Lecture14:

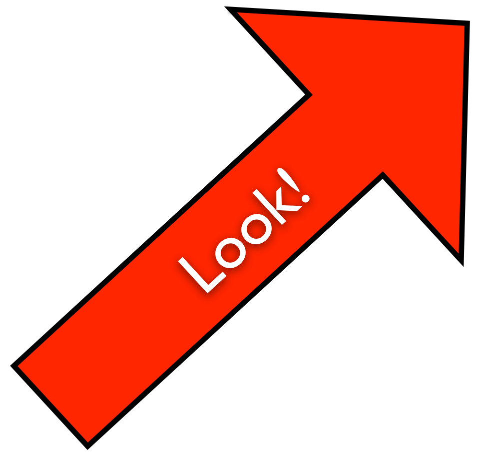
Plasma Physics 1

APPH E6101x
Columbia University

This Lecture: Inhomogeneous Plasma

- Trivelpiece-Gould Modes (cylindrical plasma, surface waves, $\omega < \omega_p$)
- Langmuir probes: Child-Langmuir Law, Bohm Sheath Criteria
- Langmuir probes in magnetized plasma

JOURNAL OF APPLIED PHYSICS VOLUME 30, NUMBER 11 NOVEMBER, 1959



Space Charge Waves in Cylindrical Plasma Columns*

A. W. TRIVELPIECE† AND R. W. GOULD
California Institute of Technology, Pasadena, California

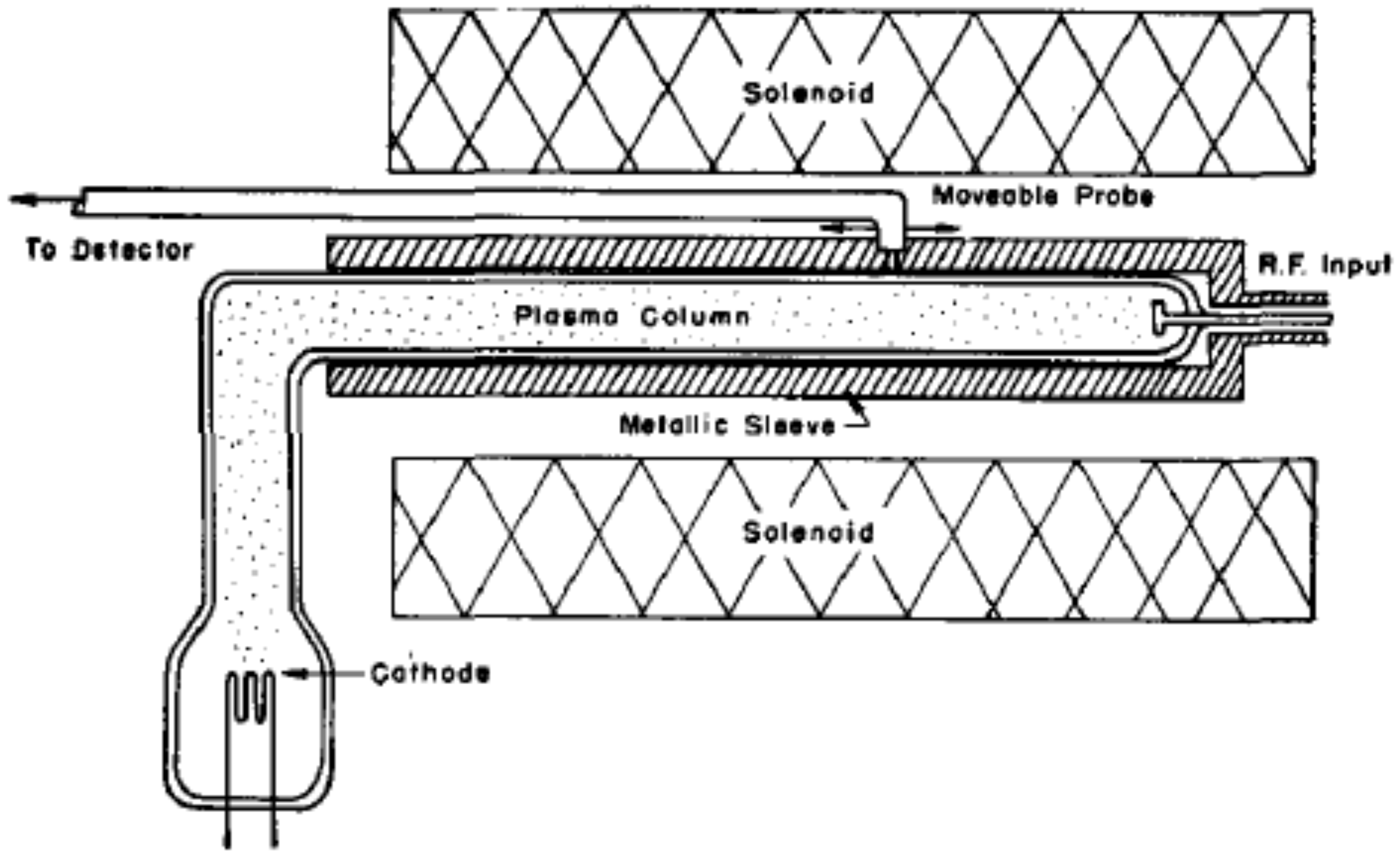


FIG. 10. Schematic of apparatus to measure phase and attenuation characteristics of space charge waves in a plasma.

TABLE I. Pertinent dimensions of experimental apparatus used in space charge wave propagation experiment.

Diameter of plasma	0.328 in.
Diameter of tube containing plasma	0.410 in.
Diameter of slotted wave guide	0.410 and 0.750 in.
Length of plasma column	25 cm
Signal frequency range	10 to 4000 mc
Cyclotron frequency range	0 to 5000 mc
Plasma frequency range	500 to 5000 mc
Temperature of mercury in tube	300±0.1°K
Empty wave guide cutoff frequency (approx)	25 000 mc
Pressure of mercury at 300°K (approx)	2 microns
Mean free path of plasma electrons (approx)	5 cm

Assuming wave solutions $\exp[j(\omega t-\beta z)]$

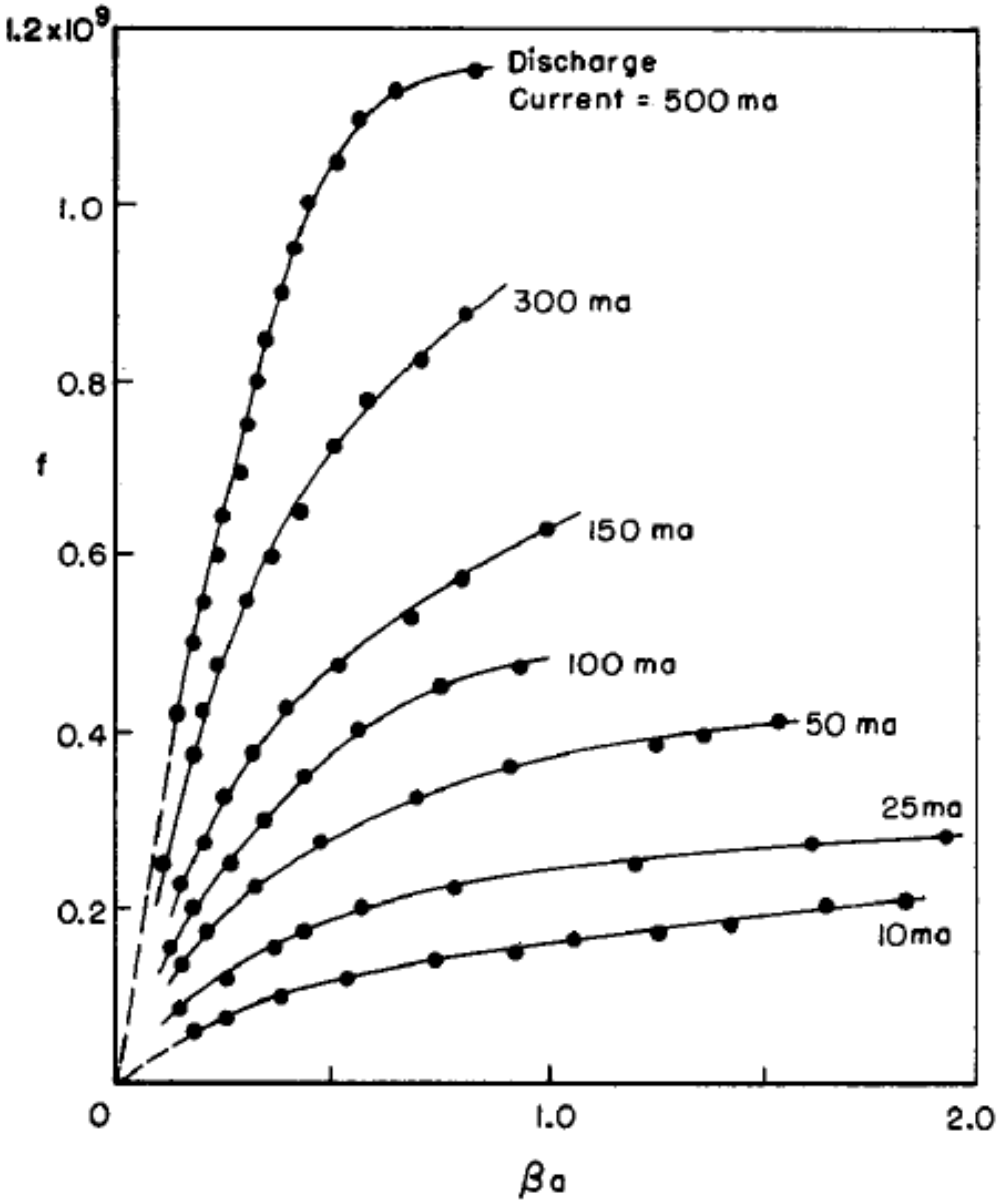


FIG. 13. Measured phase characteristics of plasma space charge waves for no magnetic field for $a=0.52$ cm, $b=0.62$ cm, $K=4.6$.

TG Modes: Low Frequency Surface Waves

INSIDE PLASMA $(r < a)$:

$$\bar{E} = -\nabla \Phi \qquad \nabla \cdot (\epsilon_0 K \bar{E}) = 0$$

$$\Rightarrow \quad \nabla^2 \Phi = 0$$

$$K = 1 - \frac{\omega_p^2}{\omega^2}$$

Plasma Dielectric ~ constant

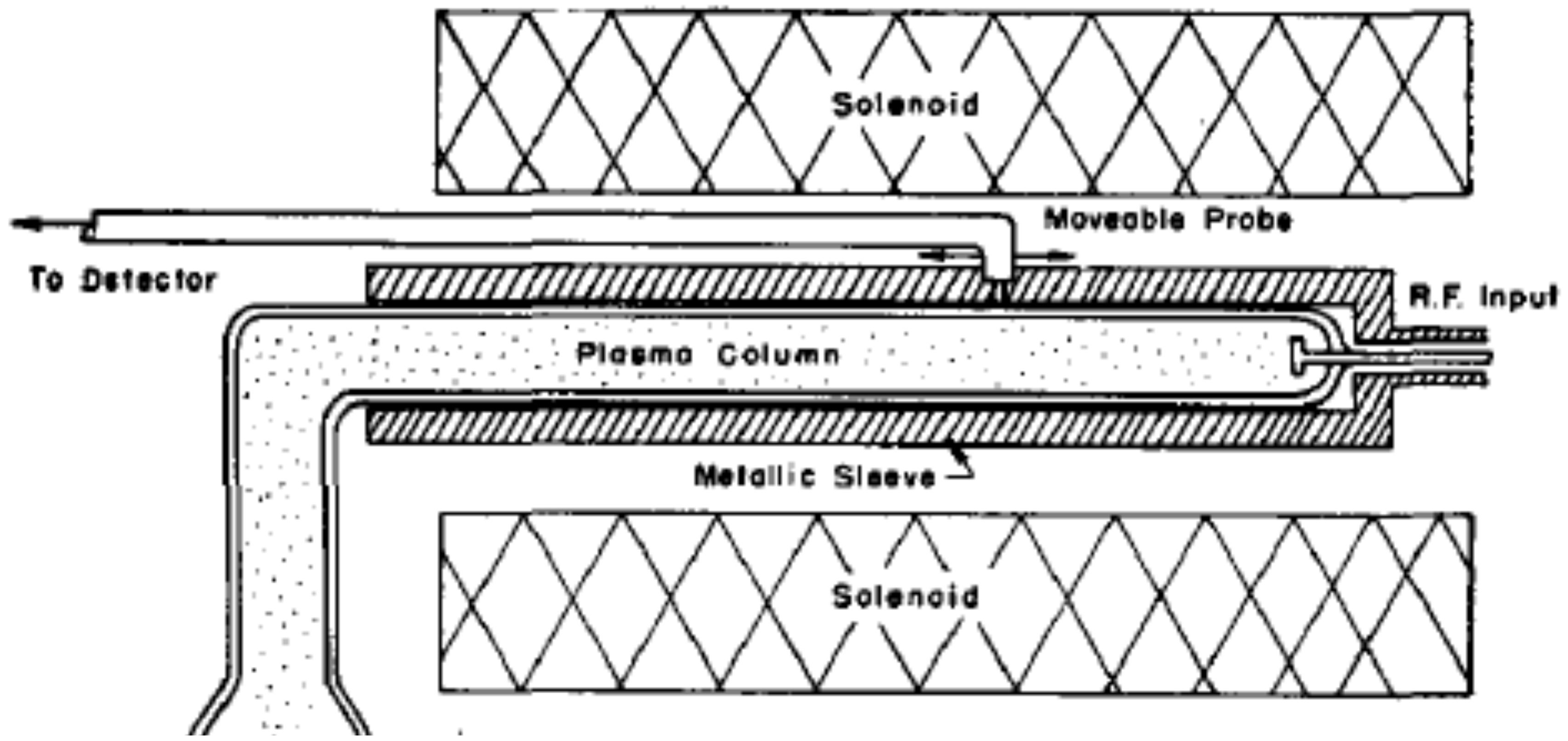
OUTSIDE PLASMA $(r > a)$:

$$\bar{E} = -\nabla \Phi \qquad \nabla \cdot (\epsilon_0 \bar{E}) = 0$$

$$\nabla^2 \Phi = 0$$

BOUNDARY CONDITIONS $(r = a)$:

$$\hat{n} \cdot (\epsilon_0 K \bar{E}_{in} - \epsilon_0 \bar{E}_{out}) = 0$$

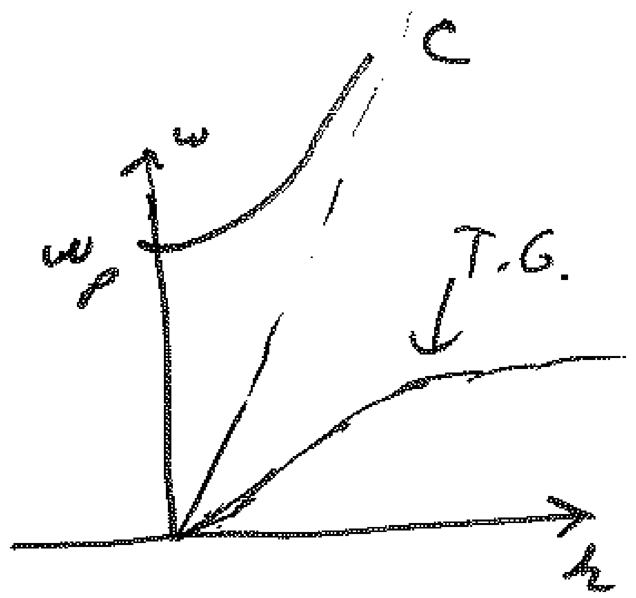


TG Modes: Low Frequency Surface Waves

Boundary Conditions $(r=a)$:

$$\hat{n}\cdot\left(\epsilon_0 K \bar{E}_{IN} - \epsilon_0 \bar{E}_{OUT}\right) = 0$$

