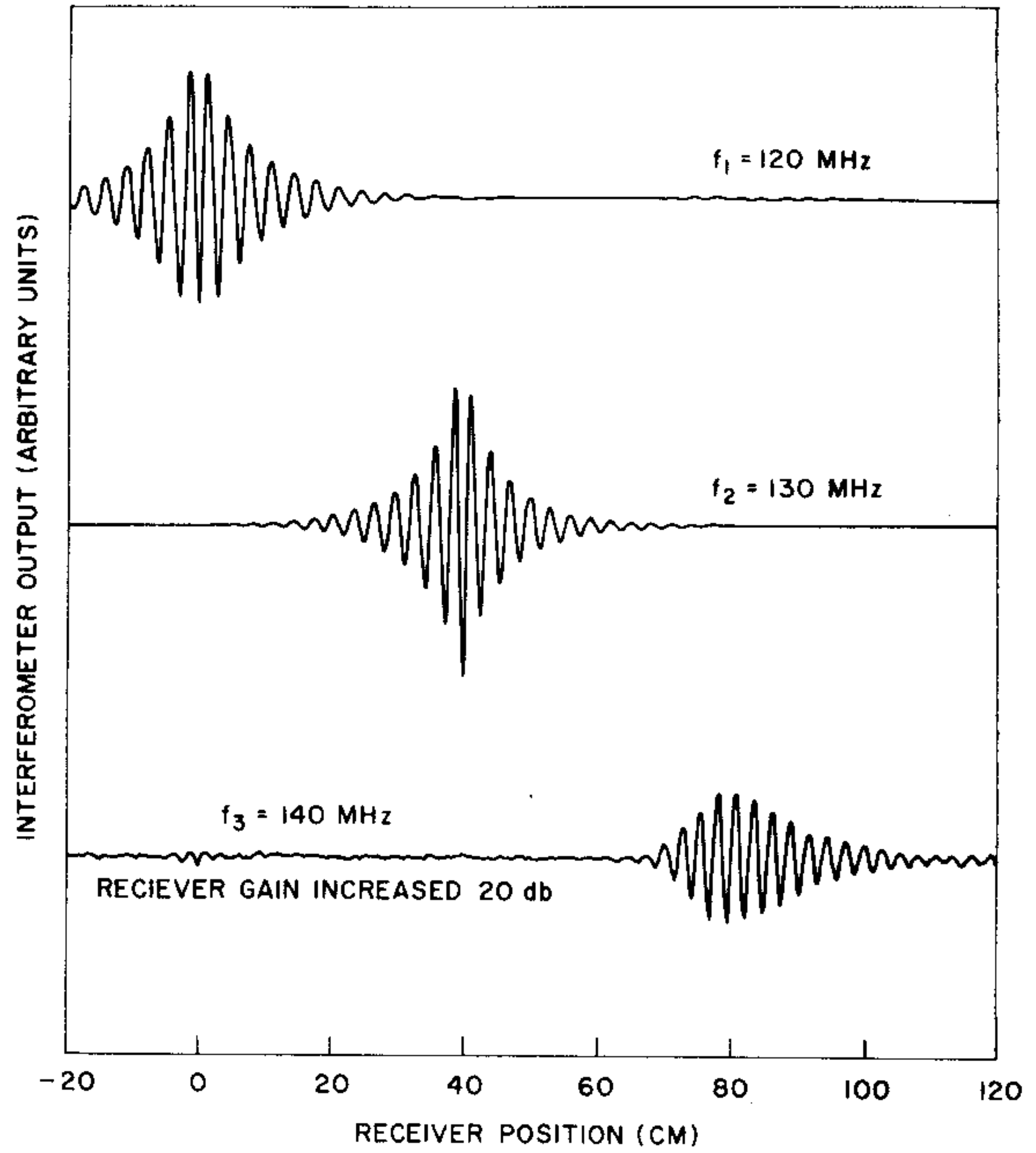

FIG. 2. Third-order echo. The transmitter probes are at 0 and 40 cm. Upper curve, receiver tuned to f_1; second curve, receiver tuned to f_2; third curve receiver tuned to f_3 and gain increased 20 dB.

Quasilinear Theory (Heating)

11.1.1 The Quasi-linear Diffusion Equation

$$\gamma = \pi \frac{k}{|k|} \frac{\omega_p^2}{k^2 \partial Đ_r/\partial\omega} \frac{\partial F_0}{\partial v_z}\bigg|_{v_z = \omega/k}. \tag{9.2.43}$$

$\sim (e/m)^2\, \tau_{cor}\, |E^2|$

$$\frac{\partial \mathscr{E}(k,t)}{\partial t} = 2\gamma(k,t)\mathscr{E}(k,t), \tag{11.1.43}$$

("D" is dispersion relation)

$$\frac{\partial}{\partial t}\langle f_s\rangle(v_z,t) = \frac{\partial}{\partial v_z}\left[D_q(v_z,t)\frac{\partial}{\partial v_z}\langle f_z\rangle(v_z,t)\right], \tag{11.1.44}$$

$$D_q(v_z,t) = \frac{2}{\epsilon_0}\left(\frac{e_s}{m_s}\right)^2 \int_{-\infty}^{\infty} \frac{\mathscr{E}(k,t)\gamma(k,t)}{[\omega_r(k,t) - kv_z]^2 + \gamma^2(k,t)}\,\mathrm{d}k, \tag{11.1.47}$$

See also Lectures 5 & 6

("D" is diffusion coefficient)

Old Notes on Wave Energy Density

http://sites.apam.columbia.edu/courses/apph6102y/Wave-Kinetic-Equation.pdf

$$\frac{\partial}{\partial t}\langle w_k \rangle + \frac{\partial}{\partial \bar{r}} \cdot \left(\bar{V}_g \langle w_k \rangle \right) - 2\omega_I \langle w_k \rangle = -\frac{1}{2} \mathrm{Re}\left\{ E^{*} \cdot J_{EXT} \right\}$$

