Experimental Test of Quasilinear Theory*

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The shape and amplitude of the electron-plasma wave spectrum resulting from a "gentle bump" on the tail of the electron velocity distribution of a plasma is measured and found to be in good agreement with quasilinear theory.

Nonlinear theories of unstable plasmas are usually concerned with two principal problems: estimating the final wave spectrum and calculating the effect of the waves on the particles. The simplest instability that may be followed analytically to its nonlinear limit is a "gentle bump" on the tail of the electron velocity distribution. The amplitude and shape of the spectrum as well as the change in the velocity distribution may be calculated from quasilinear theory.[1,2]

In this Letter we report an experiment designed to test the validity of this theory by measuring the electron-plasma wave spectrum resulting from the injection of an electron beam of sufficiently low density and large velocity spread to satisfy the assumptions of quasilinear theory. In prior beam-plasma experiments the initial velocity spread of the beam electrons was not sufficient to meet the requirements.[3-5]

When the beam electrons form a "gentle bump" on the plasma electron velocity distribution, the dispersion relation is determined by the plasma electrons, and the only effect of the beam is to cause exponential growth of the waves (or plasma noise) with phase velocities corresponding to the positive slope of the beam distribution. As the waves become sufficiently large they cause a diffusion of the beam electrons in velocity. The growth rate of the waves therefore decreases and eventually goes to zero when the velocity distribution becomes flattened in the beam region. Hence, we can calculate the dispersion relation and initial growth rate from linear theory and the equilibrium spectrum from the quasilinear theory.

To determine the dispersion relation, we consider a long cylinder of collisionless plasma in a strong magnetic field aligned with the axis. The plasma density is a function of radius only. In the limit that the phase velocity of the waves is small compared with the velocity of light, the electric field may be derived from a scalar potential, and for the radially symmetric modes the dispersion relation may be found from

$$\frac{d^2\Phi(r)}{dr^2}+\frac{1}{r}\frac{d\Phi(r)}{dr}-k^2\left[1-\frac{\omega_p^2(r)}{k^2v_e^2}W\frac{-\omega}{kv_e}\right]\Phi(r)=0, \tag{1}$$

where Φ is the wave potential, k and ω are the complex wave number and angular frequency of the wave, $\omega_p^2(r)$ is the radially dependent plasma frequency, v_e the mean thermal velocity of the plasma electrons, and W is related to the plasma dispersion function[7] by $W(x)=\frac{1}{2}Z'(-x/\sqrt{2})$. Using the experimentally observed density profile, the numerical solution of Eq. (1) yields the potential profile $\Phi(r)$ and the complex wave number as a function of frequency in the absence of the beam.

When the beam in injected into the plasma the real part of the dispersion relation is unchanged and the growth due to the beam is given by[8]

$$\alpha=\frac{\pi v_e^3}{v_g W_R'}\,\frac{\int_0^\infty r\,dr\,\Phi^2(r)[\partial f_b(v,r)/\partial v]_{v=\omega/k}}{\int_0^\infty r\,dr\,\Phi^2(r)N(r)}, \tag{2}$$

where α is the imaginary part of the wave number, v_g is the group velocity, $N(r)$ the plasma density, W_R' the derivative of the real part of the W function, and f_b the beam-velocity distribution function. The growth rate depends on the ratio of the beam density integrated over its cross-sectional area to the plasma density integrated over the plasma cross section. Both low density and small cross section in the beam contribute to the required weakness of the instability.