$$\Phi_{IN} = \Phi_{OUT}$$



$$\nabla^2 \Phi = 0 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) - h^2 \Phi = 0$$

$$\Phi(r,z,t) \propto e^{-j(\omega t - hz)}$$

$$\left.\begin{array}{ll} \Phi(r) \sim I_0(hr) & \text{inside} \\ K_0(hr) & \text{outside} \end{array}\right\} r=a$$

$$\left(1-\frac{\omega_p^2}{\omega^2}\right) I_0' C_1 = K_0' C_2$$

$$I_0 C_1 = K_0 C_2$$

$$1-\frac{\omega_p^2}{\omega^2} = \frac{I_0(ha)\,K_0'(ha)}{I_0'(ha)\,K_0(ha)}$$

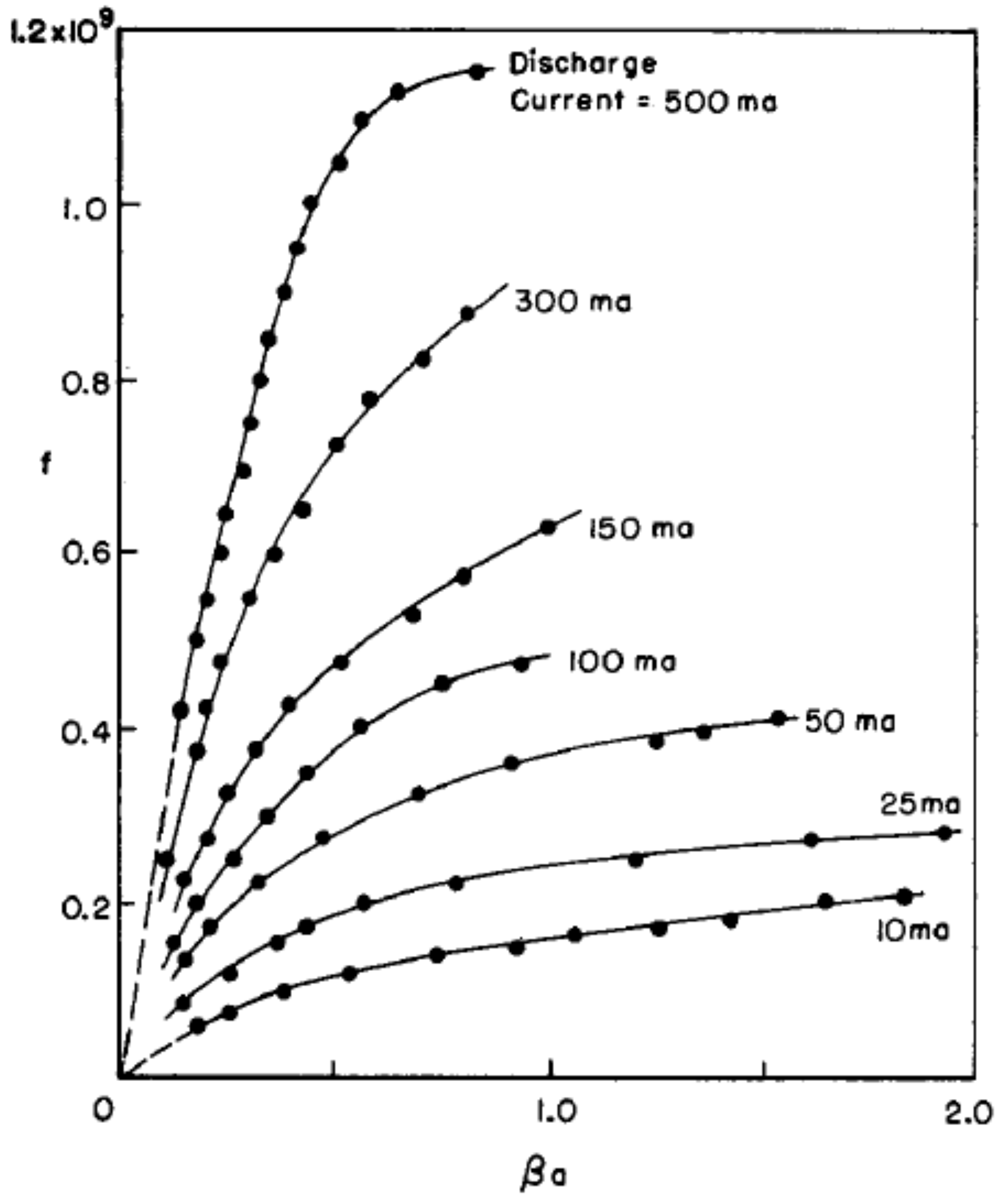


FIG. 13. Measured phase characteristics of plasma space charge waves for no magnetic field for $a=0.52$ cm, $b=0.62$ cm, $K=4.6$.

Understanding Langmuir probe current-voltage characteristics

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(Received 26 February 2007; accepted 14 July 2007)

I give several simple examples of model Langmuir probe current-voltage (I-V) characteristics that help students learn how to interpret real I-V characteristics obtained in a plasma. Students can also create their own Langmuir probe I-V characteristics using a program with the plasma density, plasma potential, electron temperature, ion temperature, and probe area as input parameters. Some examples of Langmuir probe I-V characteristics obtained in laboratory plasmas are presented and analyzed. A few comments are made advocating the inclusion of plasma experiments in the advanced undergraduate laboratory.
[DOI: 10.1119/1.2772282] Am. J. Phys. **75** (12), December 2007

(read this!)

Probes are Frequent Diagnostics for Plasmas

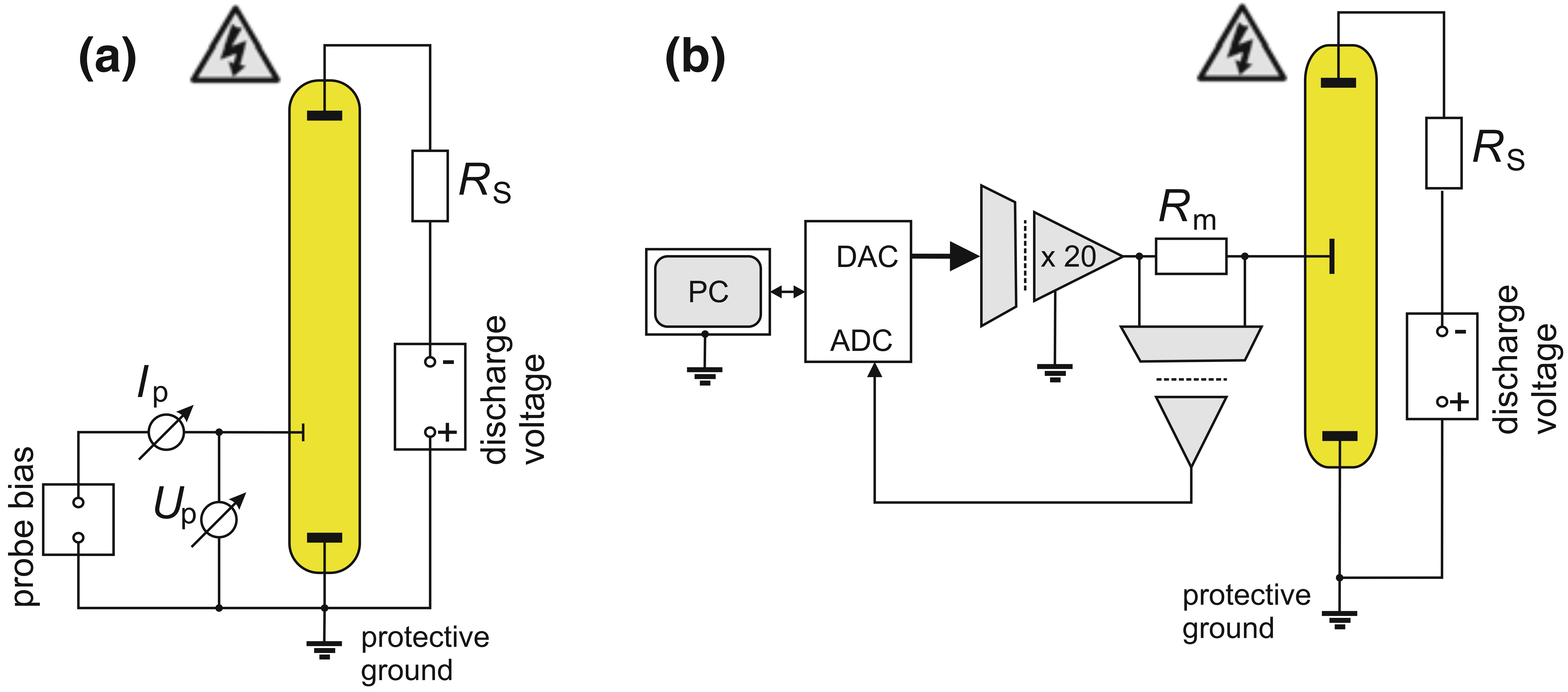

Fig. 7.6 **a** Arrangement for a plane Langmuir probe in a dc-discharge. The probe is biased with a voltage U_p with respect to a proper reference electrode. **b** Computer-controlled Langmuir probe circuit. A digital-to-analog converter (DAC) with subsequent amplifier provides a probe bias, between -100 and $+100$ V. The probe current is measured with a series resistor R_m and an isolation amplifier, and finally A-D converted for numerical processing

Although physicists knew that V_f and V_P were not the same, they thought that the difference was probably small, and in any case, they had no way of either estimating the difference or of measuring the actual plasma potential. Irving Langmuir and Harold Mott-Smith of the General Electric Research Laboratory in the 1920s were the first to provide a quantitative understanding of the difference between V_f and V_P. They developed an experimental method for determining the plasma potential and also showed how it was possible to use the probe (now known as a "Langmuir" probe) to determine the plasma density and the electron temperature as well.[2] Langmuir's method consists of obtaining the current-voltage (I-V) characteristic of the probe as the applied bias voltage V_B, is swept from a negative to a positive potential.

OCTOBER, 1926 PHYSICAL REVIEW VOLUME 28

THE THEORY OF COLLECTORS IN GASEOUS DISCHARGES

By H. M. Mott-Smith and Irving Langmuir

Abstract

When a cylindrical or spherical electrode (collector) immersed in an ionized gas is brought to a suitable potential, it becomes surrounded by a symmetrical space-charge region or "sheath" of positive or of negative ions (or electrons). Assuming that the gas pressure is so low that the proportion of ions which collide with gas molecules in the sheath is negligibly small, the current taken by the collector can be calculated in terms of the radii of the collector or sheath, the distribution of velocities among the ions arriving at the sheath boundary and the total drop of potential in the sheath. The current is independent of the actual distribution of potential in the sheath provided this distribution satisfies certain conditions.

The curves of Fig. 4 illustrate the characteristics of the three forms of collectors with a Maxwellian distribution.

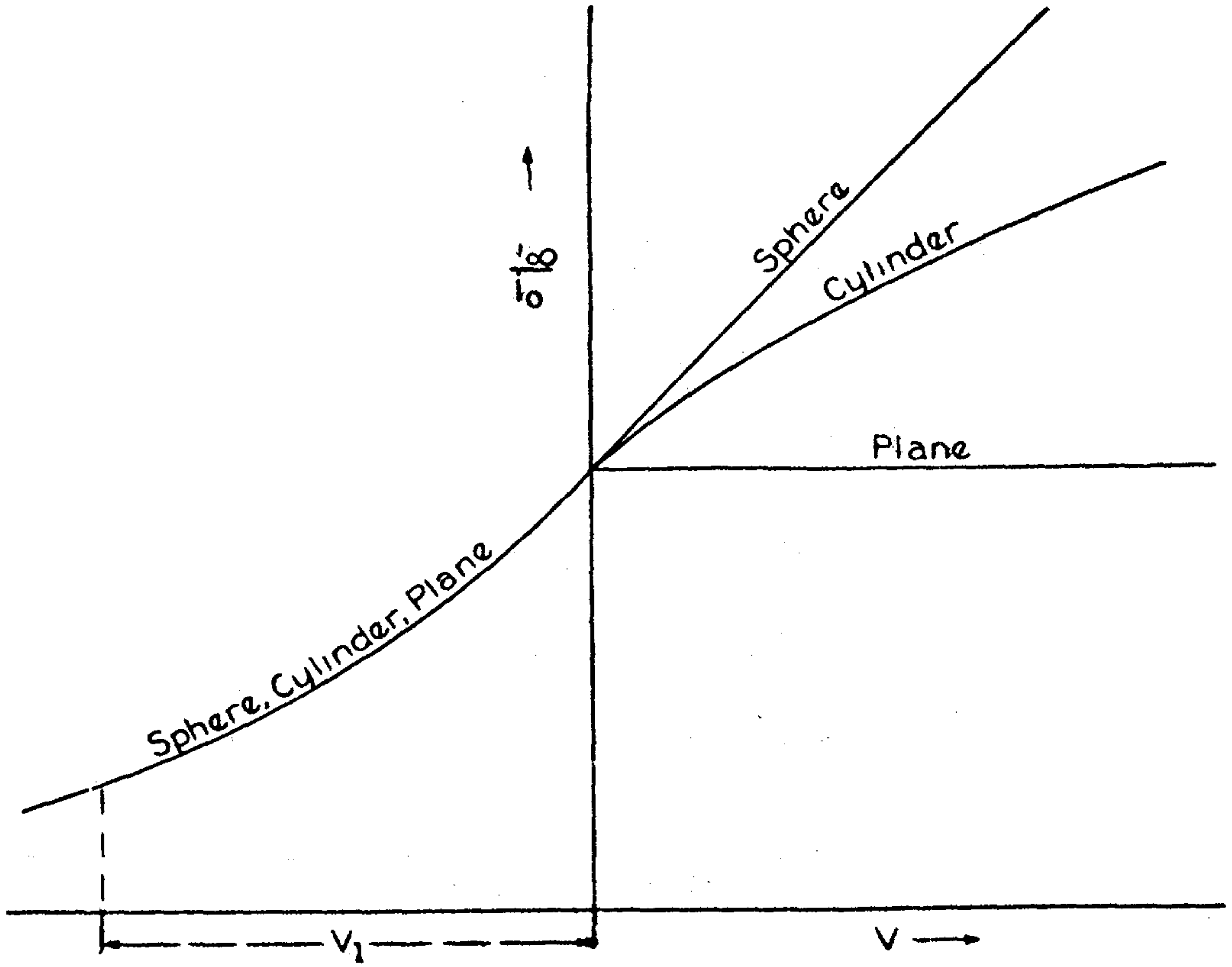

FIG. 4.

II. MODEL LANGMUIR PROBE CURRENT-VOLTAGE CHARACTERISTICS

A. Ion and electron currents to a Langmuir probe

1. The ion current

$$I_i(V_B) = \begin{cases} -I_{is}\exp[e(V_P - V_B)/kT_i], & V_B \geq V_P, \\ -I_{is}, & V_B < V_P, \end{cases} \qquad (1)$$

$$I_{is} = \frac{1}{4} e n_i v_{i,th} A_{\mathrm{probe}},$$

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$$I_{is} = \frac{1}{4} e n_i v_{i,th} A_{\text{probe}},$$

!!!