Local Power Density

$$P = \mathbf{J} \cdot \mathbf{E}$$

$$P = \frac{1}{2} \Re \left\{ \mathbf{J}^* \cdot \mathbf{E} \right\}$$

Kinetic processes in plasma heating by resonant mode conversion of Alfvén wave

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(Received 17 May 1976)

$$n_0(dT_i/dt) = \tfrac{1}{2} \mathrm{Re}(\mathbf{J} \cdot \mathbf{E}^*)_i$$

Electrostatic Dispersion Relation for a Multi-Component Plasma

See Lecture 4

$$Đ(k,p) = 1 - \sum_s \frac{\omega_{ps}^2}{k^2} \int_C \frac{\partial F_{s0}/\partial v_z}{v_z - \mathrm{i}p/k} \, \mathrm{d}v_z = 0, \qquad (9.4.1)$$

$$Đ(k,\omega) = 1 - \frac{\omega_{p}^2}{k^2} \int_{-\infty}^{\infty} \frac{\partial F_{0}/\partial v_z}{(v_z - \omega/k)} \, \mathrm{d}v_z = 0. \qquad (9.1.13)$$

Electrostatic Dispersion Relation for IAW

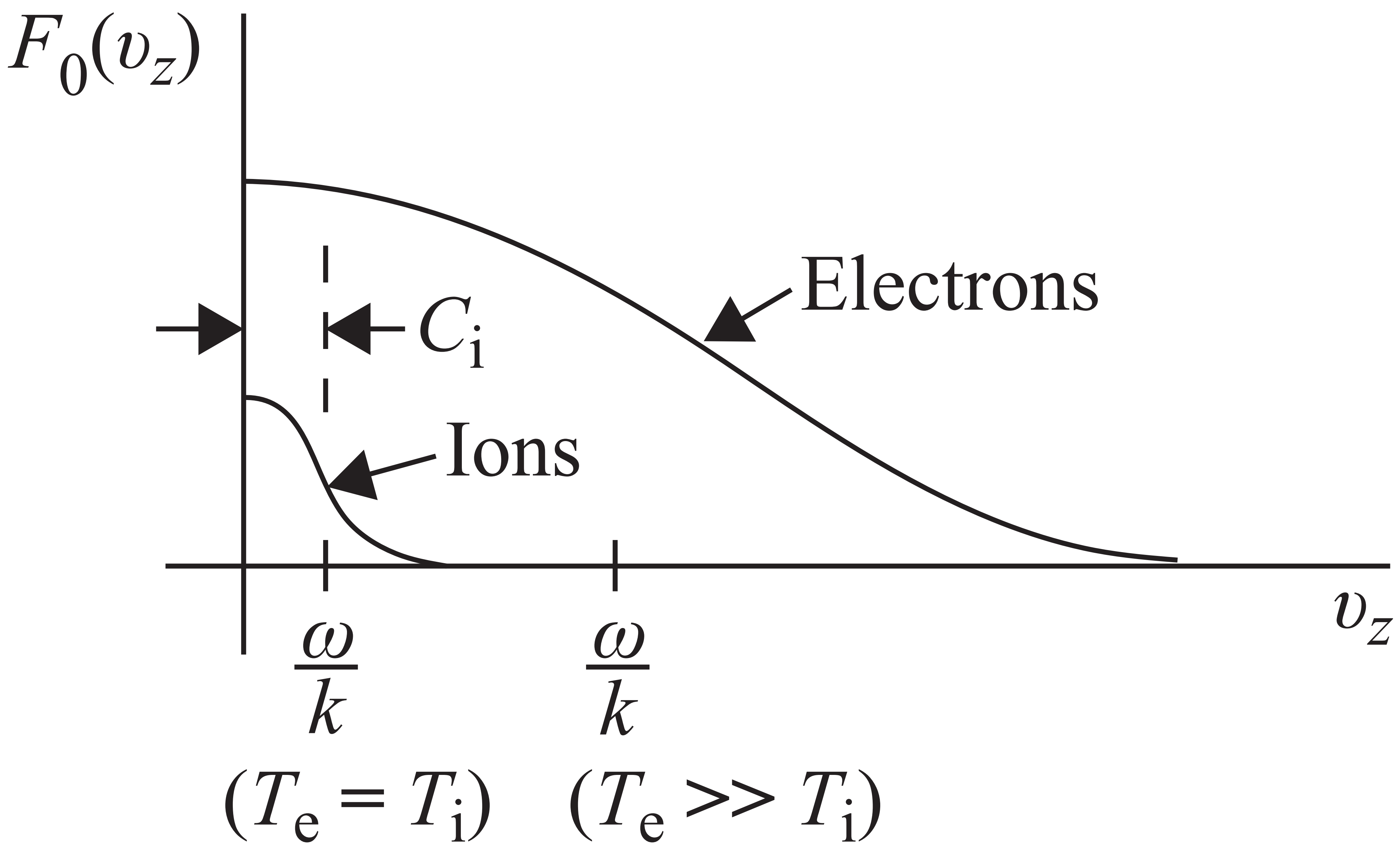

Figure 9.17 When $T_e \gg T_i$, the ion acoustic wave is weakly damped because the phase velocity, ω/k, is much greater than the ion thermal speed, C_i. When $T_e = T_i$, the damping is very large because $\omega/k \simeq C_i$.

Electrostatic Dispersion Relation for IAW

$$Đ(k,p) = 1 + \frac{1}{(k\lambda_{\mathrm{De}})^2}\left\{1 - \frac{1}{2}\left(\frac{T_{\mathrm{e}}}{T_{\mathrm{i}}}\right)\frac{1}{x_{\mathrm{i}}^2}\left(1 - \mathrm{i}\frac{2y_{\mathrm{i}}}{x_{\mathrm{i}}}\right)\right.$$
$$\left. + \mathrm{i}\frac{k}{|k|}\sqrt{\pi}x_{\mathrm{i}}\left[\sqrt{\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}}\sqrt{\frac{T_{\mathrm{i}}}{T_{\mathrm{e}}}} + \left(\frac{T_{\mathrm{e}}}{T_{\mathrm{i}}}\right)\mathrm{e}^{-x_{\mathrm{i}}^2}\right]\right\} = 0, \qquad (9.4.23)$$

$$\frac{\omega}{k} = \pm\sqrt{\frac{\kappa T_{\mathrm{e}}}{m_{\mathrm{i}}}}\frac{1}{(1 + k^2\lambda_{\mathrm{De}}^2)^{1/2}} \qquad (9.4.24)$$

and

$$\frac{\gamma}{\omega} = -\sqrt{\frac{\pi}{8}}\left[\sqrt{\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}} + \left(\frac{T_{\mathrm{e}}}{T_{\mathrm{i}}}\right)^{3/2}\exp\left(-\frac{T_{\mathrm{e}}}{2T_{\mathrm{i}}}\frac{1}{(1 + k^2\lambda_{\mathrm{De}}^2)}\right)\right]\frac{1}{(1 + k^2\lambda_{\mathrm{De}}^2)^{3/2}}. \qquad (9.4.25)$$

The Current-Driven Ion Acoustic Instability

When a current is present in a plasma, the electrons have a net drift relative to the ions. If the relative drift between the electrons and the ions is sufficiently large, a double hump occurs in the equivalent reduced distribution function, as shown in Figure 9.31. The ion acoustic mode can then be driven unstable. This instability is called the current-driven ion acoustic instability.