Drummond[9] has considered the quasilinear development of spatially growing waves in a plasma column immersed in a strong magnetic field and finds a local diffusion equation similar to the one-dimensional case. When the diffusion equation is integrated over the cross section of the plasma and combined with the equation for the rate of change in wave energy, we find the power spectrum at saturation to be given by

$$P(\nu)=C\frac{\nu}{k^2}\left(1-\frac{v_p}{v_g}\right)\int_{-\infty}^{v_p}dv\, v[g_f(v)-g_i(v)], \qquad (3)$$

where $P(\nu)$ is the power per unit frequency, C is a constant, ν and v_p are the wave frequency and phase velocity, respectively, v is the electron-velocity component parallel to the magnetic field, and $g_{i,f}$ are the initial and final distribution functions integrated over the plasma cross section and perpendicular velocities.[10] The final distribution is flat and given by the appropriate construction on the initial distribution. The total power in the wave spectrum is equal to the decrease in particle energy flux caused by the velocity diffusion process and is equal to

$$P=\int_{v_1}^{v_2}dv\, mv^3/2[g_f(v)-g_i(v)], \qquad (4)$$

where m is the mass of an electron and the limits of integration are over the flattened region of $g_f(v)$.

These equations are the quasilinear predictions of the shape and amplitude of the wave spectrum from a "gentle bump," and it is these results that the experiment is designed to check.

The plasma is produced by ionization of hydrogen gas in a coaxial stub microwave cavity, and it drifts along magnetic field lines down a 250-cm aluminum tube 10 cm in diameter. The plasma is terminated by a plate with a $\frac{3}{4}$-cm hole behind which the electron gun is mounted. The plate is biased to reflect electrons with velocities less than the slowest of those that come from the gun. The tube acts as a waveguide beyond cutoff for electromagnetic propagation at the wave frequencies used, and has four longitudinal slots equispaced around the circumference along which antenna probes may be moved. The complete assembly is contained in a vacuum chamber and maintained at a pressure of less than 10^{-5} Torr by diffusion and Ti sublimation pumps. Axial magnetic field coils are mounted around the vacuum chamber and provide a magnetic field of about 1 kG. The electron gun is a simple diode with a large-aperture (2.5-cm diam) plate mounted inside a soft-iron cylinder. The result is a beam distribution with a large spread in parallel energy, which when injected into the plasma makes a bump on the tail of the parallel electron-velocity distribution. The energy distribution of the beam is measured directly using a large-aperture gridded energy analyzer.

The dispersion curve is determined from measurements of wavelength as a function of the frequency of the plasma wave. The wavelength can be observed directly by adding a constant-phase reference signal from the transmitter to the received signal. As the receiving probe is moved, the interference pattern is plotted on an X-Y recorder, displaying the wavelength. The temperature and density of the plasma are inferred by computer using a program that solves Eq. (1) to obtain the best least-squares fit to the experimental points. The beam may be turned on and the dispersion data repeated using coherent detection to find the test wave in the noise spectrum. The wavelengths may be compared with those measured without the beam to show that the beam does not change the dispersion relation. The results are shown in Fig. 1.

The dispersion curve is the most sensitive indicator of the transition from gentle bump to two-stream instability. The invariance of the dispersion curve with the beam establishes empirically that the bump is sufficiently broad. The transition has also been examined theoretically by O'Neil and Malmberg.[11] They obtain a parameter s depending on the beam and plasma properties and show that for $s > 1.64$, the dispersion curves are bumplike. In this experiment, $s \geq 1.8$ for all beams, and their results confirm that the dispersion curves are definitely quasilinear in this case.