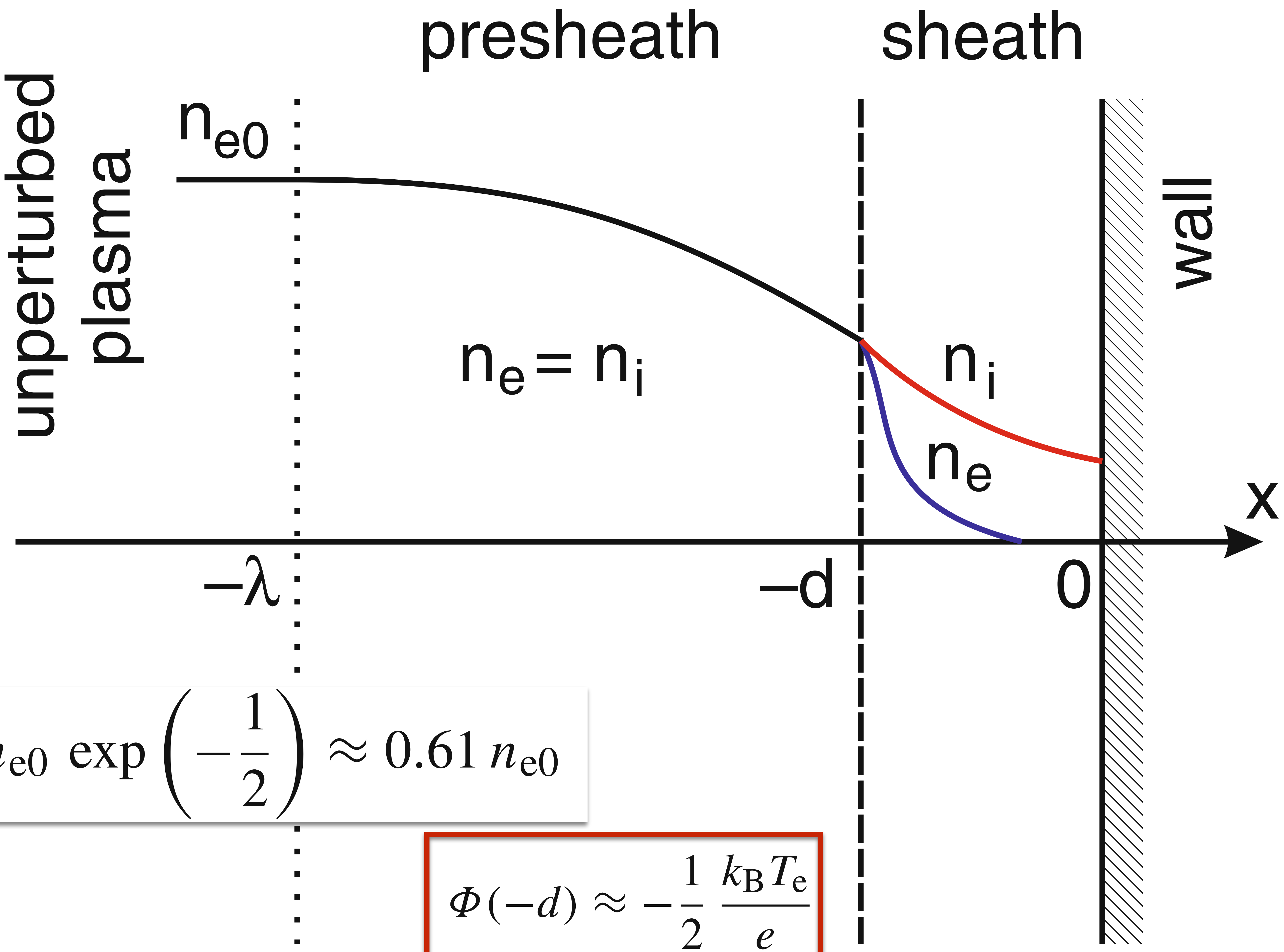
ing area. When $T_e \gg T_i$,[11] the ion saturation current is not determined by the ion thermal speed, but rather is given by the Bohm ion current[3,4,12]

$$I_{is} = I_{\text{Bohm}} = 0.6 e n_i \sqrt{\frac{kT_e}{m_i}} A_{\text{probe}}. \quad (3)$$

The fact that the ion current is determined by the electron temperature when $T_e \gg T_i$ is counterintuitive and requires some explanation. The physical reason for the dependence $I_{is} \sim (kT_e/m_i)^{1/2}$ has to do with the formation of a sheath around a negatively biased probe.[12,13] If an electrode in a plasma has a potential different from the local plasma potential, the electrons and ions distribute themselves spatially around the electrode in order to limit, or shield, the effect of this potential on the bulk plasma. A positively biased electrode acquires an electron shielding cloud surrounding it, while a negatively biased electrode acquires a positive space charge cloud. For a negatively biased electrode, the characteristic shielding distance of the potential disturbance is the electron Debye length[14]

Probe Theory

$$\boxed{v_\mathrm{i}(-d) = v_\mathrm{B}}$$



$$n_\mathrm{i}(-d) = n_\mathrm{e}(-d) = n_\mathrm{e0}\, \exp\left(-\frac{1}{2}\right) \approx 0.61\, n_\mathrm{e0}$$

$$\boxed{\Phi(-d) \approx -\frac{1}{2}\frac{k_\mathrm{B} T_\mathrm{e}}{e}}$$

In the vicinity of a negatively biased probe, both the electron and ion densities decrease as the particles approach the probe, but not at the same rate. The electron density decreases because electrons are repelled by the probe. In contrast, the ions are accelerated toward the probe, and due to the continuity of the current density, the ion density decreases. A positive space charge sheath can form only if the ion density exceeds the electron density at the sheath edge, and for the ion density to decrease more slowly than the electron density, the ions must approach the sheath with a speed exceeding the *Bohm velocity* $u_B = (kT_e/m_i)^{1/2}$.[13,15] To achieve this speed, the ions must acquire an energy corresponding to a potential drop of $0.5(kT_e/e)$, which occurs over a long distance in the plasma. The factor of 0.6 in Eq. (3) is due to the reduction in the density of the ions in the *presheath*, which is the region over which the ions are accelerated up to the Bohm speed.

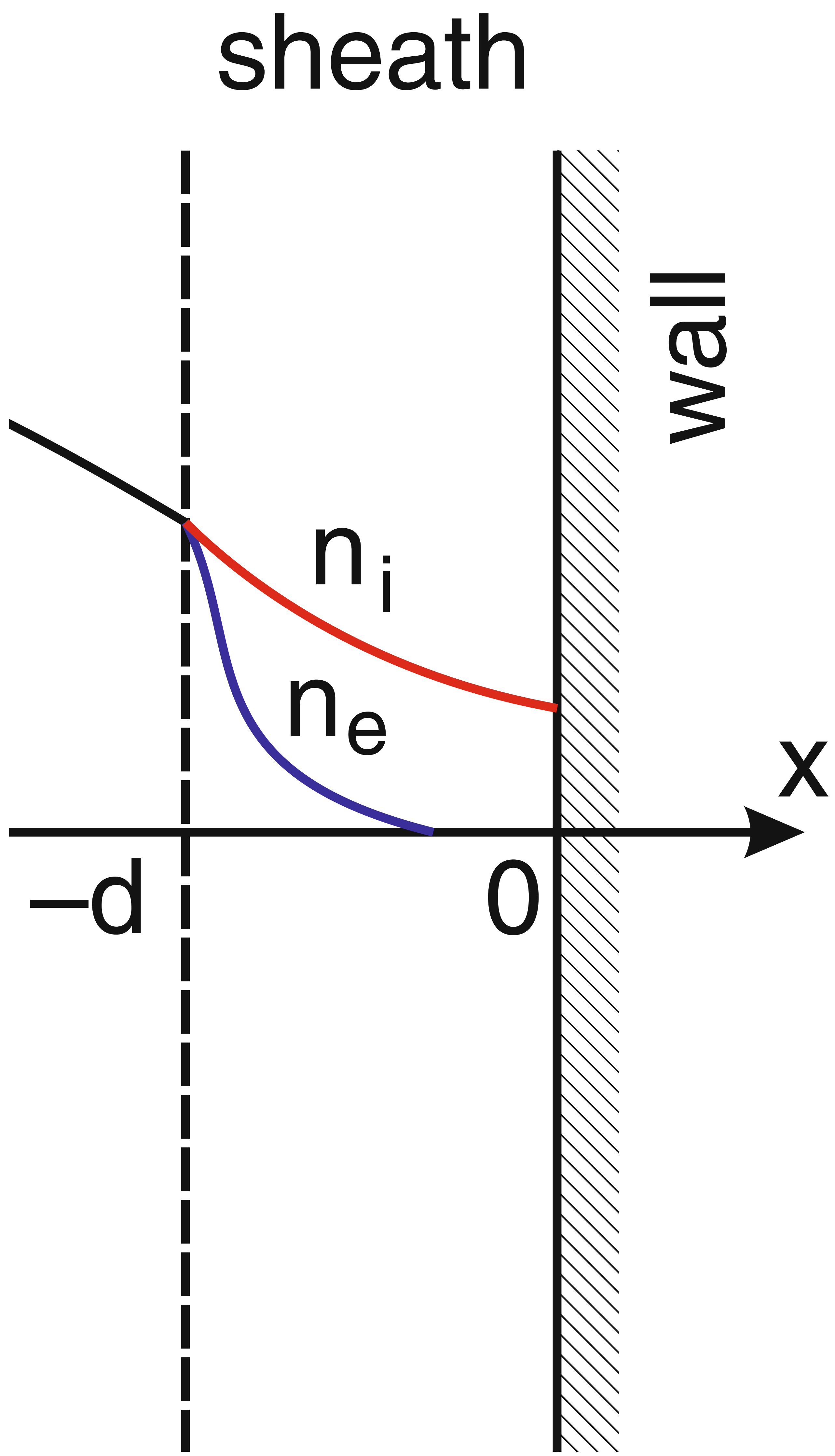
Assuming only ions…

$$\frac{1}{2}m_\mathrm{i}u_\mathrm{i}^2(x) + e\Phi(x) = \frac{1}{2}m_\mathrm{i}u_\mathrm{i}^2(-d) + e\Phi(-d)$$

$$u_\mathrm{i}(x) = \left[u_0^2 - \frac{2e\Phi(x)}{m_\mathrm{i}}\right]^{1/2}$$

$$n_\mathrm{i}(x)u_\mathrm{i}(x) = n_\mathrm{i}(-d)u_0$$

$$n_\mathrm{i}(x) = n_\mathrm{i}(-d)\left[1 - \frac{2e\Phi(x)}{m_\mathrm{i}u_0^2}\right]^{-1/2} \approx \left(-\frac{2e\Phi(x)}{m_\mathrm{i}u_0^2}\right)^{-1/2}$$



Assuming only ions...

$$\Phi'' \approx -\frac{en_{\mathrm{i}}(-d)}{\varepsilon_0}\left(-\frac{2e\Phi(x)}{m_{\mathrm{i}}u_0^2}\right)^{-1/2}$$

$$\frac{1}{2}\left[\Phi'^2(x) - \Phi'^2(-d)\right] = \frac{en_{\mathrm{i}}(-d)u_0}{\varepsilon_0}\left(\frac{2m_{\mathrm{i}}}{e}\right)^{1/2} \times \left\{[-\Phi(x)]^{1/2} - [-\Phi(-d)]^{1/2}\right\}$$

$$en_{\mathrm{i}}(-d)u_0 = j_{\mathrm{i}}$$

Assuming only ions... Child-Langmuir Law

$$\Phi'(x) = 2\left(\frac{m_\mathrm{i}}{2e}\right)^{1/4}\left(\frac{j_\mathrm{i}}{\varepsilon_0}\right)^{1/2}[-\Phi(x)]^{1/4}$$

$$\frac{4}{3}\Phi^{3/4} = 2\left(\frac{m_\mathrm{i}}{2e}\right)^{1/4}\left(\frac{j_\mathrm{i}}{\varepsilon_0}\right)^{1/2}(x+d)$$

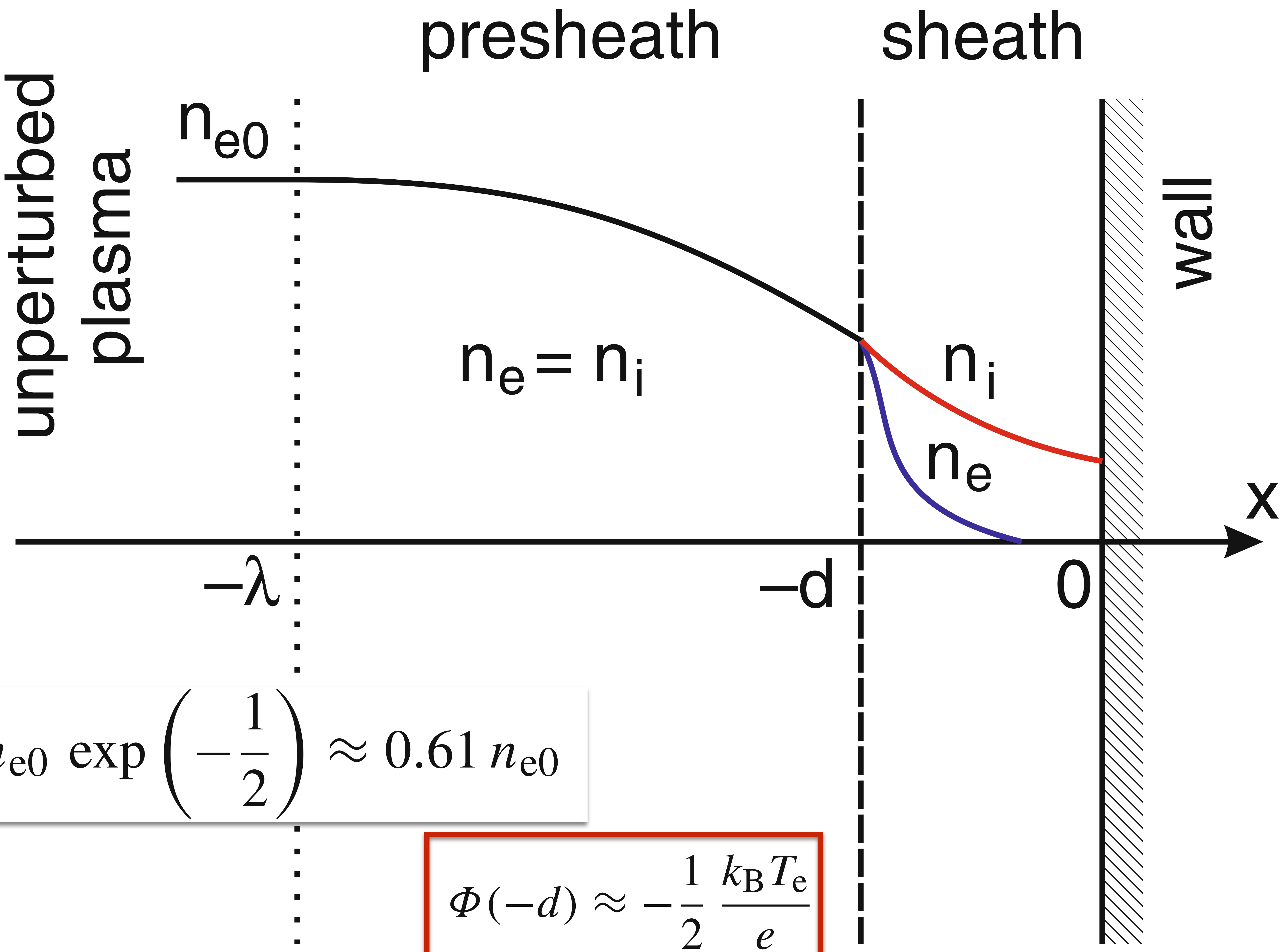
$$\Phi(x) = \left(\frac{3}{2}\right)^{4/3}\left(\frac{m_\mathrm{i}}{2e}\right)^{1/3}\left(\frac{j_\mathrm{i}}{\varepsilon_0}\right)^{2/3}(x+d)^{4/3}$$

$$U^{3/2} = \frac{9}{4}\left(\frac{m_\mathrm{i}}{2e}\right)^{1/2}\left(\frac{j_\mathrm{i}}{\varepsilon_0}\right)d^2$$

$$\boxed{j_\mathrm{i} = \frac{4}{9}\varepsilon_0\left(\frac{2e}{m_\mathrm{i}}\right)^{1/2}\frac{U^{3/2}}{d^2}}$$

