Lecture 2:
Plasma Physics I

APPH E6101x
Columbia University
Fall, 2023

Outline

- Review from Last Week: Homework #1
- Irving Langmuir
- Langmuir “oscillations” (*i.e.* electrostatic plasma waves)
- Plasma “sound” waves
- Homework #2 due next Monday

Alexander Piel

Plasma Physics

An Introduction to Laboratory, Space, and Fusion Plasmas

Second Edition

Last Week: Plasma Parameters

- $n(r,t)$ – density - plasma frequency, ω_p
- $T(r,t)$ – temperature, $v_{th} = (kT/m)^{1/2}$
- λ_D – Debye length, v_{th}/ω_p
- N_D - plasma parameter, $(4\ \lambda_D{}^3/3)\ n >> 1$

Next Week:
In Class Homework

From Fitzpatrick

2. The perturbed electrostatic potential $\delta\Phi$ due to a charge q placed at the origin in a plasma of Debye length λ_D is governed by

$$\left(\nabla^2 - \frac{2}{\lambda_D^2}\right)\delta\Phi = -\frac{q\,\delta(\mathbf{r})}{\epsilon_0}.$$

Show that the nonhomogeneous solution to this equation is

$$\delta\Phi(r) = \frac{q}{4\pi\,\epsilon_0\,r}\,\exp\left(-\frac{\sqrt{2}\,r}{\lambda_D}\right).$$

Demonstrate that the charge density of the shielding cloud is

$$\delta\rho(r) = -\frac{2\,q}{4\pi\,r\,\lambda_D^2}\,\exp\left(-\frac{\sqrt{2}\,r}{\lambda_D}\right),$$

and that the net shielding charge contained within a sphere of radius r, centered on the origin, is

$$Q(r) = -q\left[1 - \left(1 + \frac{\sqrt{2}\,r}{\lambda_D}\right)\exp\left(-\frac{\sqrt{2}\,r}{\lambda_D}\right)\right].$$

$\boxed{\nabla\cdot\nabla \text{ in spherical coordinates}}$

Problems *From Piel (answers in back)*

2.1 Prove that the electron Debye length can be written as

$$\lambda_{\mathrm{De}} = 69\,\mathrm{m}\left[\frac{T\,(\mathrm{K})}{n_{\mathrm{e}}(\mathrm{m}^{-3})}\right]^{1/2}$$

2.2 Calculate the electron and ion Debye length
(a) for the ionospheric plasma ($T_{\mathrm{e}} = T_{\mathrm{i}} = 3000\,\mathrm{K}$, $n = 10^{12}\,\mathrm{m}^{-3}$).
(b) for a neon gas discharge ($T_{\mathrm{e}} = 3\,\mathrm{eV}$, $T_{\mathrm{i}} = 300\,\mathrm{K}$, $n = 10^{16}\,\mathrm{m}^{-3}$).

2.3 Consider an infinitely large homogeneous plasma with $n_{\mathrm{e}} = n_{\mathrm{i}} = 10^{16}\,\mathrm{m}^{-3}$. From this plasma, all electrons are removed from a slab of thickness $d = 0.01\,\mathrm{m}$ extending from $x = -d$ to $x = 0$ and redeposited in the neighboring slab from $x = 0$ to $x = d$. (a) Calculate the electric potential in this double slab using Poisson's equation. What are the boundary conditions at $x = \pm d$? (b) Draw a sketch of space charge, electric field and potential for this situation. What is the potential difference between $x = -d$ and $x = d$?

$\boxed{\text{“Double Layers”}}$

2.4 Show that the equation for the shielding contribution (2.24) results from (2.21) and (2.23).

2.5 Derive the relationship between the coupling parameter for ion-ion interaction Γ Eqs. (2.15) and N_{D} (2.33) under the assumption that $T_{\mathrm{e}} = T_{\mathrm{i}}$.

2.6 Show that the second Lagrange multiplier in Eq. (2.6) is $\lambda = (k_{\mathrm{B}}T)^{-1}$.
Hint: Start from

$$\frac{1}{T} = \frac{\partial S}{\partial \lambda}\frac{\partial \lambda}{\partial U}$$

and use $\sum n_i = 1$.

$\boxed{\text{Maximum Entropy (\textit{careful of definitions})}}$

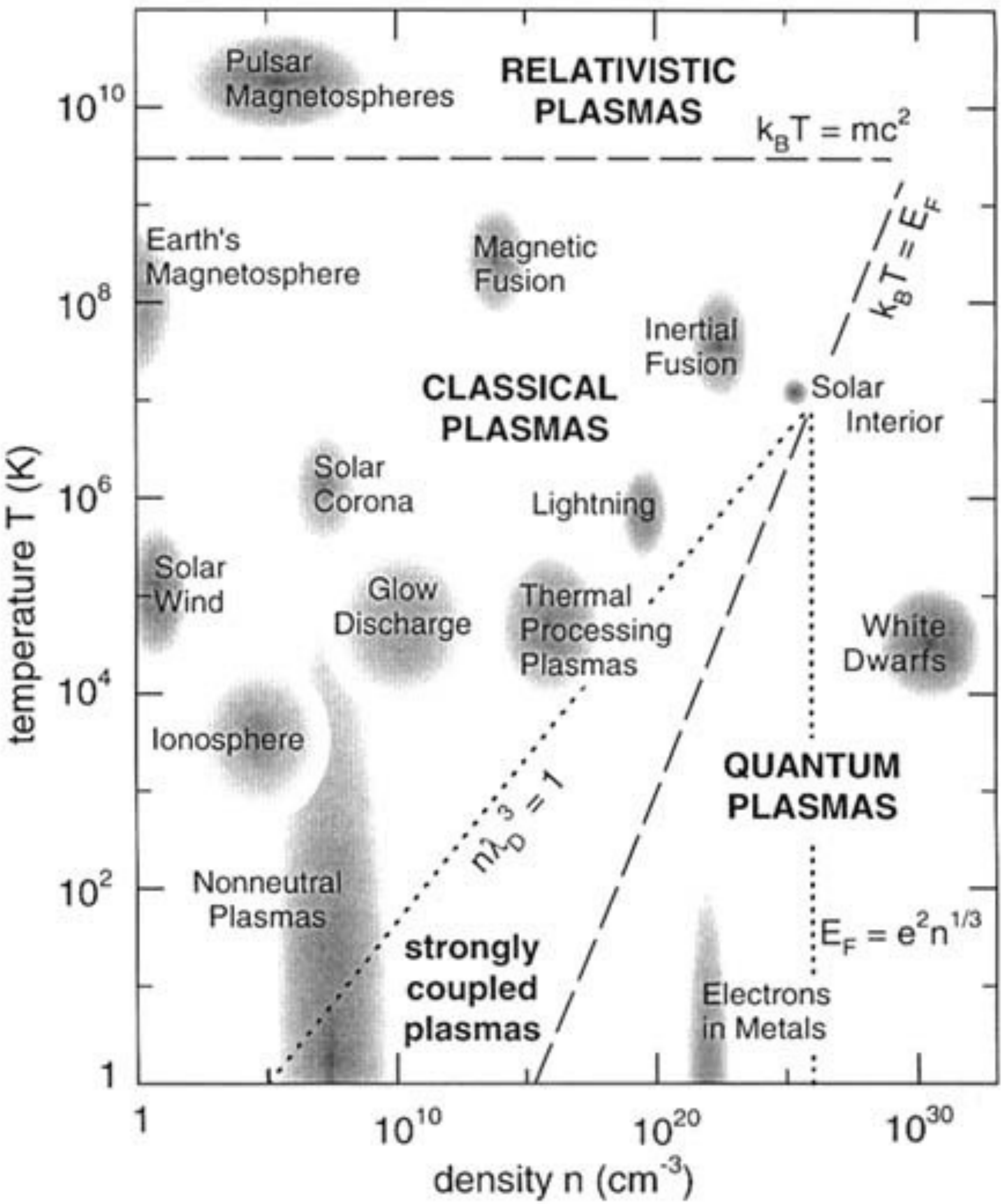


FIGURE S.1 Plasmas that occur naturally or can be created in the laboratory are shown as a function of density (in particles per cubic centimeter) and temperature (in kelvin). The boundaries are approximate and indicate typical ranges of plasma parameters.

Distinct plasma regimes are indicated:

- For thermal energies greater than that of the rest mass of the electron ($k_BT>mc^2$), relativistic effects are important.
- At high densities, where the Fermi energy is greater than the thermal energy ($E_F>k_BT$), quantum effects are dominant.
- In strongly coupled plasmas (i.e., $n\lambda_D^3 < 1$, where λ_D is the Debye screening length), the effects of the Coulomb interaction dominate thermal effects; and
- When $E_f>e^2n^{1/3}$, quantum effects dominate those due to the Coulomb interaction, resulting in nearly ideal quantum plasmas.
- At temperatures less than about 105 K, recombination of electrons and ions can be significant, and the plasmas are often only partially ionized.

http://www.nap.edu/catalog/4936/plasma-science-from-fundamental-research-to-technological-applications

The Equations of Plasma Physics

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \tag{5.1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{5.2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{5.3}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) . \tag{5.4}$$

Distribution Function

$$f^{(\alpha)}(\mathbf{r}.\mathbf{v}, t) = \sum_k \delta(\mathbf{r} - \mathbf{r}_k(t))\delta(\mathbf{v} - \mathbf{v}_k(t)) , \tag{9.4}$$

$$N^{(\alpha)} = \iint f^{(\alpha)}(\mathbf{r}, \mathbf{v}, t)\, \mathrm{d}^3 r\, \mathrm{d}^3 v , \tag{9.5}$$

$$\rho_m(\mathbf{r}, t) = \sum_\alpha m^{(\alpha)} n^{(\alpha)}(\mathbf{r}, t) \tag{9.7}$$

$$\rho(\mathbf{r}, t) = \sum_\alpha q^{(\alpha)} n^{(\alpha)}(\mathbf{r}, t) . \tag{9.8}$$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_r f + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v f = 0 . \tag{9.13}$$

Particle Motion

$$m\dot{\mathbf{v}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) , \tag{3.1}$$