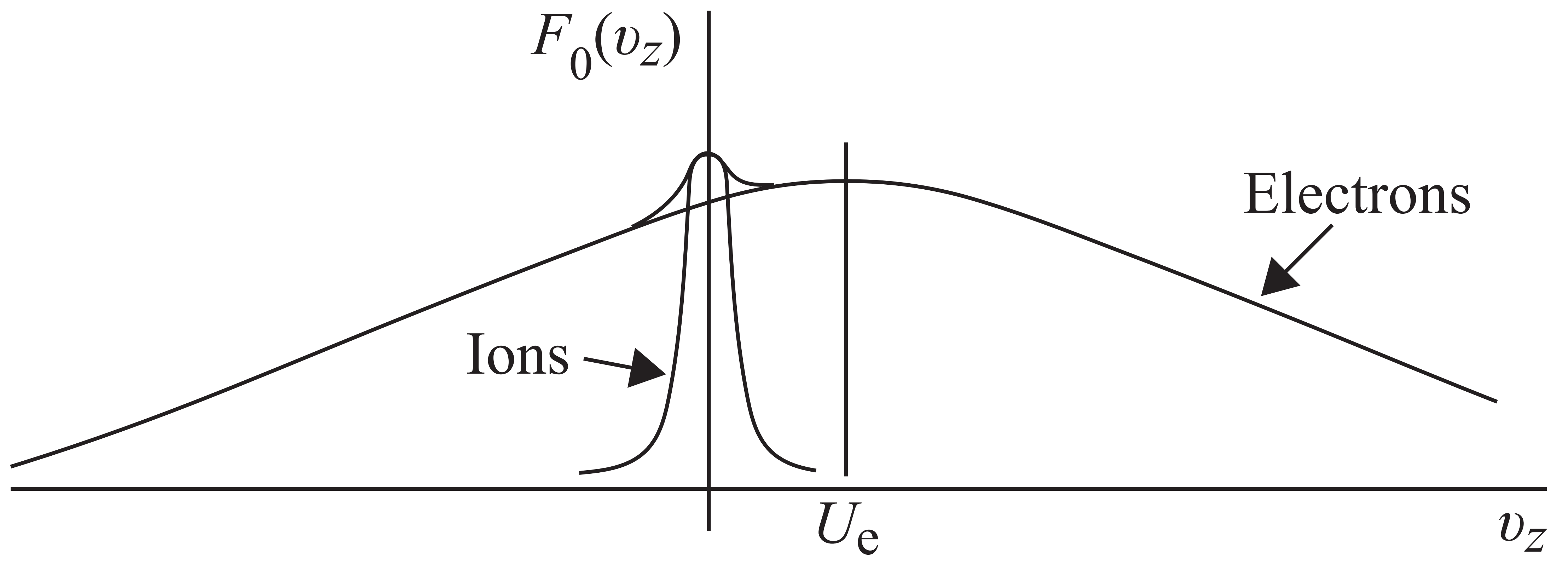

Figure 9.31 When a current is present, a double hump is produced in the equivalent reduced velocity distribution function. This double hump can lead to the growth of ion acoustic waves.

$$\gamma \approx -\left(C_{\mathrm{i}} - U_{\mathrm{e}} \frac{k}{|k|} \left(\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\right)^{1/2}\right) |k|. \tag{9.5.29}$$

As can be seen, if the electron drift velocity, U_{e}, is sufficiently large, the growth rate can become positive, indicating instability. Note that the unstable waves occur