Probe Theory

$$\boxed{v_\mathrm{i}(-d) = v_\mathrm{B}}$$



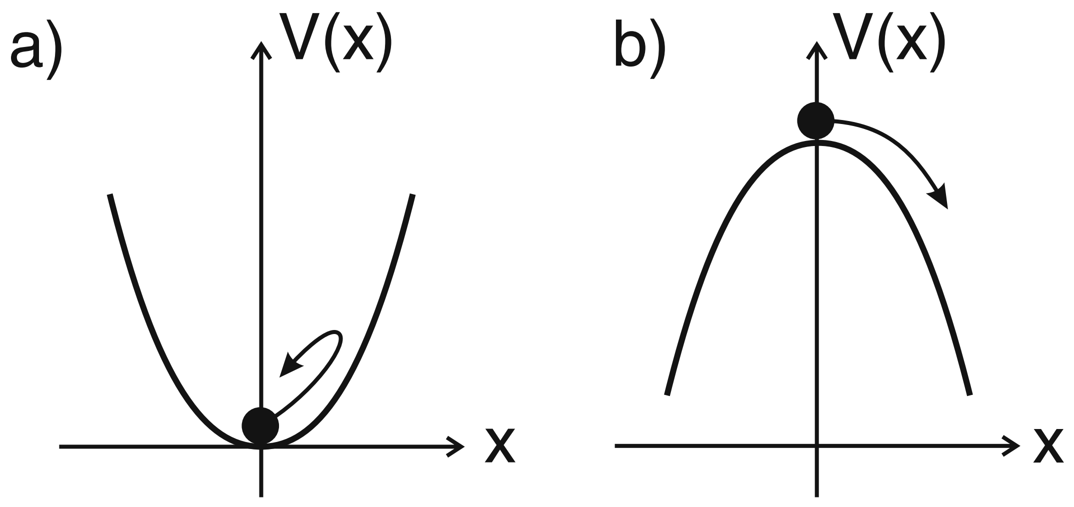
$$n_\mathrm{i}(-d) = n_\mathrm{e}(-d) = n_\mathrm{e0}\,\exp\left(-\frac{1}{2}\right) \approx 0.61\,n_\mathrm{e0}$$

$$\boxed{\Phi(-d) \approx -\frac{1}{2}\,\frac{k_\mathrm{B}T_\mathrm{e}}{e}}$$

Bohm Sheath Criterion

Bohm criterion represents a *sound barrier*

Fig. 7.2 (**a**) Stable mechanical equilibrium, $V''(0) > 0$. (**b**) Unstable mechanical equilibrium, $V''(0) < 0$



Sagdeev Potential / Pseudopotential

$$\frac{d^2\Phi}{dx^2} = -\frac{\rho}{\varepsilon_0} = f(\Phi) = -\frac{d\Psi}{d\Phi}$$

Table 7.1 Analogy between mechanical stability and sheath stability

Mechanical stability		Sheath stability	
Particle trajectory	$x(t)$	Electric potential distribution	$\Phi(x)$
Time	t	Space coordinate	x
Mechanical potential	$V(x)$	Pseudopotential	$\Psi(\Phi)$

$$f(\Phi) = \frac{e n_e(-d)}{\varepsilon_0}\left[\exp\left(\frac{e\Phi}{k_B T_e}\right) - \left(1 - \frac{2e\Phi}{m_i u_0^2}\right)^{-1/2}\right]$$

$$m\frac{d^2x}{dt^2} = -\frac{dV}{dx}$$

$$\boxed{m_i u_0^2 \geq k_B T_e}$$

$$-\left.\frac{df}{d\Phi}\right|_{\Phi=0} = \frac{e}{k_B T_e} - \frac{e}{m_i u_0^2} \leq 0$$

B. Examples of Langmuir probe characteristic

1. The ideal Langmuir probe characteristic

Table I. Parameters of a typical laboratory plasma used to construct an ideal Langmuir probe volt-ampere characteristic.

Parameter	Symbol	Value	Units
Ion species	Ar^+		
Ion mass	m_i	6.7×10^{-26}	kg
Electron density	n_e	1.0×10^{16}	m^{-3}
Ion density	n_i	1.0×10^{16}	m^{-3}
Electron temperature	T_e	2.0	eV
Ion temperature	T_i	0.1	eV
Plasma potential	V_P	1.0	V
Probe diameter	d_{probe}	3.0	mm

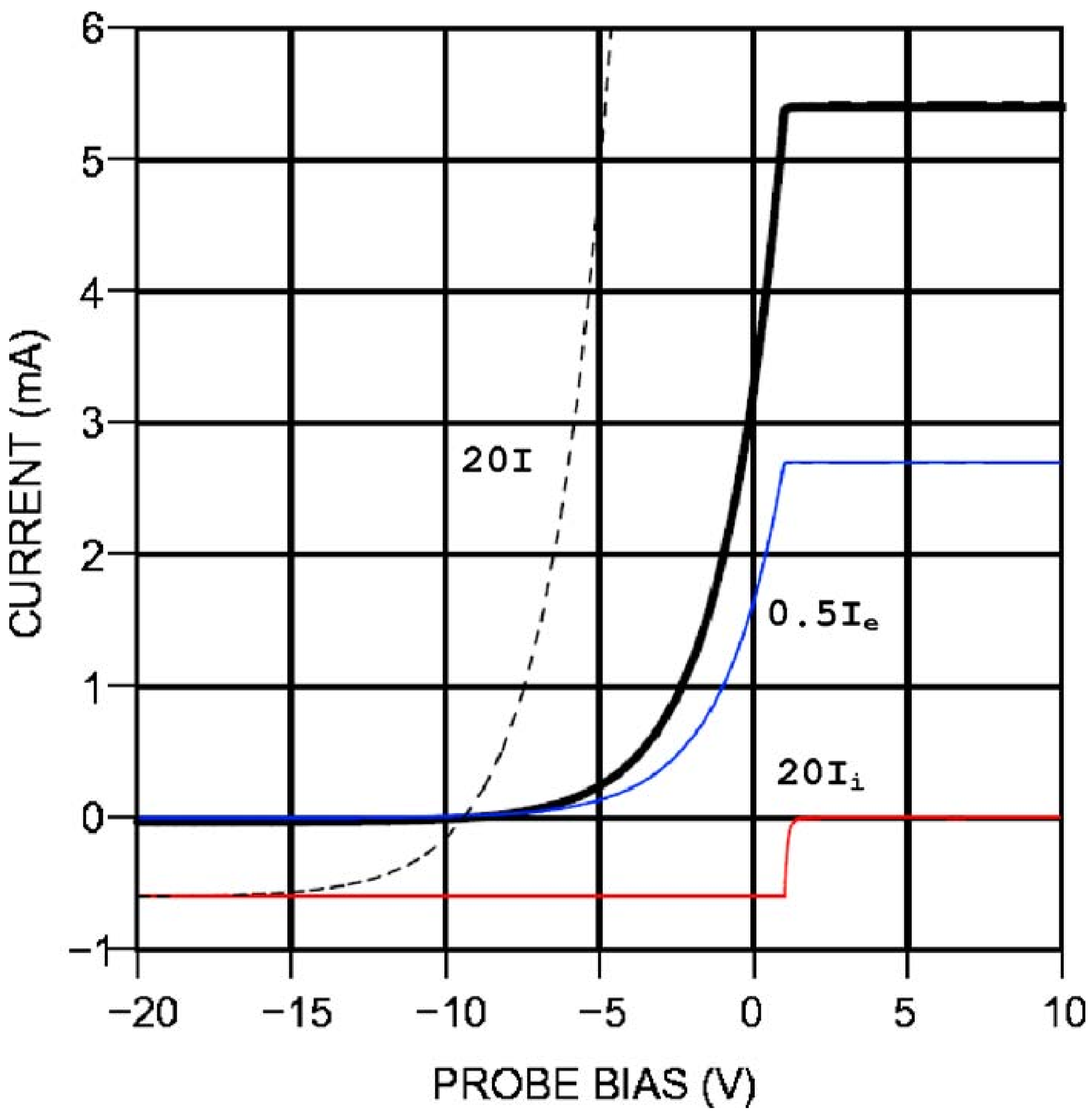


Fig. 2. Ideal Langmuir probe current-voltage characteristic (heavy line) for a model plasma with the parameters listed in Table I. The individual electron and ion currents that are used to construct the full characteristic are also shown. The dotted line is the full probe characteristic magnified by a factor of 20 so that the probe floating potential, V_f (the probe voltage where $I=0$) can be easily determined.

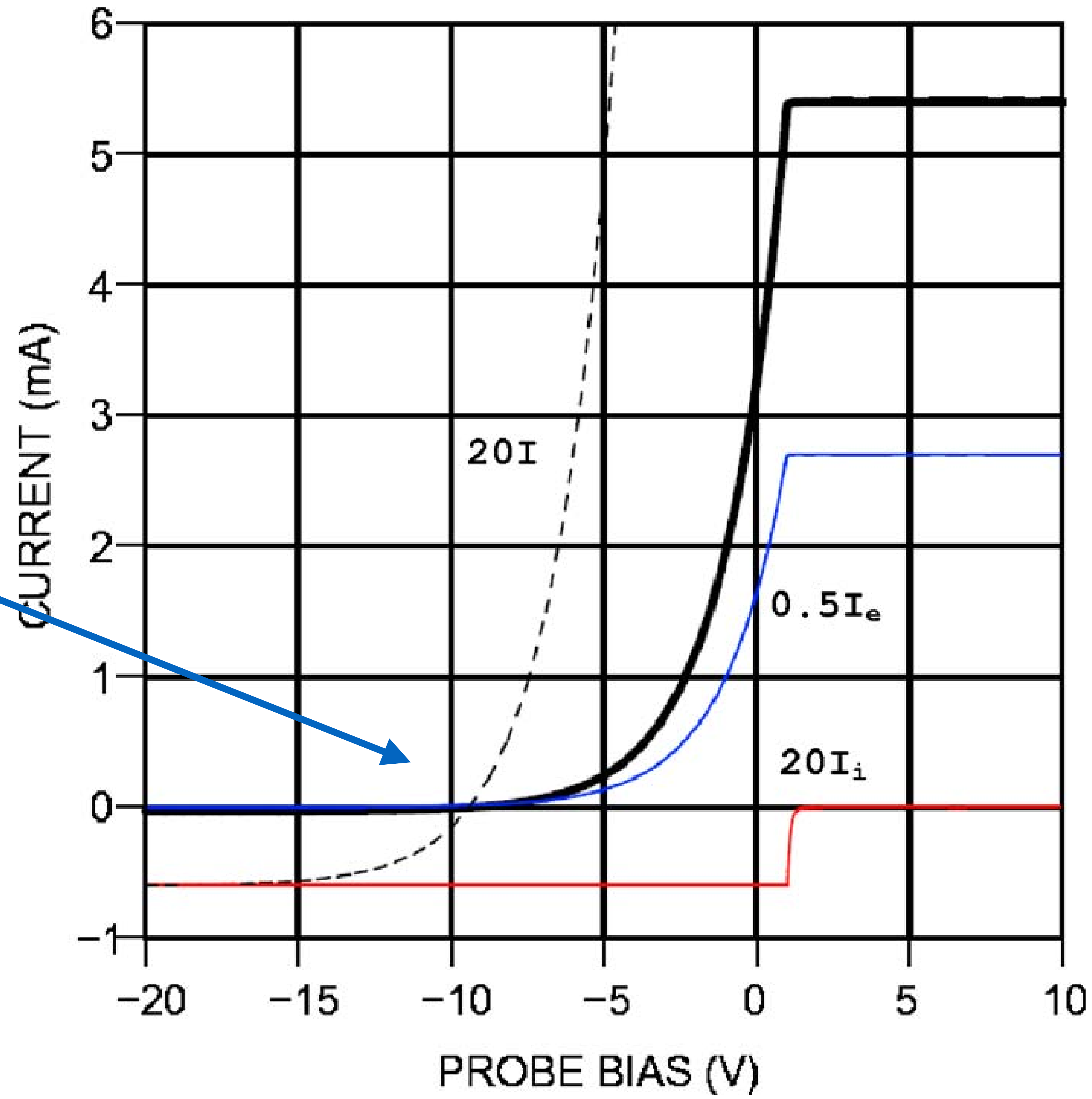
B. Examples of Langmuir probe characteristic

1. The ideal Langmuir probe characteristic

$$I_{es} \exp[e(V_f - V_P)/kT_e] = I_{is},$$

or

$$V_f = V_P + \left(\frac{kT_e}{e}\right)\ln\left(0.6\sqrt{\frac{2\pi m_e}{m_i}}\right).$$



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2. Probe I-V characteristic for a positive ion (+)/negative ion (–) plasma with $m_+ = m_-$ and $T_+ = T_-$