Debye Length:
The small scale of electric fluctuations

$$f(W = \text{ENERGY}) \propto e^{-W/kT}$$

THERMODYNAMIC EQUILIBRIUM

$$kT = \tfrac{1}{2} m V_{TH}^2$$

$$W = \tfrac{1}{2} m v^2 + q\Phi$$

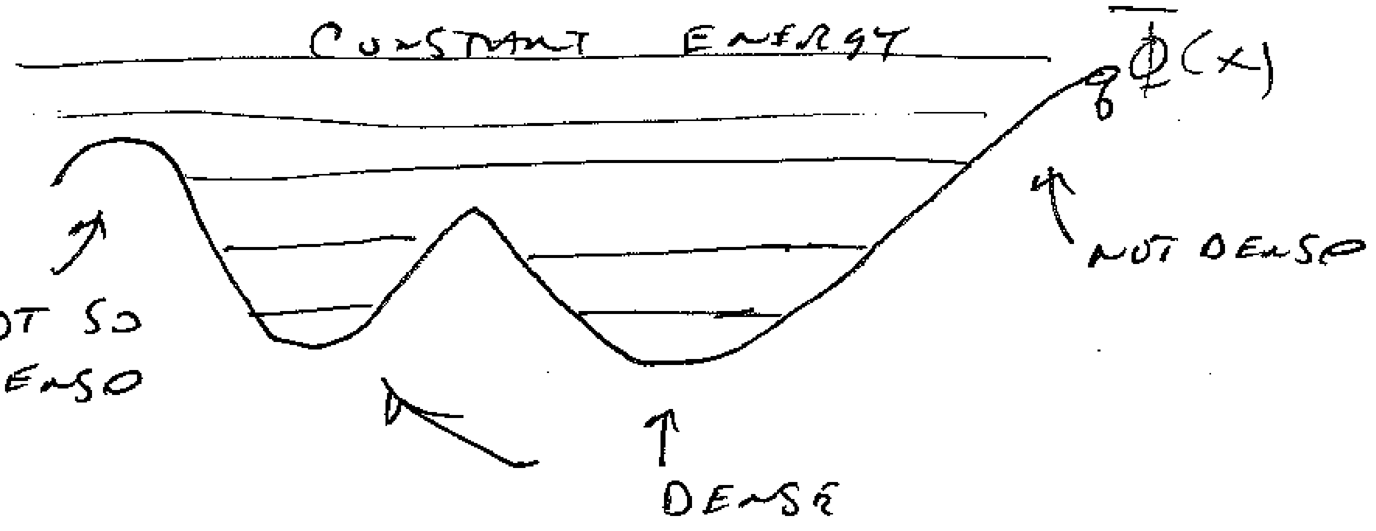
KINETIC

POTENTIAL

$$f(\bar{v}, \bar{x})$$

VELOCITY

SPACE



$$n_e(\bar{x}) \propto e^{-q_e \Phi(x)/kT_e}$$

$$n_i(\bar{x}) \propto e^{-q_i \Phi(x)/kT_i}$$

$$n_e(\bar{x}) - n_i(\bar{x}) \approx -e\,\Phi(x)\, n_0 \left(\frac{1}{kT_i} + \frac{1}{kT_e}\right)$$

$\Phi(r) = ?$

$$\nabla \cdot E = \rho/\epsilon_0$$

$$= \frac{Q}{\epsilon_0}\delta(\bar{r}) - \frac{e\, n_e(x)}{\epsilon_0} + \frac{e\, n_i(x)}{\epsilon_0}$$

$$\bar{E} = -\nabla\Phi$$

$$\nabla^2\Phi = -\rho/\epsilon_0 = -\frac{Q}{\epsilon_0}\delta(r) + \Phi(r) \underbrace{\left[\frac{e^2 n_0}{\epsilon_0 kT_e} + \frac{e^2 n_0}{\epsilon_0 kT_i}\right]}_{\lambda_D^{-2}}$$

NOTE: λ_D is INDEPENDENT of MASS!

Debye Length:
Potential near a change within a plasma

$$\nabla^2 \Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) = \frac{\partial^2 \Phi}{\partial r^2} + \frac{2}{r}\frac{\partial \Phi}{\partial r}$$

$$\iiint d^3r \; \nabla \cdot \nabla \Phi = -\frac{Q}{\epsilon_0}$$

$$\iint d\bar{A} \cdot \nabla \Phi \;\Rightarrow\; \Phi = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$$

BUT WITH PLASMA SHIELDING, WE CAN FIND A SOLUTION

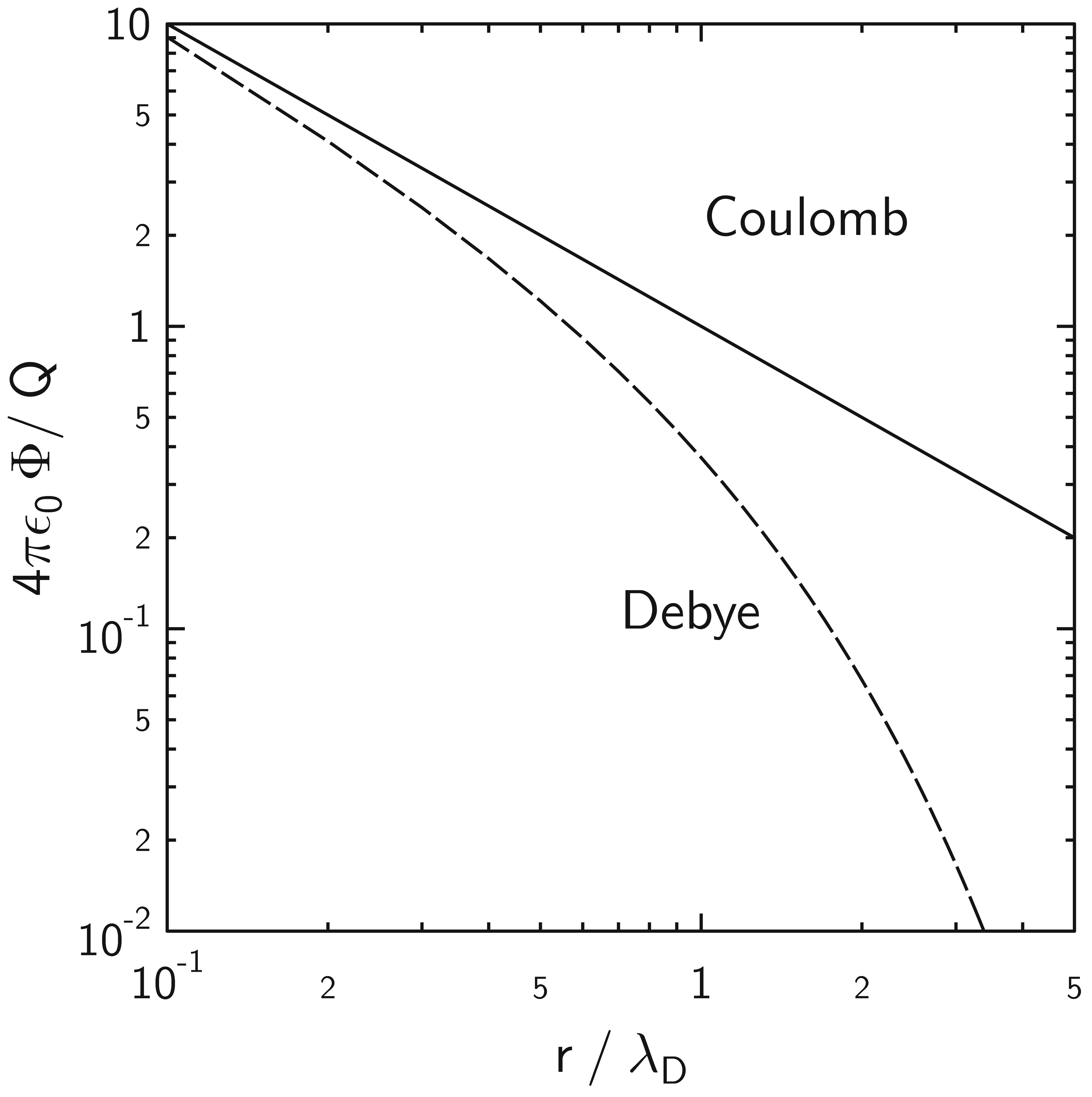
$$\Phi(r) \sim \frac{Q}{4\pi\epsilon_0}\frac{1}{r} f(r)$$

$$\frac{\partial \Phi}{\partial r} \sim -\frac{Q}{4\pi\epsilon_0 r^2} f(r) + \frac{Q}{4\pi\epsilon_0}\frac{1}{r} f'$$

$$\frac{\partial^2 \Phi}{\partial r^2} = \frac{2Q}{4\pi\epsilon_0 r^3} f(r) - \frac{Q}{4\pi\epsilon_0 r^2} f' - \frac{Q}{4\pi\epsilon_0 r^2} f' + \frac{Q}{4\pi\epsilon_0 r} f''$$

$$\boxed{f'' = \frac{1}{\lambda_D^2} f}$$

$$\Phi = \frac{Q}{4\pi\epsilon_0}\frac{1}{r} e^{-r/\lambda_D}$$



Plasma Parameter

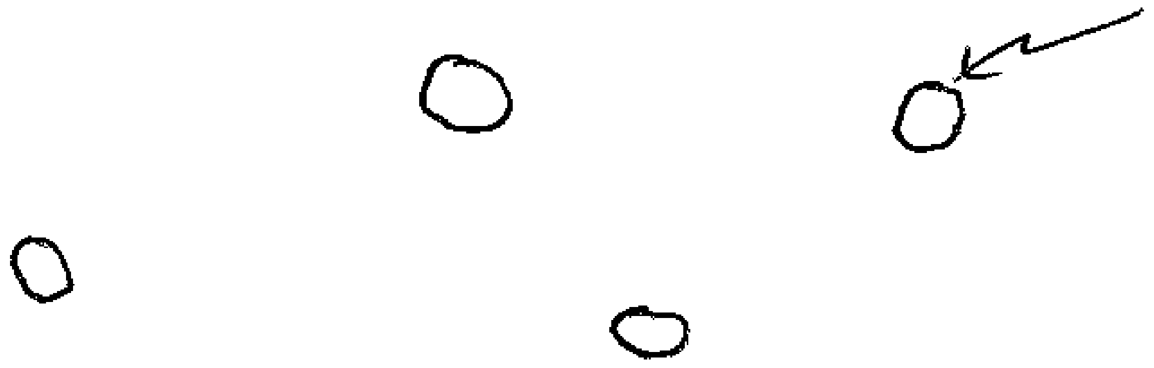
$N_D = \frac{4}{3}\pi\lambda_D^3 n \gg 1$ FOR A "USUAL" PLASMA

$N_D = \left(\frac{1}{3\Gamma_i}\right)^{3/2}$ $\Gamma_i \sim \dfrac{q^2/4\pi\epsilon_0\langle d\rangle}{kT_i}$ $\left(\frac{4\pi}{3}\langle d\rangle^3 \sim \frac{1}{n_i}\right)$

WHEN $\Gamma_i \sim$ LARGE, THEN "PLASMA" IS STRONGLY COUPLED

USUALLY ONLY FOR CASES WHERE THERE EXISTS LARGE q SMALL T_i COMPONENTS WITHIN PLASMA

LIKE DUSTY PLASMA



DUST 10-1000 nm DIA

$Q \sim 10^4 - 10^6 e$

$T_i \sim 0.01$ ROOM TEMPERATURE

N_D or Λ: Plasma Parameter

Table 1.1 Key parameters for some typical weakly coupled plasmas.

