Lecture11: More Plasma Waves
Plasma Physics 1

APPH E6101x
Columbia University

Last Lecture

- Introduction to plasma waves
  - Basic review of electromagnetic waves in various media (conducting, dielectric, gyrotropic, …)
  - Basic waves concepts (especially plane waves)
- Electromagnetic waves in unmagnetized plasma
- Electrostatic waves in unmagnetized plasma

This Lecture

- Wave energy density and its relationship to the dispersion function, $D(\omega)$
- Measurement of electrostatic plasma waves
- Waves in a (cold) magnetized plasma

The Sounds of Interstellar Space

https://science.nasa.gov/science-research/planetary-science/01nov_ismsounds/

NASA Science Editorial Team

OCT 31, 2013

ARTICLE

Nov. 1, 2013: Scifi movies are sometimes criticized when explosions in the void make noise. As the old saying goes, “in space, no one can hear you scream.” Without air there is no sound.

But if that’s true, what was space physicist Don Gurnett talking about when he stated at a NASA press conference in Sept. 2013 that he had heard "the sounds of interstellar space?"

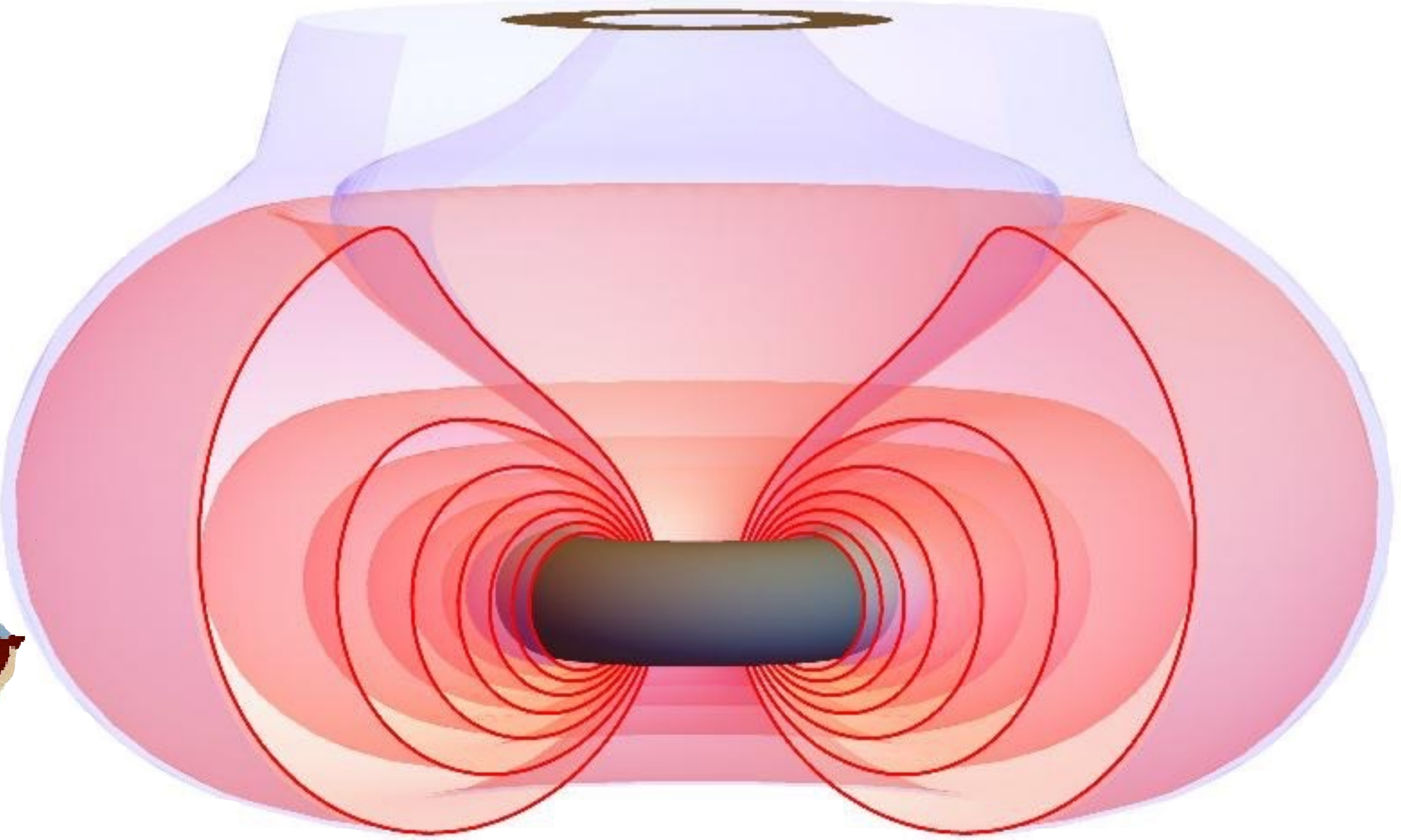
It turns out that space *can* make music … if you know how to listen.

Selected Sounds of Space

These are some of Professor Don Gurnett's favorite sounds of space, recorded by University of Iowa instruments on a variety of spacecraft over the past 60 years.