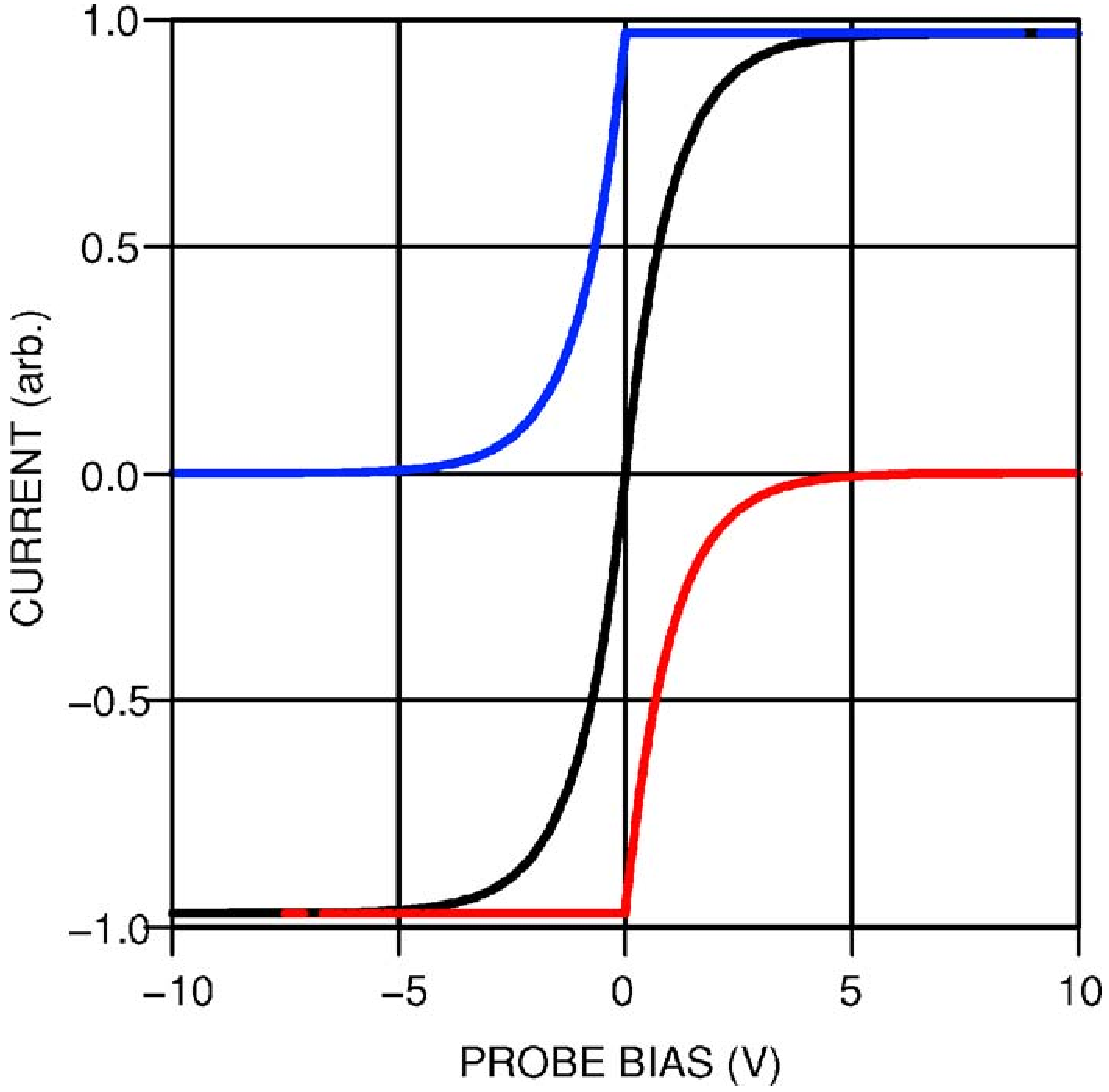
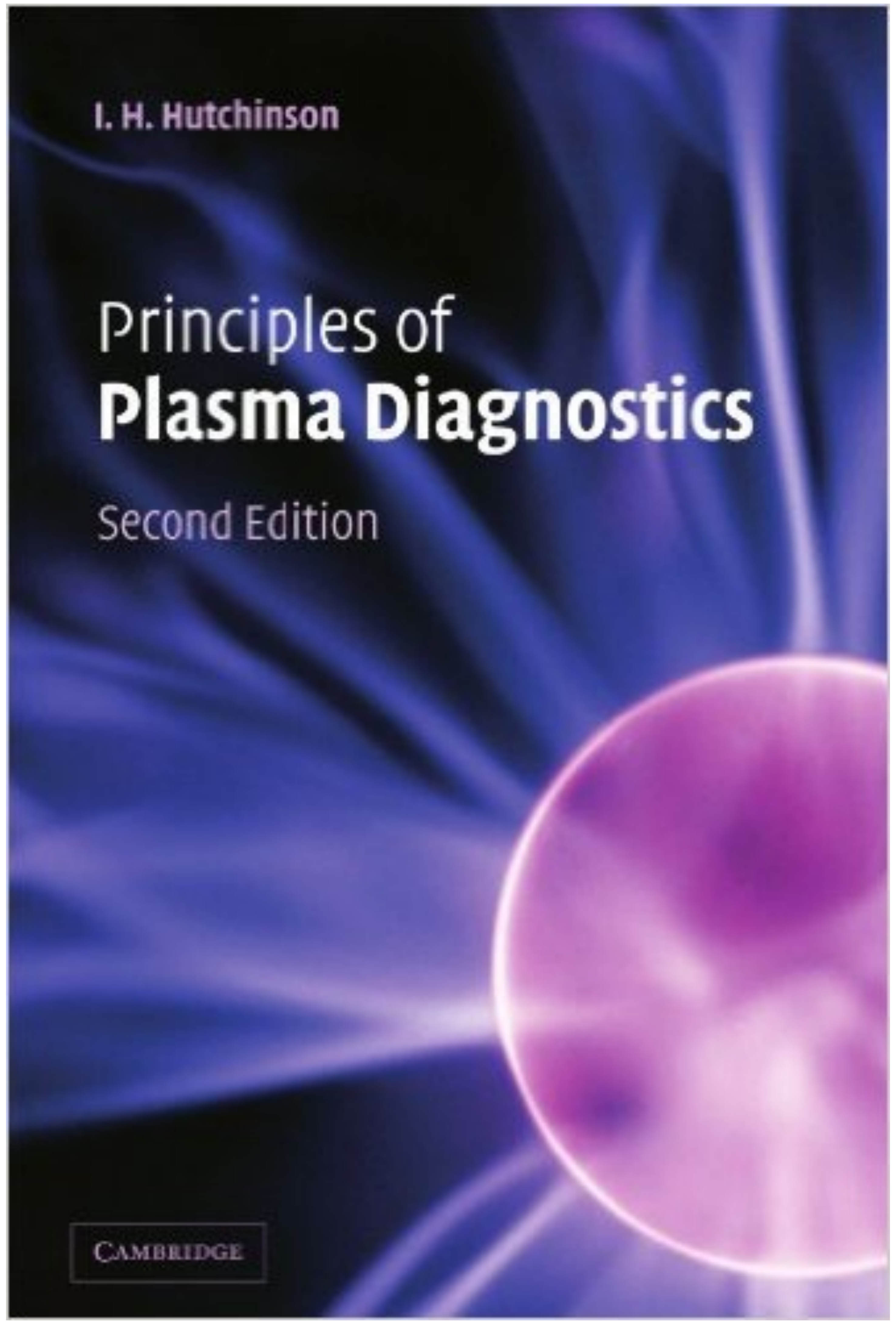


Fig. 3. Langmuir probe I-V characteristic for a plasma with positive and negative ions of equal mass and temperatures. The positive ion and negative ion currents are also shown.

C. Effect of sheath expansion on probe characteristics



Probes in collisionless plasmas without magnetic fields 63

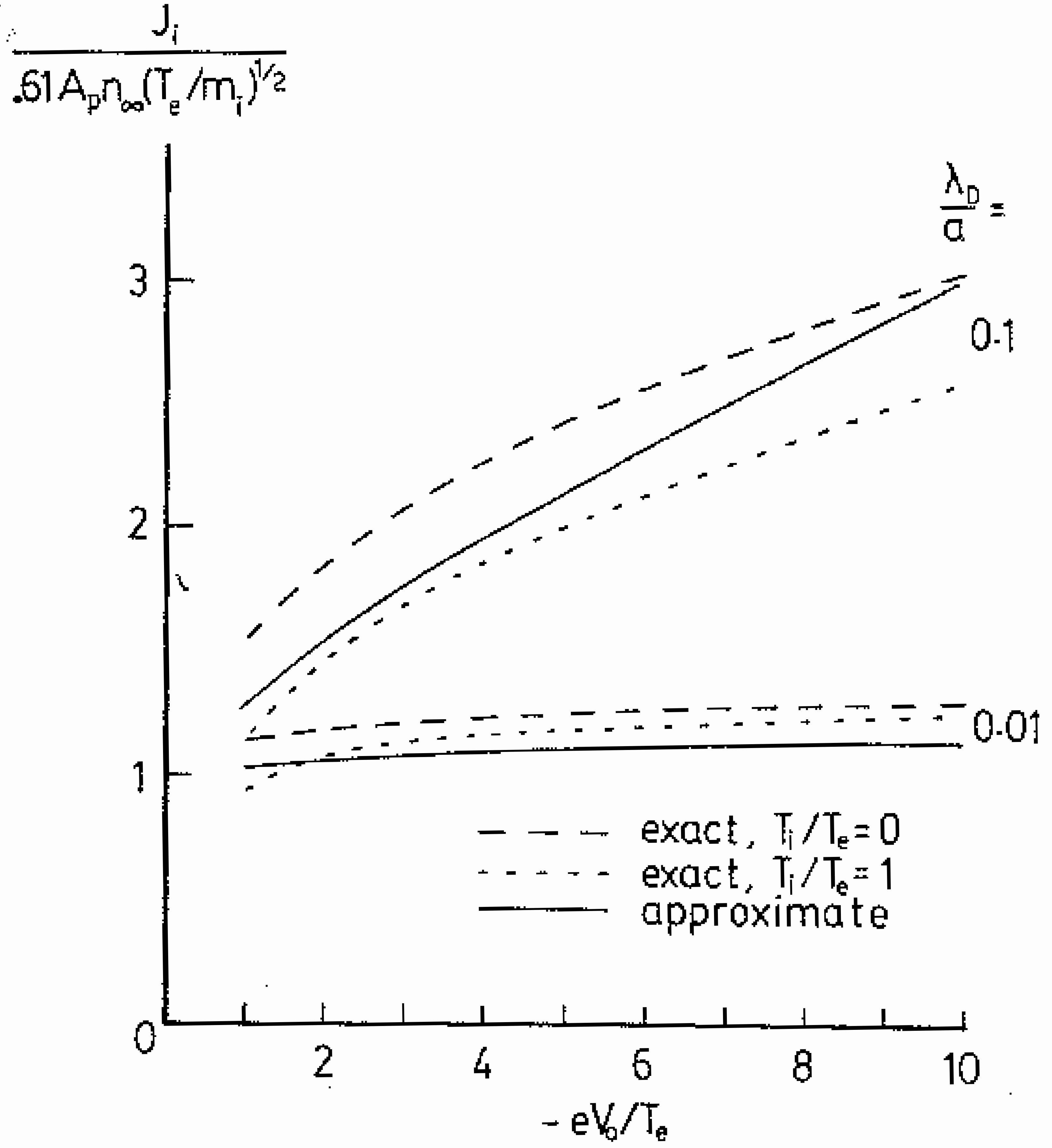
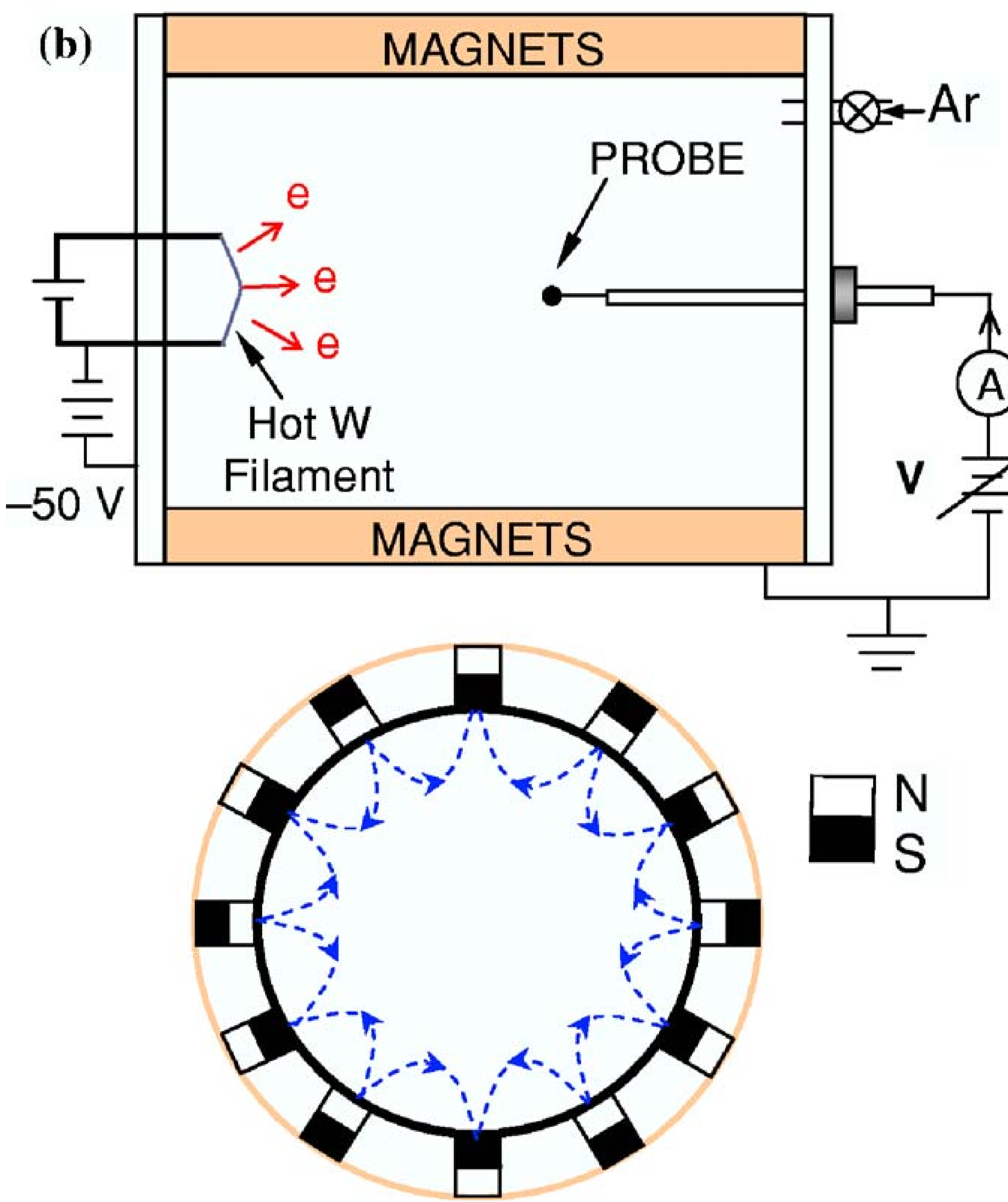


Fig. 3.4. Comparison of the approximate sheath analysis with "exact" numerical results of Laframboise (1966) for a spherical probe.



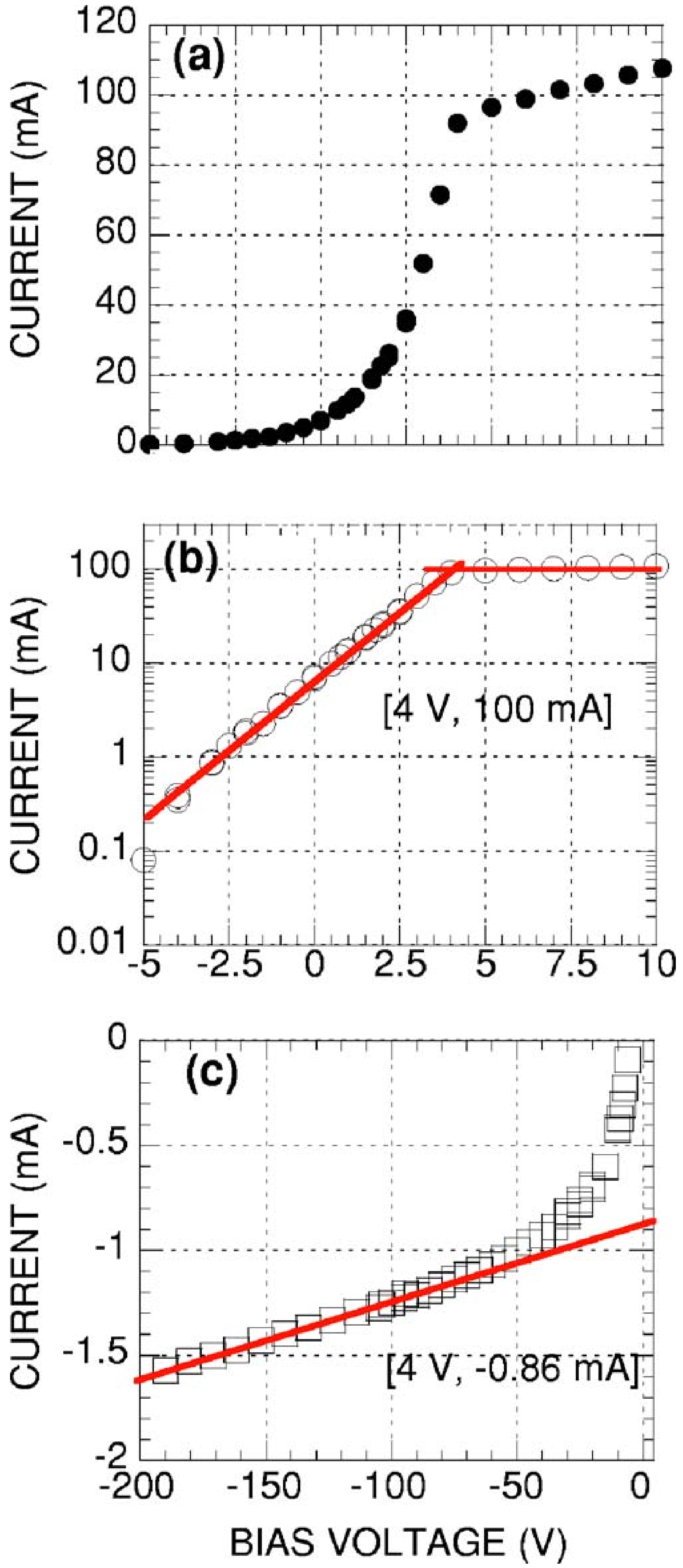


Fig. 5. Langmuir probe I-V characteristic obtained in a multidipole plasma in argon at a pressure of 0.5 mTorr. (a) Electron current. (b) log $I(V_B)$ versus V_B. The semilog plot of the electron current provides a clear demarcation of the plasma potential and electron saturation current. T_e is found from the slope of the exponentially decreasing portion. (c) Expanded scale view of the ion current used to find I_{is}.

