Lecture 14:

Plasma Physics 1

APPH E6101x
Columbia University
This Lecture: Inhomogeneous Plasma

- Trivelpiece-Gould Modes (cylindrical plasma, surface waves, $\omega < \omega_p$)
- Langmuir probes: Child-Langmuir Law, Bohm Sheath Criteria
- Langmuir probes in magnetized plasma
Space Charge Waves in Cylindrical Plasma Columns

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Fig. 10. Schematic of apparatus to measure phase and attenuation characteristics of space charge waves in a plasma.

Table I. Pertinent dimensions of experimental apparatus used in space charge wave propagation experiment.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of plasma</td>
<td>0.325 in.</td>
</tr>
<tr>
<td>Diameter of tube containing plasma</td>
<td>0.410 in.</td>
</tr>
<tr>
<td>Diameter of slotted wave guide</td>
<td>0.410 and 0.750 in.</td>
</tr>
<tr>
<td>Length of plasma column</td>
<td>25 cm</td>
</tr>
<tr>
<td>Signal frequency range</td>
<td>10 to 4000 mc</td>
</tr>
<tr>
<td>Cyclotron frequency range</td>
<td>0 to 5000 mc</td>
</tr>
<tr>
<td>Plasma frequency range</td>
<td>500 to 5000 mc</td>
</tr>
<tr>
<td>Temperature of mercury in tube</td>
<td>300±0.1°C</td>
</tr>
<tr>
<td>Empty wave guide cutoff frequency</td>
<td>25,000 mc</td>
</tr>
<tr>
<td>Pressure of mercury at 300°C (approx)</td>
<td>2 microns</td>
</tr>
<tr>
<td>Mean free path of plasma electrons (approx)</td>
<td>5 cm</td>
</tr>
</tbody>
</table>

Assuming wave solutions \[ j(\omega t - \beta z) \]

Fig. 13. Measured phase characteristics of plasma space charge waves for no magnetic field for \( a = 0.52 \) cm, \( b = 0.62 \) cm, \( K = 4.6 \).
TG Modes: Low Frequency Surface Waves

Inside Plasma (r < a):

\[ E = -\nabla \Phi \quad \nabla \cdot (\varepsilon_0 k \bar{E}) = 0 \]

\[ \kappa = 1 - \frac{\omega_p^2}{\omega^2} \]

\[ \Rightarrow \nabla^2 \Phi = 0 \]

Outside Plasma (r > a):

\[ E = -\nabla \Phi \quad \nabla \cdot (\varepsilon_0 \bar{E}) = 0 \]

\[ \nabla^2 \Phi = 0 \]

Boundary Conditions (r = a):

\[ \bar{M} \cdot (\varepsilon_0 k \bar{E}_n - \varepsilon_0 \bar{E}_m) = 0 \]

Plasma Dielectric ~ constant
TG Modes: Low Frequency Surface Waves

\[ \nabla^2 \Phi = 0 = \frac{1}{\alpha} \left( \frac{2 \Phi}{2 \eta} \right) - \kappa^2 \Phi = 0 \]

\[ \Phi(1, z, t) \propto \epsilon \]

\[ \Phi(\eta) \sim I_0(\eta) \text{ inside} \]

\[ K_0(\eta) \text{ outside} \]

\[ n = a \]

\[ (1 - \frac{\omega^2}{\omega^2}) I_0' C_1 = K_0' C_2 \]

\[ I_0 C_1 = K_0 C_2 \]

\[ 1 - \frac{\omega^2}{\omega^2} = \frac{I_0(ka) K_0'(ka)}{I_0'(ka) K_0(ka)} \]

Fig. 13. Measured phase characteristics of plasma space charge waves for no magnetic field for \( a = 0.52 \text{ cm}, b = 0.62 \text{ cm}, K = 4.6. \)
Understanding Langmuir probe current-voltage characteristics