https://space.physics.uiowa.edu/plasma-wave/space-audio/

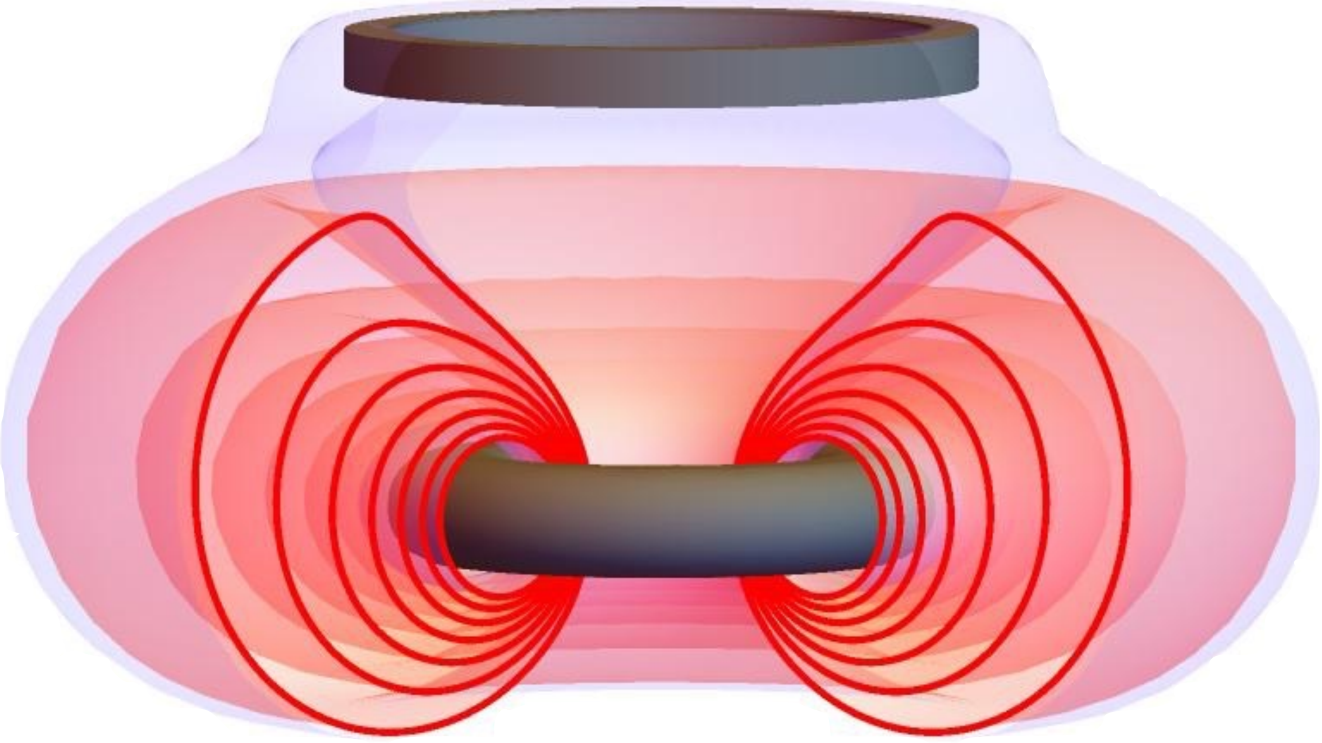
Laboratory Magnetospheres



3.6 m

Levitated Dipole Experiment (LDX)

(1.2 MA · 0.41 MA m^2 · 550 kJ · 565 kg)
Nb_3Sn · 3 Hours Float Time
24 kW ECRH

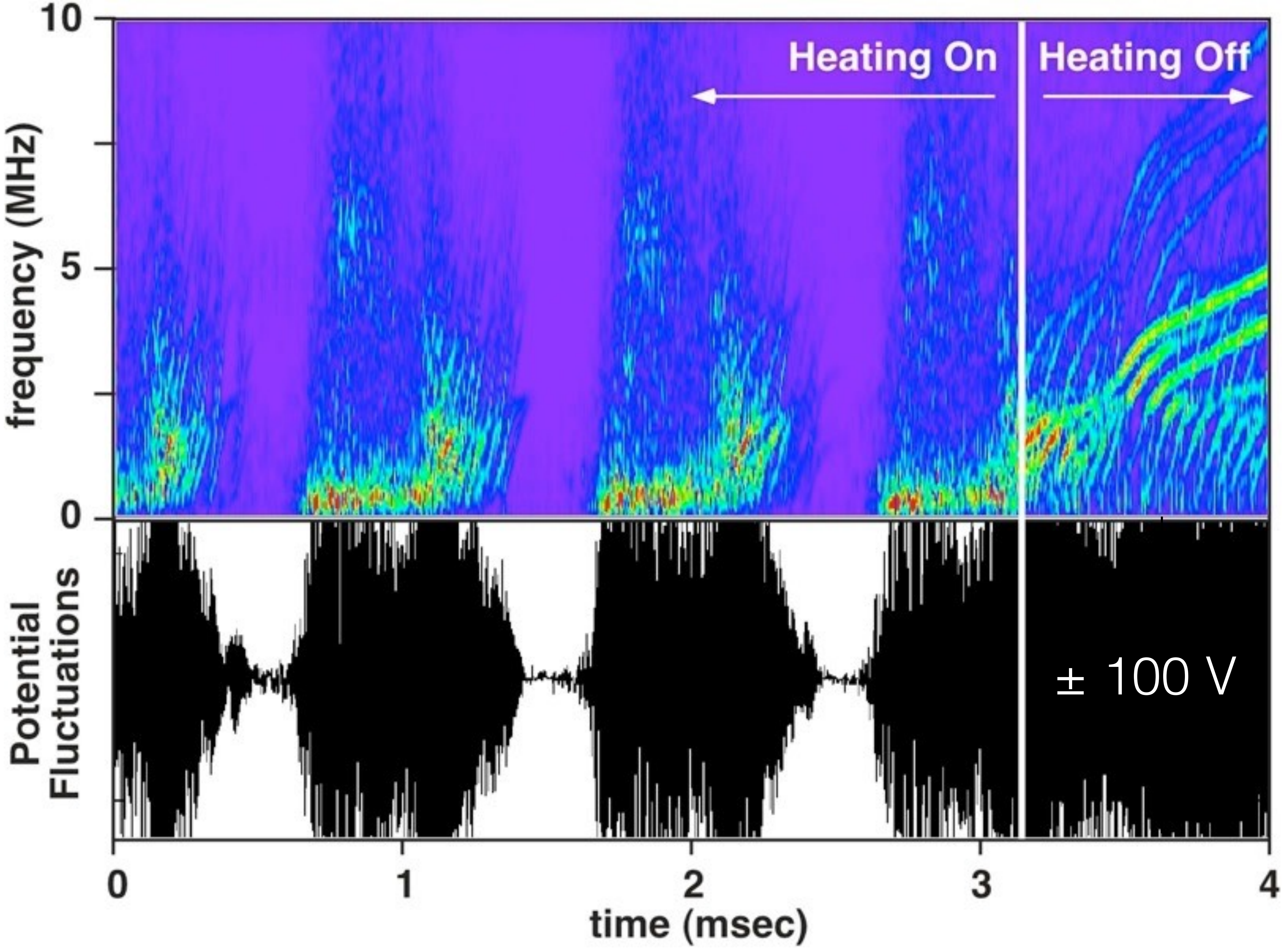


1.8 m

Ring Trap 1 (RT-1)

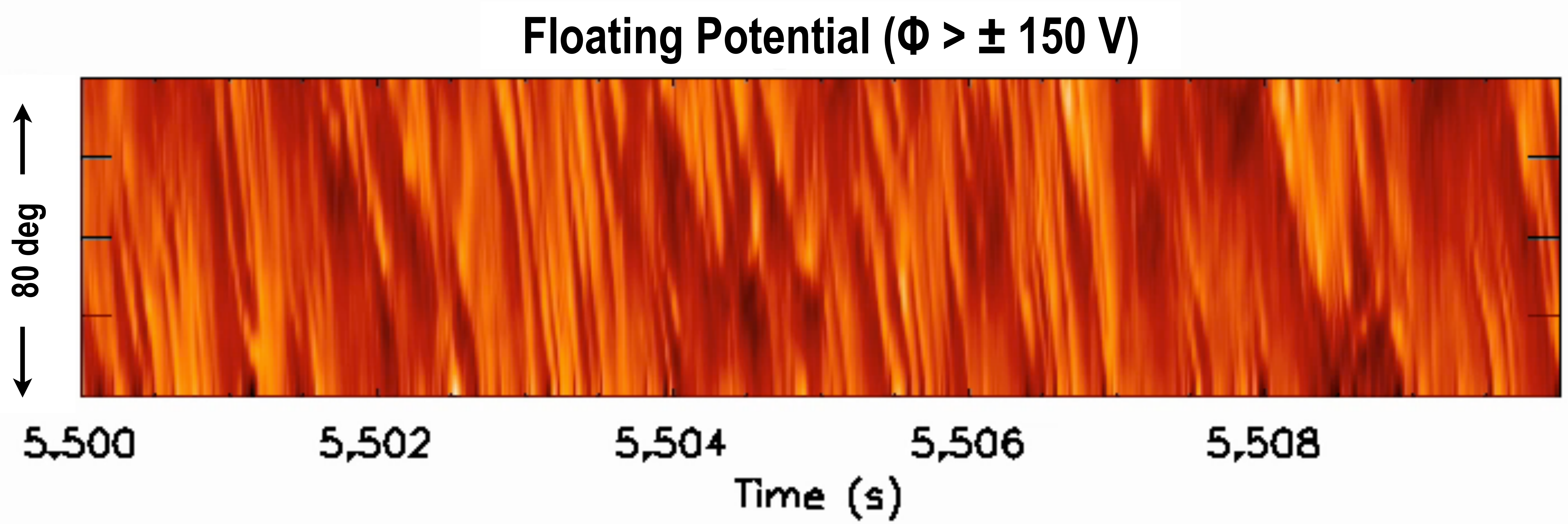
(0.25 MA · 0.17 MA m^2 · 22 kJ · 112 kg)
Bi-2223 · 6 Hours Float Time
50 kW ECRH