Plasma	n(m^{-3})	T(eV)	Π(sec^{-1})	λ_D(m)	Λ
Solar wind (1AU)	10^7	10	2×10^5	7×10^0	5×10^{10}
Tokamak	10^{20}	10^4	6×10^{11}	7×10^{-5}	4×10^8
Interstellar medium	10^6	10^{-2}	6×10^4	7×10^{-1}	4×10^6
Ionosphere	10^{12}	10^{-1}	6×10^7	2×10^{-3}	1×10^5
Inertial confinement	10^{28}	10^4	6×10^{15}	7×10^{-9}	5×10^4
Solar chromosphere	10^{18}	2	6×10^{10}	5×10^{-6}	2×10^3
Arc discharge	10^{20}	1	6×10^{11}	7×10^{-7}	5×10^2

Plasma Parameter

$N_D = \frac{4}{3}\pi \lambda_D^3 n \gg 1$ FOR A "USUAL" PLASMA

$$N_D = \left(\frac{1}{3\Gamma_i}\right)^{3/2}$$

$$\Gamma_i \sim \frac{q^2/4\pi\epsilon_0 \langle d\rangle}{kT_i} \qquad \left(\frac{4\pi}{3}\langle d\rangle^3 \sim \frac{1}{n_i}\right)$$

WHEN $\Gamma_i \sim$ LARGE, THEN "PLASMA" IS STRONGLY COUPLED

USUALLY ONLY FOR CASES WHERE THERE EXISTS LARGE q SMALL T_i COMPONENTS WITHIN PLASMA

LIKE DUSTY PLASMA

DUST 10-1000 nm DIA

$Q \sim 10^4 - 10^6 e$

$T_i \sim 0.01$ ROOM TEMPERATURE

$$\Gamma_i = \frac{1}{3} N_{Di}^{-2/3} .$$

Coupling Parameter

$$\Gamma_i = \frac{e^2}{4\pi\varepsilon_0 a_{WS} k_B T_i} .$$

$$n_i \frac{4\pi}{3} a_{WS}^3 = 1 .$$

a_{WS} = Wigner-Seitz radius

Plasma Frequency: "Fast" Electron Motion of Plasma

$$\nabla \cdot \bar{E} = \rho/\epsilon_0$$

$$\rho = \rho_i + \rho_e \approx -e\tilde{n}_e$$ (IONS DON'T MOVE)

$$\frac{\partial f}{\partial t} + \bar{v}\cdot\bar{\nabla} f + \frac{q}{m}\bar{E}\cdot\nabla_v f = 0$$

REMEMBER $(\bar{x}, \bar{v}, t)$ IS A 6+1 DIMENSIONAL SPACE

$$\iiint d^3v\,(\ldots) \Longrightarrow \quad \frac{\partial n}{\partial t} + \bar{\nabla}\cdot\bar{v}n = 0$$

NO PRESSURE

$$\bar{v}n = \iiint d^3v\,\bar{v}f \qquad m n \frac{d\bar{v}}{dt} = q n \bar{E}$$

$\Rightarrow$ LINEARIZE

$$n = n_0 + \tilde{n} \qquad (\tilde{n}/n \lll 1)$$

$$\frac{\partial \tilde{n}}{\partial t} + n_0 \bar{\nabla}\cdot\tilde{v} \approx 0$$

BUT $\nabla \cdot \frac{\partial \bar{v}}{\partial t} = \frac{q}{m}\nabla\cdot\bar{E}$

$$\frac{\partial^2 \tilde{n}}{\partial t^2} + n_0 \bar{\nabla}\cdot\frac{\partial \bar{v}}{\partial t} \approx 0$$

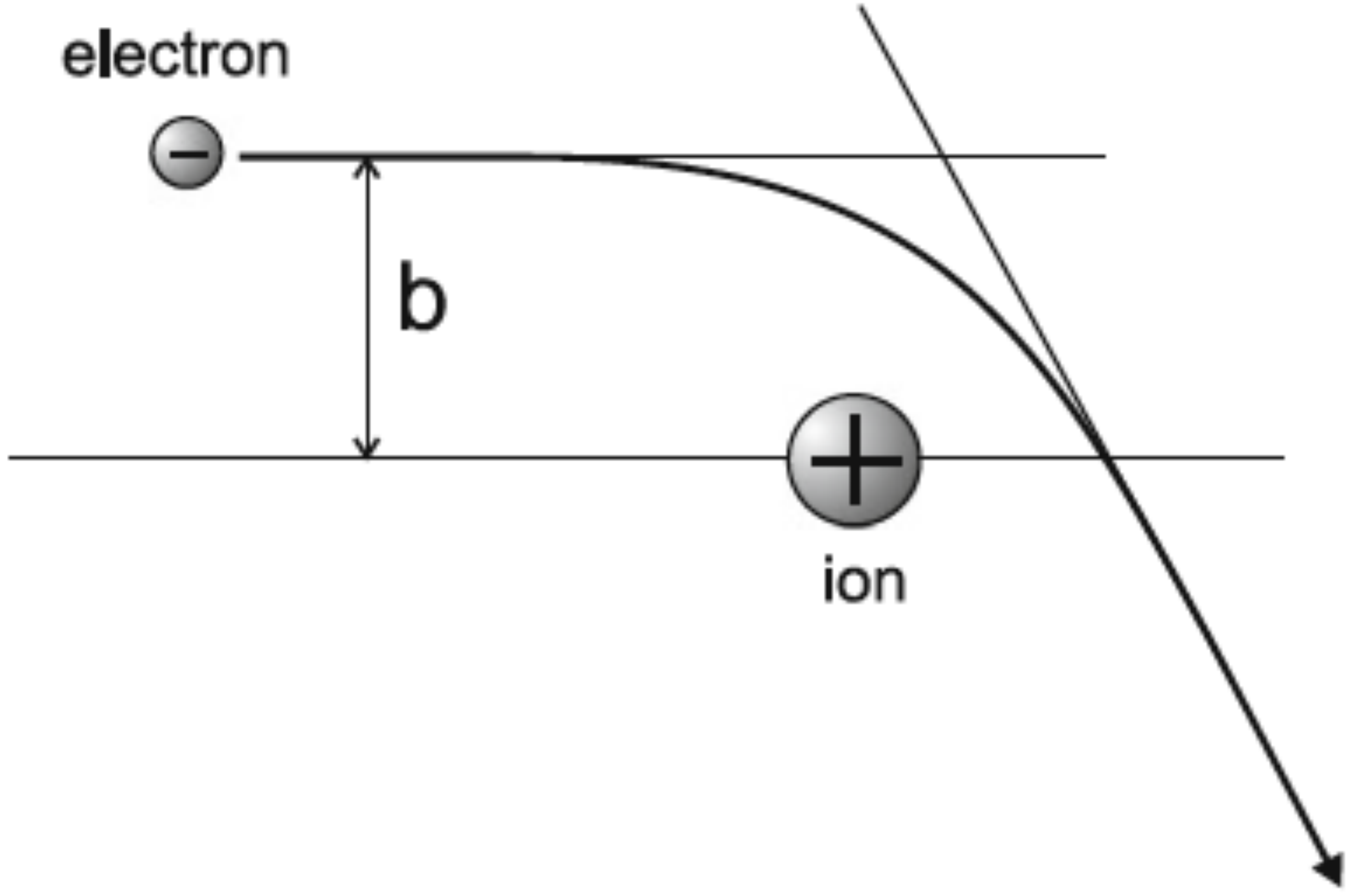
$$\boxed{\frac{\partial^2 \tilde{n}}{\partial t^2} + \frac{q^2 n_0}{m_0 \epsilon_0}\tilde{n} = 0}$$

Last Week: Plasma Parameters

- $n(r,t)$ – density - plasma frequency, ω_p
- $T(r,t)$ – temperature, $v_{th} = (kT/m)^{1/2}$
- λ_D – Debye length, v_{th}/ω_p
- N_D - plasma parameter, $(4\ \lambda_D^3/3)\ n >> 1$

➡ Collisions

Collisions



$\Delta p \approx F\, \Delta t$

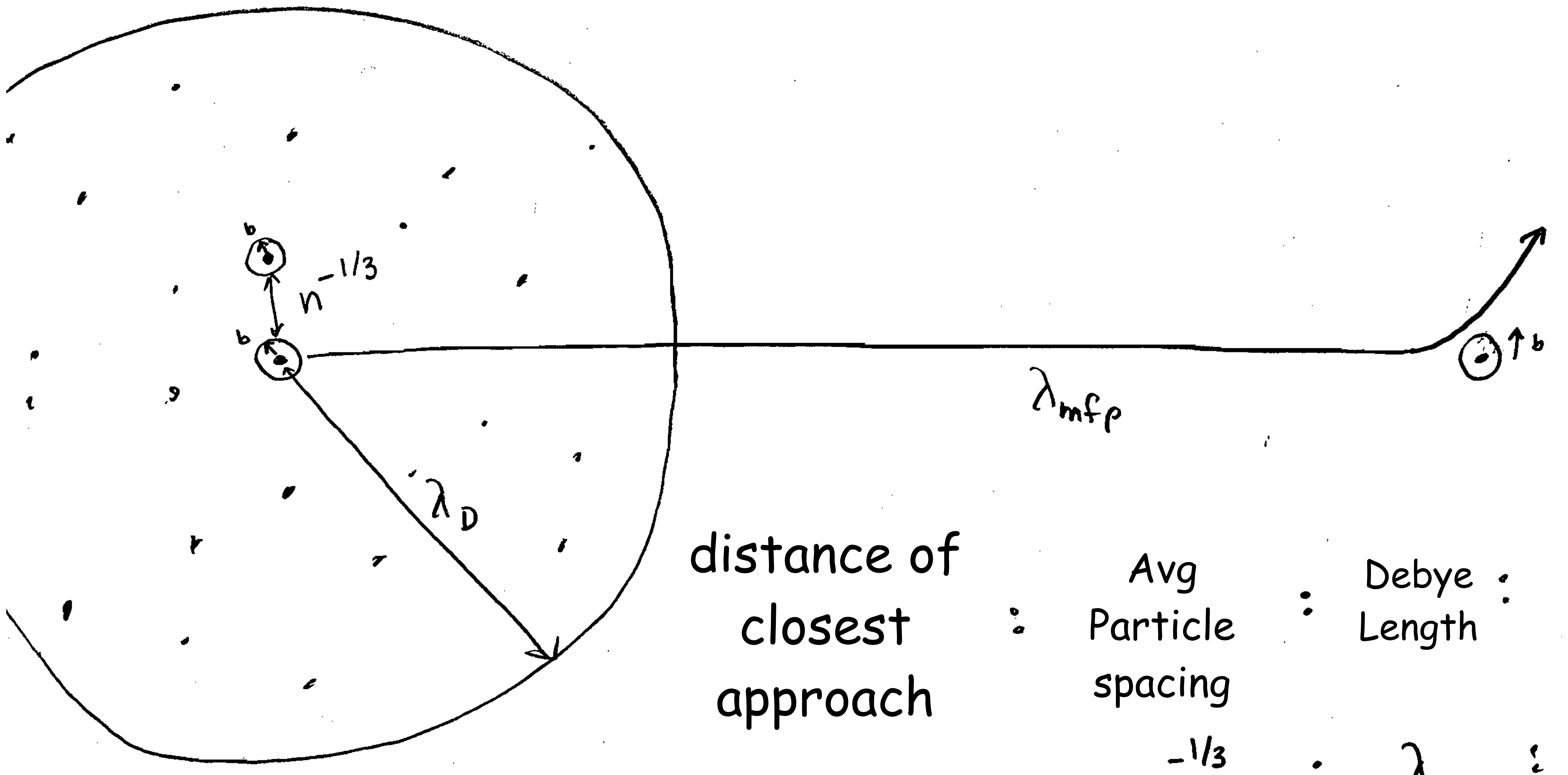
$$m_e v = \frac{e^2}{4\pi\epsilon_0 b^2}\left(\frac{b}{v}\right) = \frac{e^2}{4\pi\varepsilon_0 b v}$$

