## Lecture 6: Plasma Physics 1

APPH E6101x Columbia University

## Outline

- Collisions and mobility
  - Low-temperature plasma are "weakly" ionized
- Spitzer resistivity: "fully" ionized plasma
- Conductivity in a magnetized plasma
- ("Classical") Diffusion of plasma and magnetic field

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### **Electron-Impact Cross Sections for Ionization** and Excitation Database

### NIST Standard Reference Database 107

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Y.-K. Kim,<sup>1</sup> K.K. Irikura,<sup>2</sup> M.E. Rudd,<sup>3</sup> M.A. Ali,<sup>4</sup> and P.M. Stone<sup>1</sup> J. Chang,<sup>1</sup> J.S. Coursey,<sup>1</sup> R.A. Dragoset,<sup>1</sup> A.R. Kishore,<sup>1</sup> K.J. Olsen,<sup>1</sup> A.M. Sansonetti,<sup>1</sup> G.G. Wiersma,<sup>1</sup> D.S. Zucker,<sup>1</sup> and M.A. Zucker,<sup>1</sup>

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This is a database primarily of total ionization cross sections of molecules by electron impact. The database also includes cross sections for some atoms and energy distributions of ejected electrons for H, He, and H<sub>2</sub>. The cross sections were calculated using the Binary-Encounter-Bethe (BEB) model which combines the Mott cross section with the high-incident energy behavior of the

Q **Menu** 

https://www.nist.gov/pml/electron-impact-cross-sections-ionization-and-excitation-database

Last Update to Data Content: August 2004 Version History Disclaimer DOI: https://dx.doi.org/10.18434/T4KK5C







### **Holdings for Hydrogen**

Neutral						
Ionization	Excitation					
	<u>1s -&gt; 2p</u>					
	<u>1s -&gt; 3p</u>					
	<u>1s -&gt; 4p</u>					
<u>Total</u>	<u>1s -&gt; 5p</u>					
	<u>1s -&gt; 6p</u>					
<b>Differential</b>	<u>1s -&gt; 7p</u>					
	<u>1s -&gt; 8p</u>					
	<u>1s -&gt; 9p</u>					
	<u>1s -&gt; 10p</u>					

National Institute of Standards and Technology

### **Electron-Impact Cross Sections** for Ionization and Excitation

Table of Atoms **Table of Molecules** 

Symbol: H Atomic Weight: 1.00794(7) Ionization Energy: 13.5984 eV Ground-state Configuration: 1s Ground-state Level:  ${}^{2}S_{1/2}$ 

Excitation Energies (E) in eV					
Excitation	E				
1s-2p	10.1988				
1s-3p	12.0875				
1s-4p	12.7485				
1s-5p	13.0545				
1s-6p	13.2207				
1s-7p	13.3209				
1s-8p	13.3860				
1s-9p	13.4306				
1s-10p	13.4625				

Physical Measurement Laboratory Introduction and References

Neutral Hydrogen Total Ionization Cross-Section



Table of Ionization Cross Sections at Specific Energies (tab-delimited ASCII) Atomic Orbital Constants for BEB Calculation of the Direct Cross Section All cross sections are in  $10^{-16}$  cm<sup>2</sup> unless otherwise specified.



### **Total Ionization Cross Section**

Incident electron energy, T =eV

Calculate Cross Section

I - BEB Direct II - BEQ	YK. Kim and M.E. Rudd, Phys. Rev. A 50, 3954 (1994).(T)
III - shah87	M. B. Shah, D. S. Elliott, and H. B. Gilbody, J. Phys B 20, 3501 (1987).
IV - gryz65	M. Gryzinski, Phys. Rev. 138, A305, A322, A336 (1965).(T)
V - younger81	S. M. Younger, J. Quant. Spectrosc. Radiat. Transfer 26, 329 (1981).(T)





### **Topic Areas**

and Tools

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### NIST Standard Reference Database 64

### NIST Electron Elastic-Scattering Cross-Section Database: Version 3.2

No Charge.

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Version 3.2 of this databaseprovides values of differential elastic-scattering cross sections, totalelasticscattering cross sections, phase shifts, and transport cross sections for elements with atomic numbers from 1 to 96 and for electron energies between 50 eV and 300 keV (in steps of 1 eV). The cross sections in the database were provided by Prof. F. Salvatusing relativistic theory. Knowledge of elastic-scattering effects is important for the development of theoretical models for quantitative analysis by AES, XPS, electron microprobe analysis, and analytical electron microscopy. The software package is designed to facilitate simulations of electron transport for these and similar applications in whichelectron energies from 50 eV to 300 keV are utilized. An analysis of availableelastic-scattering cross-section data has been published by A. Jablonski, F.Salvat, and C. J. Powell J. Phys. Chem. Ref. Data 33, 409 (2004)].