Drift-Resonant (Hot Electron) Interchange Instability



Quantitative Verification of Turbulent Particle Pinch

*Using only **measured electric field** fluctuations,*
Thomas Birmingham's diffusion model is verified with levitated dipole



Edge Probe Array:

$$D = \lim_{t \to \infty} \int_0^t dt \langle \dot{\psi}(t) \dot{\psi}(0) \rangle \equiv \langle \dot{\psi}^2 \rangle \tau_c$$

$$D_\psi = R^2 \langle E_\varphi^2 \rangle \tau_c$$

Review of EM Waves

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

$$\begin{aligned}
\nabla \times (\nabla \times \mathbf{E}) &= -\nabla \times \frac{\partial \mathbf{B}}{\partial t} \\
&= -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) \\
&= -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial \mathbf{j}}{\partial t}
\end{aligned}$$

Review of EM Waves

$$\nabla \times (\nabla \times \mathbf{E}) + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu_0 \frac{\partial \mathbf{j}}{\partial t}$$

$$\mathrm{i}\mathbf{k} \times \hat{\mathbf{E}} = \mathrm{i}\omega \hat{\mathbf{B}}$$

$$\mathrm{i}\mathbf{k} \times \hat{\mathbf{B}} = -\mathrm{i}\omega\varepsilon_0\mu_0 \hat{\mathbf{E}} + \mu_0 \hat{\mathbf{j}}_0$$

$$\mathbf{k} \times (\mathbf{k} \times \hat{\mathbf{E}}) = (\mathbf{k}\mathbf{k} - k^2 \hat{\mathbf{I}})\hat{\mathbf{E}}$$

All of the plasma physics here

$$\left\{ \mathbf{kk} - k^2 \boldsymbol{I} + \frac{\omega^2}{c^2}\boldsymbol{I} + \mathrm{i}\omega\mu_0 \boldsymbol{\sigma}(\omega) \right\} \cdot \hat{\mathbf{E}} = 0$$

$$\left\{ \mathbf{kk} - k^2 \boldsymbol{I} + \frac{\omega^2}{c^2}\boldsymbol{\varepsilon}(\omega) \right\} \cdot \hat{\mathbf{E}} = 0$$

Wave Energy Density (Poynting's Theorem)

$$\frac{\partial \bar{B}}{\partial t} = -\nabla \times E \qquad \frac{\partial}{\partial t}\left(\frac{B^2}{2\mu_0}\right) = -\bar{H}\cdot\nabla\times E \qquad (\bar{B} = \mu_0 \bar{H})$$

$$\epsilon_0 \frac{\partial \bar{E}}{\partial t} = \nabla \times H + \bar{J} \qquad \frac{\partial}{\partial t}\left(\epsilon \frac{E^2}{2}\right) = E\cdot\nabla\times H + \bar{J}\cdot\bar{E}$$

ADDING

$$\underbrace{\frac{\partial}{\partial t}\left(\frac{\epsilon_0}{2}E^2\right) + \frac{\partial}{\partial t}\left(\frac{B^2}{2\mu_0}\right)}_{\text{ENERGY DENSITY}} = \underbrace{\bar{E}\cdot\nabla\times H - \bar{H}\cdot\nabla\times E}_{\substack{-\nabla\cdot(\bar{E}\times\bar{H}) \\ \text{POYNTING'S FLUX}}} + \underbrace{\bar{J}\cdot\bar{E}}_{\substack{\text{POWER} \\ \text{DISSIPATED/} \\ \text{EXCHANGED}}}$$

$$\bar{B} = \frac{\bar{k}}{\omega}\times\bar{E}$$

$$B^2 = \bar{E}\cdot\left(\frac{k^2}{\omega} - \frac{\bar{k}\bar{k}}{\omega^2}\right)\cdot\bar{E}$$

$$\frac{\partial}{\partial t} W + \nabla\cdot\bar{v}_g W = -\bar{J}\cdot\bar{E} \sim -2\gamma W$$

Electrostatic Waves

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$-\nabla \cdot \nabla \Phi = \frac{\rho}{\epsilon_0}$$

$$k^2 \Phi = \frac{\rho}{\epsilon_0} \approx -\frac{e n_e}{\epsilon_0}$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0$$

$$m\dot{v} = -q\frac{\mathrm{d}\phi}{\mathrm{d}x} - \frac{\gamma}{n}\frac{\mathrm{d}(nk_\mathrm{B}T)}{\mathrm{d}x}$$

Electron Pressure Force

Electrostatic Plasma Waves

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$-\nabla \cdot \nabla \Phi = \frac{\rho}{\epsilon_0}$$

$$k^2 \Phi = \frac{\rho}{\epsilon_0} \approx -\frac{e n_e}{\epsilon_0}$$

Particle Conservation

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0$$

$$-\mathrm{i}\omega \hat{n} + \mathrm{i}k n_0 \hat{v} = 0$$

Momentum Dynamics

$$m\dot{v} = -q\frac{\mathrm{d}\phi}{\mathrm{d}x} - \frac{\gamma}{n}\frac{\mathrm{d}(n k_\mathrm{B} T)}{\mathrm{d}x}$$

$$-\mathrm{i}\omega m \hat{v} = -\mathrm{i}kq\hat{\phi} - \mathrm{i}k\gamma k_\mathrm{B} T \hat{n}$$

Electron Pressure Force

Electrostatic Plasma Waves

$$\omega = \left(\omega_{\mathrm{pe}}^2 + \frac{3}{2}k^2 v_{\mathrm{Te}}^2\right)^{1/2} = \omega_{\mathrm{pe}}\left(1 + 3k^2\lambda_{\mathrm{De}}^2\right)^{1/2}$$