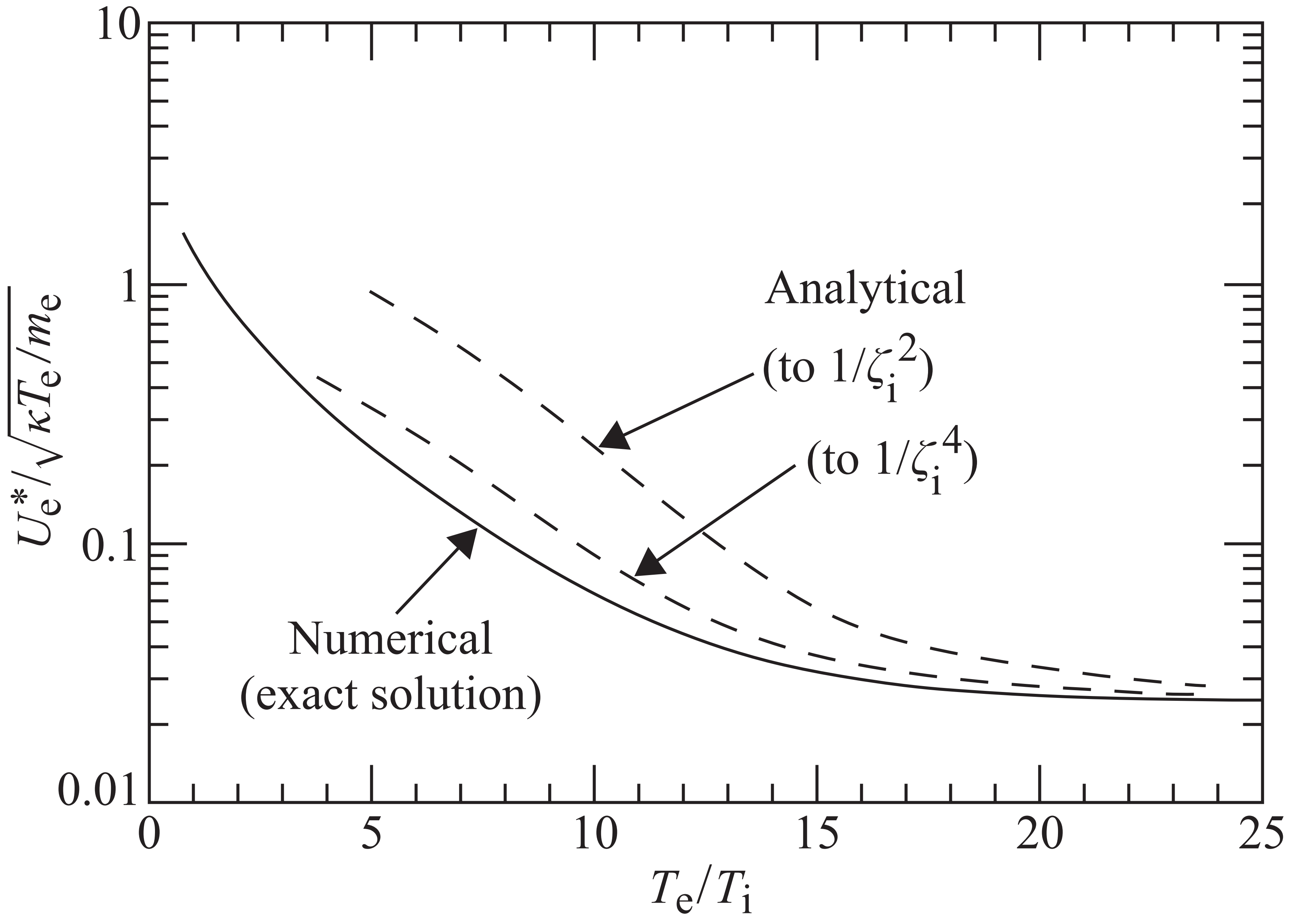

Figure 9.32 A comparison of the approximate analytical solution for the threshold electron drift velocity required to produce unstable ion acoustic waves with the results of an exact numerical calculation.

Cyclotron Damping

$$\gamma = \pi \frac{1}{\partial Đ_{\mathrm{r}}/\partial\omega} \sum_s \frac{\omega_{\mathrm{ps}}^2}{\omega^2} \left[-\frac{\omega}{|k_{\|}|} \int_0^\infty F_{s0} 2\pi v_\perp \mathrm{d}v_\perp \right.$$

$$\left. \left. - \frac{k_{\|}}{|k_{\|}|} \int_0^\infty \left(v_{\|} \frac{\partial F_{s0}}{\partial v_\perp} - v_\perp \frac{\partial F_{s0}}{\partial v_{\|}} \right) \pi v_\perp^2 \mathrm{d}v_\perp \right] \right|_{v_{\|} = v_{\|\mathrm{Res}}} . \qquad (10.3.42)$$

Quasilinear diffusion coefficients in a finite Larmor radius expansion for ion cyclotron heated plasmas

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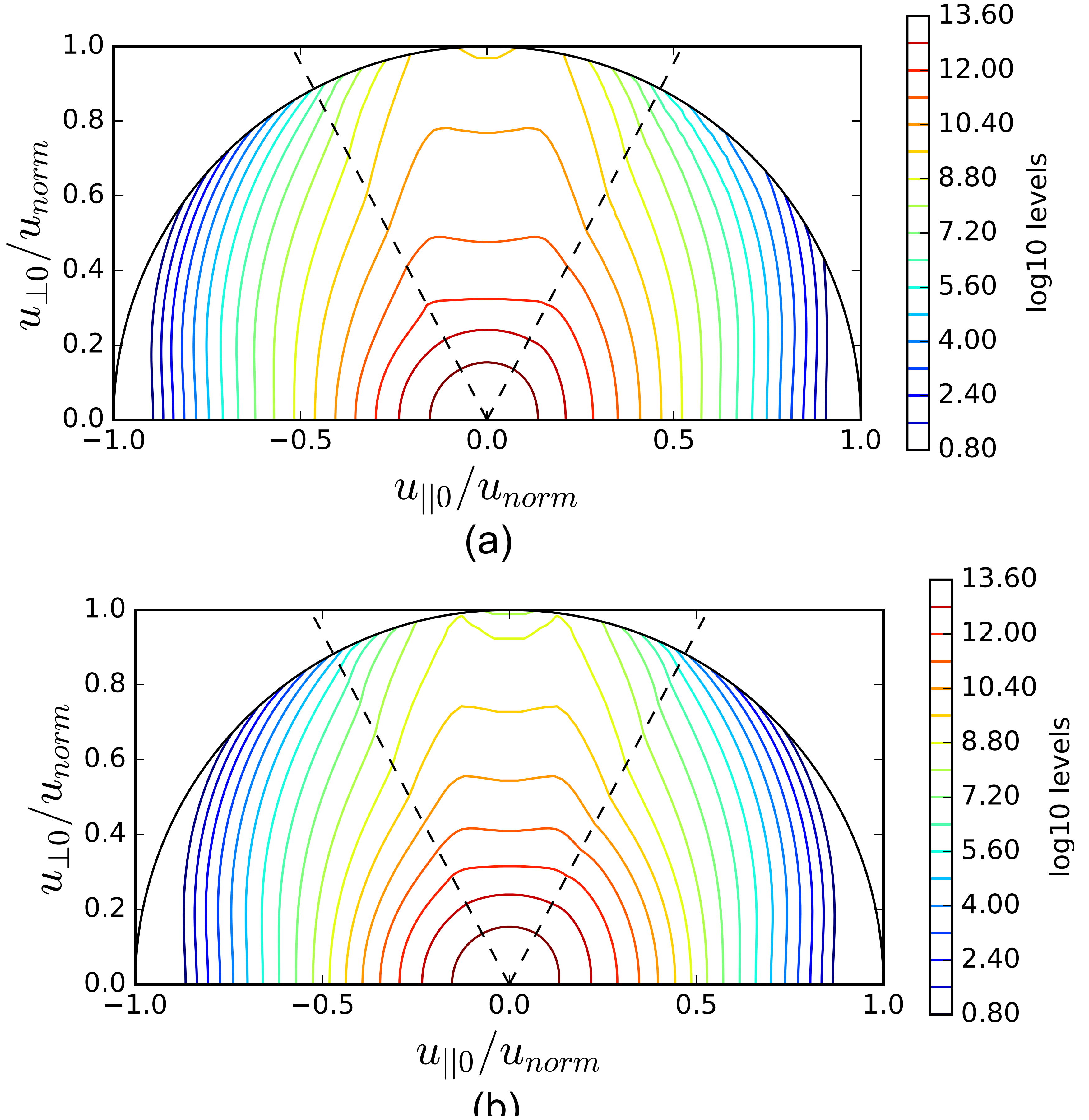
In this paper, a reduced model of quasilinear velocity diffusion by a small Larmor radius approximation is derived to couple the Maxwell's equations and the Fokker Planck equation self-consistently for the ion cyclotron range of frequency waves in a tokamak. The reduced model ensures the important properties of the full model by Kennel-Engelmann diffusion, such as diffusion directions, wave polarizations, and H-theorem. The kinetic energy change ($\dot{W}$) is used to derive the reduced model diffusion coefficients for the fundamental damping (n = 1) and the second harmonic damping (n = 2) to the lowest order of the finite Larmor radius expansion. The quasilinear diffusion coefficients are implemented in a coupled code (TORIC-CQL3D) with the equivalent reduced model of the dielectric tensor. We also present the simulations of the ITER minority heating scenario, in which the reduced model is verified within the allowable errors from the full model results. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4982060]