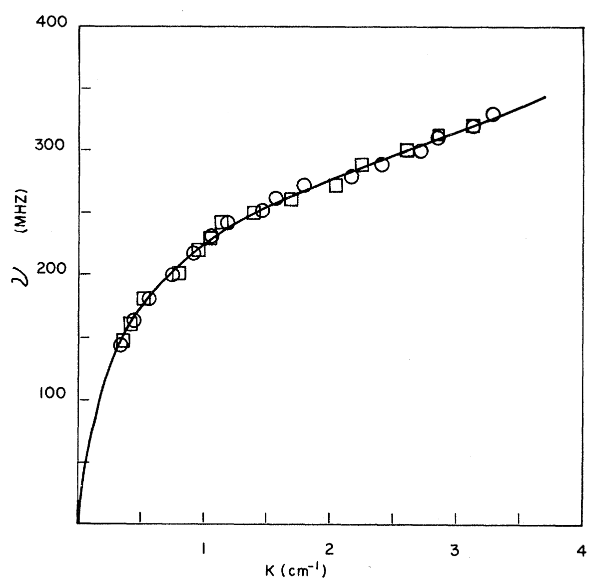

FIG. 1. Dispersion curve. The solid line is calculated without beam, circles are observations without beam, squares are observations with a beam current of 2.6 mA. The electron temperature is 14 eV. The central electron density is 1.3×10^9 electron/cm^3 with a half-width of approximately 3 cm.

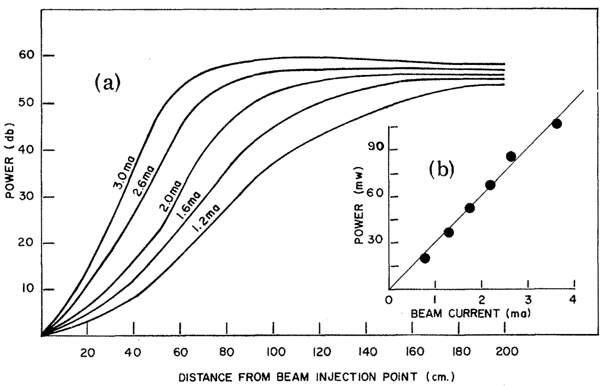

FIG. 2. (a) Wave growth and saturation versus distance. (b) Saturation power level versus beam current.

The spatial growth of the noise as a result of the beam is measured by connecting the receiving antenna directly to a broad-band amplifier and a sampling rf voltmeter. The log of the voltmeter output is plotted on the Y axis and distance from beam injection point on the X axis of the recorder. The results [Fig. 2(a)] have the qualitative features expected from the theory: slow exponential growth to reach a quasiequilibrium saturation level without overshoot.

To determine the wave power in the spectrum we must know the absolute coupling constant of the probes. No precise way of obtaining this is available, but the total coupling through the plasma for a pair of probes can easily be measured. For three probes, three transmitter-receiver pairs are possible. This gives three equations for the three unknown single-probe coupling coefficients. A calibrated receiver is positioned 200 cm from the beam injection point and the wave power is measured as a function of beam current. The wave power is linearly proportional to the beam current [Fig. 2(b)] as predicted by Eq. (4). For our beam distribution, we calculate that 14% of the beam power should be converted to wave power. We observe approximately 10% conversion, which is excellent agreement for a measurement of absolute power level.[12]

The shape of the equilibrium spectrum is measured using an adjustable narrow-band (3%) filter in the receiving line. The power and coupling constant are measured at a series of fixed frequencies. The amplitude of the spectrum as calculated from Eq. (3) is normalized to the measured value at one point and the comparison of theory and experiment is shown in Fig. 3(a). The shape and width of the spectrum are found to be in excellent agreement with theory.