Electron Pressure Force

Electron Pressure Force

Electrostatic Ion Sound Waves

Ion Pressure Force

$$-\,\mathrm{i}\omega m_\mathrm{i}\hat{v}_\mathrm{i} = e\hat{E} - \frac{\mathrm{i}k}{n_{\mathrm{i}0}}(\gamma_\mathrm{i} k_\mathrm{B} T_\mathrm{i})\hat{n}_\mathrm{i}$$

$$\hat{n}_\mathrm{i} = \frac{ek}{-\mathrm{i}\omega^2 m_\mathrm{i} + \mathrm{i}k^2\gamma_\mathrm{i} k_\mathrm{B} T_\mathrm{i}}\hat{E}$$

$$0 = -e\hat{E} - \frac{\mathrm{i}k}{n_{\mathrm{e}0}}(k_\mathrm{B} T_\mathrm{e})\hat{n}_\mathrm{e}$$

$$\hat{n}_\mathrm{e} = \frac{-e}{\mathrm{i}k k_\mathrm{B} T_\mathrm{e}}\hat{E}\,,$$

Look!
No electron acceleration

Electron Pressure Force

$$k^2\Phi = \frac{\rho}{\epsilon_0} \approx \frac{e}{\epsilon_0}(n_i - n_e)$$

$$\mathrm{i}k\hat{E} = \left(\frac{n_{\mathrm{i}0}e^2}{\varepsilon_0 m_\mathrm{i}}\right)\frac{k}{-\mathrm{i}\omega^2 + \mathrm{i}k^2\gamma_\mathrm{i} k_\mathrm{B} T_\mathrm{i}/m_\mathrm{i}}\hat{E} + \left(\frac{n_{\mathrm{e}0}e^2}{\varepsilon_0 k_\mathrm{B} T_\mathrm{e}}\right)\frac{1}{\mathrm{i}k}\hat{E}$$

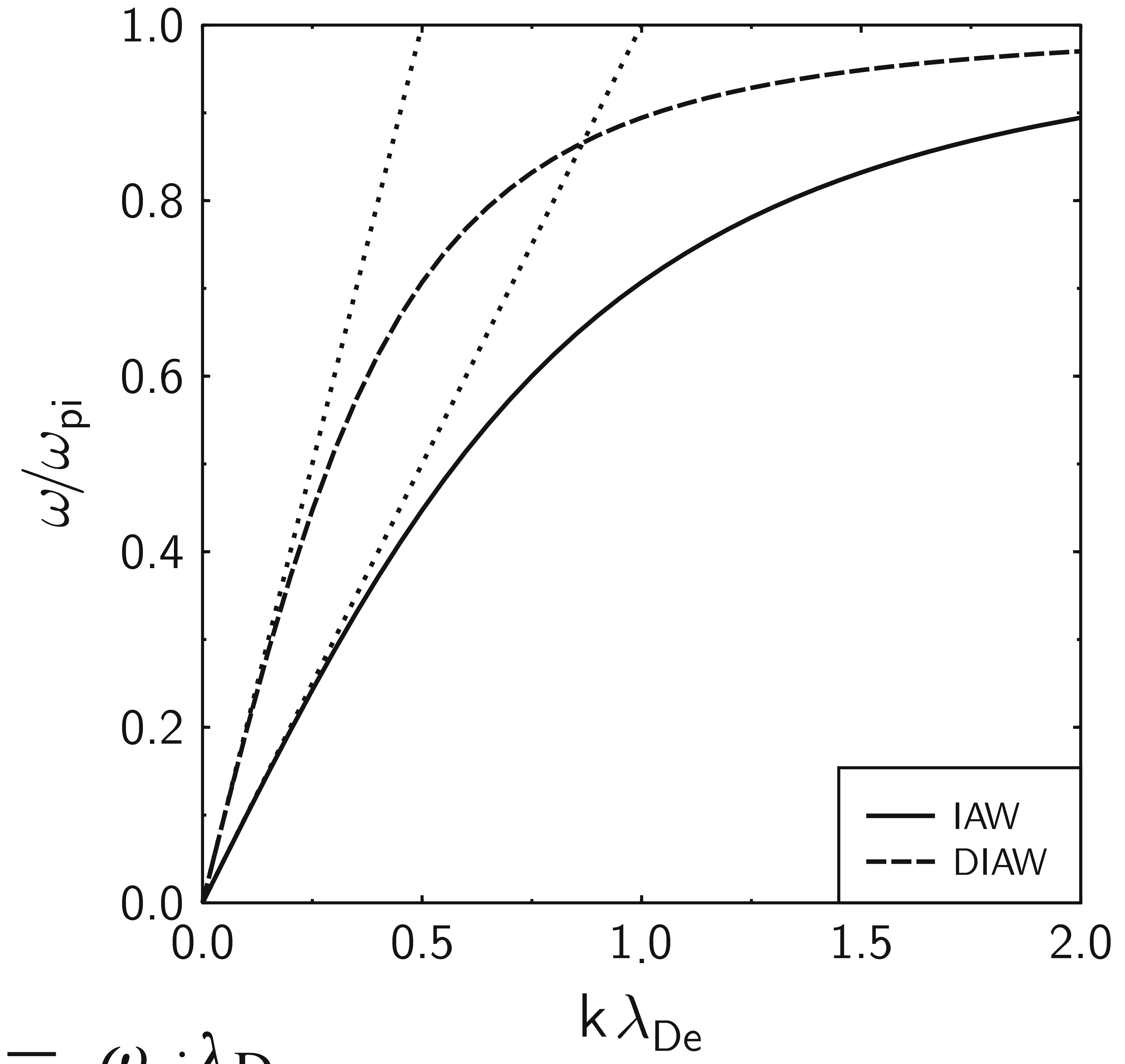
Electrostatic Ion Sound Waves

$$\varepsilon(k,\omega) = 1 - \frac{\omega_{\mathrm{pi}}^2}{\omega^2 - k^2\gamma_{\mathrm{i}} k_{\mathrm{B}} T_{\mathrm{i}}/m_{\mathrm{i}}} + \frac{1}{k^2\lambda_{\mathrm{De}}^2}$$

$$\omega^2 = k^2\left(\frac{\gamma_{\mathrm{i}} k_{\mathrm{B}} T_{\mathrm{i}}}{m_{\mathrm{i}}} + \frac{\omega_{\mathrm{pi}}^2\lambda_{\mathrm{De}}^2}{1+k^2\lambda_{\mathrm{De}}^2}\right)$$

$$\omega \approx \frac{k\,C_{\mathrm{s}}}{\sqrt{1+k^2\lambda_{\mathrm{De}}^2}}$$

$T_{\mathrm{e}} \gg T_{\mathrm{i}}$ $\qquad$ $C_{\mathrm{s}} = \omega_{\mathrm{pi}}\lambda_{\mathrm{De}}$



Energy Density for Electrostatic Waves

$$m\frac{d\bar{v}}{dt} = q\bar{E}e^{-j(\omega t - k\cdot r)}$$

$$-j\omega\hat{V} = \frac{q}{m}E \rightarrow \bar{V} = \frac{j}{\omega}\frac{q}{m}\bar{E}$$