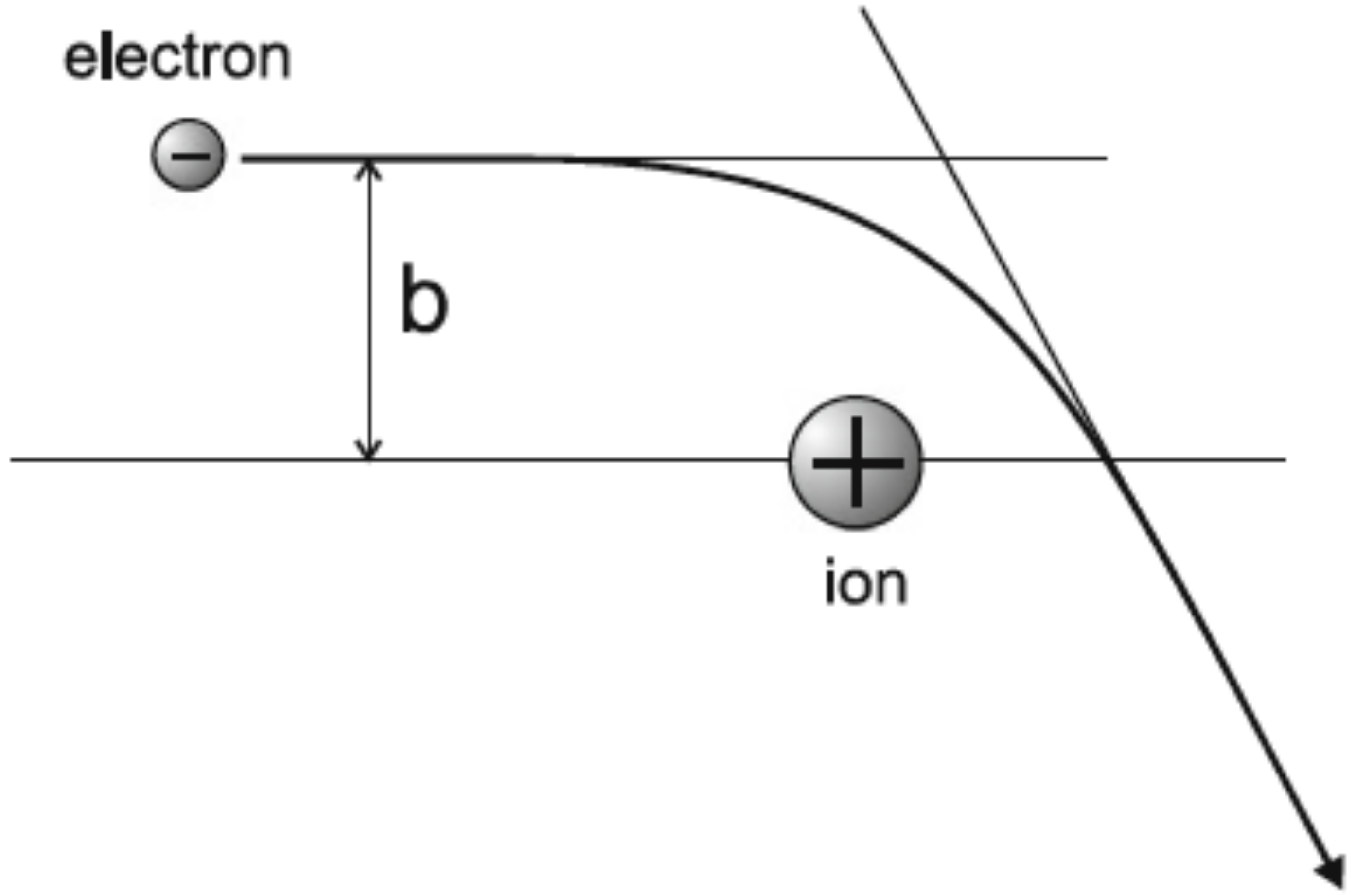
$\sigma \approx \pi\, b^2$

$$\nu_{ei} = n\sigma_{\pi/2} v = \frac{ne^4}{16\pi\varepsilon_0^2 m_e^2 v^3} \times \mathrm{Log}(4\pi\Lambda)$$

Collisions

Review Fundamental Length Scales of a Plasma





distance of closest approach	:	Avg Particle spacing	:	Debye Length	:	Mean Free Path for Collisions
b	:	$n^{-1/3}$	:	λ_D	:	λ_{mfp}
Λ^{-1}	:	$\Lambda^{-1/3}$	:	1	:	$\frac{\Lambda}{\ln\Lambda}$
10^{-6}	:	10^{-2}	:	1	:	$\frac{10^6}{15} \sim 10^5$

$\Lambda \sim n\lambda_D^3$

$=$ # of particles in Debye Sphere

$$\left.\begin{array}{l}\lambda_D \sim 10^{-2}\ \text{cm} \\ \lambda_{mfp} \sim 100\ \text{m}\end{array}\right\}\ \text{TFTR}$$

$$m_e v = \frac{e^2}{4\pi\epsilon_0 b^2}\left(\frac{b}{v}\right) = \frac{e^2}{4\pi\varepsilon_0 bv}$$

$$\nu_{ei} = n\sigma_{\pi/2}v = \frac{ne^4}{16\pi\varepsilon_0^2 m_e^2 v^3} \times \text{Log}(4\pi\Lambda)$$

Greg Hammett (PPPL)
http://w3.pppl.gov/~hammett/talks/

80 Years of Plasma: A Collection of Reviews and Research Articles Celebrating the 80th Anniversary of Langmuir's Coining of the Word 'Plasma'

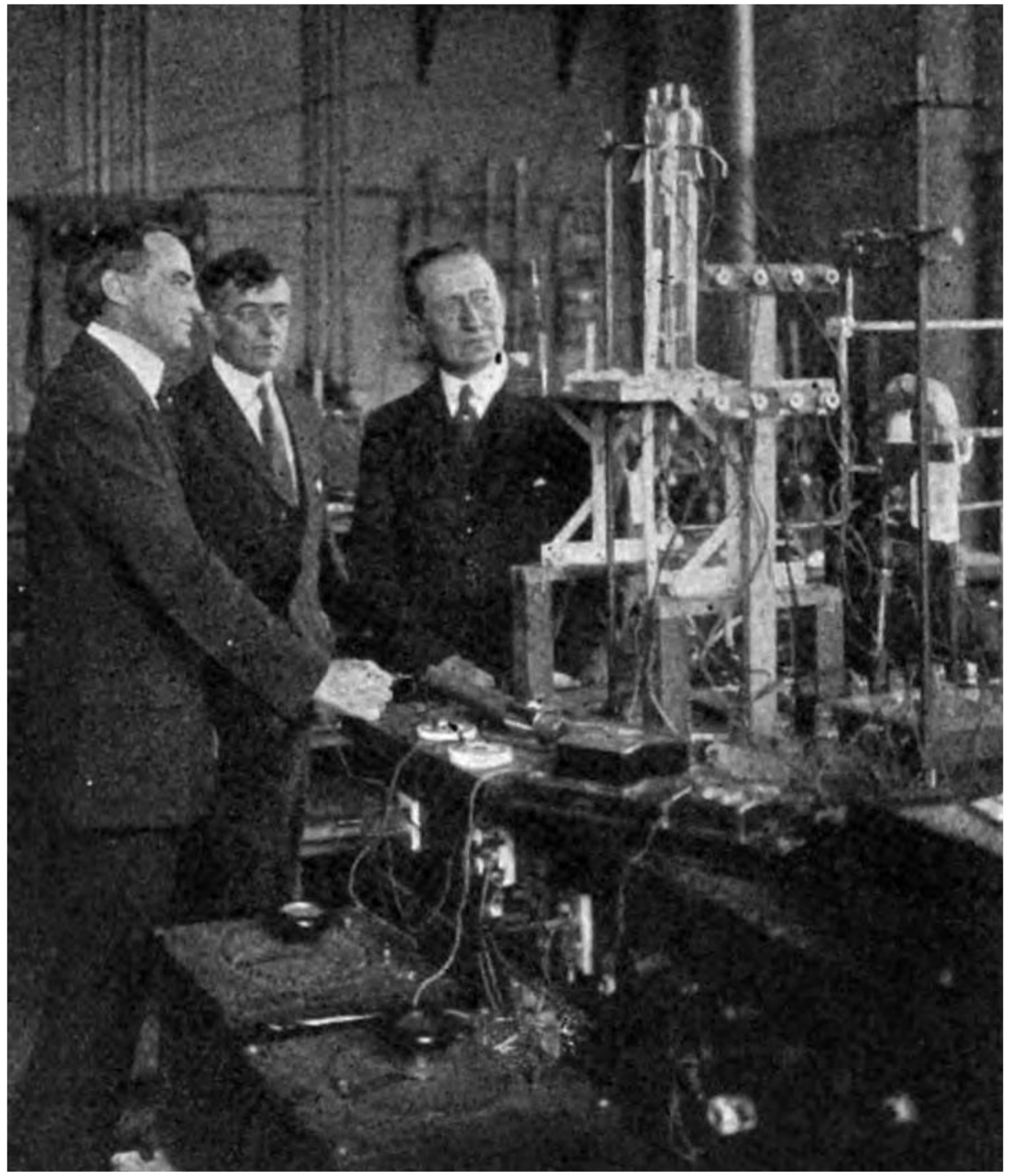
Irving Langmuir proposed the term 'plasma' in a paper in 1928 (*Proc. Natl Acad. Sci*. USA 14 627–637) to describe a 'region containing balanced charges of ions and electrons'. There does not appear to be any record of the thinking behind this proposal, so it is difficult to be definitive. One idea is that since the Greek word 'plasma' was used to describe a mouldable fluid, 'neon' lighting, with its almost limitless ability to provide colourful shapes, provided the inspiration. Another relates to the prior medical use in relation to blood with its variety of different 'corpuscles' and that the essential description of the positive column required one to recognize at least the role of the separate species of electrons, ions and gas atoms.

H M Mott-Smith, one of Langmuir's co-workers, makes clear his recollection that Langmuir was struck by the analogy between 'the way blood plasma carries around red and white corpuscles and germs' and the way that the ' . . . "equilibrium" part of the discharge acted as a sort of sub-stratum carrying particles of special kinds, like high-velocity electrons from thermionic filaments, molecules and ions of gas impurities'.

Plasma Sources Science and Technology, Volume 18, Number 1, February 2009;
http://dx.doi.org/10.1088/0963-0252/18/1/010201

Irving Langmuir (1881–1957) condensed c.v.