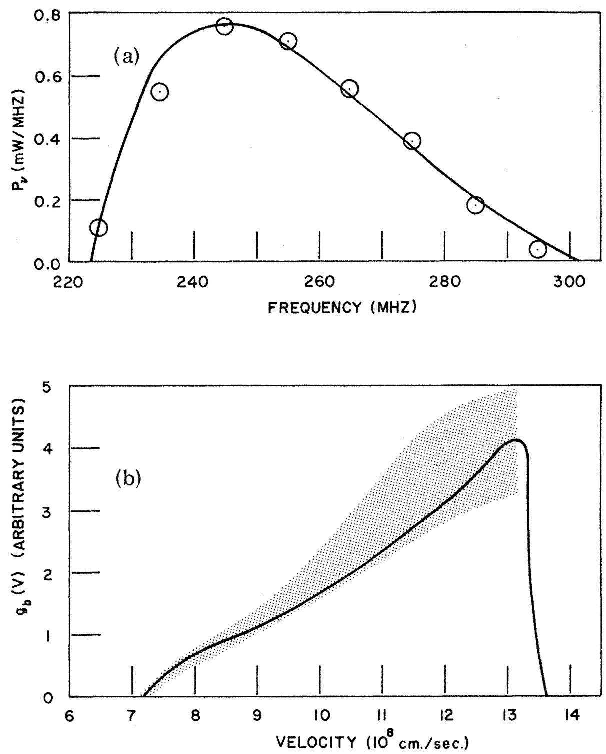

FIG. 3. (a) Shape of wave spectrum. Solid line is theory, circles are experimental values. Beam current is 2 mA. (b) Inferred beam-velocity distribution versus energy analyzer distribution. The solid curve is obtained from an electronically differentiated output of the analyzer.

We have also measured the growth rates of the spectral components by using the adjustable filter and plotting the log of the receiver power versus distance for a number of frequencies. These growth rates were used in Eq. (2) to find the averaged beam-velocity distribution function assuming that the wave potential is constant over the beam diameter; the beam is sufficiently small to justify this easily. This inferred beam distribution is compared with the direct measurement using an energy analyzer. The results are shown in Fig. 3(b), where the width of the shaded region is an indication of the estimated error in determining the inferred distribution. No "normalization" procedure is used in this comparison.

In summary, we find that the quasilinear theory correctly predicts the manner in which the electron-plasma wave spectrum from a "gentle bump" grows and saturates. The theory gives the proper dependence of saturation level as a function of beam current, the shape of the spectrum, and the magnitude of the equilibrium power level.

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Fast-Electron Spectroscopy of Surface Excitations

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New theoretical results are presented for the probability of exciting surface oscillations (optical phonons or plasmons) by fast electrons reflected from the surface of a thin crystal film. Both specular and Bragg reflections are considered and the effect of the finite slab thickness is included. The theory explains successfully the energy-loss spectra measured by Powell on metallic surfaces and recent measurements by Ibach on ZnO surfaces.

Recently the authors have proposed a new semiclassical theory of the characteristic energy-loss spectra of fast electrons in solids.[1] In this approach, the electron is treated as a classical particle on the well-defined trajectory $\vec{r}(t)$ and acts as a time-dependent perturbation, linearly coupled to the quantized field of elementary excitations (e.g., optical phonons or plasmons). The response of the system can be calculated exactly to give the field excitation probability and hence the energy-loss spectrum. The trajectory could be chosen arbitrarily so that one could consider reflection cases (both specular and Bragg reflections) as well as the transmission case treated in the dielectric theory[2,3] of energy-loss spectra.

In this Letter we present general formulas for the loss probability function appropriate to the specularly or Bragg reflected electron at the surface of a slab of arbitrary thickness. Application of the theory to the inelastic scattering by surface optical phonons in ZnO and surface plasmons in metals leads to scattering probabilities in excellent agreement with recent experimental data obtained by Ibach[4] and Powell.[5]

Let $\omega_\pm(\vec{k})$ be the frequencies of the odd (even) modes of long-wavelength surface excitations (either phonons or plasmons) in a slab of thickness $2a$. The dispersion relation for these modes can be implicitly written as[6]

$$\sinh 2ka = \pm 2\epsilon(\omega)/[\epsilon^2(\omega)-1] \tag{1}$$

[which is equivalent to the relation (3.23a) of Ref.6], where $\vec{k}$ is a two-dimensional wave vector parallel to the surface and $\epsilon(\omega)$ is the frequency-dependent dielectric function of the material. The probability that the electron loses an energy $\hbar\omega$ is found to be[1]

$$P(\omega)=\frac{P_0}{2\pi}\int_{-\infty}^{+\infty} dt\, e^{i\omega t} \exp\left[\int d^2k\,(Q_+ e^{-i\omega_+ t}+Q_- e^{-i\omega_- t})\right], \tag{2}$$