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I give several simple examples of model Langmuir probe current-voltage (I-V) characteristics that help students learn how to interpret real I-V characteristics obtained in a plasma. Students can also create their own Langmuir probe I-V characteristics using a program with the plasma density, plasma potential, electron temperature, ion temperature, and probe area as input parameters. Some examples of Langmuir probe I-V characteristics obtained in laboratory plasmas are presented and analyzed. A few comments are made advocating the inclusion of plasma experiments in the advanced undergraduate laboratory. © 2007 American Association of Physics Teachers. [DOI: 10.1119/1.2772282] Am. J. Phys. 75 (12), December 2007

(read this!)
We will show below, how the probe characteristic can be used to determine the electron density and electron temperature of a plasma. The fundamental electric circuit of a Langmuir probe measurement is shown in Fig. 7.6a. A small plane electrode is inserted into a gas discharge. The discharge tube is typically operated from a high-voltage supply via a current-limiting series resistor $R_s$. The probe is biased by an external voltage that is applied between the probe and a suitable electrode. For reasons of lab safety, this electrode must be properly grounded. Likewise, the power supply must be able to operate in a mode where the negative output is the "hot lead" and the positive output grounded. In this case, the anode (positive electrode) was chosen because the voltage drop in the anode layer is usually much smaller than that in the cathode (negative electrode) layer (see Chap. 11). A voltmeter gives the probe bias voltage $U_p$ and a current meter the probe current $I_p$.

A modern realization of the circuit for recording probe characteristic with a computer is shown in Fig. 7.6b. The bias voltage is generated by a digital-to-analog converter (DAC), which delivers a voltage between $-5$ and $+5$ V and is amplified 20 times by a high-voltage operational amplifier. To protect the DAC and the computer from any unwanted plasma currents, an optically-isolated operational amplifier is used. The probe current is sensed as the voltage drop (less than 1 V) across a small series resistor $R_m$ by a second optically-isolated operational amplifier. The current signal is then read out by the computer via an analog-to-digital converter (ADC). Finally, the probe bias is corrected by the computer for the voltage drop across the series resistor, and the probe characteristic can be displayed and stored. Again, the probe circuit is closed by a connection between the ground terminal of the high-voltage opamp and the reference electrode that is connected to protective ground of the lab electrics. For your

**Fig. 7.6**  
(a) Arrangement for a plane Langmuir probe in a dc-discharge. The probe is biased with a voltage $U_p$ with respect to a proper reference electrode.  
(b) Computer-controlled Langmuir probe circuit. A digital-to-analog converter (DAC) with subsequent amplifier provides a probe bias, between $-100$ and $+100$ V. The probe current is measured with a series resistor $R_m$ and an isolation amplifier, and finally A-D converted for numerical processing.
Although physicists knew that $V_f$ and $V_P$ were not the same, they thought that the difference was probably small, and in any case, they had no way of either estimating the difference or of measuring the actual plasma potential. Irving Langmuir and Harold Mott-Smith of the General Electric Research Laboratory in the 1920s were the first to provide a quantitative understanding of the difference between $V_f$ and $V_P$. They developed an experimental method for determining the plasma potential and also showed how it was possible to use the probe (now known as a “Langmuir” probe) to determine the plasma density and the electron temperature as well.\(^2\) Langmuir’s method consists of obtaining the current-voltage (I-V) characteristic of the probe as the applied bias voltage $V_B$, is swept from a negative to a positive potential.
THE THEORY OF COLLECTORS IN GASEOUS DISCHARGES

BY H. M. MOTT-SMITH AND IRVING LANGMUIR

ABSTRACT

When a cylindrical or spherical electrode (collector) immersed in an ionized gas is brought to a suitable potential, it becomes surrounded by a symmetrical space-charge region or “sheath” of positive or of negative ions (or electrons). Assuming that the gas pressure is so low that the proportion of ions which collide with gas molecules in the sheath is negligibly small, the current taken by the collector can be calculated in terms of the radii of the collector or sheath, the distribution of velocities among the ions arriving at the sheath boundary and the total drop of potential in the sheath. The current is independent of the actual distribution of potential in the sheath provided this distribution satisfies certain conditions.
The curves of Fig. 4 illustrate the characteristics of the three forms of collectors with a Maxwellian distribution.
II. MODEL LANGMUIR PROBE CURRENT-VOLTAGE CHARACTERISTICS

