Lecture 14:

Plasma Physics 1

APPH E6101x
Columbia University
This Lecture

• Langmuir probes
• Langmuir probes in magnetized plasma
Understanding Langmuir probe current-voltage characteristics

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I give several simple examples of model Langmuir probe current-voltage (I-V) characteristics that help students learn how to interpret real I-V characteristics obtained in a plasma. Students can also create their own Langmuir probe I-V characteristics using a program with the plasma density, plasma potential, electron temperature, ion temperature, and probe area as input parameters. Some examples of Langmuir probe I-V characteristics obtained in laboratory plasmas are presented and analyzed. A few comments are made advocating the inclusion of plasma experiments in the advanced undergraduate laboratory. © 2007 American Association of Physics Teachers.
Although physicists knew that $V_f$ and $V_p$ were not the same, they thought that the difference was probably small, and in any case, they had no way of either estimating the difference or of measuring the actual plasma potential. Irving Langmuir and Harold Mott-Smith of the General Electric Research Laboratory in the 1920s were the first to provide a quantitative understanding of the difference between $V_f$ and $V_p$. They developed an experimental method for determining the plasma potential and also showed how it was possible to use the probe (now known as a “Langmuir” probe) to determine the plasma density and the electron temperature as well.\(^2\) Langmuir’s method consists of obtaining the current-voltage (I-V) characteristic of the probe as the applied bias voltage $V_B$, is swept from a negative to a positive potential.
THE THEORY OF COLLECTORS IN GASEOUS DISCHARGES

By H. M. Mott-Smith and Irving Langmuir

Abstract

When a cylindrical or spherical electrode (collector) immersed in an ionized gas is brought to a suitable potential, it becomes surrounded by a symmetrical space-charge region or "sheath" of positive or of negative ions (or electrons). Assuming that the gas pressure is so low that the proportion of ions which collide with gas molecules in the sheath is negligibly small, the current taken by the collector can be calculated in terms of the radii of the collector or sheath, the distribution of velocities among the ions arriving at the sheath boundary and the total drop of potential in the sheath. The current is independent of the actual distribution of potential in the sheath provided this distribution satisfies certain conditions.
The curves of Fig. 4 illustrate the characteristics of the three forms of collectors with a Maxwellian distribution.
II. MODEL LANGMUIR PROBE CURRENT-VOLTAGE CHARACTERISTICS

A. Ion and electron currents to a Langmuir probe

1. The ion current

\[ I_i(V_B) = \begin{cases} 
- I_{is} \exp[e(V_P - V_B)/kT_i], & V_B \geq V_P, \\
- I_{is}, & V_B < V_P, 
\end{cases} \]  \quad (1)

\[ I_{is} = \frac{1}{4} en_i v_{i,th} A_{probe}, \]
II. MODEL LANGMUIR PROBE CURRENT-VOLTAGE CHARACTERISTICS

A. Ion and electron currents to a Langmuir probe

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\end{cases}
\]

(1)

where,

\[
I_{is} = \frac{1}{4} en_i v_{i,th} A_{\text{probe}},
\]

The fact that the ion current is determined by the electron thermal speed, but rather is given by the Bohm ion current\(^\text{11}\)

\[
I_{is} = I_{\text{Bohm}} = 0.6en_i \sqrt{\frac{kT_e}{m_i}} A_{\text{probe}}.
\]

(3)
The fact that the ion current is determined by the electron temperature when $T_e \gg T_i$ is counterintuitive and requires some explanation. The physical reason for the dependence $I_{is} \sim (kT_e/m_i)^{1/2}$ has to do with the formation of a sheath around a negatively biased probe.\(^{12,13}\) If an electrode in a plasma has a potential different from the local plasma potential, the electrons and ions distribute themselves spatially around the electrode in order to limit, or shield, the effect of this potential on the bulk plasma. A positively biased electrode acquires an electron shielding cloud surrounding it, while a negatively biased electrode acquires a positive space charge cloud. For a negatively biased electrode, the characteristic shielding distance of the potential disturbance is the electron Debye length\(^{14}\)
The Child-Langmuir Law

\[ n_i(-d) = n_e(-d) = n_{e0} \exp\left(-\frac{1}{2}\right) \approx 0.61 n_{e0} \]