B. A positive ion/negative ion plasma in a Q machine

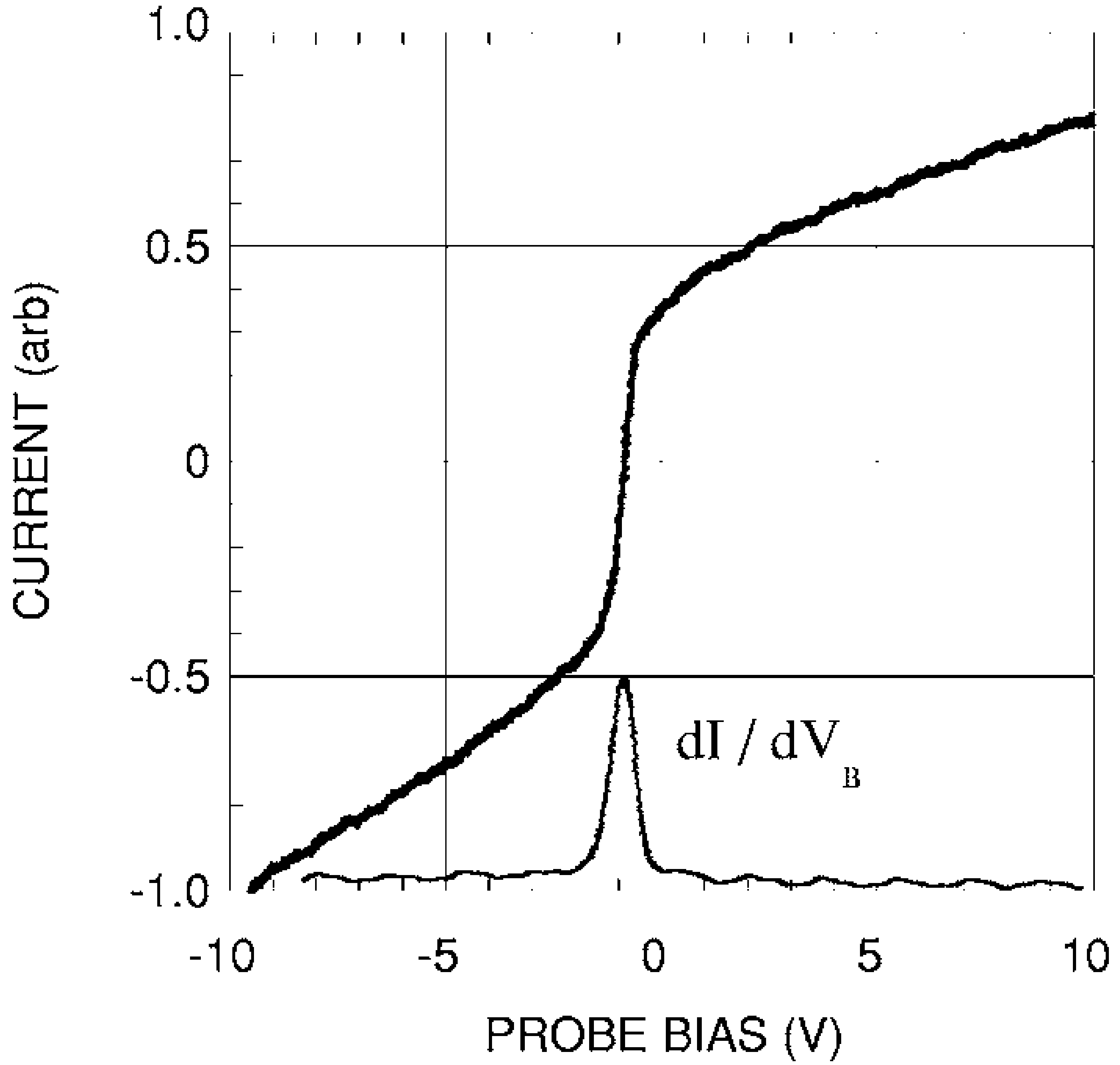


Fig. 6. Langmuir probe I-V characteristic obtained in a singly ionized potassium plasma produced in a Q machine. SF_6 gas was introduced into the plasma to form a negative ion plasma by electron attachment. A substantial fraction of the electrons became attached to the heavy SF_6 molecules resulting in a nearly symmetric probe characteristic with $I_{+s} \approx I_{-s}$. The lower curve is the derivative of the probe current, dI/dV_B. The plasma potential is the value of the V_B for which dI/dV_B is a maximum.

Problem 1. It is not uncommon to find in low pressure plasma discharges that there are two distinct Maxwellian distributions of electrons—a cold and hot distribution with temperatures T_{ec} and T_{eh}, respectively. Extend the analysis of Sec. II to include a two-temperature electron distribution. In this case the electron probe current is written as $I_e(V_B) = I_{ec}(V_B) + I_{eh}(V_B)$. Take the respective densities of the cold and hot components to be n_{ec} and n_{eh} with $n_e = n_{ec} + n_{eh}$. To simplify the analysis, introduce the parameter $f_{eh} \equiv n_{eh}/n_e$ as the fraction of hot electrons, so that $n_{ec}/n_e = 1 - f_{eh}$. An interesting issue arises as to what value of T_e to use in calculating the Bohm ion current. It was shown[26] that the appropriate T_e is the harmonic average of T_{ec} and T_{eh}:

$$\frac{1}{T_e} = \left(\frac{n_{ec}}{n_e}\right)\frac{1}{T_{ec}} + \left(\frac{n_{eh}}{n_e}\right)\frac{1}{T_{eh}}. \tag{A1}$$

After you have produced a Langmuir I-V plot, replot the electron current as a semilog plot to see more clearly the effect of the two-temperature electron distribution.

Problem 2. In plasmas produced in hot-filament discharges, the effect of the ionizing (primary) electrons on the probe I-V trace can be observed, particularly at neutral pressures below $\sim 10^{-4}$ Torr. Extend the probe analysis to include the presence of these energetic primary electrons, which can be modeled as an isotropic monoenergetic distribution. Express the total electron current as $I_{et}(V_B) = I_e(V_B) + I_{ep}(V_B)$, where $I_e(V_B)$ is the contribution from the bulk electrons, and $I_{ep}(V_B)$ is the primary electron contribution, which for an isotropic monoenergetic distribution is[3]

$$I_{ep} = \begin{cases} I_{ep}^{*} \equiv \frac{1}{4} e n_{ep} v_{ep} A_{\text{probe}}, & V_B > V_P, \\ I_{ep}^{*}\left[1 - \frac{2e(V_P - V_B)}{m_e v_{ep}^2}\right], & \left(V_p - \frac{m_e v_{ep}^2}{2e}\right) \le V_B \le V_P, \\ 0, & V_B \le \left(V_P - \frac{m_e v_{ep}^2}{2e}\right), \end{cases} \tag{A2}$$

where n_{ep} is the density of primary electrons, and $v_{ep} = \sqrt{2E_p/m_e}$ is the speed of the primary electrons with energy E_p. To produce an I-V plot, assume that the primary electrons are accelerated through a potential drop $\sim$50–60 V, and the density is in the range of $(0.001–0.1)n_e$.

Plasma Phys. Control. Fusion 36 (1994) 1595–1628. Printed in the UK

REVIEW ARTICLE

Tokamak plasma diagnosis by electrical probes

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Langmuir probes remain the most reported edge diagnostic in the tokamak literature, primarily because it is relatively easy to measure the voltage–current characteristics of an object inserted into a plasma. Reviews of the theoretical and

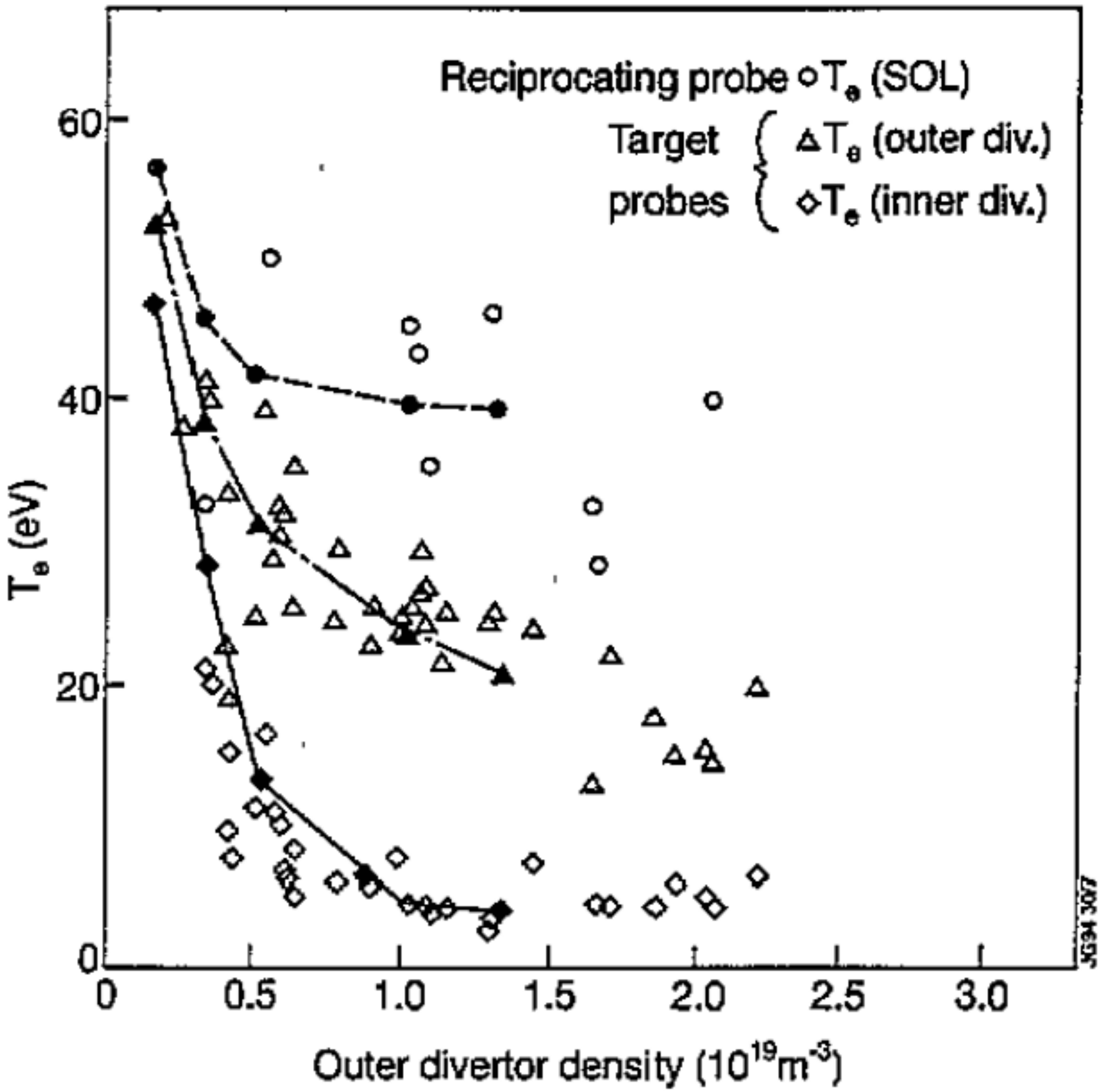


Figure 1. Comparison of T_e measured with a reciprocating Langmuir probe in the SOL of JET near the stagnation point with that measured with fixed probes located in the divertor targets. Full curves show predictions of the EDGE1D fluid code.

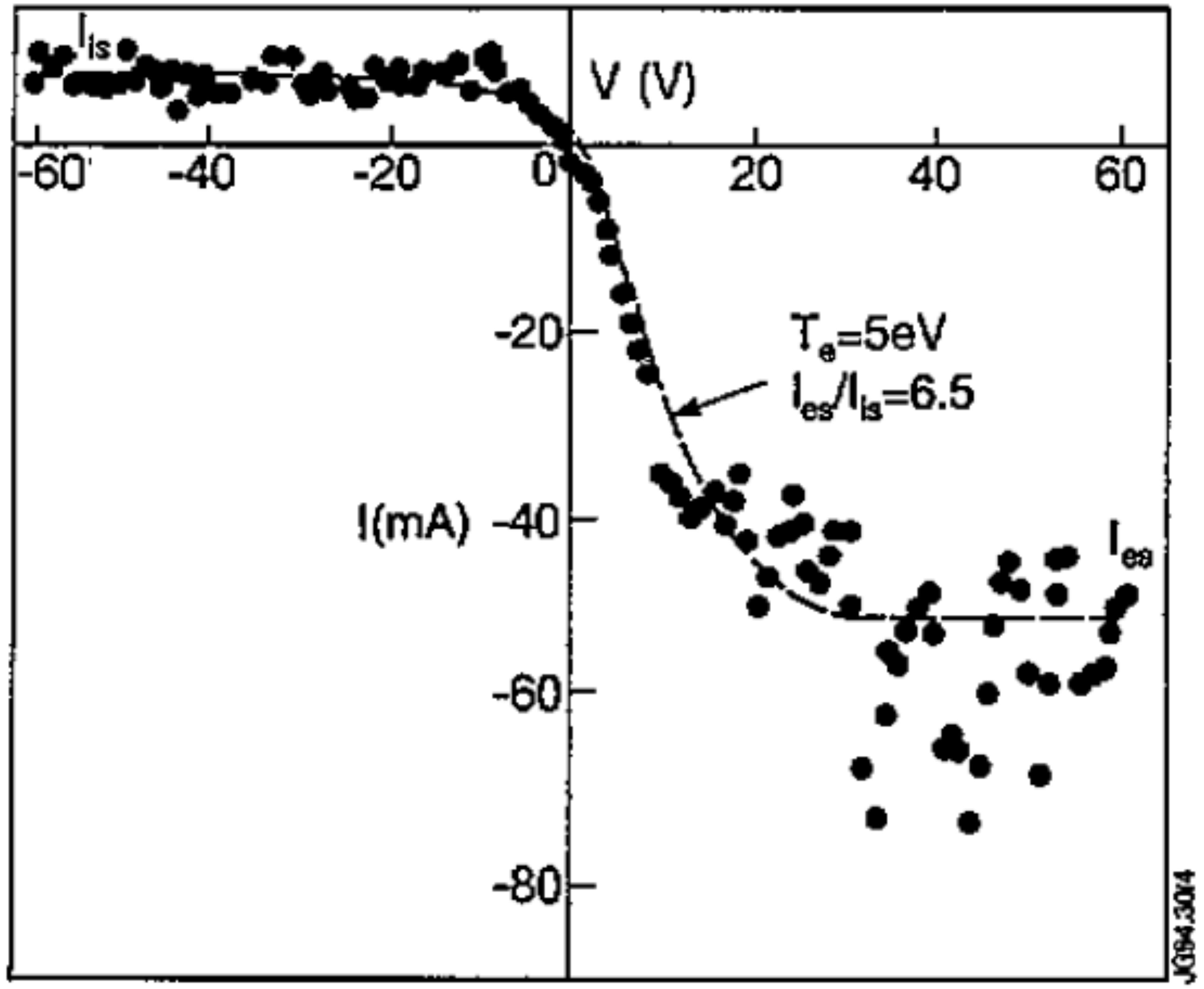


Figure 2. Langmuir probe characteristic recorded in T10 showing low electron to ion saturation current ratio.

to the point in the characteristic where electron saturation occurs. In the absence of a magnetic field the ratio between the electron and ion saturation currents $R_{ei} = I_{es}/I_{is}$ should be around 50 but in tokamaks it is found to be much lower, as can be seen from figure 2 (Gunther *et al* 1990). It has also become apparent that when the angle between the magnetic field and the surface is small the ion current does not saturate.