!!

$$\left\langle \tfrac{1}{2} m n V^2 \right\rangle = \tfrac{1}{2} m n_0 \left\langle \tfrac{1}{2} V^* V \right\rangle$$

$$= \tfrac{1}{2} m n_0 \frac{1}{2}\frac{q^2}{m^2}\frac{|E|^2}{\omega^2}$$

$$= \frac{1}{2}\cdot\frac{\omega_p^2}{\omega^2}\epsilon_0 \frac{1}{2}|E|^2$$

For High Frequency waves $\sim \omega_{pe}$

Plasma Energy Density $\sim \dfrac{\omega_p^2}{\omega^2} \times$ Electric Energy Density

For Low Frequency Sound waves $\lesssim \omega_{pi}$

Plasma Energy $\sim \dfrac{\omega_{pi}^2}{\omega^2} \times$ Electric Energy

$\gg$ Electric Energy

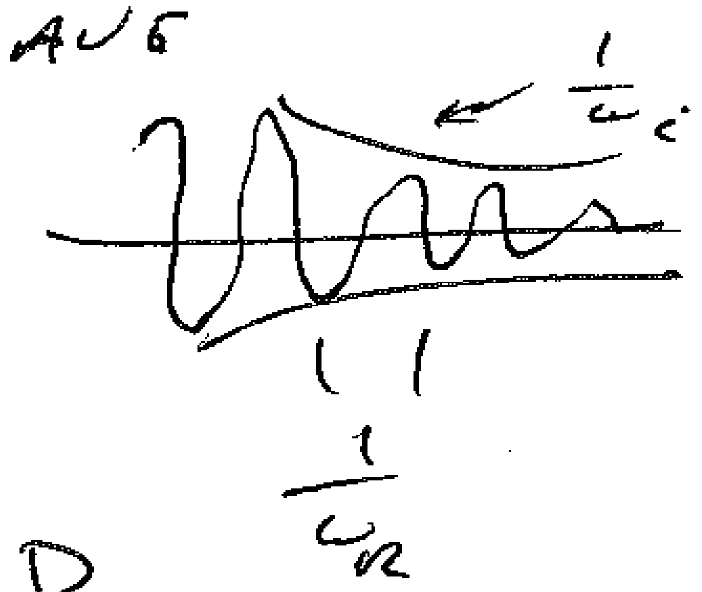
For $\omega \ll \omega_{pi}$

Damping and Dispersion

$D(\omega, k) = 0$ $\qquad\qquad \omega = \omega_R + i\,\omega_I$

WAVES USUALLY HAVE

$|\omega_R| \gg |\omega_I|$



$$D(\omega, k) = 0 \simeq D_R(\omega_R, k) + i\,D_I + i\,\omega_I \left.\frac{\partial D_R}{\partial \omega}\right|_{\omega_R} + \dots$$

$$\therefore \quad \omega_I = -\left.\frac{D_I}{\partial D_R/\partial \omega}\right|_{\omega_R}$$

WITH $\quad D_R(\omega_R, k) = 0$

$D_I \sim$ ENERGY LOSS RATE

$\frac{\partial D_R}{\partial \omega} \sim$ ENERGY DENSITY OF WAVE

Slowly-Varying Wave

$$\bar{E}(r,t) = \mathrm{Re}\left\{ \hat{E}(\bar{r},t)\, e^{-j(\omega t - \bar{k}\cdot\bar{r})} \right\}$$

SLOWLY VARYING AMPLITUDE/PHASE

FAST WAVE OSCILLATIONS

$$\frac{1}{2}\left| E^2(r,t) \right| = \frac{1}{2} \mathrm{Re}\left\{ \hat{E}^* \cdot \hat{E} \right\}$$

BUT WHAT ABOUT PLASMA CURRENT?

$$\bar{J}(\bar{r},t) = \iiint dr' \int dt'\ \bar{\bar{\sigma}}(\bar{r}',t') \cdot \bar{E}(\bar{r}-r', t-t')$$

EXPAND THIS NEAR $(\bar{r},t)$

$$\bar{E}(\bar{r}-r', t-t') \approx \left[\hat{E}(\bar{r},t) - t' \frac{\partial \hat{E}}{\partial t} - \bar{r}' \cdot \frac{\partial \hat{E}}{\partial \bar{r}} + \dots \right] e^{-j(\omega t - \bar{k}\cdot\bar{r})}$$

$$\bar{\bar{\sigma}}(\bar{r}',t') = \hat{\sigma}^*(r',t')\, e^{+j(\omega t' - \bar{k}\cdot\bar{r}')}$$

Slowly-Varying Wave

$$E(\bar{r}-r', t-t') = \left[\bar{E}(r,t) - t'\frac{\partial \bar{E}}{\partial t} - \bar{r}' \cdot \frac{\partial E}{\partial r}\right] e^{-j(\omega t - \bar{k}\cdot\bar{r})} \times e^{+j(\omega t' - \bar{k}\cdot\bar{r})}$$

BUT

$$\iiint dr' \int dt' \, \bar{\sigma}(r', t') \, e^{j(\omega t - \bar{k}\cdot\bar{r})} = \bar{\sigma}(\omega, k)$$

$$\iiint dr' \int dt \; t' \bar{\sigma}(r', t) \, e^{j(\omega t - k\cdot r)} = i \frac{\partial \bar{\sigma}}{\partial \omega}$$

$$\iiint dr' \int dt \; r' \bar{\sigma}(r, t) \, e^{j(\omega t - k\cdot r)} = -i \frac{\partial \bar{\sigma}}{\partial k}$$

$$\therefore \quad \boxed{J(\omega, k) = \bar{\sigma}(k) \cdot \bar{E}(\omega, k) + i \frac{\partial \bar{\sigma}}{\partial \omega} \cdot \frac{\partial E_k}{\partial t} - i \frac{\partial \bar{\sigma}}{\partial \bar{k}} \cdot \frac{\partial \bar{E}}{\partial \bar{r}} + \cdots}$$

FOR SLOWLY VARYING AMPLITUDES

Electrostatic Wave Energy Conservation

$$\epsilon_0 \frac{\partial \bar{E}}{\partial t} = -\bar{J}$$

$$\epsilon_0 \frac{\partial \hat{E}}{\partial t} - j\omega\epsilon_0 \hat{E} = -\hat{\sigma}\hat{E} + i\frac{\partial \sigma}{\partial \omega}\frac{\partial \hat{E}}{\partial t} - i\frac{\partial \sigma}{\partial k}\cdot\frac{\partial \hat{E}}{\partial r} + \cdots$$

$$\sigma = i\sigma_I + \sigma_R$$

RESISTIVE PART (DAMPING)

REACTIVE PART

FOR WAVES $|\sigma_I| \gg |\sigma_R|$

IMAGINARY PART:

$$0 = -j\omega\epsilon_0\left(1 - \frac{\sigma_i}{\omega\epsilon_0}\right)\hat{E}$$

