Plasma 2
Lecture 1: Introduction

APPH E6102y
Columbia University
Welcome to the APPH E6102y class information site.

This is the second semester of a two-semester sequence in plasma physics. Plasma physics is the study of "luminous matter", matter that has been heated sufficiently or prepared specially in order to be ionized. In plasma long-range electromagnetic forces are more important than short range forces. Plasma dynamics is dominated by "collective" motion of large populations of neighboring particles. Electric and electromagnetic waves propagate at speeds that resonate with particles and allow energy and momentum exchange. Plasma motion and the self-consistent electric and magnetic fields exhibit beautiful nonlinear physics.

Plasma is studied in the laboratory and in space. Most of the visible universe is in the plasma state. Laboratory generated plasma are used to studied the fundamental properties of high-temperature matter, and they are employed for many valuable applications like surface processing and lighting. Integrated circuits are manufactured using plasma processing, and plasma displays are status symbols of today's world of entertainment. Controlled fusion energy research reflects the remarkable success of plasma physics. The controlled release of more than 10 MW of fusion power has occurred within the strong confining fields of tokamak devices, and the world is now building the first experimental fusion power source, called ITER.


APPH 6102 requires prior experience with plasma physics. The formal prerequisites are APPH E6101 Plasma physics. The goal of this course is to provide a working understanding of plasma physics and prepare students for research.
(No/Optional) Textbook

Textbook

There will be no textbook for the course. Instead, we will make frequent use of journal publications.

For those who want a well-rounded textbook, I recommend Introduction to Plasma Physics (2nd Edition) by Don Gurnett (University of Iowa) and Amitava Bhattacharjee (Princeton University). Don Gurnett is a well known space plasma physicists and plasma wave expert. Bhattacharjee has worked in many areas of magnetized plasma physics, including magnetic fusion, plasma astrophysics, theory, and high-performance computation.

Since Columbia's plasma physics program has a focus on fusion energy, I also recommend an introductory textbook by Garry McCracken and Peter Stott: Fusion: The Energy of the Universe. McCracken has made pioneering studies of tokamak plasma confinement. He's an expert on plasma wall interactions and worked at Alcator CMOD and JET. Peter Stott is also a leading expert on tokamak fusion confinement, having worked at PPPL, JET, and most recently working on ITER diagnostic systems.

These books are available as ebooks for Columbia University students: see CLIO. Occasionally, I will present numerical illustrations of plasma physics using Mathematica. Mathematica is available to all students through Columbia University.

Most frequently, I will distribute published journal articles that illustrate the scientific progress and discoveries in the field of plasma physics.
Grading

A student's grade for the course will be based on two research and writing assignments:

**one midterm paper and one final paper**

These papers must follow the style used for publication in *Physics of Plasmas*. You will clearly describe *in your own words* a plasma physics topic or question and then review the topic or present an answer to the question.
Radiation from laser-microplasma-waveguide interactions in the ultra-intense regime
by L. Yi, A. Pukhov, and B. Shen
INTERACTION OF SOLITONS IN A COLLISIONLESS PLASMA AND RECURRENCE OF INITIAL STATES
By: ZABUSKY, NJ; KRUSKAL, MD
PHYSICAL REVIEW LETTERS Volume: 15 Issue: 6 Pages: 240-8 Published: 1965 (Times Cited: 1,990)

RELAXATION OF TOROIDAL PLASMA AND GENERATION OF REVERSE MAGNETIC-FIELDS
By: TAYLOR, JB
PHYSICAL REVIEW LETTERS Volume: 33 Issue: 19 Pages: 1139-1141 Published: 1974 (Times Cited: 1,323)

ABSORPTION OF ULTRA-INTENSE LASER-PULSES
By: WILKS, SC; KRUEER, WL; TABAK, M; et al.
PHYSICAL REVIEW LETTERS Volume: 69 Issue: 9 Pages: 1383-1386 Published: AUG 31 1992 (Times Cited: 1,312)

PLASMA CRYSTAL - COULOMB CRYSTALLIZATION IN A DUSTY PLASMA
By: THOMAS, H; MORFILL, GE; DEMMEL, V; et al.
PHYSICAL REVIEW LETTERS Volume: 73 Issue: 5 Pages: 652-655 Published: AUG 1 1994 (Times Cited: 1,243)