Wave-Particle Resonant Radial Diffusion

Theory of mode-induced beam particle loss in tokamaks

2958 Phys. Fluids **26** (10), October 1983

R. B. White, R. J. Goldston, K. McGuire, A. H. Boozer, D. A. Monticello, and W. Park
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(Received 17 March 1983; accepted 28 June 1983)

Large-amplitude rotating magnetohydrodynamic modes are observed to induce significant high-energy beam particle loss during high-power perpendicular netural beam injection on the poloidal divertor experiment (PDX). A Hamiltonian formalism for drift orbit trajectories in the presence of such modes is used to study induced particle loss analytically and numerically. Results are in good agreement with experiment.

The Hamiltonian for the guiding-center motion is

$$H = \tfrac{1}{2}B^2\rho_\parallel^2 + \mu B + \Phi,$$

$$\chi = \phi/\epsilon, \quad \rho_c = \rho_\parallel + \alpha,$$

$$\theta_0 = \theta - \phi/q, \quad \psi = r^2/2,$$

$$\dot{\chi} = \frac{\partial H}{\partial \rho_c}, \quad \dot{\rho}_c = -\frac{\partial H}{\partial \chi},$$

$$\dot{\theta}_0 = \frac{\partial H}{\partial \psi}, \quad \dot{\psi} = -\frac{\partial H}{\partial \theta_0}.$$