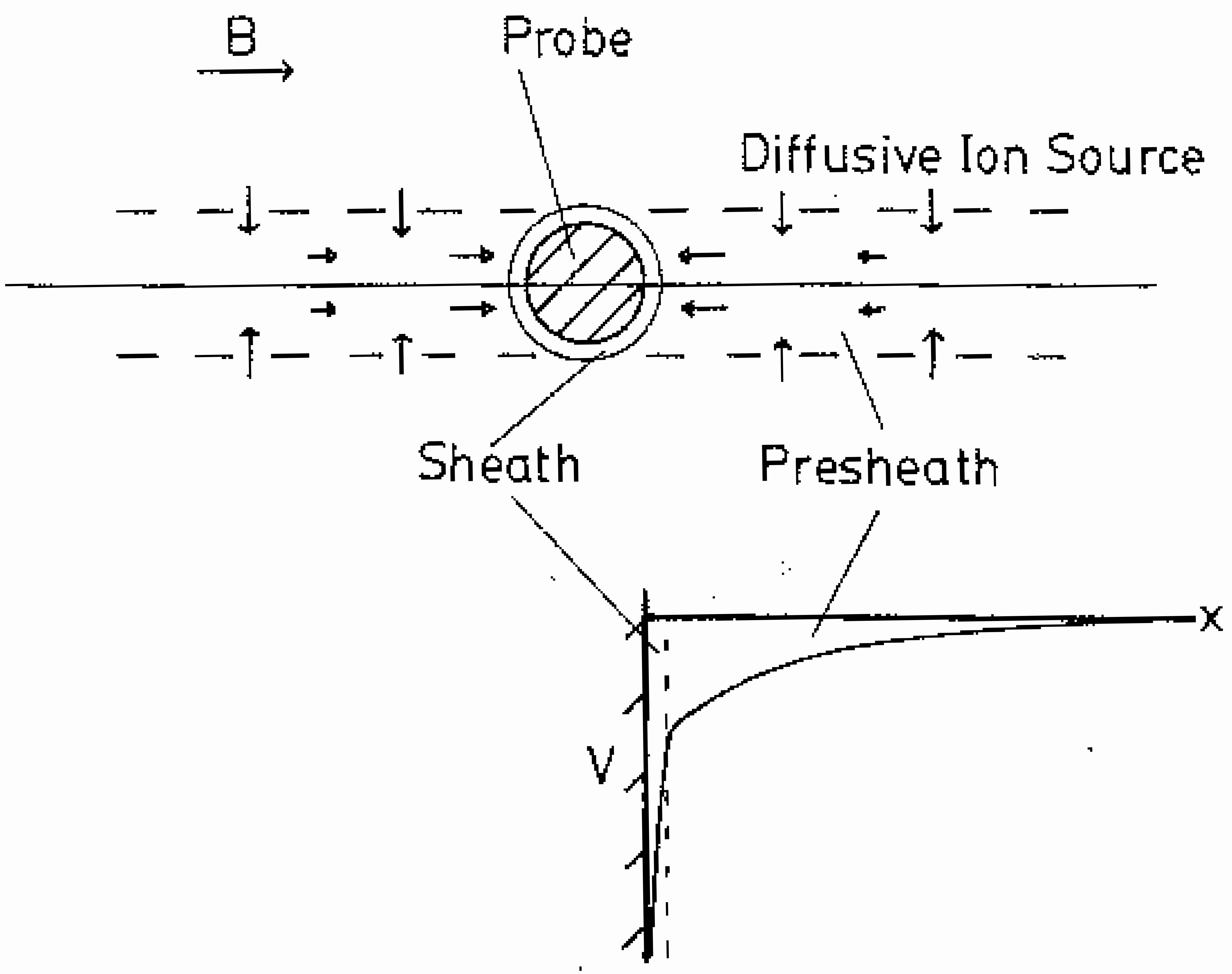


Fig. 3.6. Schematic representation of sheath and presheath in a strong magnetic field.

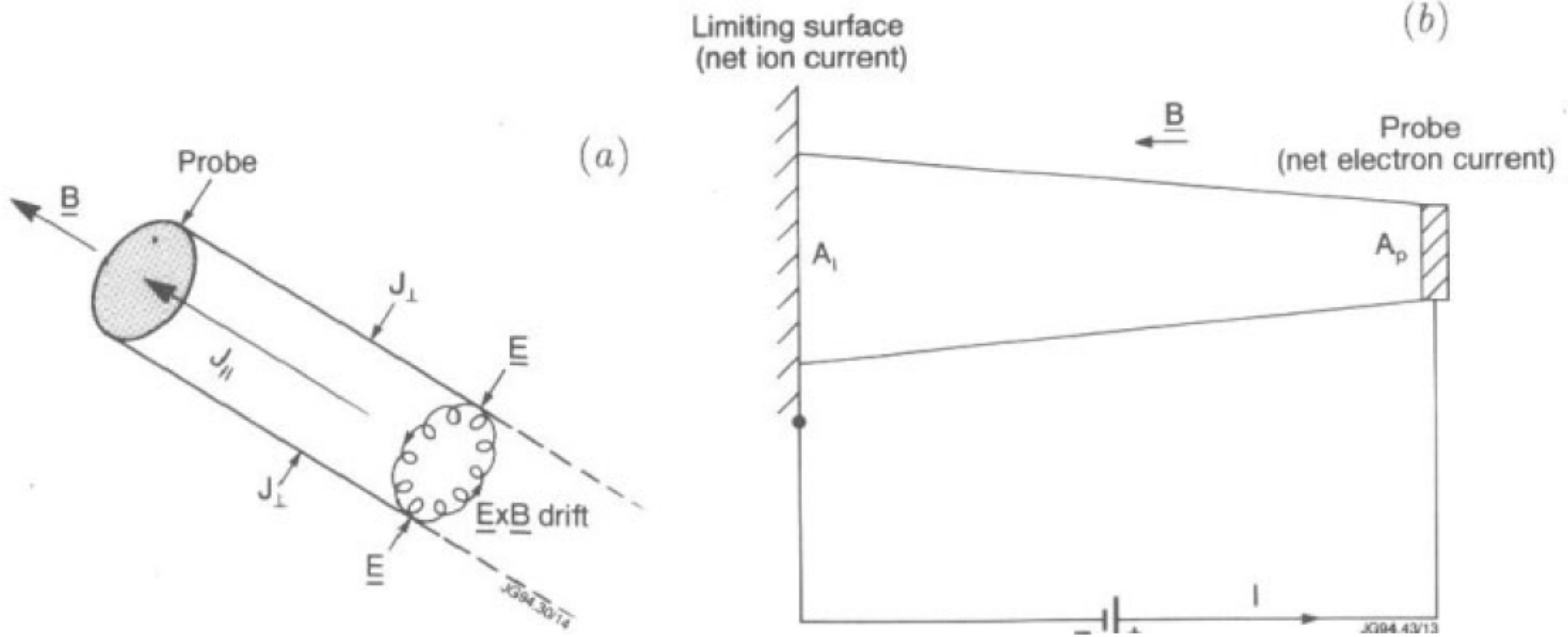


Figure 5. (*a*) In infinite non-viscous magnetized plasmas there is no mechanism for drawing a cross-field current. Ions and electrons would merely undergo $\boldsymbol{E} \times \boldsymbol{B}$ drifts around the probe axis. Gunther has included the effects of viscosity in calculating the electron collection length (*b*) In electron collection Gunther predicts that the collection region is so long that it will normally connect with a limiting surface and therefore behave like a double probe.

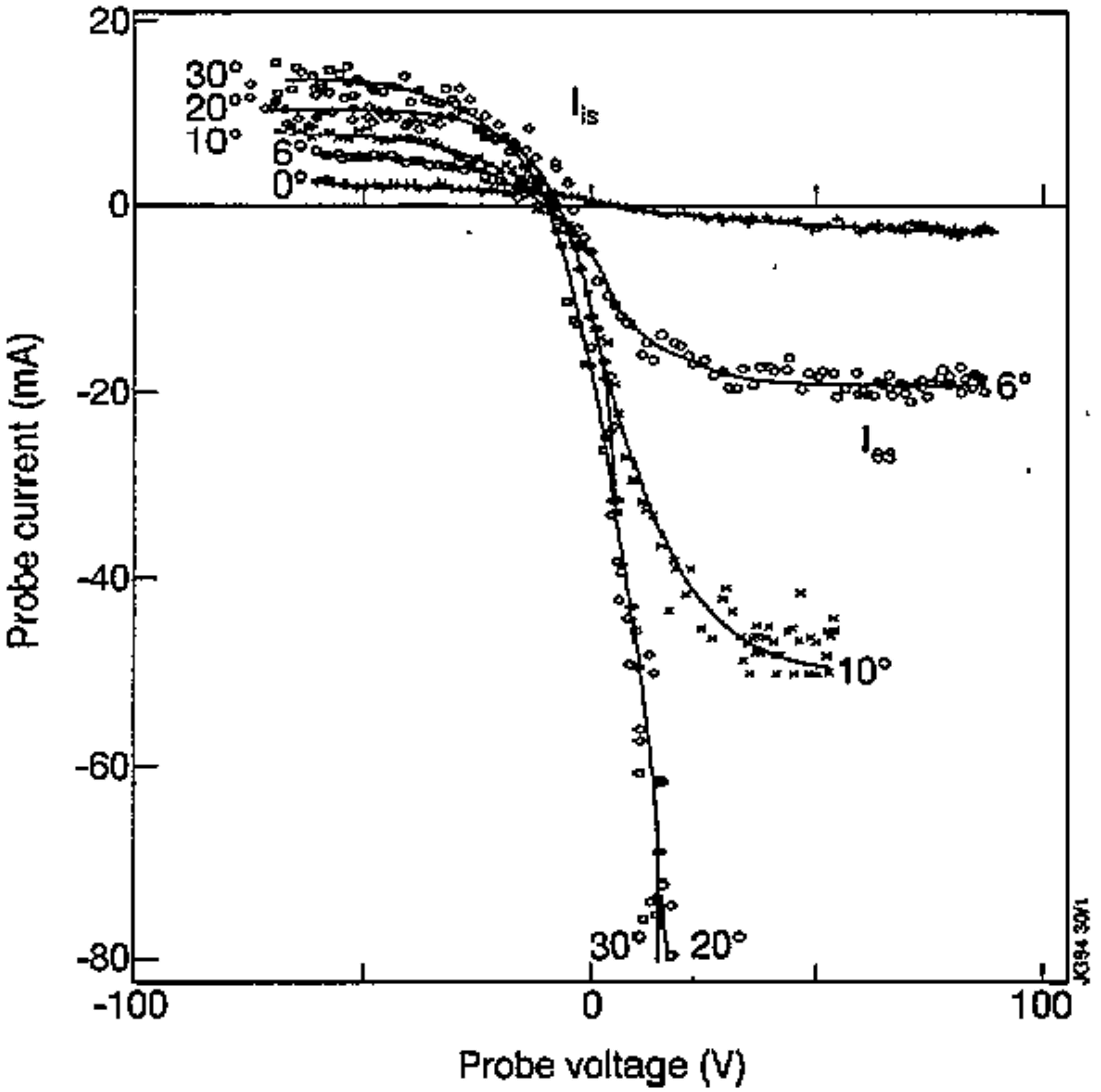


Figure 8. Probe characteristics measured on DITE with a tilting probe at various angles. When $\theta = 0°$ the magnetic field lies parallel to the probe surface.

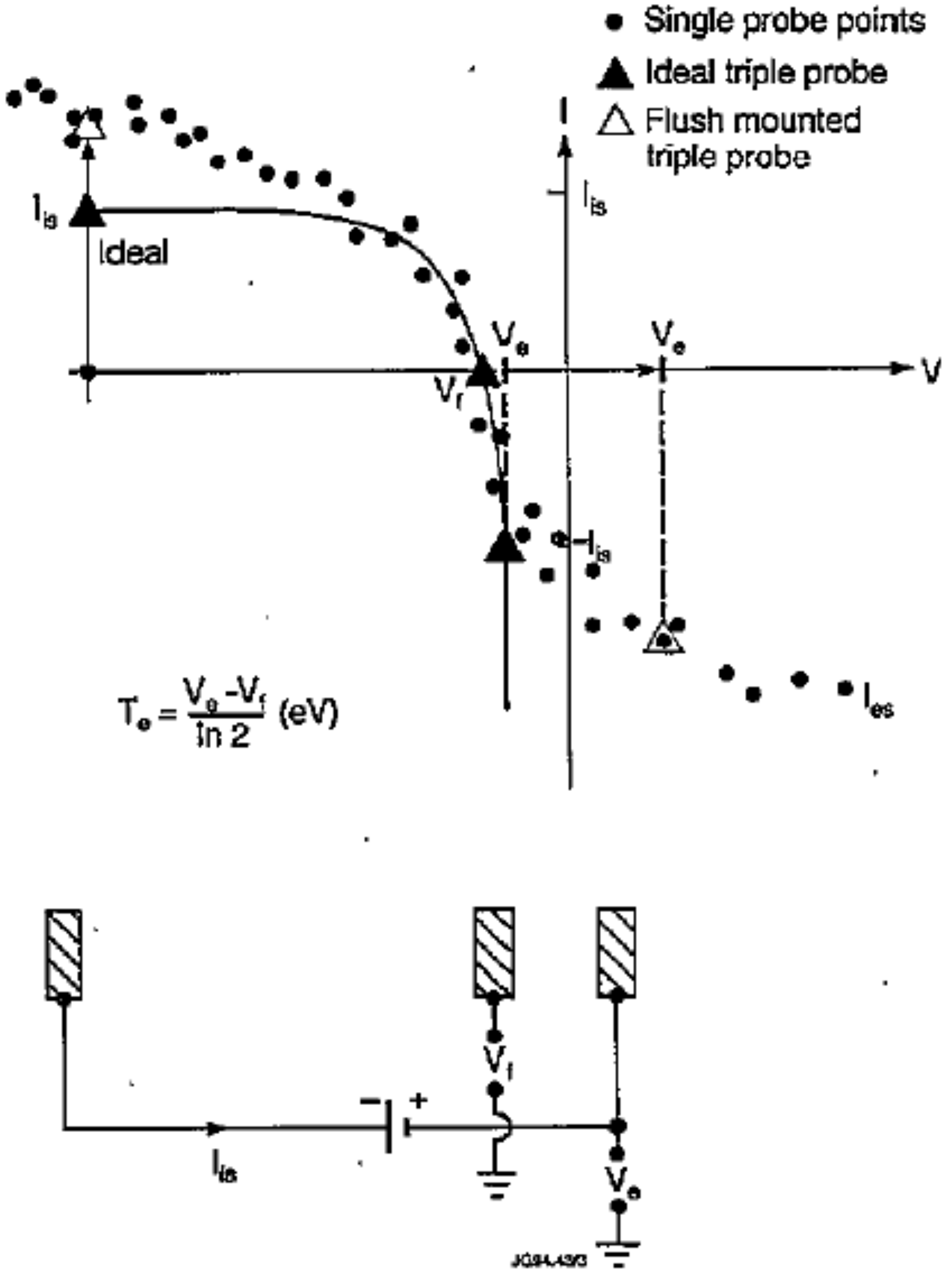
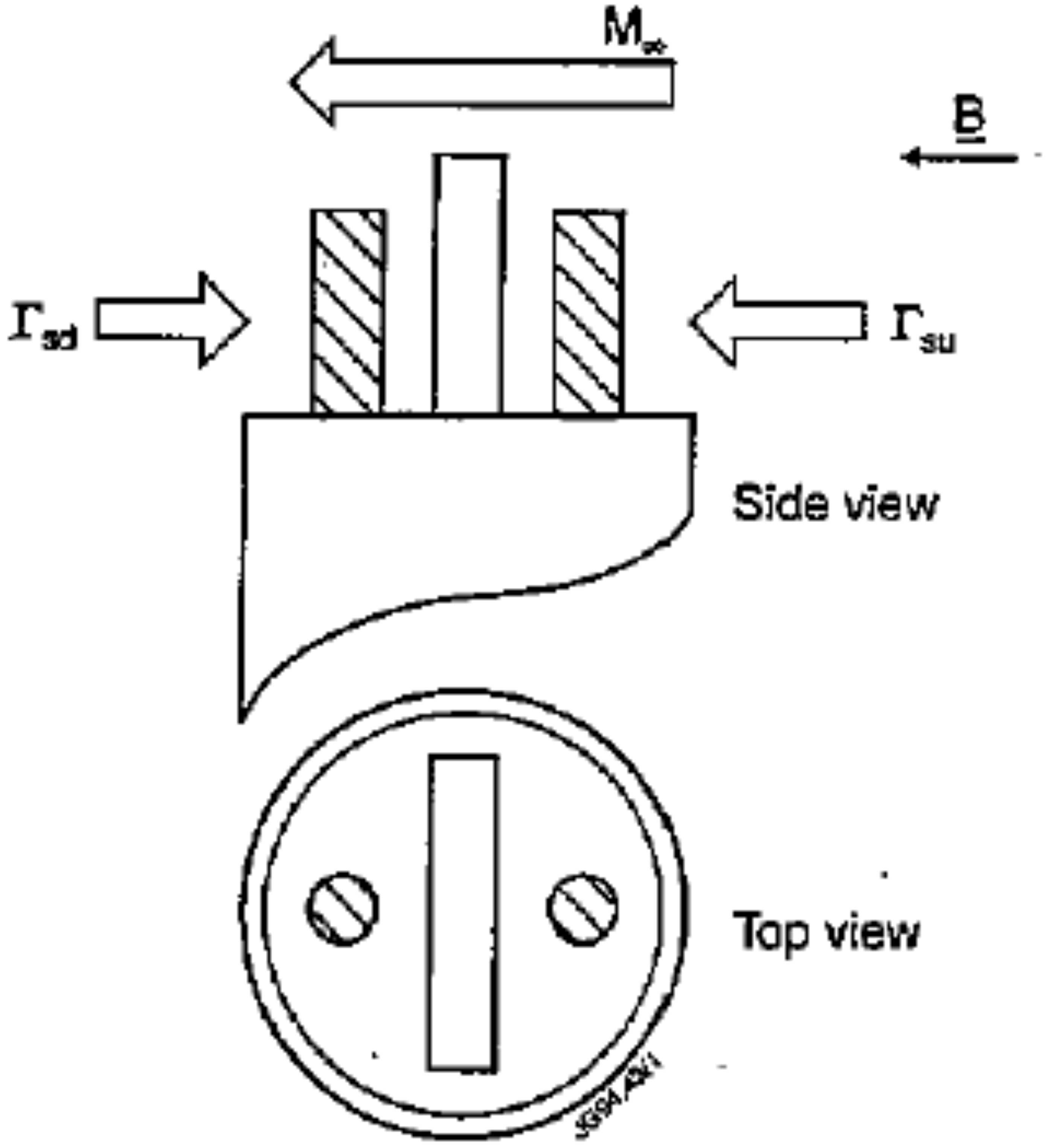


