More Consequences of Large $N_D$

- Most interactions are “distant encounters” with small-angle collisions (very few “close encounters”, very few large-angle collisions, and very very few three-body collisions)
- Mean potential energy is much less than mean kinetic energy
- Energy density of electrostatic fluctuations are much less than the plasma’s kinetic energy density
"Close" and "Distant" Encounters

**Close Encounters**

\[ N_D = m \lambda_0^2 \]

\[ b = \text{closest\ product} \]

\[ \frac{b^2}{b \lambda_0} \sim 3 \hbar T \]

\[ b \sim \frac{1}{3} \frac{e^2/\lambda_0}{\hbar T} \sim \frac{1}{5} (m \lambda_0^2) \]

**Distant Encounters**

\[ \lambda_0 = \frac{1}{\hbar} \frac{b_{2T}}{e^2/\lambda_0} \]

**Distant Encounters**

\[ \frac{b}{b_0} \sim \frac{1}{3} \frac{m^{1/3}}{(m \lambda_0^2)} \sim \frac{1}{N_0^{2/3}} \ll 1 \]

- Most two-particle collisions are distant encounters.
- 3-body large-angle collisions are very unlikely.
## Potential and Kinetic Energy

| "Mean" Potential Energy per Particle | $\sim \frac{e^2}{\varepsilon_0} < 1$ |
| "Mean" Kinetic Energy per Particle | $\sim \frac{3}{2} kT$ |
| Potential Kinetic | $\sim \frac{2}{3} \frac{m^{1/3}}{m\lambda_0^2} \sim \frac{2}{3} \frac{1}{\lambda_0^{2.5}} < < 1$ |
Fluctuation and Kinetic Energy

IN THERMODYNAMIC EQUILIBRIUM EACH NORMAL MODE WILL HAVE $\frac{1}{2} L \Delta T$ OF ENERGY.

Number of modes in a box with side $L$:

$$\left(\frac{L}{2\pi\lambda_D}\right)^3 \propto \text{N of modes}$$

Total fluctuation energy per unit volume:

$$\frac{1}{2} \frac{1}{2} L \Delta T \left(\frac{1}{2\pi\lambda_D}\right)^3$$

Total kinetic energy per unit volume:

$$\frac{3}{2} a_k^2 \frac{1}{2} L \Delta T$$

$$\frac{\text{Fluctuation Energy}}{\text{Kinetic Energy}} \sim \frac{1}{24\pi^3 \text{N}_0} \ll 1$$
Outline (Piel, Ch. 4)

- Distribution function
- Electron ionization cross-sections
- Other cross sections
- Electron/Ion neutral collisions
- Coulomb Collisions (next week)
- Mobility and electrical conductivity
Distribution Function

Isotropic and Maxwellian

\[ f(\mathbf{v}) \propto e^{-m\mathbf{v} \cdot \mathbf{v} / 2kT} \]

\[ f(\mathbf{v}) = n \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-m\mathbf{v} \cdot \mathbf{v} / 2kT} \]

\[ n = \iiint d^3v \, f(\mathbf{v}) \]
Flow, Temp, Pressure

\[ \bar{v} = \frac{1}{n} \iiint d^3v \, v \, f(v) \]

\[ \hat{x} \cdot P = \iiint d^3v \, m[\hat{x} \cdot (v - \bar{v})]^2 \, f(v) \]

\[ = nkT \]
Distribution Function

Bi-Isotropic

\[ f(v_\perp, v_\parallel) = n \left( \frac{m}{2\pi kT_\parallel} \right)^{3/2} \left( \frac{T_\parallel}{T_\perp} \right) e^{-mv_\perp^2/2kT_\perp} e^{-mv_\parallel^2/2kT_\parallel} \]

\[ n = \int_0^\infty \int_{-\infty}^{\infty} 2\pi v_\perp dv_\parallel f(v_\perp, v_\parallel) \]
\[ f(\mu, E) = \frac{n}{kT} \frac{\sqrt{2}}{\sqrt{\pi mkT}} e^{-E/kT} \]

\[ n = \int_0^\infty \int_0^{E/B} \frac{d\mu B \, dE}{\sqrt{(2/m)(E - \mu B)}} \, f(\mu, E) \]
This is a database primarily of total ionization cross sections of molecules by electron impact. The database also includes cross sections for some atoms and energy distributions of ejected electrons for H, He, and H₂. The cross sections were calculated using the Binary-Encounter-Bethe (BEB) model, which combines the Mott cross section with the high-incident energy behavior of the Bethe cross section. Selected experimental data are included. Electron-impact excitation cross sections are also included for some selected atoms.