Energetic Particle Orbits

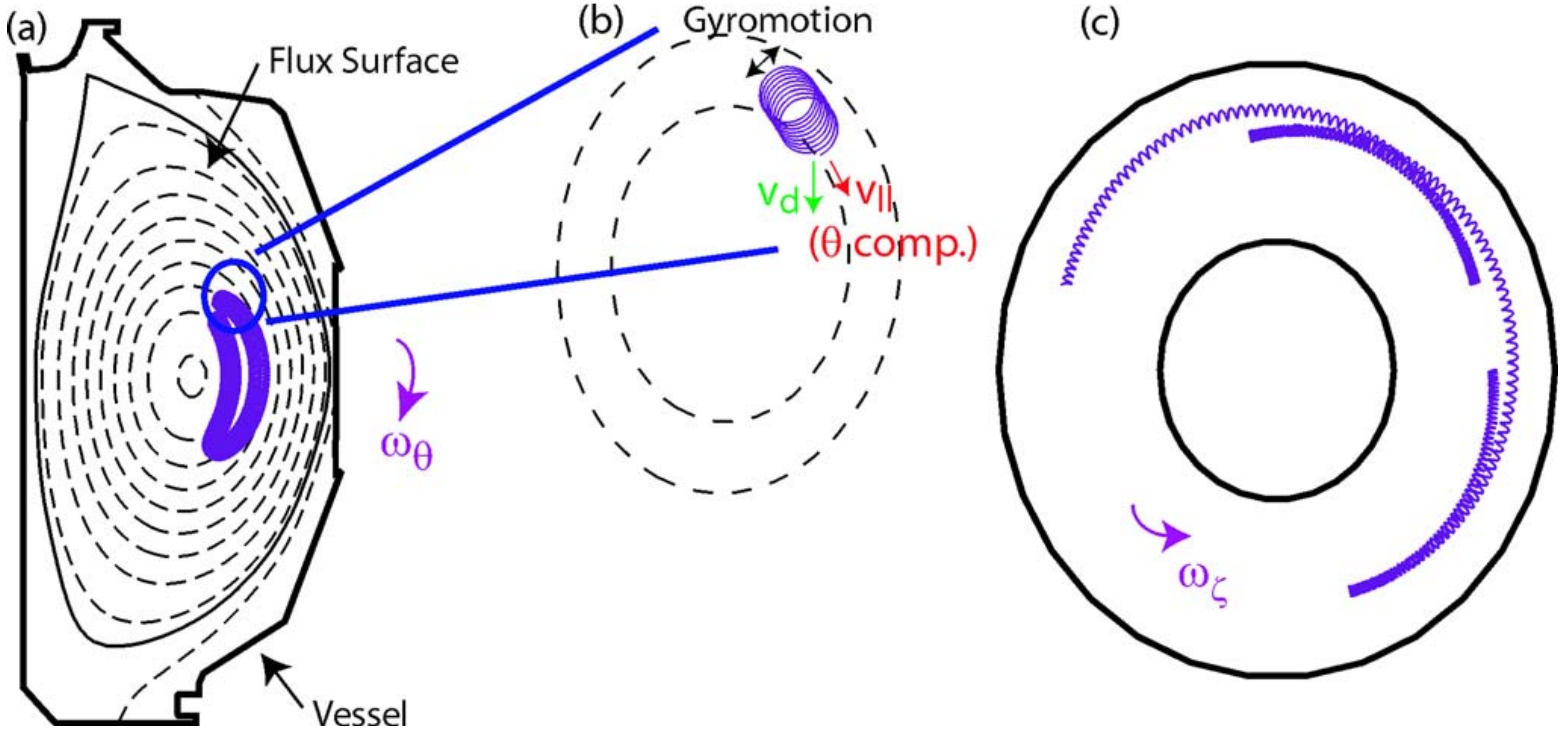
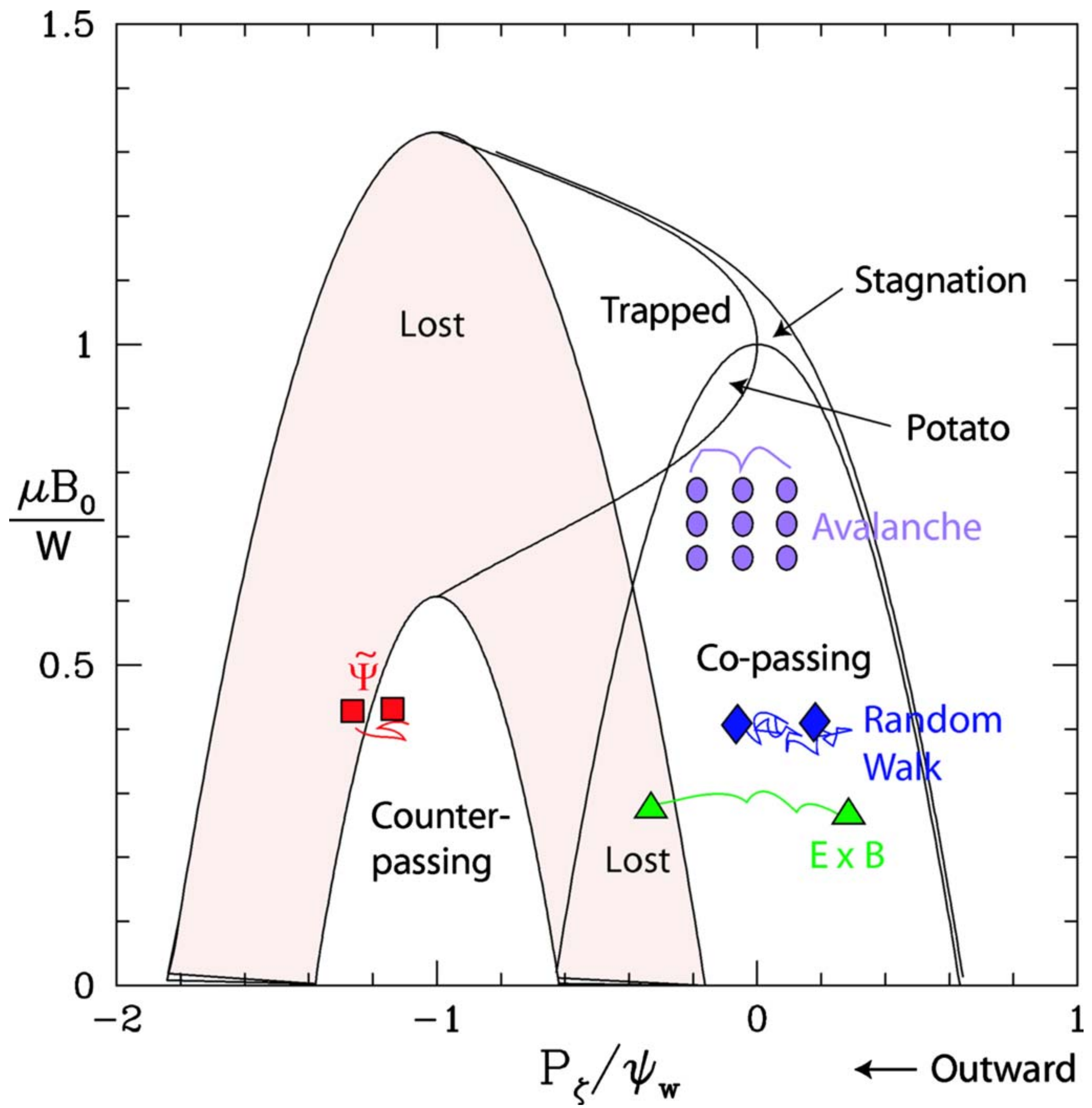


FIG. 9. (Color online) Projection of the orbit of an 80-keV deuterium beam ion in the DIII-D tokamak. (a) Elevation. The dashed lines represent the magnetic flux surfaces. The particle orbits poloidally with a frequency ω_θ. (b) Detail of the beginning of the orbit. The rapid gyromotion, parallel drift along the flux surface, and vertical drift velocity are indicated. (c) Plan view of the orbit. The particle precesses toroidally with a frequency ω_ζ.



Unperturbed Orbits

First consider deeply trapped particles. The unperturbed orbit can be approximated by uniform precession in ϕ $[\phi = \Omega t + \omega_p(\psi)t\,]$ and periodic bounce motion in θ $(\theta \approx -\theta_b \sin \omega_p t\,)$. For trapped particles the average toroidal precession rate is given by

$$\omega_p \approx \epsilon\rho^2 q/2r \tag{18}$$

and the bounce frequency by

$$\omega_b \approx \epsilon\rho\sqrt{\epsilon r}/q, \tag{19}$$

where ρ is the on-axis gyroradius. The variation of ψ and the periodic motion in ϕ about the average precession in the unperturbed orbit are unessential complications and can be

Perturbation

$$\Phi_1 = \sum_{m,n} \frac{\alpha_{nm}(m\Phi_0' + \omega_{nm})}{\epsilon(n - m/q)} \sin(n\phi - m\theta - \omega_{nm} - \delta_{nm}), \tag{14}$$

where the prime refers to differentiation with respect to ψ.

gy. Consider first the radial position. The radial motion ψ is given by

$$\dot{\psi} = -(\rho_\parallel^2 B + \mu)\epsilon r \sin\theta + \sum_{m,n} \frac{m\alpha_{nm}}{\epsilon}\left(\frac{(\omega_{nm} + m\Phi_0')}{(n - m/q)} - \dot{\phi}\right) \times \cos(n\phi - m\theta - \omega_{nm} t + \delta_{nm}), \tag{17}$$

where Eq. (10) has been used to substitute $\dot{\phi}$ for $\epsilon\rho_\parallel B^2$.