SPONTANEOUSLY GROWING TRANSVERSE WAVES IN A PLASMA DUE TO AN ANISOTROPIC VELOCITY DISTRIBUTION
By: WEIBEL, ES
PHYSICAL REVIEW LETTERS Volume: 2 Issue: 3 Pages: 83-84 Published: 1959 (Times Cited: 1,123)

Plasma expansion into a vacuum
By: Mora, P
PHYSICAL REVIEW LETTERS Volume: 90 Issue: 18 Article Number: 185002 Published: MAY 9 2003 (Times Cited: 617)

DIRECT OBSERVATION OF COULOMB CRYSTALS AND LIQUIDS IN STRONGLY COUPLED RF DUSTY PLASMAS
By: CHU, JH; I, L
PHYSICAL REVIEW LETTERS Volume: 72 Issue: 25 Pages: 4009-4012 Published: JUN 20 1994 (Times Cited: 1,048)

TRANSPORT OF DUST PARTICLES IN GLOW-DISCHARGE PLASMAS
By: BARNES, MS; KELLER, JH; FORSTER, JC; et al.
PHYSICAL REVIEW LETTERS Volume: 68 Issue: 3 Pages: 313-316 Published: JAN 20 1992 (Times Cited: 582)
REGIME OF IMPROVED CONFINEMENT AND HIGH-BETA IN NEUTRAL-BEAM-HEATED DIVERTOR DISCHARGES OF THE ASDEX TOKAMAK
By: WAGNER, F; BECKER, G; BEHRINGER, K; et al.
PHYSICAL REVIEW LETTERS Volume: 49 Issue: 19 Pages: 1408-1412 Published: 1982 (Times Cited: 1,504)

ELECTRON HEAT-TRANSPORT IN A TOKAMAK WITH DESTROYED MAGNETIC SURFACES
By: RECHESTER, AB; ROSENBLUTH, MN
PHYSICAL REVIEW LETTERS Volume: 40 Issue: 1 Pages: 38-41 Published: 1978 (Times Cited: 994)

ENHANCED CONFINEMENT AND STABILITY IN DIII-D DISCHARGES WITH REVERSED MAGNETIC SHEAR
By: STRAIT, EJ; LAO, LL; MAUEL, ME; et al.
PHYSICAL REVIEW LETTERS Volume: 75 Issue: 24 Pages: 4421-4424 Published: DEC 11 1995 (Times Cited: 514)

H-MODE BEHAVIOR INDUCED BY CROSS-FIELD CURRENTS IN A TOKAMAK
By: TAYLOR, RJ; BROWN, ML; FRIED, BD; et al.
PHYSICAL REVIEW LETTERS Volume: 63 Issue: 21 Pages: 2365-2368 Published: NOV 20 1989 (Times Cited: 491)

STUDIES OF INTERNAL DISRUPTIONS AND M = 1 OSCILLATIONS IN TOKAMAK DISCHARGES WITH SOFT-X-RAY TECHNIQUES
By: VONGOELER, S; STODIEK, W; SAUTHOFF, N
PHYSICAL REVIEW LETTERS Volume: 33 Issue: 20 Pages: 1201-1203 Published: 1974 (Times Cited: 462)

CONFINING A TOKAMAK PLASMA WITH RF-DRIVEN CURRENTS
By: FISCH, NJ
PHYSICAL REVIEW LETTERS Volume: 41 Issue: 13 Pages: 873-876 Published: 1978 (Times Cited: 461)

Electron temperature gradient turbulence
By: Dorland, W; Jenko, F; Kotschenreuther, M; et al.
Recent “Hot” Articles

“Hot” articles rank within the top 1% of all articles published in physics.