Born:	31 January 1881, parents Charles and Sadie Comings Langmuir
Schooling:	Brooklyn Public Schools 1887–92
	Paris, Boarding School 1892–5
	Philadelphia, Chestnut Hill Academy 1895–6
	Pratt Institute 1897–8
University:	Columbia University, BMetEng 1899–1903
	Goetingen University, PhD 1904–6
Teaching:	Stevens Institute of Technology 1906–9
Research:	General Electric Laboratory 1909–50
	Associate Director 1929–50
	Consultant 1950–7



GE (1922)

Nobel Prize in Chemistry (1932) for "for his discoveries and investigations in surface chemistry"

OSCILLATIONS IN IONIZED GASES

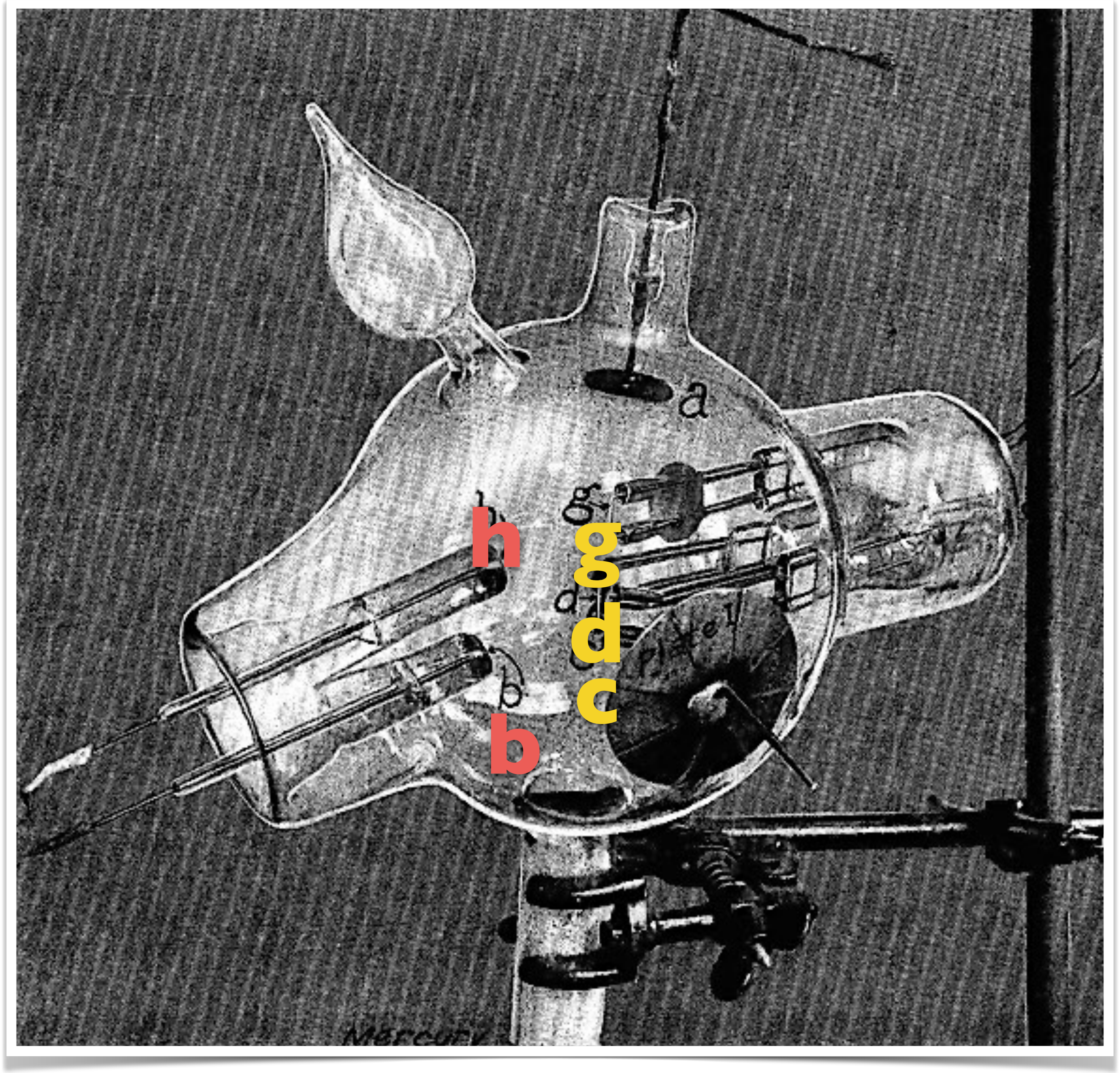
BY LEWI TONKS AND IRVING LANGMUIR

ABSTRACT

A simple theory of electronic and ionic oscillations in an ionized gas has been developed. *The electronic oscillations* are so rapid (ca. 10^9 cycles) that the heavier positive ions are unaffected. They have a natural frequency $\nu_e=(ne^2/\pi m)^{1/2}$ and, except for secondary factors, do not transmit energy. *The ionic oscillations* are so slow that the electron density has its equilibrium value at all times. They vary in type according to their wave-length. The oscillations of shorter wave-length are similar to the electron vibrations, approaching the natural frequency $\nu_p=\nu_e(m_e/m_p)^{1/2}$ as upper limit. The oscillations of longer wave-length are similar to sound waves, the velocity approaching the value $v=(kT_e/m_p)^{1/2}$. The transition occurs roughly (i.e. to 5% of limiting values) within a 10-fold wave-length range centering around $2(2)^{1/2}\pi\lambda_D$, λ_D being the "Debye distance." While the theory offers no explanation of the cause of the observed oscillations, the frequency range of the most rapid oscillations, namely from 300 to 1000 megacycles agrees with that predicted for the oscillations of the ultimate electrons. Another observed frequency of 50 to 60 megacycles may correspond to oscillations of the beam electrons. Frequencies from 1.5 megacycles down can be attributed to positive ion oscillations. The correlation between theory and observed oscillations is to be considered tentative until simpler experimental conditions can be attained.

II. EXPERIMENTS

A. Discharge tubes. On the experimental side we have worked with two tubes, both containing filamentary cathodes used as electron sources, collectors so placed as to receive a portion of the direct beam of primary electrons from a filament, and an anode off to one side to maintain the discharge. The two tungsten filaments in the first tube used were supported near the middle of the 18 em spherical bulb by long glass-covered leads. Their exposed portions were about 1.1 em long, parallel and about 0.5 em apart. At a distance of 4.2 em from them was the collector, a circular disk backed by mica and 1.1 cm in diameter.



The second tube was similar except that it contained three vertical tungsten filaments, ***g***, ***c***, and ***d***. Fig. 1, ***g*** above, and ***c*** and ***d*** about 2.5 cm below it. These two were 0.4 em apart and all three lay in the same plane. Their diameter was 0.025 em and their vertical and active portions were 1.1 cm long. Opposite them and about 4 em away were supported the 1.1 cm disk collectors ***h*** and ***b***. The primary electrons are somewhat deflected by the magnetic field of the heating current, and the collectors are inclined as shown in order to give perpendicular incidence of the primaries on them.

An appendix containing a little mercury extended from the bottom of the bulb and was immersed in a water bath, the temperature of which controlled the mercury pressure.

B. Detection of oscillations. The oscillations were detected with a zincite-tellurium detector X and galvanometer arranged, for most of the work, in a circuit as shown in Fig. 2. The detector was supported by a spring suspension to shield it from mechanical shocks which were found to destroy its sensitivity. The high frequency potential was applied across the two points X and Y, Y being grounded to one side of Fig. 2. Crystal detector circuit a filament and also to the metal-screen cage surrounding the apparatus. The inductance L was often only a 10 to 15 em length of copper wire. The two condensers, which were of 0.0025 pf each, shunted the galvanometer and 60-cycle crystal-calibrating circuits for the high-frequency oscillations, but at the same time allowed known 60-cycle voltages to be conveniently impressed on the crystal for calibrating purposes at frequent intervals.

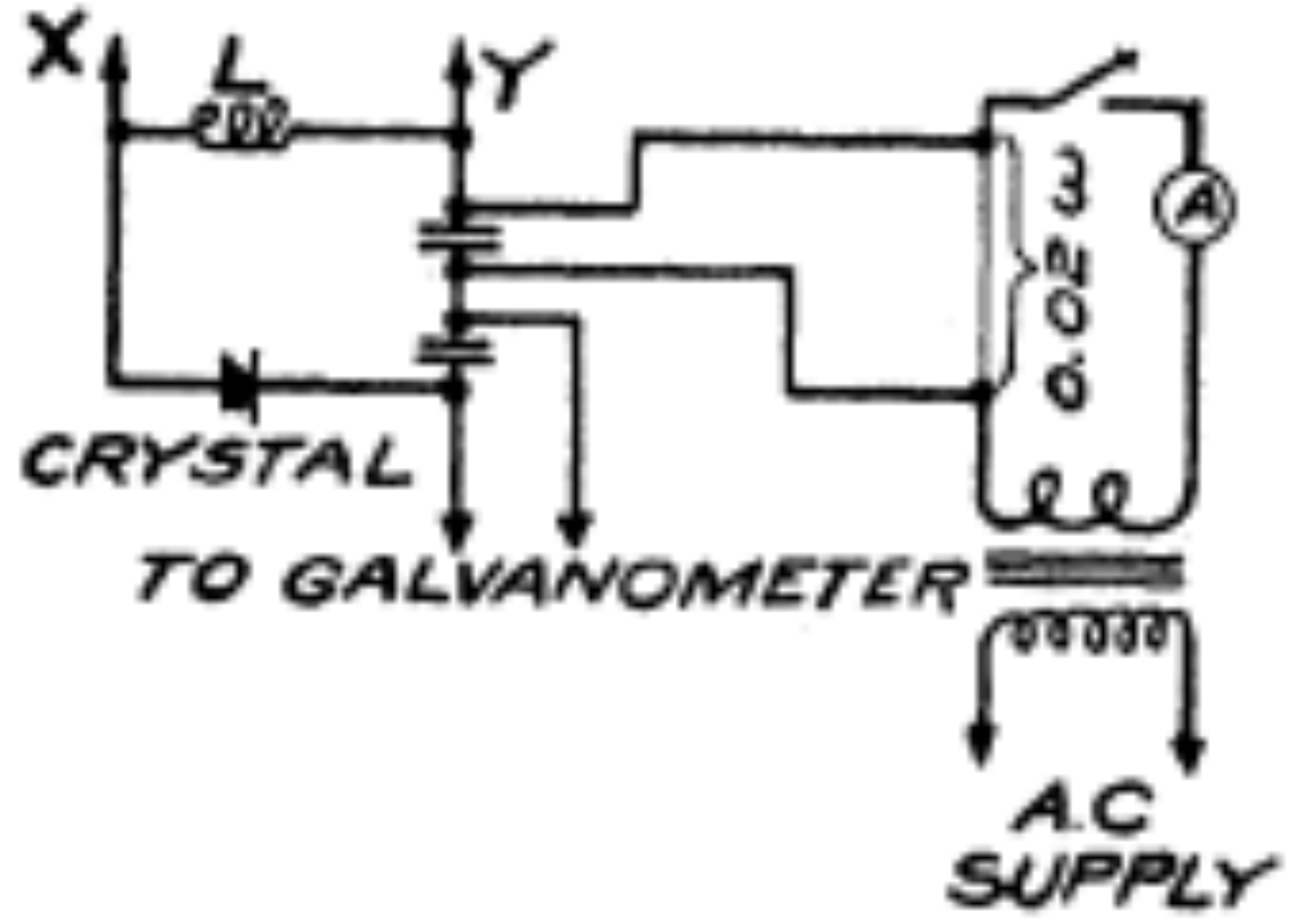


Fig. 2. Crystal detector circuit.