Following features:

- •

graphical display of differential elastic-scattering cross sections in different coordinate systems graphical display of the dependence of transport cross sections on electron energy



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### http://www.nrl.navy.mil/ppd/content/nrl-plasma-formulary





### Weakly Ionized Plasmas

Collision frequency for scattering of charged particles of species  $\alpha$  by neutrals is  $\sigma_s^{\alpha|0} (kT_\alpha/m_\alpha)^{1/2} = V_{th}/\lambda_{mfn}$ 

$$\nu_{\alpha} = n_0 \sigma_s^{\alpha}$$

where  $n_0$  is the neutral density and  $\sigma_s^{\alpha \setminus 0}$  is the cross section, typically  $\sim 5 \times 10^{-15} \text{ cm}^2$  and weakly dependent on temperature.

of motion for an *average electron* 

 $m\dot{\bar{v}} = -$ 

This average electron now moves in -E-direction. The quantity  $v_{\rm m} = 1/\tau_{\rm coll}$ is the effective collision frequency for momentum transfer.

The elastic scattering of electrons on atoms is almost isotropic [68]. Therefore, on average, the electron loses its mean momentum  $m_e \bar{\mathbf{v}}_e$  and we can write the equation

$$-eE - m\bar{v}v_{\rm m} \,. \tag{4.25}$$

# $v_{\rm d} = -\frac{e}{m\nu_{\rm m}}E = -\mu_{\rm e}E$

## The mobilities of electrons and ions are defined as

 $\mu_{\rm e} = \frac{e}{m_{\rm e}\nu_{\rm m,e}} \quad ; \quad \mu_{\rm i} = \frac{e}{m_{\rm i}\nu_{\rm m,i}}.$ 



## $j = j_e + j_i = n[(-e)v_{de} + ev_{di}] = ne(\mu_e + \mu_i)E = \sigma E$



## **Electrical Conductivity**

 $\sigma_{e,i} = ne\mu_{e,i} = \frac{ne^2}{m_{e,i}\nu_{m}}$ 

## **Collisional Diffusion**



## $\boldsymbol{\Gamma}_{e,i} = n_{e,i} \bar{\mathbf{v}}_{e,i} = -D \nabla n_{e,i}$



Einstein relation is an unexpected result of Brownian motion from 1904-5.

### PHYSICAL REVIEW

### **Transport Phenomena in a Completely Ionized Gas\***

LYMAN SPITZER, JR., AND RICHARD HÄRM Princeton University Observatory, Princeton, New Jersey (Received November 10, 1952)

H x v. The coefficients of electrical and thermal conductivity have been computed for completely ionized gases with a wide variety of mean ionic charges. The effect of mutual electron encounters is considered as a problem of diffusion in velocity space, taking into account a term which previously had been neglected. The appropriate integro-differential equations are then solved numerically. The resultant conductivities are very close to the less extensive results obtained with the higher approximations on the Chapman-Cowling method, provided the Debye shielding distance is used as the cutoff in summing the effects of two-body encounters.

> $\eta_{s}$  $-\frac{1}{(4\pi\varepsilon)}$

transverse Spitzer resistivity

electrical conductivities

$$\frac{\pi e^2 m_e^{1/2}}{(k_B T)^{3/2}} \ln \Lambda \sum_{\substack{log [12\pi N_b] \\ p_{lASMA} \\ PANAMACETA}} \frac{\log [12\pi N_b]}{k_B T}$$

$$\begin{aligned} \gamma_{\mu} &= \frac{1}{2} \gamma_{\mu} \\ \eta_{\perp} &= 1.15 \times 10^{-14} Z \ln \Lambda T^{-3/2} \operatorname{sec} \\ &= 1.03 \times 10^{-2} Z \ln \Lambda T^{-3/2} \Omega \operatorname{cm} \end{aligned}$$

$$= 1.96\sigma_{\perp}; \ \sigma_{\perp} = ne^{2}\tau_{e}/m_{e} \ \left| \begin{array}{c} v_{m} = \frac{1}{\zeta_{m}} \end{array} \right|$$