Active control of type-I edge-localized modes with n=1 perturbation fields in the JET tokamak
By: Liang, Y.; Koslowski, H. R.; Thomas, P. R.; et al.
PHYSICAL REVIEW LETTERS Volume: 98 Issue: 26 (Times Cited: 301) Published: JUN 29 2007

First Observation of Edge Localized Modes Mitigation with Resonant and Nonresonant Magnetic Perturbations in ASDEX Upgrade
By: Suttrop, W.; Eich, T.; Fuchs, J. C.; et al.
Group Author(s): ASDEX Upgrade Team
PHYSICAL REVIEW LETTERS Volume: 106 Issue: 22 (Times Cited: 218) Published: JUN 2 2011

Observations of plasmons in warm dense matter
By: Glenzer, S. H.; Landen, O. L.; Neumayer, P.; et al.
PHYSICAL REVIEW LETTERS Volume: 98 Issue: 6 (Times Cited: 301) Published: FEB 9 2007

Radiation-Pressure Acceleration of Ion Beams Driven by Circularly Polarized Laser Pulses
By: Henig, A.; Steinke, S.; Schnuerer, M.; et al.
PHYSICAL REVIEW LETTERS Volume: 103 Issue: 24 (Times Cited: 291) Published: DEC 11 2009

Generating high-current monoenergetic proton beams by a circularly polarized laser pulse in the phase-stable acceleration regime
By: Yan, X. Q.; Lin, C.; Sheng, Z. M.; et al.
PHYSICAL REVIEW LETTERS Volume: 100 Issue: 13 (Times Cited: 274) Published: APR 4 2008

Dynamics of spin-1/2 quantum plasmas
By: Marklund, Mattias; Brodin, Gert
PHYSICAL REVIEW LETTERS Volume: 98 Issue: 2 (Times Cited: 268) Published: JAN 12 2007
Most Cited from *Nuclear Fusion*

**RECONSTRUCTION OF CURRENT PROFILE PARAMETERS AND PLASMA SHAPES IN TOKAMAKS**
By: LAO, LL; STJOHN, H; STAMBAUGH, RD; et al.
*Nuclear Fusion* Volume: 25 Issue: 11 Pages: 1611-1622 Published: 1985 (Times Cited: 951)

**NEOCLASSICAL TRANSPORT OF IMPURITIES IN TOKAMAK PLASMAS**
By: HIRSHMAN, SP; SIGMAR, DJ
*Nuclear Fusion* Volume: 21 Issue: 9 Pages: 1079-1201 Published: 1981 (Times Cited: 900)

**FAST-WAVE HEATING OF A 2-COMPONENT PLASMA**
By: STIX, TH
*Nuclear Fusion* Volume: 15 Issue: 5 Pages: 737-754 Published: 1975 (Times Cited: 582)

Plasma-material interactions in current tokamaks and their implications for next step fusion reactors
By: Federici, G; Skinner, CH; Brooks, JN; et al.
*Nuclear Fusion* Volume: 41 Issue: 12R Special Issue: SI Pages: 1967-2137 Published: DEC 2001 (Times Cited: 804)

**MEASUREMENTS OF MICROTURBULENCE IN TOKAMAKS AND COMPARISONS WITH THEORIES OF TURBULENCE AND ANOMALOUS TRANSPORT**
By: LIEWER, PC
*Nuclear Fusion* Volume: 25 Issue: 5 Pages: 543-621 Published: 1985 (Times Cited: 670)

**A NEW LOOK AT DENSITY LIMITS IN TOKAMAKS**
By: GREENWALD, M; TERRY, JL; WOLFE, SM; et al.
*Nuclear Fusion* Volume: 28 Issue: 12 Pages: 2199-2207 Published: DEC 1988 (Times Cited: 530)

**PLASMA BOUNDARY PHENOMENA IN TOKAMAKS**
By: STANGEBY, PC; MCCRACKEN, GM
*Nuclear Fusion* Volume: 30 Issue: 7 Pages: 1225-1379 Published: JUL 1990 (Times Cited: 512)
Some “Most Cited” from *Physics of Plasmas*

**COLLISIONLESS DAMPING OF NONLINEAR PLASMA OSCILLATIONS**
By: ONEIL, T  
*Physics of Fluids* Volume: 8 Issue: 12 Pages: 2255-& Published: **1965** (Times Cited: **681**)

**VELOCITY SPACE DIFFUSION FROM WEAK PLASMA TURBULENCE IN A MAGNETIC FIELD**
By: KENNEL, CF; ENGELMANN, F  
*Physics of Fluids* Volume: 9 Issue: 12 Pages: 2377+- Published: **1966** (Times Cited: **719**)

**A PERTURBATION THEORY FOR STRONG PLASMA TURBULENCE**
By: DUPREE, TH  
*Physics of Fluids* Volume: 9 Issue: 9 Pages: 1773-& Published: **1966** (Times Cited: **611**)