REAL PART:

$$\epsilon_0 \frac{\partial \hat{E}}{\partial t}\left(1 - \frac{\partial \sigma_i}{\partial \omega}\frac{1}{\epsilon_0}\right) = -\sigma_R\hat{E} + \frac{\partial \sigma_I}{\partial k}\cdot\frac{\partial \hat{E}}{\partial r}$$

$$\frac{\partial}{\partial t}\left(\frac{\epsilon_0}{2}\frac{|\hat{E}|^2}{2}\left(1 - \frac{1}{\epsilon_0}\frac{\partial \sigma_i}{\partial \omega}\right)\right) = -\frac{\sigma_R}{\epsilon_0}\left(\epsilon_0\frac{|E|^2}{2}\right) - \nabla\cdot\left(-\frac{\partial \sigma_I}{\partial k}\right)\frac{|E|^2}{2}\frac{1}{\epsilon_0}$$

Electrostatic Wave Energy Conservation

DEFINE W = ENERGY DENSITY OF WAVE

$$= \frac{\epsilon_0}{2} \frac{|E|^2}{2} \frac{\partial}{\partial \omega}(\omega D) \qquad \omega D \equiv \omega\left(1 - \frac{\sigma_i}{\omega \epsilon_0}\right)$$

$$V_g = \frac{-\frac{\partial}{\partial k}(\omega D)}{\frac{\partial}{\partial \omega}(\omega D)} = \frac{\partial \omega}{\partial k} = \text{GROUP VELOCITY}$$

γ = DAMPING RATE

$$= \frac{\sigma_R}{\epsilon_0}$$

$$\frac{\partial}{\partial t}(W) + \nabla \cdot \bar{V}_g W = -\gamma W$$

(This is an important slide.)

Electrostatic Wave Energy Conservation

PLASMA WAVE

$$\omega D(\omega) = \omega\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)$$

$$\frac{\partial}{\partial\omega}(\omega D) = 1 + \frac{\omega_p^2}{\omega^2}$$

ELECTRIC ENERGY

ELECTRON ENERGY

ION SOUND WAVE

$$\omega D = \omega\left(1 - \frac{\omega_{pi}^2}{\omega^2} + \frac{1}{k^2\lambda_D^2}\right)$$

$$\frac{\partial}{\partial\omega}(\omega D) \simeq 1 + \frac{1}{k^2\lambda_D^2} + \frac{\omega_{pi}^2}{\omega^2}$$

$$\simeq 1 + \frac{\omega_{pi}^2}{\omega^2}$$

ELECTRIC

ION ENERGY

John Malmberg and Chuck Wharton

The first experimental measurement of Landau Damping

John Malmberg

(obit, Nov 1992)

Prof. Malmberg joined UCSD from General Atomics in 1969 as a professor of physics. Much of his work revolved around theoretical and experimental investigations of fully ionized gases or plasmas. The field could offer insights into how stars work and how to ignite and control thermonuclear reactions to produce fusion energy--the power that drives the sun.

A plasma is the fourth state of matter, with solids, liquids and gases making up the other three. Most of the matter in the Universe is in the plasma state; for example, the matter of stars is composed of plasmas.

In recent years, Prof. Malmberg had been experimenting with pure electron plasmas that were trapped in a magnetic bottle. By contrast with electrically neutral plasmas that contain an equal number of positive and negative electrons, pure electron plasmas are rare in nature.

Before joining UCSD, Prof. Malmberg was director of the Plasma Turbulence group at General Atomics, where he carried out some of the first and most important experiments to test the basic principals of plasma physics. Perhaps his most important experiment involved the confirmation of the phenomenon called "Landau damping," where electrons surf on a plasma wave, stealing energy from the wave and causing it to damp (decrease in amplitude).

For his pioneering work in testing the basic principals of plasma, and for his more recent work with electron plasmas, Prof. Malmberg was named the recipient of the American Physical Society's James Clerk Maxwell Prize in Plasma Physics in 1985.

Chuck Wharton

(emeritus, *Cornell*)

Professor Wharton was a staff member at the University of California Lawrence Radiation Laboratory at Livermore and Berkeley, California from 1950-1962. While in this position, he spent a year (1959-60) as engineer-scientist at the Max-Planck Institute for Physics in Munich, Germany, and also as a lecturer at the International Summer Course in Plasma Physics at Riso, Denmark. For the next five years he was a staff member of the Experimental Physics Group at General Atomics in San Diego, California. He joined the EE faculty as a full professor in 1967.

In 1973 he received the Humboldt Prize awarded by the Alexander von Humboldt Foundation. He was elected a fellow of the American Physical Society in 1973. In 1976 he was elected a fellow of the IEEE "in recognition of contributions to the understanding of plasmas and to the development of plasma diagnostic techniques." In 1979 he was given the award, Socio Onorario, by the International School of Plasma Physics (Milan, Italy).

Charles (Chuck) taught undergraduate courses in electromagnetic theory, plasma physics, and electrical sciences laboratory. His research was primarily in the area of plasma-physics diagnostics, in which he is a recognized world authority, and in plasma interactions and heating with waves and beams with applications to controlled thermonuclear fusion.

VOLUME 13, NUMBER 6 PHYSICAL REVIEW LETTERS 10 AUGUST 1964

COLLISIONLESS DAMPING OF ELECTROSTATIC PLASMA WAVES*

J. H. Malmberg and C. B. Wharton
John Jay Hopkins Laboratory for Pure and Applied Science,
General Atomic Division of General Dynamics Corporation, San Diego, California
(Received 6 July 1964)

VOLUME 17, NUMBER 4 PHYSICAL REVIEW LETTERS 25 JULY 1966

DISPERSION OF ELECTRON PLASMA WAVES*

J. H. Malmberg and C. B. Wharton
John Jay Hopkins Laboratory for Pure and Applied Science,
General Atomic Division of General Dynamics Corporation, San Diego, California
(Received 31 May 1966)

Description of the Experimental Device

The machine which produces the plasma has been described in detail elsewhere.[4] The plasma is produced in a duoplasmatron-type hydrogen arc source and drifts from it into a long uniform magnetic field of a few hundred gauss. The entire machine is steady state. The resulting plasma has, in a typical case, a radius of 7 mm, a length of 230 cm, a density of 5×10^8 electrons/cm^3, and a temperature of 12 ± 3 eV as measured by Langmuir probes. The background pressure is 1.7×10^{-5} Torr (mostly H_2). Hence, the Debye length is about 1 mm, the electron mean free path for electron-ion collisions is of the order of 1000 meters and for electron-neutral collisions is about 40 meters. The plasma is surrounded by a stainless steel tube 3.8 cm in radius which acts as a waveguide beyond cutoff to reduce electromagnetic coupling between probes. The plasma density depends somewhat on distance from the source.