\[ \Phi(-d) \approx -\frac{1}{2} \frac{k_B T_e}{e} \]
In the vicinity of a negatively biased probe, both the electron and ion densities decrease as the particles approach the probe, but not at the same rate. The electron density decreases because electrons are repelled by the probe. In contrast, the ions are accelerated toward the probe, and due to the continuity of the current density, the ion density decreases. A positive space charge sheath can form only if the ion density exceeds the electron density at the sheath edge, and for the ion density to decrease more slowly than the electron density, the ions must approach the sheath with a speed exceeding the Bohm velocity \( u_B = (kT_e/m_i)^{1/2} \).\(^{13,15}\) To achieve this speed, the ions must acquire an energy corresponding to a potential drop of \(0.5(kT_e/e)\), which occurs over a long distance in the plasma. The factor of 0.6 in Eq. (3) is due to the reduction in the density of the ions in the presheath, which is the region over which the ions are accelerated up to the Bohm speed.
2. The electron current

\[ I_e(V_B) = \begin{cases} I_{es} \exp\left[-\frac{e(V_P - V_B)}{kT_e}\right], & V_B \leq V_P, \\ I_{es}, & V_B > V_P. \end{cases} \]  \hspace{1cm} (5)

The electron saturation current \( I_{es} \) is given by

\[ I_{es} = \frac{1}{4}en_e v_{e,th} A_{probe}, \]  \hspace{1cm} (6)
B. Examples of Langmuir probe characteristic

1. The ideal Langmuir probe characteristic

![Image of ideal Langmuir probe current-voltage characteristic]

Table I. Parameters of a typical laboratory plasma used to construct an ideal Langmuir probe volt-ampere characteristic.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion species</td>
<td>$\text{Ar}^+$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ion mass</td>
<td>$m_i$</td>
<td>$6.7 \times 10^{-26}$</td>
<td>kg</td>
</tr>
<tr>
<td>Electron density</td>
<td>$n_e$</td>
<td>$1.0 \times 10^{16}$</td>
<td>m$^{-3}$</td>
</tr>
<tr>
<td>Ion density</td>
<td>$n_i$</td>
<td>$1.0 \times 10^{16}$</td>
<td>m$^{-3}$</td>
</tr>
<tr>
<td>Electron temperature</td>
<td>$T_e$</td>
<td>2.0</td>
<td>eV</td>
</tr>
<tr>
<td>Ion temperature</td>
<td>$T_i$</td>
<td>0.1</td>
<td>eV</td>
</tr>
<tr>
<td>Plasma potential</td>
<td>$V_p$</td>
<td>1.0</td>
<td>V</td>
</tr>
<tr>
<td>Probe diameter</td>
<td>$d_{\text{probe}}$</td>
<td>3.0</td>
<td>mm</td>
</tr>
</tbody>
</table>

Fig. 2. Ideal Langmuir probe current-voltage characteristic (heavy line) for a model plasma with the parameters listed in Table I. The individual electron and ion currents that are used to construct the full characteristic are also shown. The dotted line is the full probe characteristic magnified by a factor of 20 so that the probe floating potential, $V_f$ (the probe voltage where $I=0$) can be easily determined.
B. Examples of Langmuir probe characteristic

1. The ideal Langmuir probe characteristic

\[ I_{es} \exp[e(V_f - V_p)/kT_e] = I_i, \]

or

\[ V_f = V_p + \left( \frac{kT_e}{e} \right) \ln\left( 0.6 \sqrt{\frac{2\pi m_e}{m_i}} \right). \]

Fig. 2. Ideal Langmuir probe current-voltage characteristic (heavy line) for a model plasma with the parameters listed in Table I. The individual electron and ion currents that are used to construct the full characteristic are also shown. The dotted line is the full probe characteristic magnified by a factor of 20 so that the probe floating potential, \( V_f \) (the probe voltage where \( I=0 \)) can be easily determined.
2. **Probe I-V characteristic for a positive ion (+)/negative ion (−) plasma with** \( m_+ = m_- \) **and** \( T_+ = T_- \)