A. Ion and electron currents to a Langmuir probe

1. The ion current

\[ I_i(V_B) = \begin{cases} 
- I_{is} \exp\left[\frac{e(V_P - V_B)}{kT_i}\right], & V_B \geq V_P, \\
- I_{is}, & V_B < V_P, 
\end{cases} \]  

(1)

\[ I_{is} = \frac{1}{4} e n_i v_{i,th} A_{\text{probe}}, \]
II. MODEL LANGMUIR PROBE CURRENT-VOLTAGE CHARACTERISTICS

A. Ion and electron currents to a Langmuir probe

1. The ion current

\[ I_i(V_B) = \begin{cases} -I_{is} \exp[e(V_P - V_B)/kT_i], & V_B \geq V_P, \\ -I_{is}, & V_B < V_P, \end{cases} \]

\[ I_{is} = \frac{1}{4} e n_i v_{i,th} A_{probe}, \]

When \( T_e \gg T_i \), the ion saturation current is not determined by the ion thermal speed, but rather is given by the Bohm ion current \(^{3,4,12}\)

\[ I_{is} = I_{Bohm} = 0.6 e n_i \sqrt{\frac{kT_e}{m_i}} A_{probe}. \]
The fact that the ion current is determined by the electron temperature when \( T_e \gg T_i \) is counterintuitive and requires some explanation. The physical reason for the dependence \( I_{is} \sim (kT_e/m_i)^{1/2} \) has to do with the formation of a sheath around a negatively biased probe. \(^{12,13}\) If an electrode in a plasma has a potential different from the local plasma potential, the electrons and ions distribute themselves spatially around the electrode in order to limit, or shield, the effect of this potential on the bulk plasma. A positively biased electrode acquires an electron shielding cloud surrounding it, while a negatively biased electrode acquires a positive space charge cloud. For a negatively biased electrode, the characteristic shielding distance of the potential disturbance is the electron Debye length.\(^{14}\)
In conclusion, the ion motion in a quasineutral presheath requires that

\[ v_i(-d) = v_B \]

The fundamental electric circuit of a Langmuir probe measurement is shown in Fig. 7.3a. A small plane electrode is inserted into a gas discharge. The discharge into a plasma and studied its volt-ampere characteristic. These probes are widely used in plasma physics because of their simple construction and versatility.

In 1925, Mott-Smith and Langmuir [145] had introduced small additional electrodes into a plasma and studied its volt-ampere characteristic. These electrodes are proper grounded. Likewise, the power supply must be able to operate in a mode which do not load the probe characteristic. For reasons of lab safety, this electrode must be properly grounded. Therefore, this is a second Bohm criterion, which follows from the conditions on the electric potential? The answer is no. On its way from the plasma bulk through the plasma-wall boundary layer (see Chap. 11), a potential difference between the wall at the anode layer is usually much smaller than that in the cathode (negative electrode). In this case, the anode (positive electrode) was chosen because the voltage drop in the probe and a suitable electrode. For reasons of lab safety, this electrode must be properly grounded.

Accordingly, the plasma density at the sheath edge is reduced to

\[ \mathbf{n}_e = \frac{\mathbf{n}_e 0 \exp \left( \frac{-1}{2} \right)}{0.61 \mathbf{n}_e 0} \]

\[ \Phi(-d) \approx -\frac{1}{2} \frac{k_B T_e}{e} \]

\[ n_i(-d) = n_e(-d) = n_e 0 \exp \left( \frac{-1}{2} \right) \approx 0.61 n_e 0 \]