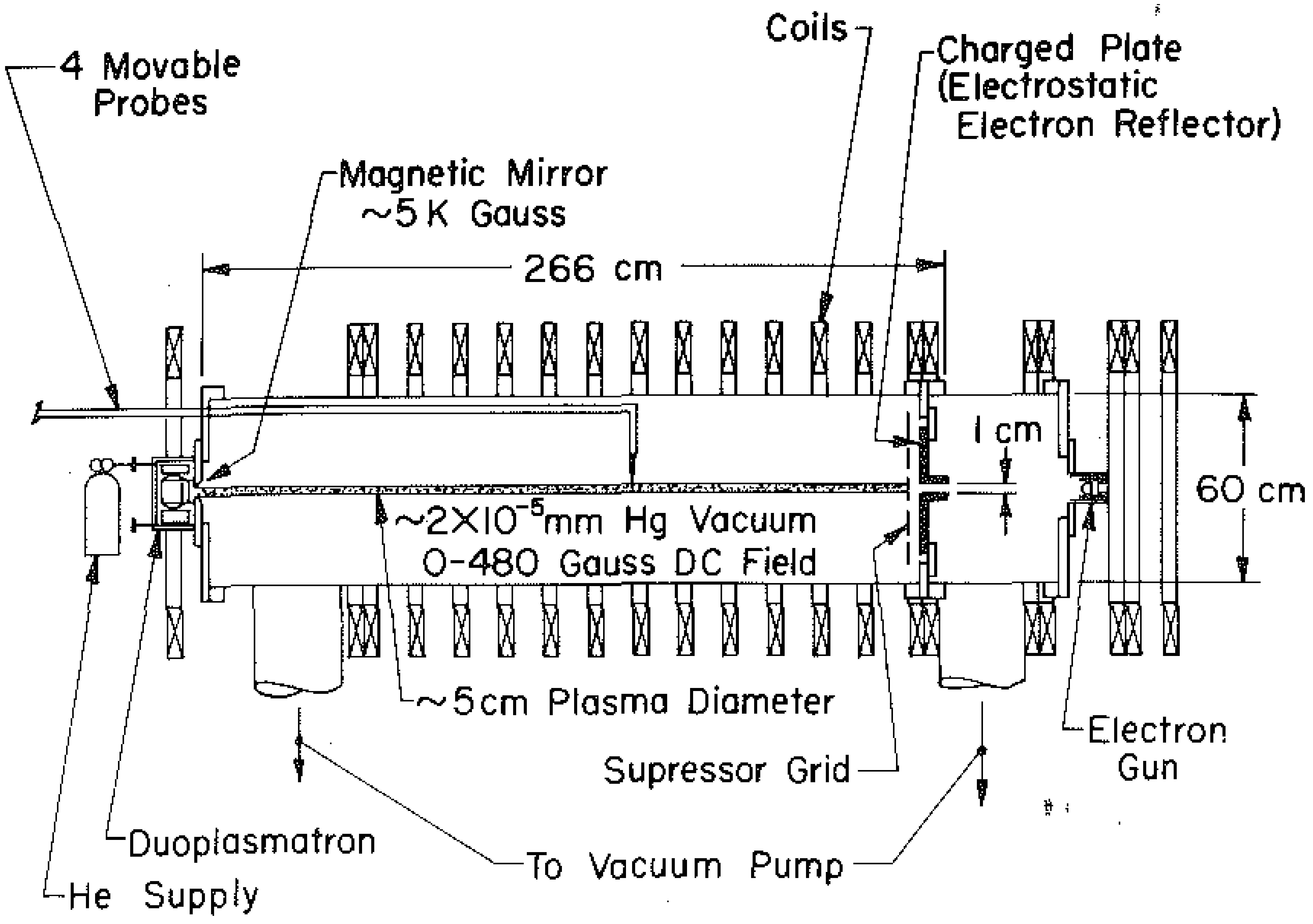


FIGURE 10.8.4
Schematic of experiment used to investigate plasma wave echoes. [*After J. H. Malmberg, et al., Proceedings of Conference on Phénomènes d'ionization dans les Gaz*, 4; 229 (1963).]

Description of the Experimental Device

If L is large compared with the Landau damping length, and if $\omega_2/(\omega_2 - \omega_1)$ is of order unity, this third electric field, which is the spatial plasma echo, appears at a position well separated from the first two electric field excitation positions. The experiment used by Malmberg *et al.*[1] to study the spatial plasma wave echoes is depicted schematically in Fig. 10.8.4. The plasma column is 180 cm long and 5 cm in diameter, with a central density of 1.5×10^8 cm^{-3}. The axial magnetic field is 300 G and can be regarded as infinite for the purposes of the experiment. The plasma has a temperature of 9.4 eV and a Debye length of 2 mm. The electron mean free path is 10^5 cm for electron-ion collisions and 4×10^4 cm for electron-neutral collisions. The plasma column is surrounded by a 5.2-cm-radius cylinder that acts as a waveguide beyond cutoff and reduces the stray electromagnetic coupling between the excitation and detection probes.

A plasma wave echo obtained with this experiment is shown in the lower trace of Fig. 10.8.5. The upper trace is the spatial distribution of the 120-MHz signal in the vicinity of the excitation probe at $x = 0$. The middle trace is the spatial distribution of the 130-MHz signal in the vicinity of the second probe at

[1] J. H. Malmberg, C. B. Wharton, R. W. Gould, and T. M. O'Neil, *Phys. Fluids*, **11**: 1147 (1968).