![Graph showing the Langmuir probe I-V characteristic for a plasma with positive and negative ions of equal mass and temperatures. The positive ion and negative ion currents are also shown.](image)

Fig. 3. Langmuir probe I-V characteristic for a plasma with positive and negative ions of equal mass and temperatures. The positive ion and negative ion currents are also shown.
C. Effect of sheath expansion on probe characteristics

Probes in collisionless plasmas without magnetic fields

![Graph](image)

Fig. 3.4. Comparison of the approximate sheath analysis with "exact" numerical results of Laframboise (1966) for a spherical probe.
Fig. 5. Langmuir probe I-V characteristic obtained in a multidipole plasma in argon at a pressure of 0.5 mTorr. (a) Electron current. (b) log $I(V_B)$ versus $V_B$. The semilog plot of the electron current provides a clear demarcation of the plasma potential and electron saturation current. $T_e$ is found from the slope of the exponentially decreasing portion. (c) Expanded scale view of the ion current used to find $I_{is}$. 
B. A positive ion/negative ion plasma in a Q machine

Fig. 6. Langmuir probe I-V characteristic obtained in a singly ionized potassium plasma produced in a Q machine. SF$_6$ gas was introduced into the plasma to form a negative ion plasma by electron attachment. A substantial fraction of the electrons became attached to the heavy SF$_6$ molecules resulting in a nearly symmetric probe characteristic with $I_{+s} \approx -I_{-s}$. The lower curve is the derivative of the probe current, $dI/dV_B$. The plasma potential is the value of the $V_B$ for which $dI/dV_B$ is a maximum.
Problem 1. It is not uncommon to find in low pressure plasma discharges that there are two distinct Maxwellian distributions of electrons—a cold and hot distribution with temperatures $T_{ec}$ and $T_{eh}$, respectively. Extend the analysis of Sec. II to include a two-temperature electron distribution. In this case the electron probe current is written as $I_e(V_B) = I_{ec}(V_B) + I_{eh}(V_B)$. Take the respective densities of the cold and hot components to be $n_{ec}$ and $n_{eh}$ with $n_e = n_{ec} + n_{eh}$. To simplify the analysis, introduce the parameter $f_{eh} \equiv n_{eh}/n_e$ as the fraction of hot electrons, so that $n_{ec}/n_e = 1 - f_{eh}$. An interesting issue arises as to what value of $T_e$ to use in calculating the Bohm ion current. It was shown\textsuperscript{26} that the appropriate $T_e$ is the harmonic average of $T_{ec}$ and $T_{eh}$:

$$\frac{1}{T_e} = \left( \frac{n_{ec}}{n_e} \right) \frac{1}{T_{ec}} + \left( \frac{n_{eh}}{n_e} \right) \frac{1}{T_{eh}}.$$  

(A1)

After you have produced a Langmuir I-V plot, replot the electron current as a semilog plot to see more clearly the effect of the two-temperature electron distribution.
Problem 2. In plasmas produced in hot-filament discharges, the effect of the ionizing (primary) electrons on the probe I-V trace can be observed, particularly at neutral pressures below $\sim 10^{-4}$ Torr. Extend the probe analysis to include the presence of these energetic primary electrons, which can be modeled as an isotropic monoenergetic distribution. Express the total electron current as $I_{et}(V_B) = I_e(V_B) + I_{ep}(V_B)$, where $I_e(V_B)$ is the contribution from the bulk electrons, and $I_{ep}(V_B)$ is the primary electron contribution, which for an isotropic monoenergetic distribution is

\[
I_{ep} = \begin{cases} 
I^*_ep & V_B > V_P, \\
\frac{1}{4} e n_{ep} v_{ep} A_{probe}, & V_B \leq V_P,
\end{cases}
\]

\[
= \begin{cases} 
I^*_ep \left[ 1 - \frac{2e(V_P - V_B)}{m_e v_{ep}^2} \right], & (V_P - \frac{m_e v_{ep}^2}{2e}) \leq V_B \leq V_P, \\
0, & V_B \leq \left( V_P - \frac{m_e v_{ep}^2}{2e} \right),
\end{cases}
\]

where $n_{ep}$ is the density of primary electrons, and $v_{ep} = \sqrt{2E_p/m_e}$ is the speed of the primary electrons with energy $E_p$. To produce an I-V plot, assume that the primary electrons are accelerated through a potential drop $\sim 50–60$ V, and the density is in the range of $(0.001–0.1)n_e$. 
REVIEW ARTICLE