 $\sigma_{\parallel}$ 

## Coulomb Collisions





## Coulomb Collisions

electron



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## Coulomb Logarithm

(b) Electron-ion collisions

$$\lambda_{ei} = \lambda_{ie} = 23 - \ln \left( n_e^{1/2} Z_e^2 \right)$$
$$= 24 - \ln \left( n_e^{1/2} T_e^2 \right)$$
$$= 30 - \ln \left( n_i^{1/2} T_i^2 \right)$$





increasing density

## Ambipolar Diffusion

$$\mu_{\rm e} = \frac{e}{m_{\rm e}\nu_{\rm m,e}} \quad ; \quad \mu_{\rm i} = \frac{1}{m_{\rm i}}$$

 $\boldsymbol{\Gamma}_{\mathrm{e,i}} = \pm n \mu_{\mathrm{e,i}} \mathbf{E} - D_{\mathrm{e,i}} \nabla n$ 

$$\frac{D}{\mu} = \frac{k_{\rm B}T}{e}$$

$$- m\mu e E - D_{o} \nabla m = m M_{i} E - D_{i}$$

$$E = -\frac{1}{n} \nabla m \left( \frac{D_{e}}{M_{o}} - \frac{h}{m} \nabla m - \frac{h$$



## Collisional Drift Velocity in (E,B) Field

 $\delta \cong \Xi_{II} - Y_{m}V_{II}$ 

 $\overline{V}_{\perp} - \left(\frac{v_{e}}{v_{m}}\right)\overline{V}_{\perp} \times \overline{V}_{\perp} = \frac{g}{m}\overline{E}_{\perp}$  $\left( \begin{array}{c} \omega_{c} \\ \nabla_{a} \\ \nabla_{a} \end{array} \right) \left( \begin{array}{c} \nabla_{a} \end{array} \right) \left( \begin{array}{c} \nabla_{a} \\ \nabla_{a} \end{array} \right) \left( \begin{array}{c} \nabla_{a} \\ \nabla_{a} \end{array} \right) \left( \begin{array}{c} \nabla_{a} \\ \nabla_{a} \end{array} \right) \left( \begin{array}{c} \end{array} \right)$ 

 $\overline{J} = q \overline{M} \overline{U} = \overline{\overline{6}} \cdot \overline{\overline{E}}$ FIND AJERAGE DRIFT WITH STATIC E, B AND COLLISIONS

 $\frac{d\overline{v}}{dt} \approx 2 = \frac{9}{m} \overline{E} + \frac{1}{v_c} \overline{V_1} \times \overline{b} - \frac{1}{v_m} \overline{V_1}$   $0 \approx \frac{9}{m} \overline{E} \times \overline{b} - \frac{1}{v_c} \overline{V_1} \overline{b} - \frac{1}{v_m} \overline{V_1} \times \overline{b}$   $1 \quad 9$ 



## Conductivity in a Magnetized Plasma

(PHO DISSENTATION)





## Hall Effect Thruster

Fig. 4.17 Hall-effect plasma thruster. The plasma channel of the SPT100ML thruster has 69 and 100 mm inner and outer diameter, and 25 mm length. The mean radial magnetic field is  $B_r = 160$  mT. The discharge is operated at  $U_d = 300$  V and  $I_d = 4.2$  A, giving a thrust of 80 mN. (Reprinted from [69] with permission. © 2004, IOP Publishing Ltd.)



Ohm's Law (Part 1)  $\overline{\mathcal{F}} = \overline{\mathcal{J}}_i + \overline{\mathcal{J}}_p = em(\overline{\mathcal{V}}_i - \overline{\mathcal{V}}_e)$ O = - en E - en Ve XB - mon Ve Ve STARICE BALANCO NO O = + en E + en V XB - Min Vie Vi Accellerin e  $e^+$ :  $0 \leq e = (\overline{v_i} - \overline{v_e}) \times \overline{B} - n(\underline{m_i} \vee_i o \overline{v_i} + \underline{m_o} \vee_o \overline{v_e})$ ADD ' 0: JXB (NO FORCE) R MOMENTUM REALLY VP=JXR  $O = 2enE + en((\overline{v}_i + \overline{v}_e) \times \overline{D}) - n[\underline{m}_i v_i \overline{v}_e - \underline{m}_e v_e \overline{v}_e]$ SOUTRACT: ademE + den V, XI - Milio F - No [milioner.]  $O = E_{+} \times V_{\perp} \times J - \frac{M_{i} V_{i0}}{207} J_{\perp}$ ~ 0 , E+レメレニア」) Ottmis Luta