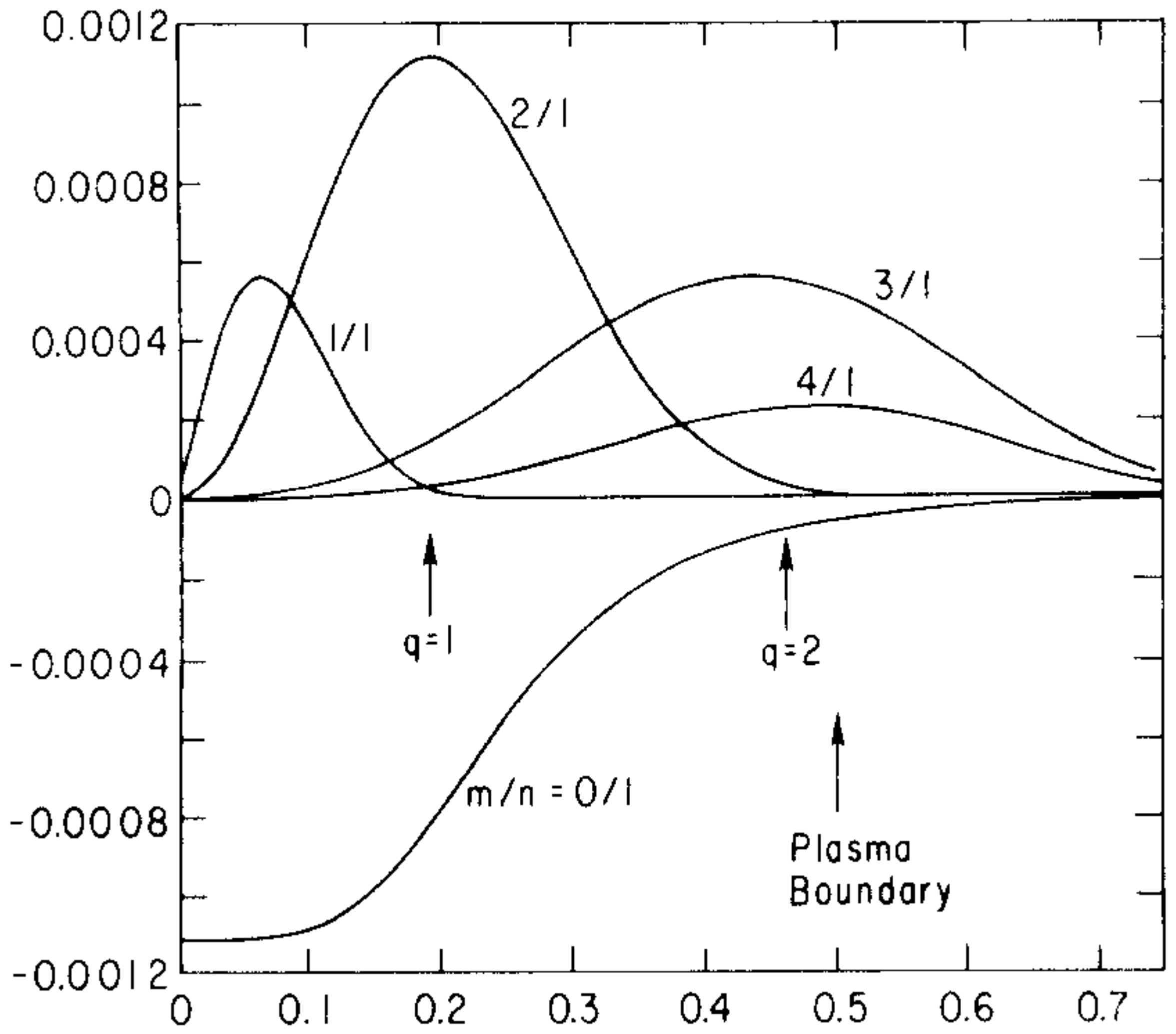


FIG. 2. Analytical approximations to the $n = 1$ MHD modes for a moderate amplitude fishbone. The mode maximum and the value at the rational surface both agree with values given by the MHD simulation. Modes with $n > 1$ were appreciably smaller.