Tokamak plasma diagnosis by electrical probes

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Langmuir probes remain the most reported edge diagnostic in the tokamak literature, primarily because it is relatively easy to measure the voltage–current characteristics of an object inserted into a plasma. Reviews of the theoretical and
Figure 1. Comparison of $T_e$ measured with a reciprocating Langmuir probe in the SOL of JET near the stagnation point with that measured with fixed probes located in the divertor targets. Full curves show predictions of the EDGE1D fluid code.
Figure 2. Langmuir probe characteristic recorded in T10 showing low electron to ion saturation current ratio.

to the point in the characteristic where electron saturation occurs. In the absence of a magnetic field the ratio between the electron and ion saturation currents $R_{ei} = I_{es}/I_{is}$ should be around 50 but in tokamaks it is found to be much lower, as can be seen from figure 2 (Gunther et al 1990). It has also become apparent that when the angle between the magnetic field and the surface is small the ion current does not saturate.
Fig. 3.6. Schematic representation of sheath and presheath in a strong magnetic field.
errors would otherwise dominate. This non-ideal behaviour of net electron collection was explained using a model developed by Stangeby (1982) in which the electron flow to the probe is restricted by momentum loss to the ions. Below floating potential when net ion current is being collected the momentum loss to the electrons can be ignored. Hence, if we restrict ourselves to fitting the characteristic below the floating potential we can use the conventional Langmuir theory to extract a value for $T_e$. The disadvantage of this is that we are basing our electron temperature measurements on about 5% of the electron distribution function, that of the high-energy tail.

Figure 4. Variation of inferred $T_e$ with normalized cut-off voltage. Raw probe data was obtained on JET with a single Langmuir probe of area 0.5 cm$^2$. Inferred $T_e$ rises as more points from the electron collection side of the characteristic are included. The distortion in the characteristics appears to be greater in strongly radiating plasmas.
Figure 5. (a) In infinite non-viscous magnetized plasmas there is no mechanism for drawing a cross-field current. Ions and electrons would merely undergo $E \times B$ drifts around the probe axis. Gunther has included the effects of viscosity in calculating the electron collection length. (b) In electron collection, Gunther predicts that the collection region is so long that it will normally connect with a limiting surface and therefore behave like a double probe.
Figure 8. Probe characteristics measured on DITE with a tilting probe at various angles. When $\theta = 0^\circ$ the magnetic field lies parallel to the probe surface.
Figure 9. Principle of the triple Langmuir probe. Also indicated is the effect of non-saturation and low electron to ion saturation current ratio on the measurement.
Figure 11. Hutchinson's model predicts that the relationship between Mach number and current ratio is sensitive to \( \alpha \) particularly at high Mach numbers.

Figure 10. Schematic of a Mach probe. Ion saturation current is measured on upstream and downstream facing pins.

\[ M = 2 \frac{1 - R}{1 + R} \]
Figure 12. (a) The Gundestrup probe developed on TdeV measures ion saturation around the circumference of a cylinder. (b) Polar plots of ion saturation are fitted using a model which has $v_\parallel$ and $v_\perp$ as variables.
4. Conclusions

Langmuir probes have for a long time been the most popular diagnostic for diagnosing tokamak edge plasmas. This has resulted from the relative simplicity of the method and the spatially localized nature of the measurements. Although some have predicted that non-perturbing spectroscopic methods would render electrical probes obsolete, there is still no sign of this happening. The development of limiter and divertor probe arrays and fast-moving probe drives has made electrical probes relevant for even the largest tokamaks in operation today. However, these developments have introduced problems. Flush-mounted Langmuir probes designed to withstand high power loads have experienced non-saturation of the ion current and low electron to ion-saturation current ratios which have made interpretation difficult. Progress is being made on understanding these effects but there is still a long way to go.
Next Lecture (Wednesday)

- Ch. 7 Problems
- Ch. 8: Instabilities