## ("Classical") Transport in a Magnetized Plasma

 $\nabla X \overline{E} = -\frac{2B}{at}$   $\nabla X \overline{S} = \mu \overline{J}$ 75-2  $\nabla x(nJ - vxB) = -\frac{2D}{2}$  $-\nabla \times \left(\frac{n}{n} \nabla \times \widehat{\Omega}\right) + \nabla \times \left(\overline{\nu} \times \overline{u}\right) = \frac{2\overline{\Omega}}{2\epsilon} / \frac{2n}{2\epsilon} + \nabla \cdot \left(\frac{n}{\overline{\nu}} \frac{n}{n} \cdot \overline{J} \times \overline{B}\right) = 0$  $2\overline{B} = (m)\overline{V}\overline{B}^{2}$ DR ~ 2ht VM  $\left(\frac{2m}{3+}+V,\frac{mh}{\left(\frac{B'}{2M_{0}}\right)},\frac{mh}{M_{0}},\frac{mh}{M_{0}}\right)$ m M. Dp - ZP Dm P - ZP Dm 2nhT D - Z/2no CONCLUSION. PARTICLES DIFFUSS MORO SLOWLY THAN MAGNETIC FLEAD

### http://www.iter.org/

- Culmination of 50 years of magnetic fusion research
- 500 MW fusion power for 7 min pulses
- EU, Japan, Russia, China, S Korea, India, USA
- *At least* 22B US\$ (14B US\$ official), the most ambitious international science project ever
- 23,000 tons (tokamak only), or \$1M/ton



## $\mathcal{P}_{\text{Fusion}} \sim 0.08 P^2 \text{ (MWm}^{-3}\text{)}$

P(plasma) = 3 n k T = 4.3 atm $n = 1.0E20 \text{ m}^3$ T = 9 keV

 $pFusion = 1.5 MW/m^3$ 

B = 5.3 TP(mag) = 110 atm

 $\beta = 4.3/110 \sim 3.9\%$ 

 $\tau = 3.7 \, s$ a ~ 2.5 m

Thousands faster than "classical"

## Fusion Gain & Ignition

For low energies  $(T \lesssim 25 \text{ keV})$  the data may be represented by  $(\overline{\sigma v})_{DD} = 2.33 \times 10^{-14} T^{-2/3} \exp(-18.76 T^{-1/3}) \,\mathrm{cm}^3 \,\mathrm{sec}^{-1}$  $(\overline{\sigma v})_{DT} = 3.68 \times 10^{-12} T^{-2/3} \exp(-19.94 T^{-1/3}) \,\mathrm{cm}^3 \,\mathrm{sec}^{-1}$ 

where T is measured in keV.

 $D + T \longrightarrow He^4(3.5 \text{ MeV}) + n(14.1 \text{ MeV})$ 



## Fusion Ignition

### $P_{DT} = 5.6 \times 10^{-13}$

Bremsstrahlung from hydrogen-like plasma:<sup>26</sup>

(30)  $P_{\rm Br} = 1.69 \times 10^{-7}$ 

where the sum is over all ionization states Z.

 $D + T \longrightarrow He^4(3.5 MeV) + n(14.1 MeV)$ 

$$^{3}n_{D}n_{T}(\overline{\sigma v})_{DT}$$
 watt cm<sup>-3</sup>

$$^{-32}N_eT_e^{1/2}\sum \left[Z^2N(Z)\right]$$
watt/cm<sup>3</sup>

## $P_{atm}T_E > 10 atm \cdot sec$ for fusion gain alon Power Conditions

$$f_{\alpha}P_{fus} + Paux = \frac{W}{\tau_E} + P_{brem} +$$

$$Q \equiv \frac{P_{fus}}{P_{aux}}$$

 $P_{fus} \propto P_{brem} \propto P_{rad} \propto n^2$ 

$$nT\tau_E = \frac{3 T^2 n^2}{(f_\alpha + 1/Q) P_{fus} - P_{brem}}$$

Thursday, February 3, 2011





but...

**Classical Confinement does not describe Magnetized Fusion Plasma** 

 $\mathsf{D} = \beta \left( \eta / \mu_0 \right)$ 

T = 9 keV $\eta \sim 1.7\text{E}-9 \text{ Ohm-m}$  $\eta/\mu_0 \sim 0.001 \text{ m}^2/\text{s}$ 

 $\tau \sim 4 a^2/D \sim 700,000 sec$ 

## Homework #4 (Ch. 4)

- Fitzpatrick: Read Chapter 3 about collisions, and discuss the meaning of "Rosenbluth Potentials" and the collision operator in Eq. 3.112
- Piel: All nine problems in Ch. 4 (answers in back of text)

## Wednesday's Lecture: "Virtual" (Prof. Mauel is away at a conference)

- Piel / Chapter 5: "Fluid" Equations
- plasma

and the equations describing the large-scale dynamics of