\[ v_i(-d) = v_B \]
In the vicinity of a negatively biased probe, both the electron and ion densities decrease as the particles approach the probe, but not at the same rate. The electron density decreases because electrons are repelled by the probe. In contrast, the ions are accelerated toward the probe, and due to the continuity of the current density, the ion density decreases. A positive space charge sheath can form only if the ion density exceeds the electron density at the sheath edge, and for the ion density to decrease more slowly than the electron density, the ions must approach the sheath with a speed exceeding the Bohm velocity $u_B = (kT_e/m_i)^{1/2}$.\textsuperscript{13,15} To achieve this speed, the ions must acquire an energy corresponding to a potential drop of $0.5(kT_e/e)$, which occurs over a long distance in the plasma. The factor of 0.6 in Eq. (3) is due to the reduction in the density of the ions in the presheath, which is the region over which the ions are accelerated up to the Bohm speed.
Assuming only ions...

\[
\frac{1}{2}m_i u_i^2(x) + e\Phi(x) = \frac{1}{2}m_i u_i^2(-d) + e\Phi(-d)
\]

\[
u_i(x) = \left[u_0^2 - \frac{2e\Phi(x)}{m_i}\right]^{1/2}
\]

\[n_i(x)u_i(x) = n_i(-d)u_0
\]

\[n_i(x) = n_i(-d)\left[1 - \frac{2e\Phi(x)}{m_i u_0^2}\right]^{-1/2} \approx \left(-\frac{2e\Phi(x)}{m_i u_0^2}\right)^{-1/2}
\]
Assuming only ions...

\[ \Phi'' \approx -\frac{en_i(-d)}{\varepsilon_0} \left( -\frac{2e\Phi(x)}{m_iu_0^2} \right)^{-1/2} \]

\[ \frac{1}{2} \left[ \Phi'^2(x) - \Phi'^2(-d) \right] = \frac{en_i(-d)u_0}{\varepsilon_0} \left( \frac{2m_i}{e} \right)^{1/2} \times \left\{ [-\Phi(x)]^{1/2} - [-\Phi(-d)]^{1/2} \right\} \]

\[ en_i(-d)u_0 = j_i \]
Assuming only ions... Child–Langmuir Law

\[
\Phi'(x) = 2 \left( \frac{m_i}{2e} \right)^{1/4} \left( \frac{j_i}{\varepsilon_0} \right)^{1/2} \left[ -\Phi(x) \right]^{1/4}
\]

\[
\frac{4}{3} \Phi^{3/4} = 2 \left( \frac{m_i}{2e} \right)^{1/4} \left( \frac{j_i}{\varepsilon_0} \right)^{1/2} (x + d)
\]

\[
\Phi(x) = \left( \frac{3}{2} \right)^{4/3} \left( \frac{m_i}{2e} \right)^{1/3} \left( \frac{j_i}{\varepsilon_0} \right)^{2/3} (x + d)^{4/3}
\]

\[
U^{3/2} = \frac{9}{4} \left( \frac{m_i}{2e} \right)^{1/2} \left( \frac{j_i}{\varepsilon_0} \right) d^2
\]

\[
j_i = \frac{4}{9} \varepsilon_0 \left( \frac{2e}{m_i} \right)^{1/2} \frac{U^{3/2}}{d^2}
\]
and ions will depend on position, and the ion drift velocity will be non-zero. This unperturbed plasma. The presheath will be quasineutral, but the densities of electrons produced as charge up negatively. This applies to the fine metal wires which Langmuir intro-

Accordingly, the plasma density at the sheath edge is reduced to

\[ n_e(-d) = n_e(-d) = n_{e0} \exp\left(-\frac{1}{2}\right) \approx 0.61 n_{e0} \]

\[ \Phi(-d) \approx -\frac{1}{2} \frac{k_B T_e}{e} \]
**Bohm Sheath Criterion**

Bohm criterion represents a *sound barrier*

---

**Fig. 7.2** (a) Stable mechanical equilibrium, \( V''(0) > 0 \). (b) Unstable mechanical equilibrium, \( V''(0) < 0 \)

\[
\begin{align*}
\frac{d^2 \Phi}{dx^2} &= -\frac{\rho}{\varepsilon_0} = f(\Phi) = -\frac{d\Psi}{d\Phi} \\