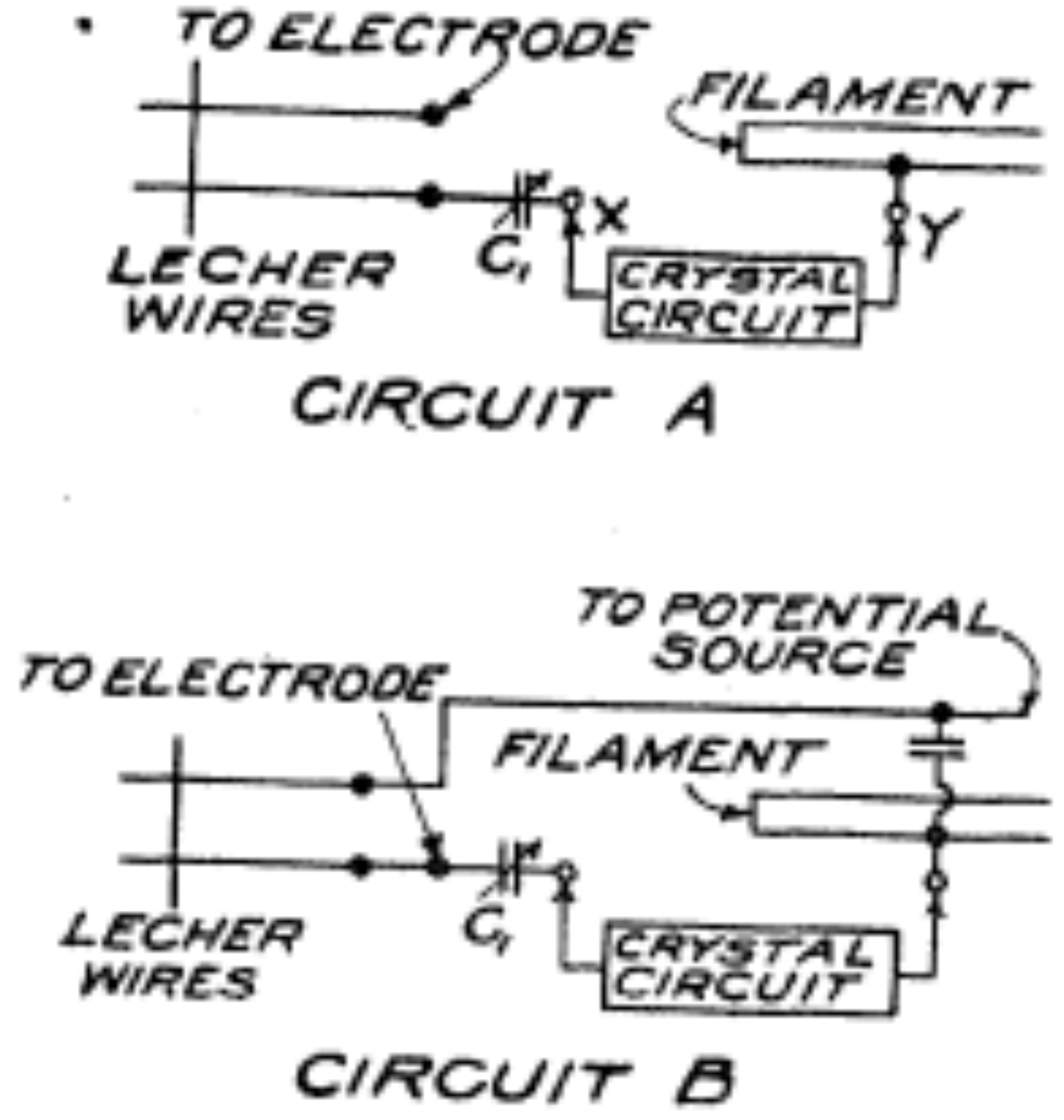


Fig. 3. Oscillation pick-up circuits.

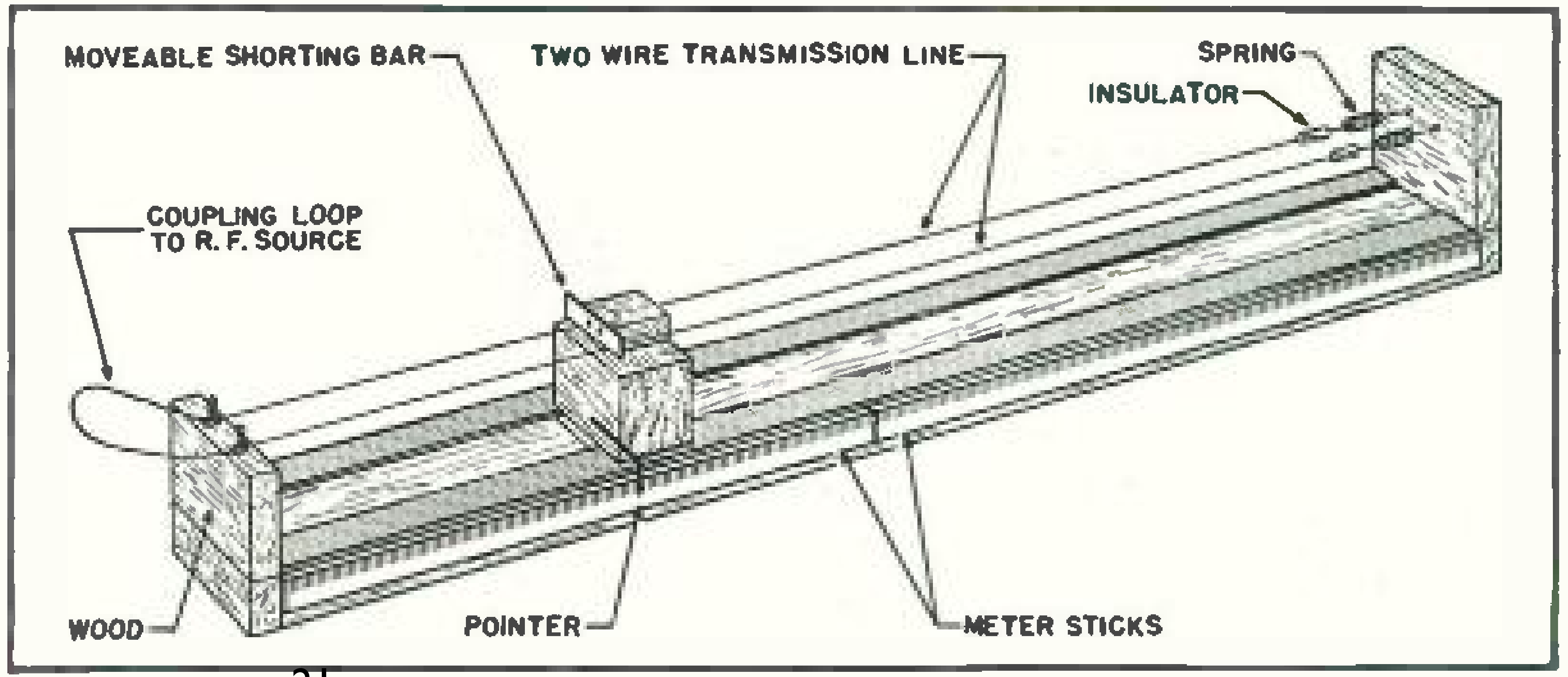
Lecher Wires (Ernst Lecher, 1888)

http://www.americanradiohistory.com/Archive-Radio-News/40s/Radio-News-1946-09.pdf

The construction of a Lecher-wire system for measuring wavelength by determining the distance between successive positions of current or voltage maximum is shown in Fig. 10. If a sensitive current-reading instrument is inserted in the shorting bar, series-resonance will be indicated by maximum current, and parallel-resonance by minimum current. When the amount of power supplied by the measured circuit is small, series-resonance (i.e., the shorting bar at half-wave positions) is indicated by maximum reaction upon the circuit, and parallel resonance (i.e., the shorting bar at odd quarter-wave positions) by minimum reaction upon the circuit. To measure frequency, the line is coupled into the circuit and the shorting bar moved to a position of resonance, with fairly loose coupling. The shorting bar is then moved to the next position of similar resonance, and the distance between the two is equal to one half-wavelength, from which frequency is known.

Under good conditions, frequency measurements may be made by the Lecher-wire method to an accuracy of the order of .1%. At the higher frequencies the accuracy decreases due to the difficulty of measuring small distances to such a high degree of accuracy. In addition to their greater accuracy, Lecher-wire wavemeters have the further advantages over tuned-circuit wavemeters that they

Fig. 10. Construction of a Lecher-wire system for measuring wavelength.



RADIO NEWS

C. Frequency range of oscillations. Oscillation frequencies as low as 10^6 and as high as 10^9 cycles per second have been observed. Over a large portion of this range it was not found possible to obtain any resonance phenomena, but oscillations were detected throughout so that we lean toward the view that these electric vibrations are irregular, thereby constituting an "electrical noise." Under certain conditions definite frequencies were observed both on the collectors and on external electrodes glued to the tube wall or placed against it.

The frequency range can roughly be divided into three parts, namely from 1 to 100 megacycles, from 100 to 300 megacycles, and from 300 to 1000 megacycles.

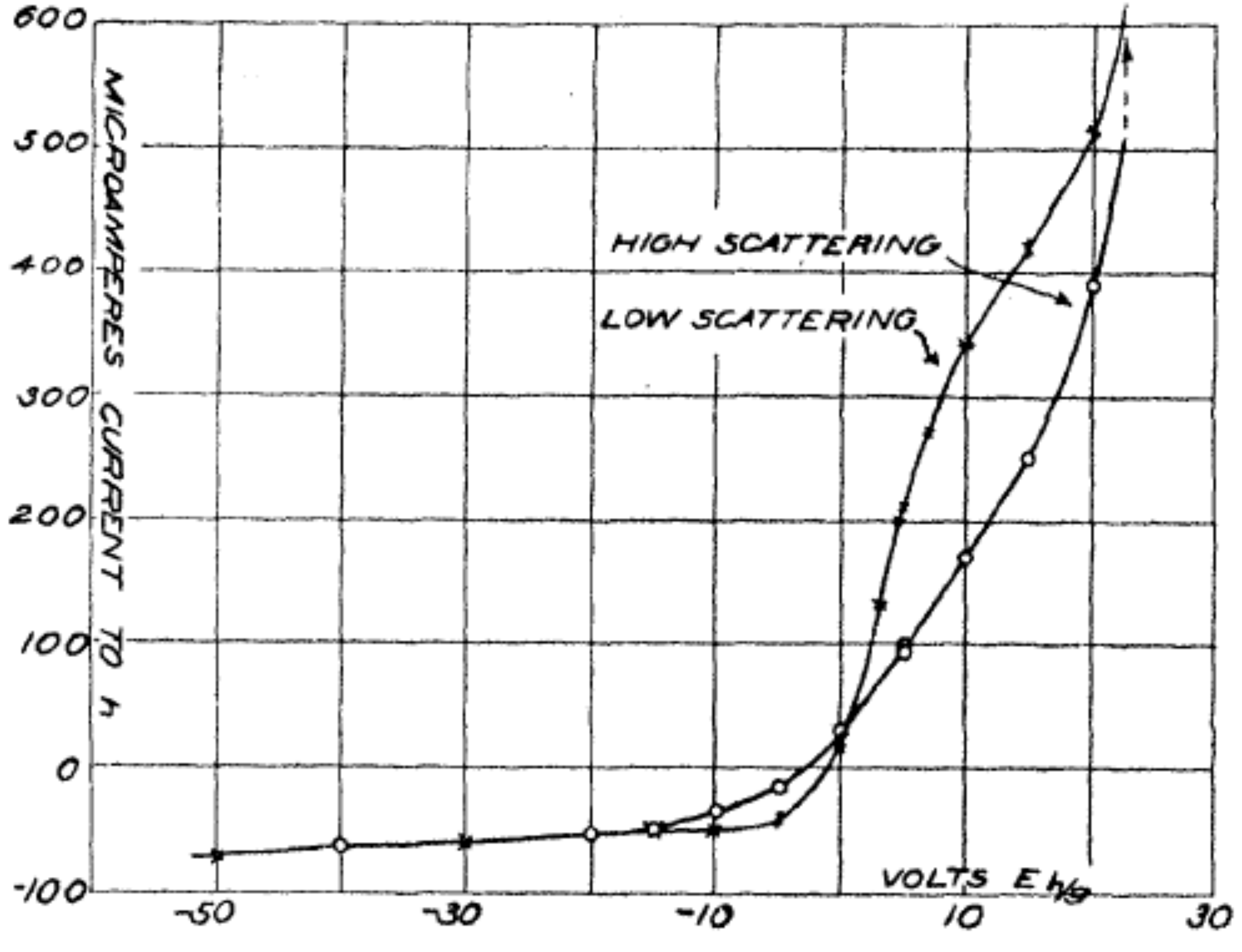


Fig. 8. Volt-ampere characteristics of a collector in the primary electron beam.