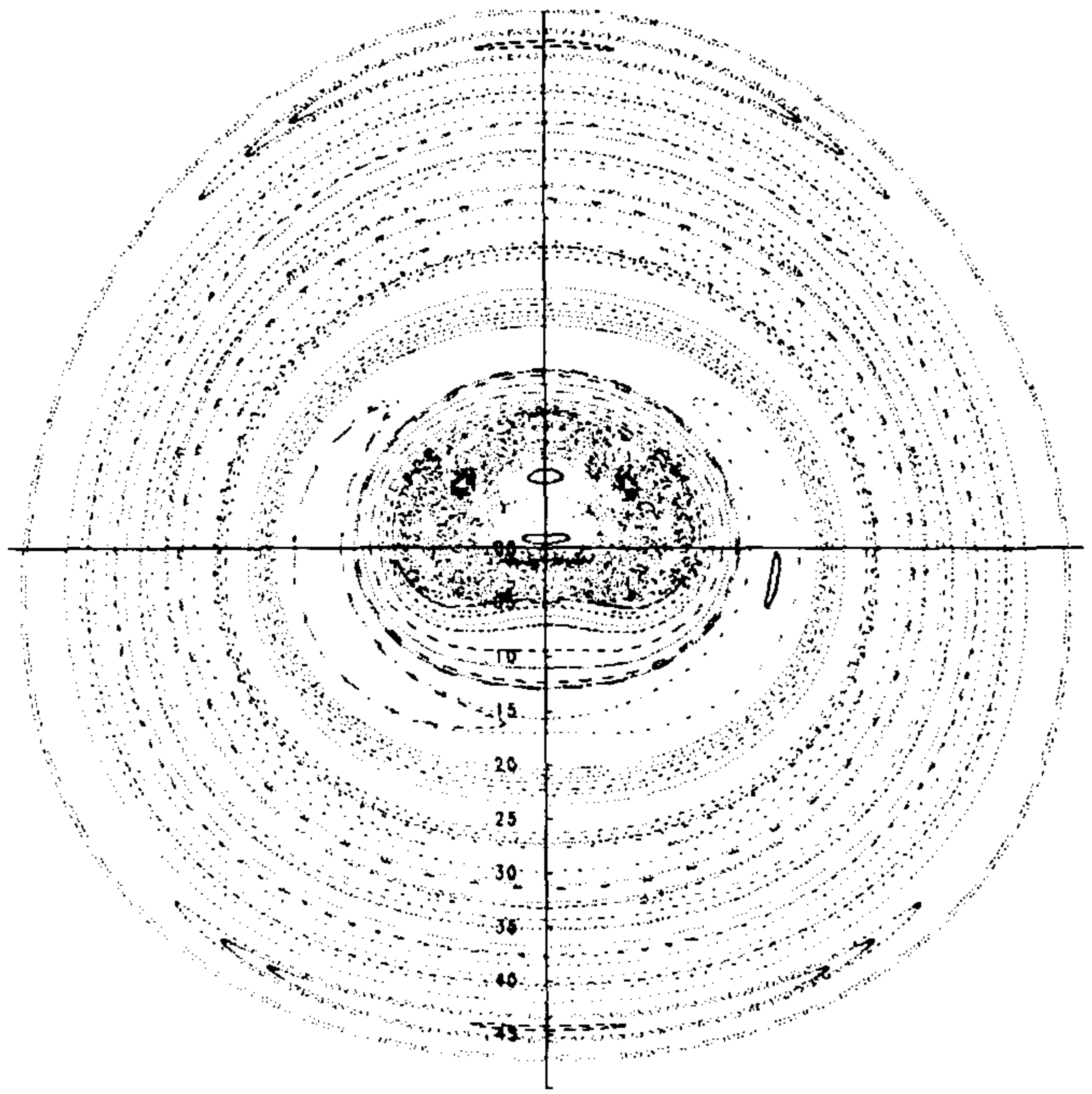


FIG. 3. The Poincaré plot of the magnetic field using the mode amplitudes approximating those given by the MHD simulation for a moderate fishbone. The $n = 1$ modes are shown in Fig. 2. Also included were $n = 2$ and $n = 3$ modes, which at this amplitude have no appreciable effect on particle loss or energy. The figure extends to the plasma edge, $r = 0.5a$.

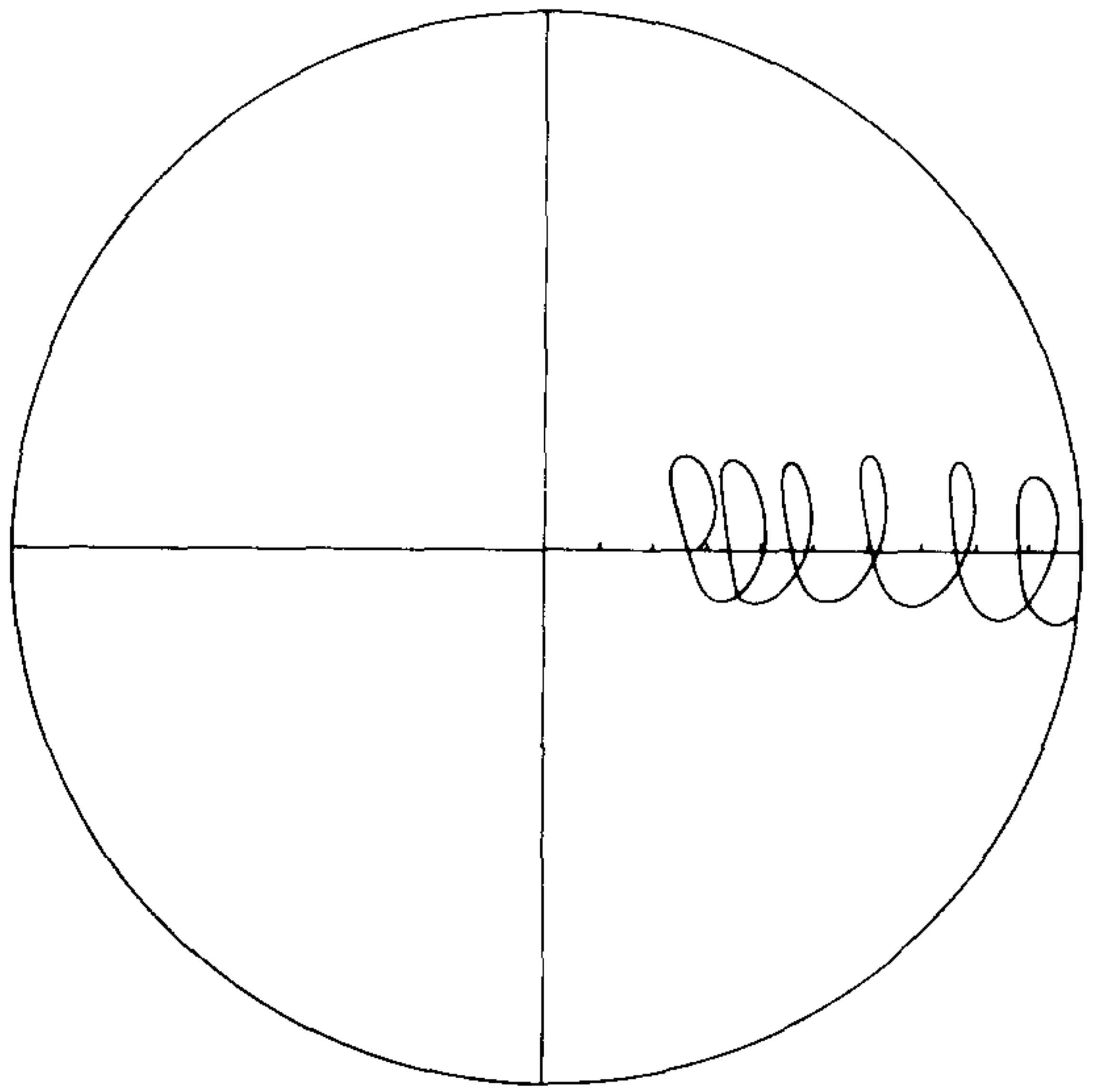


FIG. 5. A typical trajectory for a beam particle lost due to mode pumping. Here $R_T = 35$ cm and the mode frequency is 20 kHz.

Next Class

- March 7: More discussions re your "wave-particle" midterm papers
- March 12-16: ***Spring recess***
- Monday, March 19: **Midterm papers due**
- Monday, March 19: **Introduction to low-frequency drift-waves**