\text{Sagdeev Potential / Pseudopotential}
\end{align*}
\]

\[
f(\Phi) = \frac{en_e(-d)}{\varepsilon_0} \left[ \exp \left( \frac{e\Phi}{k_BT_e} \right) - \left( 1 - \frac{2e\Phi}{m_iu_0^2} \right)^{-1/2} \right]
\]

<table>
<thead>
<tr>
<th>Mechanical stability</th>
<th>Sheath stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle trajectory</td>
<td>Electric potential distribution ( \Phi(x) )</td>
</tr>
<tr>
<td>Time</td>
<td>Space coordinate ( x )</td>
</tr>
<tr>
<td>Mechanical potential ( V(x) )</td>
<td>Pseudopotential ( \Psi(\Phi) )</td>
</tr>
</tbody>
</table>

\[
m \frac{d^2 x}{dt^2} = -\frac{dV}{dx}
\]

\[
m_iu_0^2 \geq k_BT_e
\]

\[
-m \frac{df}{d\Phi} \bigg|_{\Phi=0} = \frac{e}{k_BT_e} - \frac{e}{m_iu_0^2} \leq 0
\]
B. Examples of Langmuir probe characteristic

1. The ideal Langmuir probe characteristic

Table I. Parameters of a typical laboratory plasma used to construct an ideal Langmuir probe volt-ampere characteristic.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion species</td>
<td>Ar⁺</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ion mass</td>
<td>(m_i)</td>
<td>6.7 \times 10^{-26}</td>
<td>kg</td>
</tr>
<tr>
<td>Electron density</td>
<td>(n_e)</td>
<td>1.0 \times 10^{16}</td>
<td>m⁻³</td>
</tr>
<tr>
<td>Ion density</td>
<td>(n_i)</td>
<td>1.0 \times 10^{16}</td>
<td>m⁻³</td>
</tr>
<tr>
<td>Electron temperature</td>
<td>(T_e)</td>
<td>2.0</td>
<td>eV</td>
</tr>
<tr>
<td>Ion temperature</td>
<td>(T_i)</td>
<td>0.1</td>
<td>eV</td>
</tr>
<tr>
<td>Plasma potential</td>
<td>(V_P)</td>
<td>1.0</td>
<td>V</td>
</tr>
<tr>
<td>Probe diameter</td>
<td>(d_{probe})</td>
<td>3.0</td>
<td>mm</td>
</tr>
</tbody>
</table>

Fig. 2. Ideal Langmuir probe current-voltage characteristic (heavy line) for a model plasma with the parameters listed in Table I. The individual electron and ion currents that are used to construct the full characteristic are also shown. The dotted line is the full probe characteristic magnified by a factor of 20 so that the probe floating potential, \(V_f\) (the probe voltage where \(I=0\)) can be easily determined.
B. Examples of Langmuir probe characteristic

1. The ideal Langmuir probe characteristic

\[ I_{es} \exp[e(V_f - V_P)/kT_e] = I_{is}, \]

or

\[ V_f = V_P + \left( \frac{kT_e}{e} \right) \ln \left( 0.6 \sqrt{\frac{2\pi m_e}{m_i}} \right). \]

Fig. 2. Ideal Langmuir probe current-voltage characteristic (heavy line) for a model plasma with the parameters listed in Table I. The individual electron and ion currents that are used to construct the full characteristic are also shown. The dotted line is the full probe characteristic magnified by a factor of 20 so that the probe floating potential, \( V_f \) (the probe voltage where \( I=0 \)) can be easily determined.
2. Probe I-V characteristic for a positive ion (+)/negative ion (−) plasma with \( m_+ = m_- \) and \( T_+ = T_- \)

Fig. 3. Langmuir probe I-V characteristic for a plasma with positive and negative ions of equal mass and temperatures. The positive ion and negative ion currents are also shown.
C. Effect of sheath expansion on probe characteristics

Fig. 3.4. Comparison of the approximate sheath analysis with "exact" numerical results of Laframboise (1966) for a spherical probe.
negative with respect to the plasma potential effects is presented. characteristics must be modified to account for real probe advanced laboratory course. inclusions of plasma experiments in the undergraduate ad- mas. I close in Sec. IV with comments advocating the

Fig. 1. Schematic of basic devices for producing a plasma.