**PSEUDO-3-DIMENSIONAL TURBULENCE IN MAGNETIZED NONUNIFORM PLASMA**
By: HASEGAWA, A; MIMA, K  
*Physics of Fluids* Volume: 21 Issue: 1 Pages: 87-92 Published: **1978** (Times Cited: **694**)

**INFLUENCE OF SHEARED POLOIDAL ROTATION ON EDGE TURBULENCE**
By: BIGLARI, H; DIAMOND, PH; TERRY, PW  
*Physics of Fluids B-Plasma Physics* Volume: 2 Issue: 1 Pages: 1-4 Published: **JAN 1990** (Times Cited: **1,096**)

**IGNITION AND HIGH-GAIN WITH ULTRA-POWERFUL LASERS**
By: TABAK, M; HAMMER, J; GLINSKY, ME; et al.  
*Physics of Plasmas* Volume: 1 Issue: 5 Pages: 1626-1634 Part: 2 Published: **MAY 1994** (Times Cited: **2,360**)

**LABORATORY OBSERVATION OF THE DUST-ACOUSTIC WAVE MODE**
By: BARKAN, A; MERLINO, RL; DANGELO, N  
*Physics of Plasmas* Volume: 2 Issue: 10 Pages: 3563-3565 Published: **OCT 1995** (Times Cited: **981**)

Effects of ExB velocity shear and magnetic shear on turbulence and transport in magnetic confinement devices  
By: Burrell, KH  
*Physics of Plasmas* Volume: 4 Issue: 5 Pages: 1499-1518 Part: 2 Published: **MAY 1997** (Times Cited: **975**)


Collisionless Damping of Nonlinear Plasma Oscillations

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It is well known that the linear theory of collisionless damping breaks down after a time \( \tau = (m/eEk)^4 \), where \( k \) is the wavenumber and \( E \) is the amplitude of the electric field. Jacobi elliptic functions are now used to provide an exact solution of the Vlasov equation for the resonant electrons, and the damping coefficient is generalized to be valid for times greater than \( t = \tau \). This generalized damping coefficient reduces to Landau’s result when \( t/\tau \ll 1 \); it has an oscillatory behavior when \( t/\tau \) is of order unity, and it phase mixes to zero as \( t/\tau \) approaches infinity. The above results are all shown to have simple physical interpretations.
INTRODUCTION TO
PLASMA PHYSICS

With Space, Laboratory
and Astrophysical Applications

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References

Linearized Vlasov Equation

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{e}{m} \nabla \Phi \cdot \nabla \mathbf{v} f = 0
\]  
(9.1.1)

\[
\nabla^2 \Phi = -\frac{\rho_e}{\epsilon_0} = -\frac{e}{\epsilon_0} \left[ n_0 - \int_{-\infty}^{\infty} f \, d^3 \nu \right].
\]  
(9.1.2)

\[
f(v) = f_0(v) + f_1(v).
\]  
(9.1.3)

\[
\frac{\partial f_1}{\partial t} + \mathbf{v} \cdot \nabla f_1 + \frac{e}{m} \nabla \Phi_1 \cdot \nabla \mathbf{v} f_0 = 0
\]  
(9.1.4)

\[
\nabla^2 \Phi_1 = \frac{e}{\epsilon_0} \int_{-\infty}^{\infty} f_1(v) \, d^3 \nu.
\]  
(9.1.5)
Fourier-Laplace Transform

\[ -i\omega \tilde{f} + ikv_z \tilde{f} + i\frac{e}{m} k \Phi \frac{\partial f_0}{\partial v_z} = 0 \]  
(9.1.6)

\[ k^2 \Phi = -\frac{e}{\epsilon_0} \int_{-\infty}^{\infty} \tilde{f}(v) \, d^3v. \]  
(9.1.7)

\[ \tilde{f} = \frac{-1}{(kv_z - \omega)} \frac{e}{m} k \Phi \frac{\partial f_0}{\partial v_z}, \]  
(9.1.8)

\[ k^2 \Phi = \frac{e^2}{\epsilon_0 m} k \Phi \int_{-\infty}^{\infty} \frac{\partial f_0/\partial v_z}{(kv_z - \omega)} \, d^3v. \]  
(9.1.9)