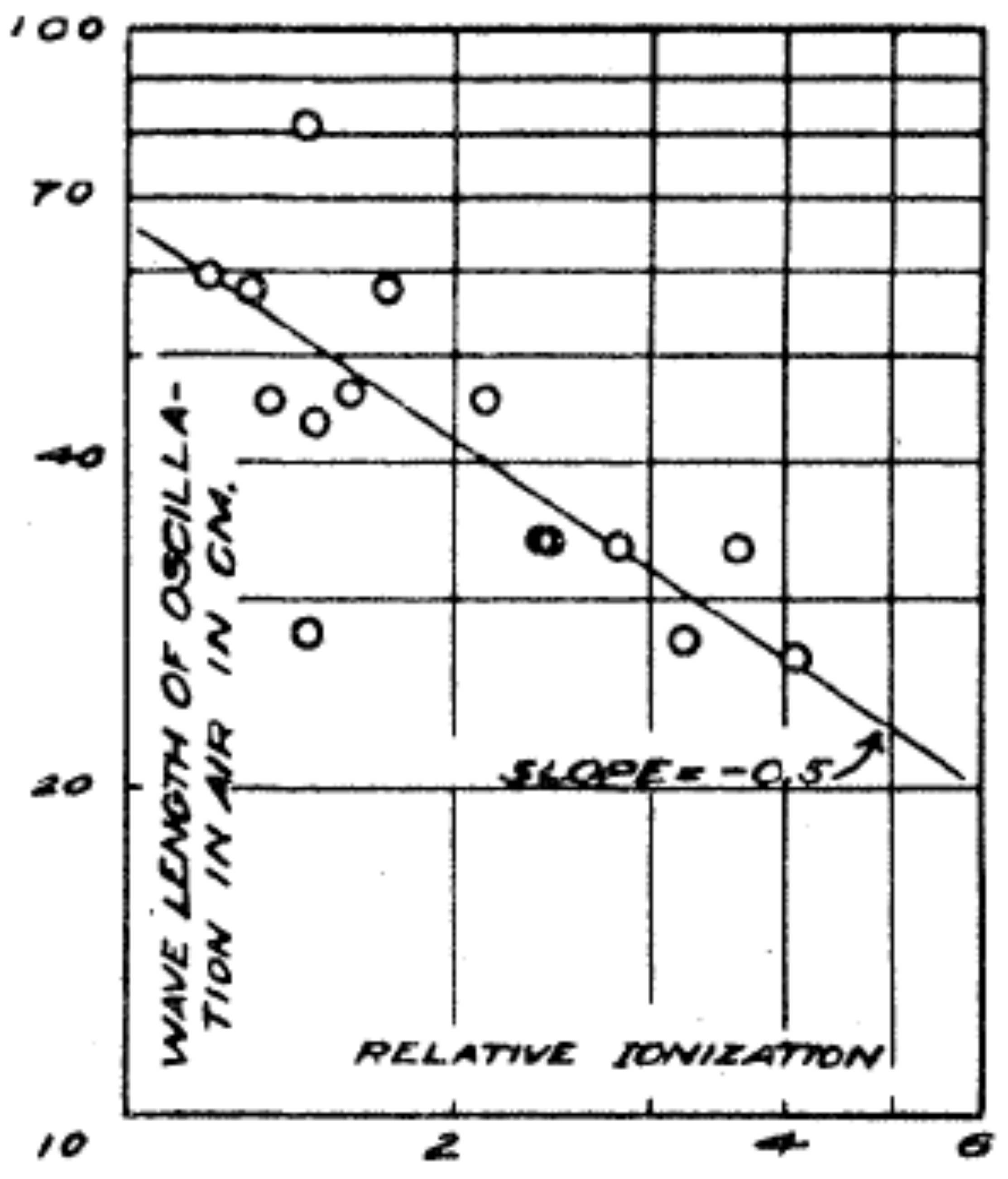


Fig. 10. Oscillation frequency and ionization density.

I. THEORY

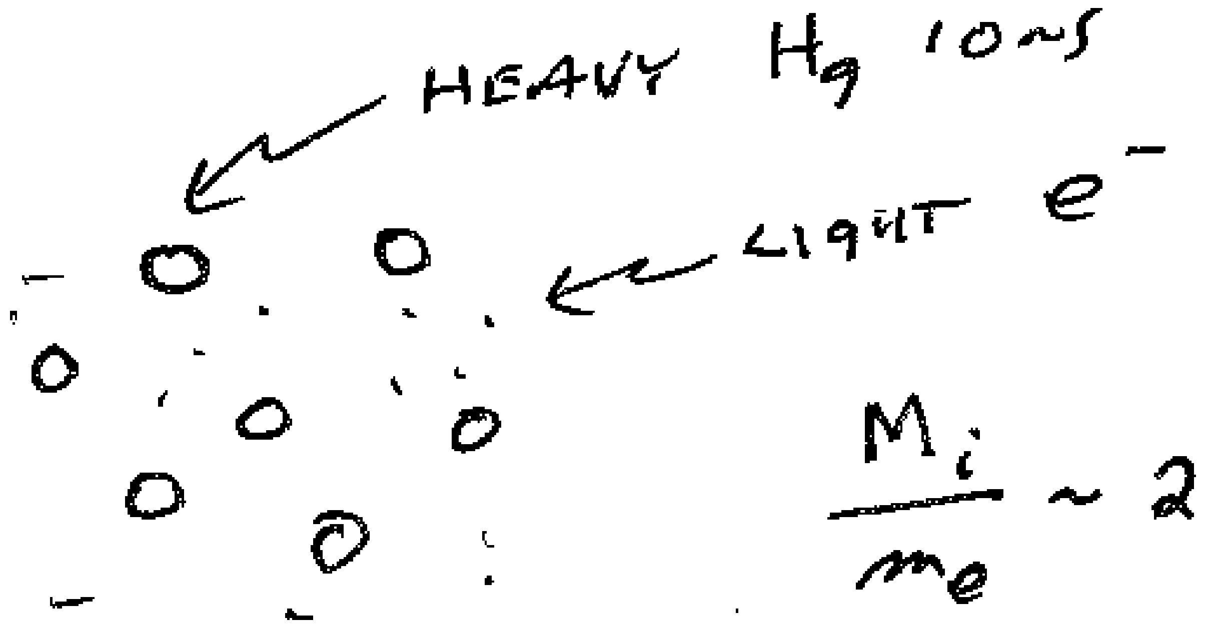
Two types of oscillation seem to be theoretically possible, first oscillations of electrons which are too rapid for the ions to follow, and second, oscillations of the ions which are so slow that the electrons continually
satisfy the Boltzmann Law.

A. Plasma-electron oscillations. When the electrons oscillate, the positive ions behave like a rigid jelly with uniform density of positive charge *ne*. Imbedded in this jelly and free to move there is an initially uniform electron distribution of charge density, -*ne*.

Slower…

B. Plasma-ion oscillations. We are now ready to discuss the slower ionic oscillations. In this case we shall assume the same type of displacement for the ions as we did for the electrons before.

Langmuir's Electrons



$$\frac{M_i}{m_e} \sim 200 \times 1{,}836 \gg 1$$

GAUSS'S LAW

$$\nabla \cdot \bar{E} = \rho/\epsilon_0 = -e\,\delta n(x)/\epsilon_0$$

$$\frac{\partial E_x}{\partial x} = + \frac{e n_o}{\epsilon_o} \frac{\partial \xi}{\partial x}$$

$$\left\{ E_x = + \left(e\, n_o/\epsilon_o\right) \xi(x) \right\}$$

BUT FORCE ON ELECTRONS...

$$m_e \frac{\partial^2 \xi}{\partial t^2} = -e E_x = -\frac{e^2 n_o}{\epsilon_o} \xi(x)$$

$$\therefore \quad \omega_p^2 = \frac{e^2 n_o}{m_e \epsilon_o}$$

$\xi(x) \sim$ ELECTRON "COLLECTIVE" DISPLACEMENT

SIGN

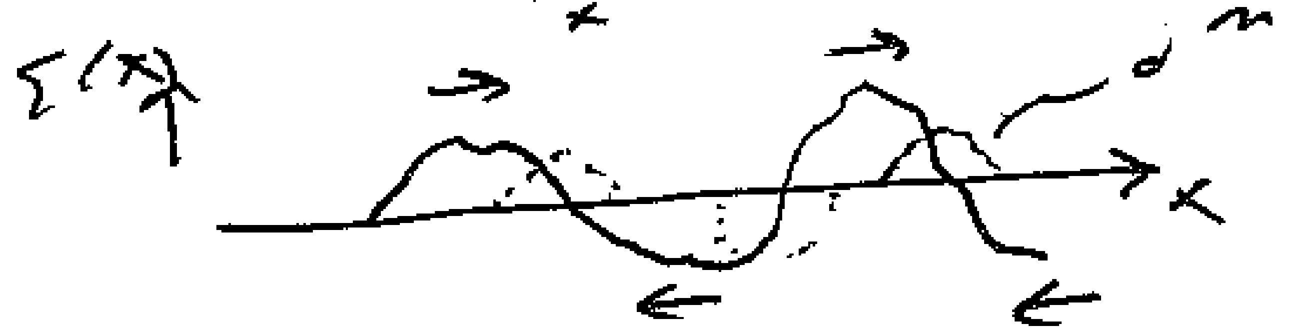
$$\delta n(x) \simeq -n_o \frac{\partial \xi}{\partial x}$$

THIS IS THE SAME AS

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \bar{v}) = 0$$

$$\delta n + \nabla \cdot (n \bar{v}\, dt) = 0$$

$$v_x\, dt = \xi$$



Electrostatic oscillations are (nearly) independent of wavelength ion: $\delta n_i = -n_0 \frac{\partial \xi_i}{\partial x}$ (ION PERTURBED DENSITY)

$\bar{E} = -\nabla \Phi$ (ELECTROSTATIC)

elec: $\delta n_e \sim n_0\left(1 - e^{-e\Phi/kT_e}\right) \approx \frac{e n_0}{kT_e}\tilde{\Phi}$

Langmuir's "Slow" Ion Oscillations

$$\nabla \cdot \bar{E} = -\nabla^2 \Phi = \rho/\epsilon_0 = \frac{e n_0}{\epsilon_0}\frac{\partial \xi_i}{\partial x} + \frac{e n_0}{kT_e}\tilde{\Phi}$$

BUT $m_p \frac{\partial^2 \xi_i}{\partial t^2} = e E_x = -e\frac{\partial \Phi}{\partial x}$

SO $m_p \frac{\partial^2}{\partial t^2}\frac{\partial \xi_i}{\partial x} = -e\frac{\partial^2 \Phi}{\partial x^2}$

$$\frac{\partial^2 \Phi}{\partial x^2} = -\frac{e n_0}{\epsilon_0}\frac{\partial \xi_i}{\partial x} - \underbrace{\frac{e n_0}{kT_e}}_{\to \frac{1}{\lambda_D^2}}\tilde{\Phi}$$

ASSUME TRAVELING PLANE WAVES: $(\tilde{\xi}_i, \tilde{\Phi}) \sim e^{-i(\omega t - kx)}$ ← *** !