Fig. 5. Langmuir probe I-V characteristic obtained in a multidipole plasma in argon at a pressure of 0.5 mTorr. (a) Electron current. (b) log $I(V_p)$ versus $V_p$. The semilog plot of the electron current provides a clear demarcation of the plasma potential and electron saturation current. $T_e$ is found from the slope of the exponentially decreasing portion. (c) Expanded scale view of the ion current used to find $I_a$. 

$A$ discharge collection. As discussed in Sec. II

$P = 1.5 \, \text{eV}$. We find that

of the ion current to the plasma potential, where

In a magnetized plasma, the discrepancy in
tainties involved in measuring the saturation currents from

This difference is a typical occurrence with Langmuir probes

A discharge can now be calculated using Eqs.

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B. A positive ion/negative ion plasma in a Q machine

Fig. 6. Langmuir probe I-V characteristic obtained in a singly ionized potassium plasma produced in a Q machine. SF$_6$ gas was introduced into the plasma to form a negative ion plasma by electron attachment. A substantial fraction of the electrons became attached to the heavy SF$_6$ molecules resulting in a nearly symmetric probe characteristic with $I_{+s} \approx I_{-s}$. The lower curve is the derivative of the probe current, $dI/dV_B$. The plasma potential is the value of the $V_B$ for which $dI/dV_B$ is a maximum.
Problem 1. It is not uncommon to find in low pressure plasma discharges that there are two distinct Maxwellian distributions of electrons—a cold and hot distribution with temperatures $T_{ec}$ and $T_{eh}$, respectively. Extend the analysis of Sec. II to include a two-temperature electron distribution. In this case the electron probe current is written as $I_e(V_B) = I_{ec}(V_B) + I_{eh}(V_B)$. Take the respective densities of the cold and hot components to be $n_{ec}$ and $n_{eh}$ with $n_e = n_{ec} + n_{eh}$. To simplify the analysis, introduce the parameter $f_{eh} = n_{eh}/n_e$ as the fraction of hot electrons, so that $n_{ec}/n_e = 1 - f_{eh}$. An interesting issue arises as to what value of $T_e$ to use in calculating the Bohm ion current. It was shown\textsuperscript{26} that the appropriate $T_e$ is the harmonic average of $T_{ec}$ and $T_{eh}$:

$$\frac{1}{T_e} = \left(\frac{n_{ec}}{n_e}\right)\frac{1}{T_{ec}} + \left(\frac{n_{eh}}{n_e}\right)\frac{1}{T_{eh}}.$$  \hspace{1cm} (A1)

After you have produced a Langmuir I-V plot, replot the electron current as a semilog plot to see more clearly the effect of the two-temperature electron distribution.
Problem 2. In plasmas produced in hot-filament discharges, the effect of the ionizing (primary) electrons on the probe I-V trace can be observed, particularly at neutral pressures below $\sim 10^{-4}$ Torr. Extend the probe analysis to include the presence of these energetic primary electrons, which can be modeled as an isotropic monoenergetic distribution. Express the total electron current as $I_{et}(V_B) = I_e(V_B) + I_{ep}(V_B)$, where $I_e(V_B)$ is the contribution from the bulk electrons, and $I_{ep}(V_B)$ is the primary electron contribution, which for an isotropic monoenergetic distribution is

$$I_{ep} = \begin{cases} 
\frac{1}{4}en_{ep}v_{ep}A_{\text{probe}}, & V_B > V_p, \\
I_{ep}^* \left[ 1 - \frac{2e(V_P - V_B)}{m_e v_{ep}^2} \right], & \left(V_p - \frac{m_e v_{ep}^2}{2e}\right) \leq V_B \leq V_P, \\
0, & V_B \leq \left(V_p - \frac{m_e v_{ep}^2}{2e}\right),
\end{cases}$$