\[ \left[ 1 - \frac{e^2}{\epsilon_0 mk^2} \int_{-\infty}^{\infty} \frac{\partial f_0/\partial v_z}{(v_z - \omega/k)} \, d^3v \right] \Phi = 0. \]  
(9.1.10)

\[ D(k, \omega) = 1 - \frac{e^2}{\epsilon_0 mk^2} \int_{-\infty}^{\infty} \frac{\partial f_0/\partial v_z}{(v_z - \omega/k)} \, d^3v = 0. \]  
(9.1.11)
Electrostatic Dispersion Relation for a Plasma

\[ D(k, \omega) = 1 - \frac{\omega_p^2}{k^2} \int_{-\infty}^{\infty} \frac{\partial F_0/\partial u_z}{(u_z - \omega/k)} \, du_z = 0. \] (9.1.13)

\[ \int_{-\infty}^{\infty} \frac{\partial F_0/\partial u_z}{(u_z - \omega/k)} \, du_z = \left[ \frac{F_0}{(u_z - \omega/k)} \right]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \frac{F_0}{(u_z - \omega/k)^2} \, du_z. \] (9.1.14)

\[ D(k, \omega) = 1 - \frac{\omega_p^2}{k^2} \int_{-\infty}^{\infty} \frac{F_0}{(u_z - \omega/k)^2} \, du_z = 0. \] (9.1.15)
I. REVIEW OF THE LINEAR THEORY OF COLLISIONLESS DAMPING

The basic equations for the problem are the Vlasov equation for the electron distribution and Poisson's equation. Dawson's model of collisionless damping also uses the equation expressing conservation of energy which is easily derived from the above two equations. The ions cannot participate in the high-frequency plasma oscillations and just form a uniform background charge. Also, the coordinates perpendicular to the propagation vector of the wave may be integrated out of the Vlasov equation at the outset; so, it is only necessary to consider the problem in one dimension.

* This work was submitted in partial fulfillment of the requirements for the Ph.D. degree, University of California, San Diego.
2 The formalism used in this section is different from that used by Dawson; however, it is quite similar to the quasi-linear formalism of W. E. Drummond and D. Pines.

To explain the mechanism of collisionless damping, Dawson divides the electron distribution into a main part and a resonant part (see Fig. 1). He shows that the main part of the distribution supports the oscillatory motion of the plasma wave and that the resonant part of the distribution damps the wave. To get the damping coefficient, he first calculates the rate of increase of the kinetic energy of the resonant electrons. By invoking the conservation of energy, he sets this rate of increase of kinetic energy equal to the rate of decrease of wave energy. The latter quantity immediately gives the damping coefficient of the wave.
In this chapter we investigate the propagation of small-amplitude waves in a hot unmagnetized plasma. Because of the shortcomings of the moment equations, the approach used is to solve the Vlasov equation directly using a linearization procedure similar to that used in the analysis of waves in cold plasmas and MHD. Although both electromagnetic and electrostatic solutions exist, the discussion in this chapter is limited to solutions that are purely electrostatic, i.e., the electric field is derivable from the gradient of a potential, \( E = -\nabla \Phi \). Electromagnetic solutions are discussed in the next chapter.

From Faraday's law it is easily verified that electrostatic waves have no magnetic component. This greatly simplifies the Vlasov equation by eliminating the \( v \times B \) force. For electrostatic waves, it is usually easier to solve for the potential rather than for the electric field. Therefore, in the following analysis, the electric field is replaced by \( E = -\nabla \Phi \) and the potential is calculated from Poisson's equation, \( \nabla^2 \Phi = -\rho q/\epsilon_0 \).

### 9.1 The Vlasov Approach

In an initial attempt to analyze the problem, we assume that normal modes of the form \( \exp(-i\omega t) \) exist and represent them by using Fourier transforms, following the same basic procedure used in Chapter 4. This is the approach used by Vlasov (1945), who first considered this problem. As we will see, the Vlasov approach encounters a mathematical difficulty that can only be resolved by reformulating the linearized problem not as a normal mode problem using Fourier transform, but as an initial value problem using the technique of Laplace transforms.

As we have done before, we start by only considering the motion of the electrons. The ions are considered to be immobile, with the same zero-order number density, \( n_0 \), as the electrons. The effects of ion motions will be considered...