**Introduction and References**

Contributions of the following colleagues are gratefully acknowledged:
- W. M. Huo, NASA Ames Research Center, Moffet Field, CA 94035-1000
- W. Hwang, Samsung Electronics, Suwon, Korea

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Access the Database:

Atoms | Molecules

NIST Standard Reference Database 107
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**Online:** August 1997 - **Last update:** August 2005

Contact
Dr. Karl Irikura
Holdings for Hydrogen

Symbol: H
Atomic Weight: 1.00794(7)
Ionization Energy: 13.5984 eV
Ground-state Configuration: 1s
Ground-state Level: \(^2S_{1/2}\)

<table>
<thead>
<tr>
<th>Ionization</th>
<th>Excitation</th>
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<tbody>
<tr>
<td>Neutral</td>
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<tr>
<td>Total</td>
<td>1s -&gt; 2p</td>
</tr>
<tr>
<td></td>
<td>1s -&gt; 3p</td>
</tr>
<tr>
<td></td>
<td>1s -&gt; 4p</td>
</tr>
<tr>
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<tr>
<td></td>
<td>1s -&gt; 10p</td>
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<tr>
<td>Differential</td>
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Excitation Energies (E) in eV

<table>
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<tr>
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<th>E</th>
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<tbody>
<tr>
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<tr>
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<td>1s-7p</td>
<td>13.3209</td>
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<tr>
<td>1s-8p</td>
<td>13.3860</td>
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<tr>
<td>1s-9p</td>
<td>13.4306</td>
</tr>
<tr>
<td>1s-10p</td>
<td>13.4625</td>
</tr>
</tbody>
</table>
Neutral Hydrogen Total Ionization Cross-Section

Total Ionization Cross Section
Incident electron energy, $T = \underline{\phantom{0}}$ eV

Calculate Cross Section

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>II - BEQ</td>
<td></td>
</tr>
</tbody>
</table>

Table of Ionization Cross Sections at Specific Energies (tab-delimited ASCII)

Atomic Orbital Constants for BEB Calculation of the Direct Cross Section

All cross sections are in $10^{-16}$ cm$^2$ unless otherwise specified.
NIST Standard Reference Database 64

NIST Electron Elastic-Scattering Cross-Section Database: Version 3.2

No Charge.

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Version 3.2 of this database provides values of differential elastic-scattering cross sections, total elastic-scattering cross sections, phase shifts, and transport cross sections for elements with atomic numbers from 96 and for electron energies between 50 eV and 300 keV (in steps of 1 eV). The cross sections in the database were provided by Prof. F. Salvat using relativistic theory. Knowledge of elastic-scattering effects is important for the development of theoretical models for quantitative analysis by AES, XPS, electron microprobe analysis, and analytical electron microscopy. The software package is designed to facilitates simulations of electron transport for these and similar applications in which electron energies from 50 eV to 300 keV are utilized. An analysis of available elastic-scattering cross-section data has been published by A. Jablonski, F. Salvat, and C. Powell J. Phys. Chem. Ref. Data 33, 409 (2004).

Following features:

- graphical display of differential elastic-scattering cross sections in different coordinate systems
- graphical display of the dependence of transport cross sections on electron energy
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Weakly Ionized Plasmas

Collision frequency for scattering of charged particles of species $\alpha$ by neutrals is

$$\nu_\alpha = n_0 \sigma_s^{\alpha|0} (kT_\alpha/m_\alpha)^{1/2} = V_{th}/\lambda_{mf}$$

where $n_0$ is the neutral density and $\sigma_s^{\alpha\|0}$ is the cross section, typically $\sim 5 \times 10^{-15}$ cm$^2$ and weakly dependent on temperature.

The elastic scattering of electrons on atoms is almost isotropic [68]. Therefore, on average, the electron loses its mean momentum $m_e \bar{v}_e$ and we can write the equation of motion for an average electron

$$m \ddot{v} = -eE - m\bar{v}v_m.$$  \hspace{1cm} (4.25)

This average electron now moves in $-E$-direction. The quantity $v_m = 1/\tau_{coll}$ is the effective collision frequency for momentum transfer.
The mobilities of electrons and ions are defined as

\[ \nu_d = -\frac{e}{m \nu_m} E = -\mu_e E \]

\[ \mu_e = \frac{e}{m_e \nu_{m,e}} ; \quad \mu_i = \frac{e}{m_i \nu_{m,i}} \]
Electrical Conductivity

\[ j = j_e j_i n \cdot -e/\nu_{de} \quad ev_{di} \quad ne. \quad e \quad i/E \quad \sigma E \]

\[ \sigma_{ei} \quad ne \quad ei \quad \frac{ne^2}{m_e i \nu_m} \]
Collisional Diffusion

\[ \Gamma_{ei} \quad n_{ei}v_{ei} \quad -D \nabla n_{ei} \]

\[ D = \frac{k_B T}{e} \quad e \quad \frac{e}{m_e \nu_m e} \]

\[ D = \frac{V_{th}^2}{\nu_m} = \frac{\lambda_{mf}^2}{\tau_m} \]
Next Lecture

- More Piel / Chapter 4: Transport
- Fusion confinement and ignition criteria