(A2)

where $n_{ep}$ is the density of primary electrons, and $v_{ep} = \sqrt{2E_p/m_e}$ is the speed of the primary electrons with energy $E_p$. To produce an I-V plot, assume that the primary electrons are accelerated through a potential drop $\sim 50–60$ V, and the density is in the range of $(0.001–0.1)n_e$. 

\[\text{Problem 2.}\]
REVIEW ARTICLE

Tokamak plasma diagnosis by electrical probes

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Langmuir probes remain the most reported edge diagnostic in the tokamak literature, primarily because it is relatively easy to measure the voltage–current characteristics of an object inserted into a plasma. Reviews of the theoretical and
**Figure 1.** Comparison of $T_e$ measured with a reciprocating Langmuir probe in the SOL of JET near the stagnation point with that measured with fixed probes located in the divertor targets. Full curves show predictions of the EDGE1D fluid code.
Figure 2. Langmuir probe characteristic recorded in T10 showing low electron to ion saturation current ratio.

to the point in the characteristic where electron saturation occurs. In the absence of a magnetic field the ratio between the electron and ion saturation currents $R_{ei} = I_{es}/I_{is}$ should be around 50 but in tokamaks it is found to be much lower, as can be seen from figure 2 (Gunther et al 1990). It has also become apparent that when the angle between the magnetic field and the surface is small the ion current does not saturate.
Fig. 3.6. Schematic representation of sheath and presheath in a strong magnetic field.
Figure 5. (a) In infinite non-viscous magnetized plasmas there is no mechanism for drawing a cross-field current. Ions and electrons would merely undergo $E \times B$ drifts around the probe axis. Gunther has included the effects of viscosity in calculating the electron collection length (b) In electron collection Gunther predicts that the collection region is so long that it will normally connect with a limiting surface and therefore behave like a double probe.
Figure 8. Probe characteristics measured on DITE with a tilting probe at various angles. When $\theta = 0^\circ$ the magnetic field lies parallel to the probe surface.
Figure 9. Principle of the triple Langmuir probe. Also indicated is the effect of non-saturation and low electron to ion saturation current ratio on the measurement.
Figure 10. Schematic of a Mach probe. Ion saturation current is measured on upstream and downstream facing pins.

Figure 11. Hutchinson's model predicts that the relationship between Mach number and current ratio is sensitive to $\alpha$ particularly at high Mach numbers.

\[ M = 2 \frac{1 - R}{1 + R}. \]
Figure 12. (a) The Gundestrup probe developed on TdeV measures ion saturation around the circumference of a cylinder. (b) Polar plots of ion saturation are fitted using a model which has $v_\parallel$ and $v_\perp$ as variables.
4. Conclusions

Langmuir probes have for a long time been the most popular diagnostic for diagnosing tokamak edge plasmas. This has resulted from the relative simplicity of the method and the spatially localized nature of the measurements. Although some have predicted that non-perturbing spectroscopic methods would render electrical probes obsolete, there is still no sign of this happening. The development of limiter and divertor probe arrays and fast-moving probe drives has made electrical probes relevant for even the largest tokamaks in operation today. However, these developments have introduced problems. Flush-mounted Langmuir probes designed to withstand high power loads have experienced non-saturation of the ion current and low electron to ion-saturation current ratios which have made interpretation difficult. Progress is being made on understanding these effects but there is still a long way to go.
9.4 Plasma Simulation with Particle Codes

- Two-stream instabilities
- PIC simulations

Integrate equation of motion, assign new coordinates:
\[ F_i \rightarrow v \rightarrow x \]

Weighting:
\[ E_p \rightarrow F_i \]

Integrate Poisson's equation on the grid:
\[ \rho_p \rightarrow E_p \]