Figure 9. Principle of the triple Langmuir probe. Also indicated is the effect of non-saturation and low electron to ion saturation current ratio on the measurement.



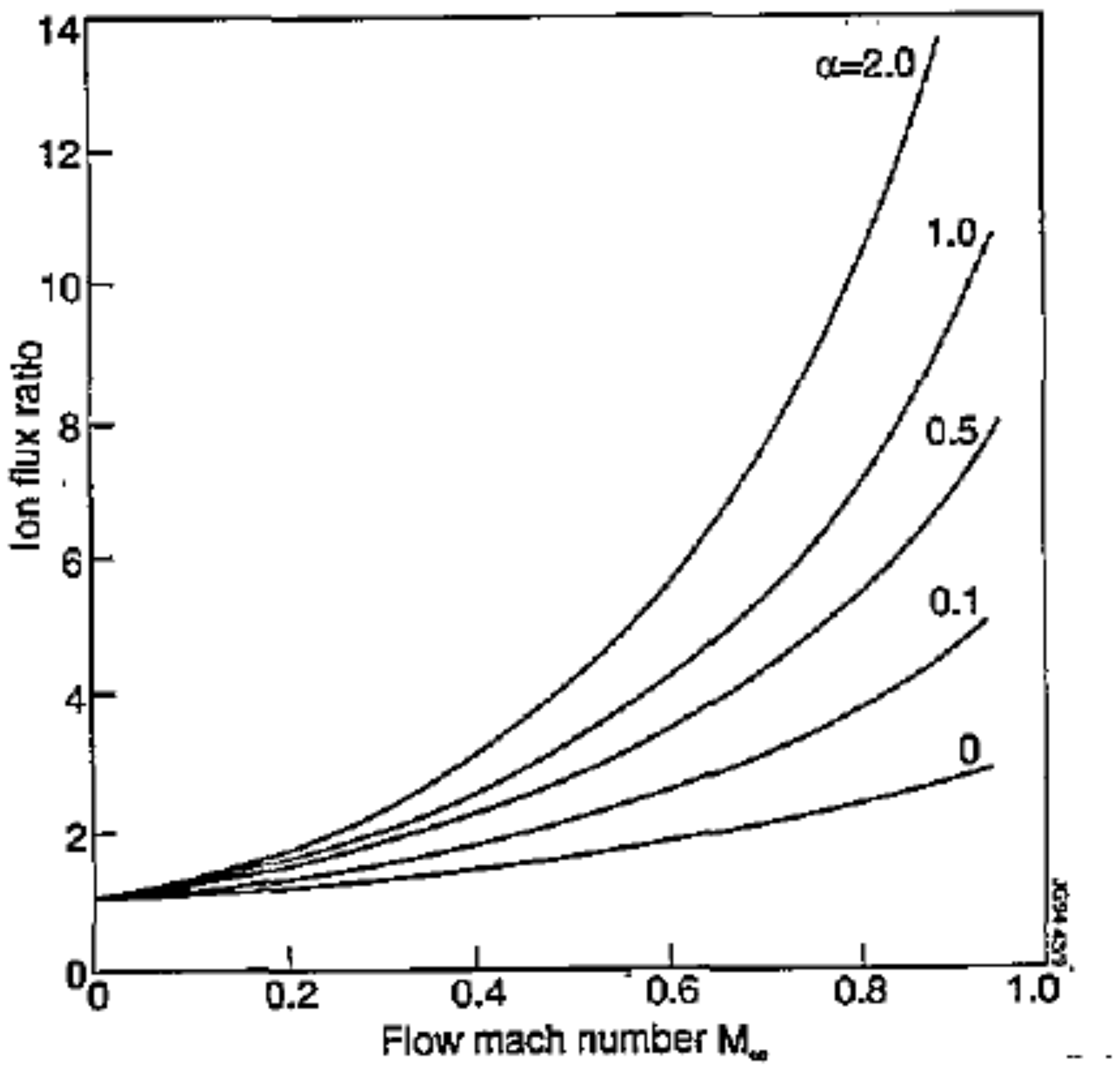


Figure 11. Hutchinson's model predicts that the relationship between Mach number and current ratio is sensitive to α particularly at high Mach numbers.

Figure 10. Schematic of a Mach probe. Ion saturation current is measured on upstream and downstream facing pins.

$$M = 2\frac{1-R}{1+R}.$$

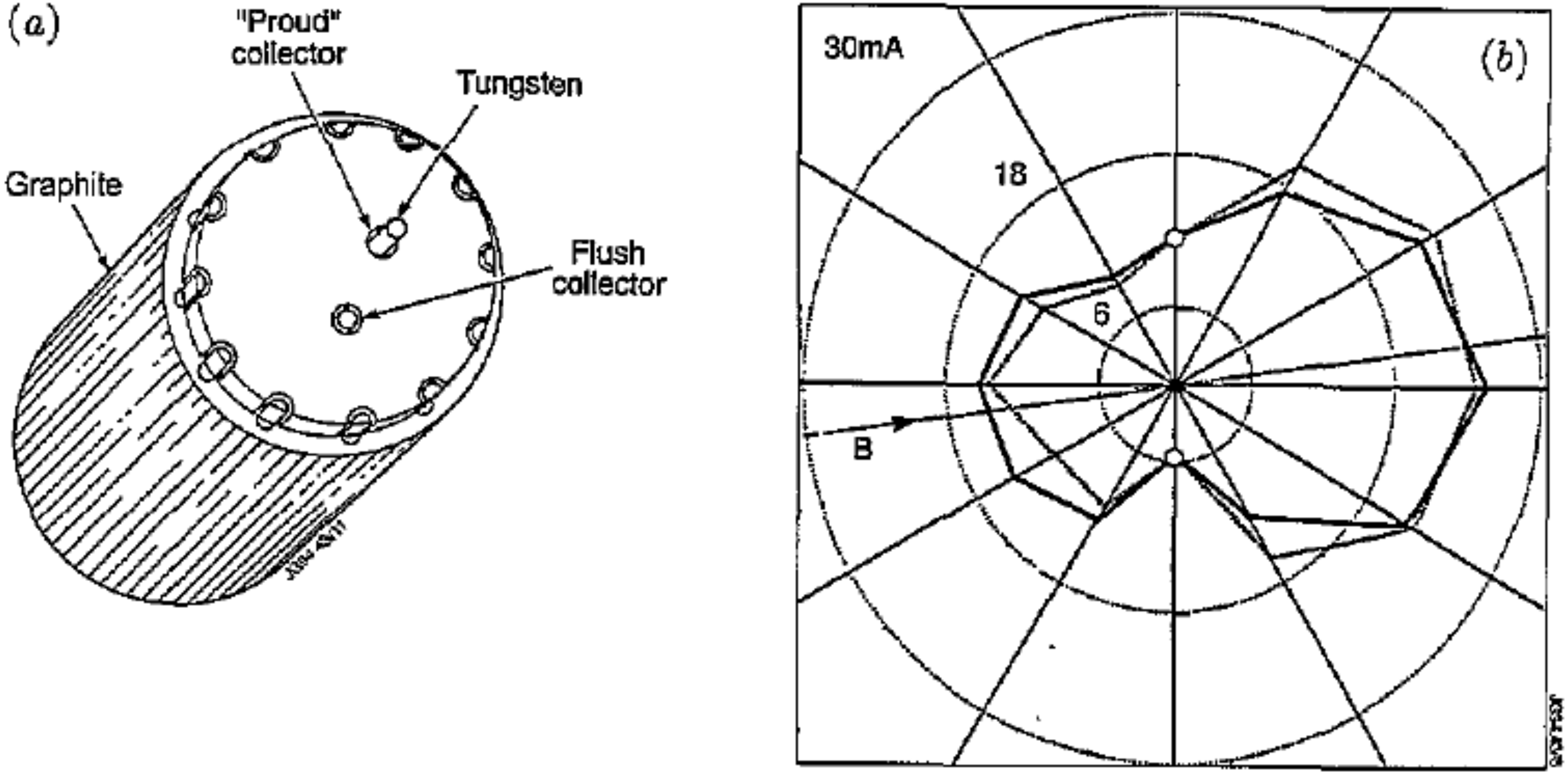


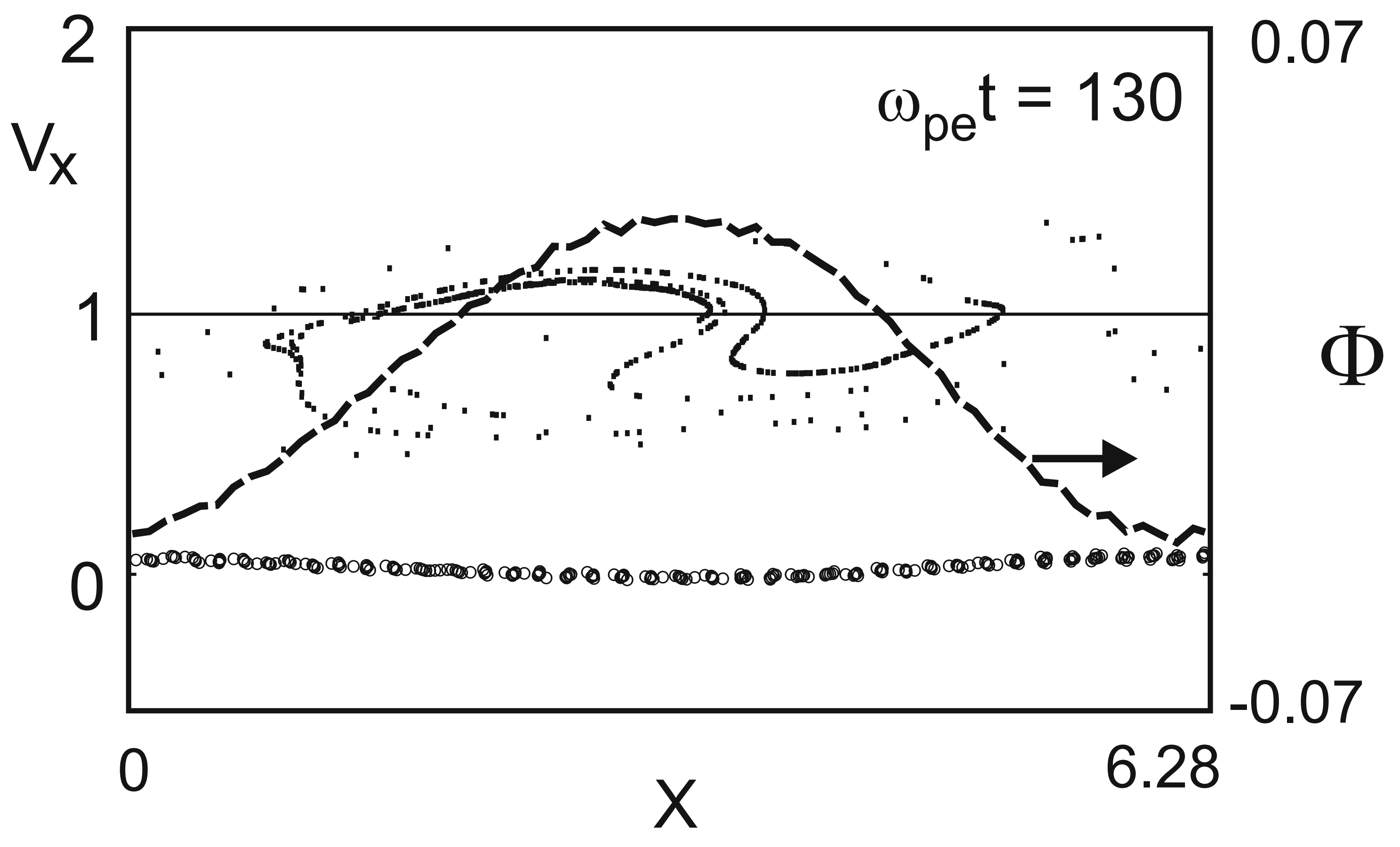
Figure 12. (*a*) The Gundestrupp probe developed on TdeV measures ion saturation around the circumference of a cylinder. (*b*) Polar plots of ion saturation are fitted using a model which has $v_\parallel$ and $v_\perp$ as variables.

4. Conclusions

Langmuir probes have for a long time been the most popular diagnostic for diagnosing tokamak edge plasmas. This has resulted from the relative simplicity of the method and the spatially localized nature of the measurements. Although some have predicted that non-perturbing spectroscopic methods would render electrical probes obsolete, there is still no sign of this happening. The development of limiter and divertor probe arrays and fast-moving probe drives has made electrical probes relevant for even the largest tokamaks in operation today. However, these developments have introduced problems. Flush-mounted Langmuir probes designed to withstand high power loads have experienced non-saturation of the ion current and low electron to ion-saturation current ratios which have made interpretation difficult. Progress is being made on understanding these effects but there is still a long way to go.

Next Lecture
(Monday November 20)

- Two-stream instabilities
- PIC simulations



9.4 Plasma Simulation with Particle Codes