THEN $\frac{\partial^2}{\partial t^2} \Rightarrow -\omega^2$ $\quad \frac{\partial^2}{\partial x^2} \Rightarrow -k^2$ $\quad \frac{\partial}{\partial x} \to ik$

$$-k^2\tilde{\Phi} = -ik\frac{e n_0}{\epsilon_0}\tilde{\xi}_i - \frac{e n_0}{kT_e}\tilde{\Phi}$$

$$-i\omega^2 k\tilde{\xi}_i = \frac{e}{m_p}k^2\tilde{\Phi}$$

$$-ik\tilde{\xi}_i = \frac{e}{m_p}\frac{k^2}{\omega^2}\tilde{\Phi}$$

$$-k^2\tilde{\Phi} = \left(\frac{e^2 n_0}{\epsilon_0 m_p}\right)\frac{k^2}{\omega^2}\tilde{\Phi} - \frac{\tilde{\Phi}}{\lambda_D^2} \Rightarrow \omega = \frac{k\omega_{pi}\lambda_D}{\sqrt{1 + k^2\lambda_D^2}}$$

APPH 6101 Plasma Physics I
Homework 2: Due 18 September, 2023.

Homework #2

Question 1

Irving Langmuir graduated from Columbia University in 1903 and received the Nobel Prize in chemistry for his investigations of surface chemistry. (Langmuir was the first to observe stable films of atoms on tungsten and platinum and formulated the first general theory of adsorbed films.) In plasma physics, he is well known as the individual who characterized ionized matter as "plasma" and for inventing the "Langmuir probe." In one of the most important early papers of plasma physics, Langmuir and his younger colleague, Lewi Tonks, were the first to observe and identify plasma oscillations (also called "Langmuir" oscillations). This was reported in *Physical Review*, **33**, p. 195 (1929). (This paper can be downloaded using Columbia University's subscription to *Physical Review*.)

http://sites.apam.columbia.edu/courses/apph6101x/Homework-2.pdf

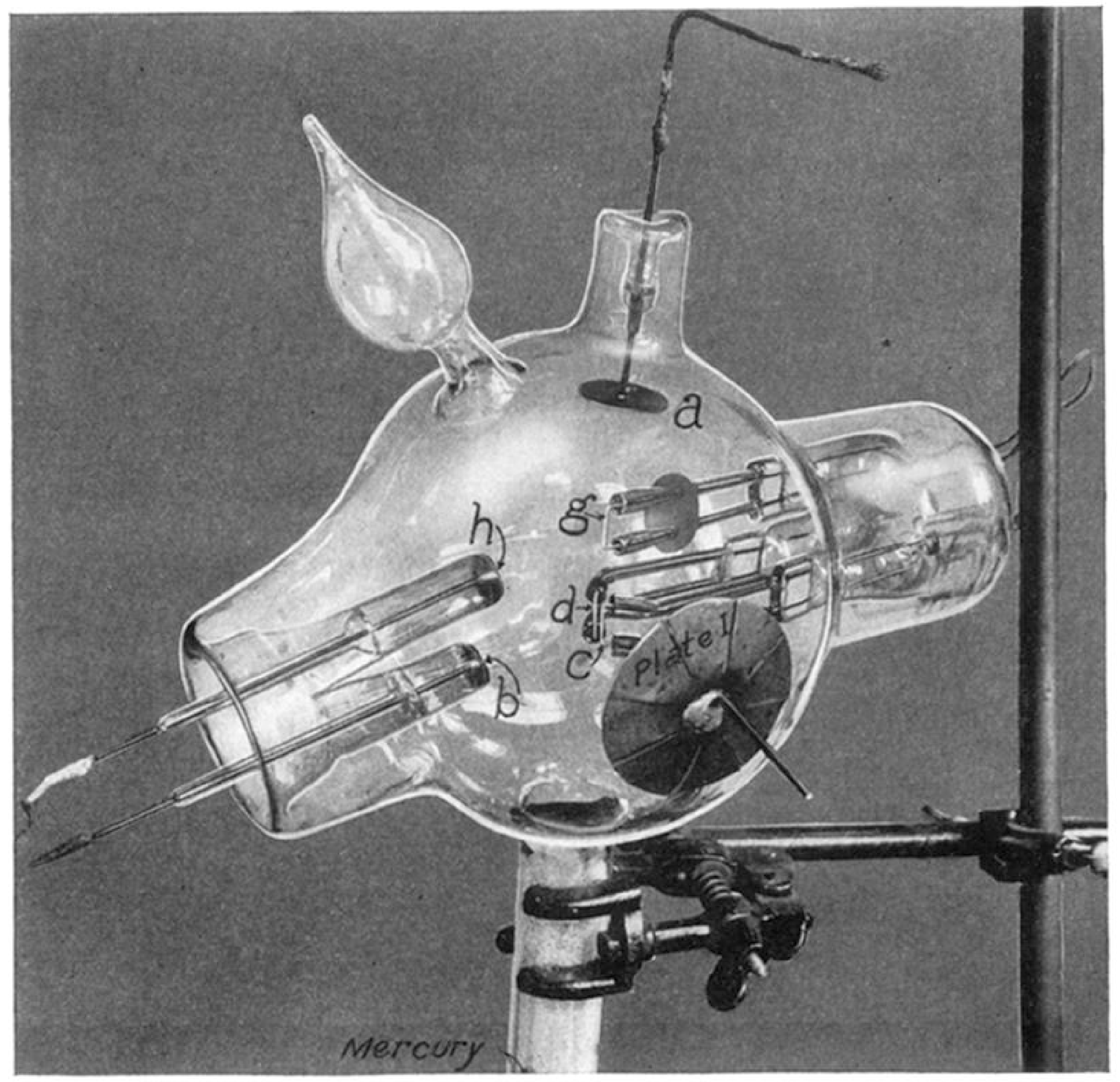


Fig. 1. Experimental tube.

Figure 1: Mercury plasma device (or gas filled lamp) built by Irving Langmuir (1881-1957, top) and Lewi Tonks (1897-1971, bottom) and used to observe plasma oscillations. Probes "b" and "h" are Langmuir probes; "c", "d", and "g" are electron-emitting tungsten filaments; "a" is a positively-biased anode; and "plate 1" is one of three external probes used to measure collective oscillations.

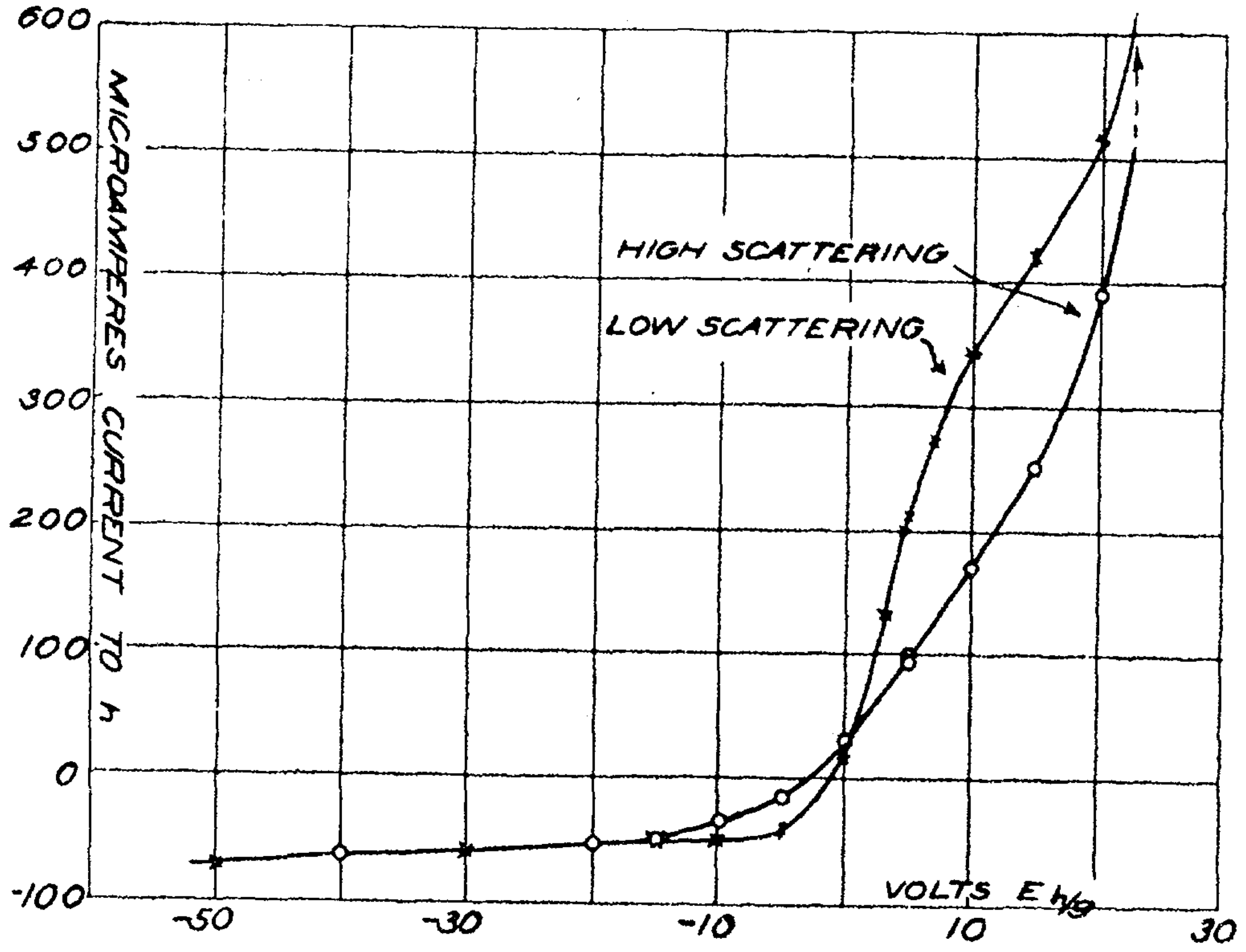


Fig. 8. Volt-ampere characteristics of a collector in the primary electron beam.

Figure 2: Measured current to a "Langmuir probe" as a function of probe bias.

Next: Chapter 3
Mechanics of Charged Particles

- Charged particle motion in inhomogeneous, static and slowly-varying electric and magnetic fields

$$m\dot{\mathbf{v}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})\,, \tag{3.1}$$

$$\mathbf{r} = \mathbf{R}(t) + \epsilon\,\boldsymbol{\rho}(\mathbf{R}, \mathbf{U}, t, \gamma),$$

$$\mathbf{v} = \mathbf{U}(t) + \mathbf{u}(\mathbf{R}, \mathbf{U}, t, \gamma),$$

Key concept: gyro-averaging