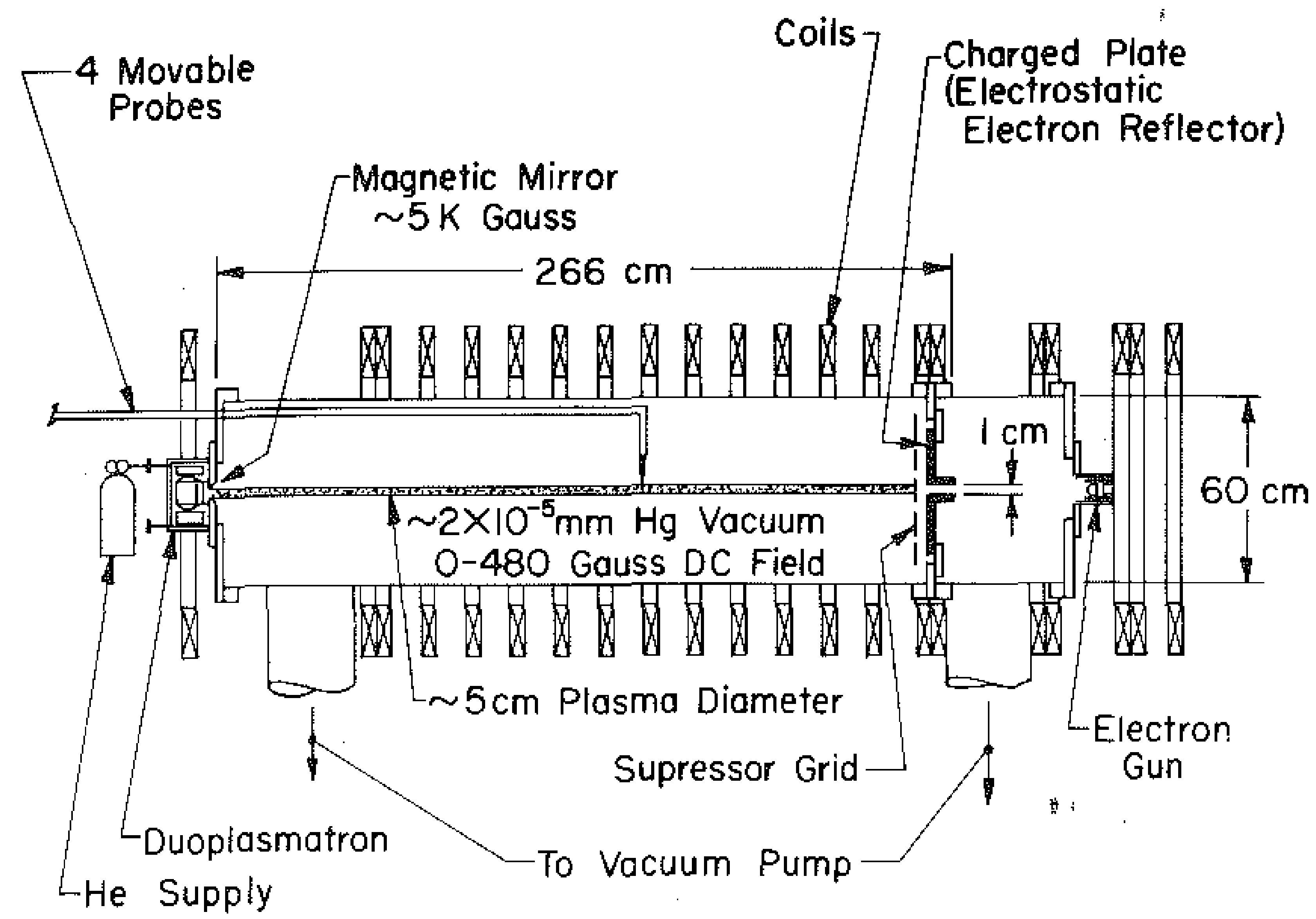


FIGURE 10.8.4
Schematic of experiment used to investigate plasma wave echoes. [*After J. H. Malmberg, et al., Proceedings of Conference on Phénomènes d'ionization dans les Gaz*, 4; 229 (1963).]

JOURNAL OF APPLIED PHYSICS VOLUME 30, NUMBER 11 NOVEMBER, 1959

Look!

Space Charge Waves in Cylindrical Plasma Columns*

A. W. TRIVELPIECE† AND R. W. GOULD
California Institute of Technology, Pasadena, California

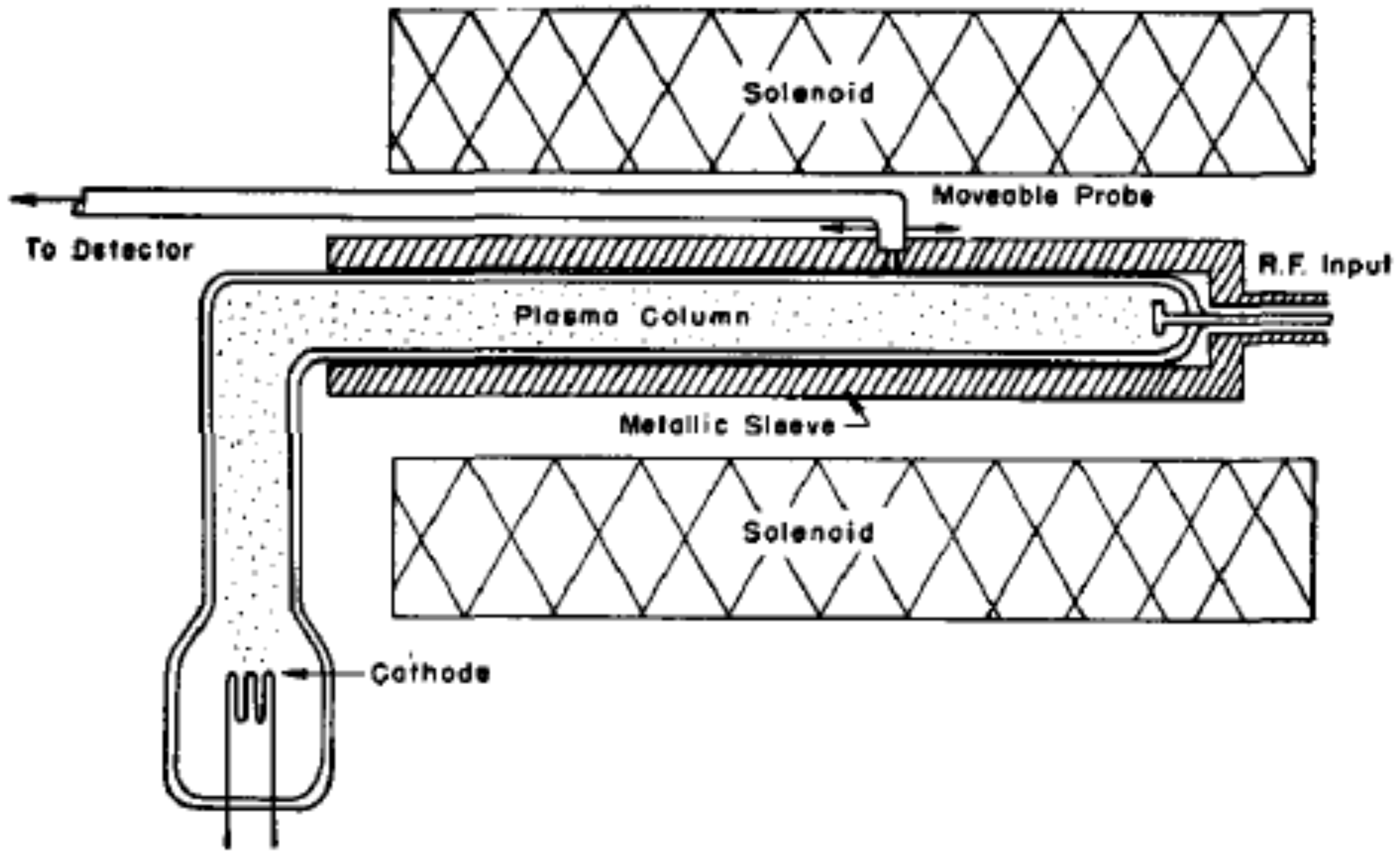


FIG. 10. Schematic of apparatus to measure phase and attenuation characteristics of space charge waves in a plasma.

TABLE I. Pertinent dimensions of experimental apparatus used in space charge wave propagation experiment.

Diameter of plasma	0.328 in.
Diameter of tube containing plasma	0.410 in.
Diameter of slotted wave guide	0.410 and 0.750 in.
Length of plasma column	25 cm
Signal frequency range	10 to 4000 mc
Cyclotron frequency range	0 to 5000 mc
Plasma frequency range	500 to 5000 mc
Temperature of mercury in tube	300±0.1°K
Empty wave guide cutoff frequency (approx)	25 000 mc
Pressure of mercury at 300°K (approx)	2 microns
Mean free path of plasma electrons (approx)	5 cm

Assuming wave solutions $\exp[j(\omega t-\beta z)]$

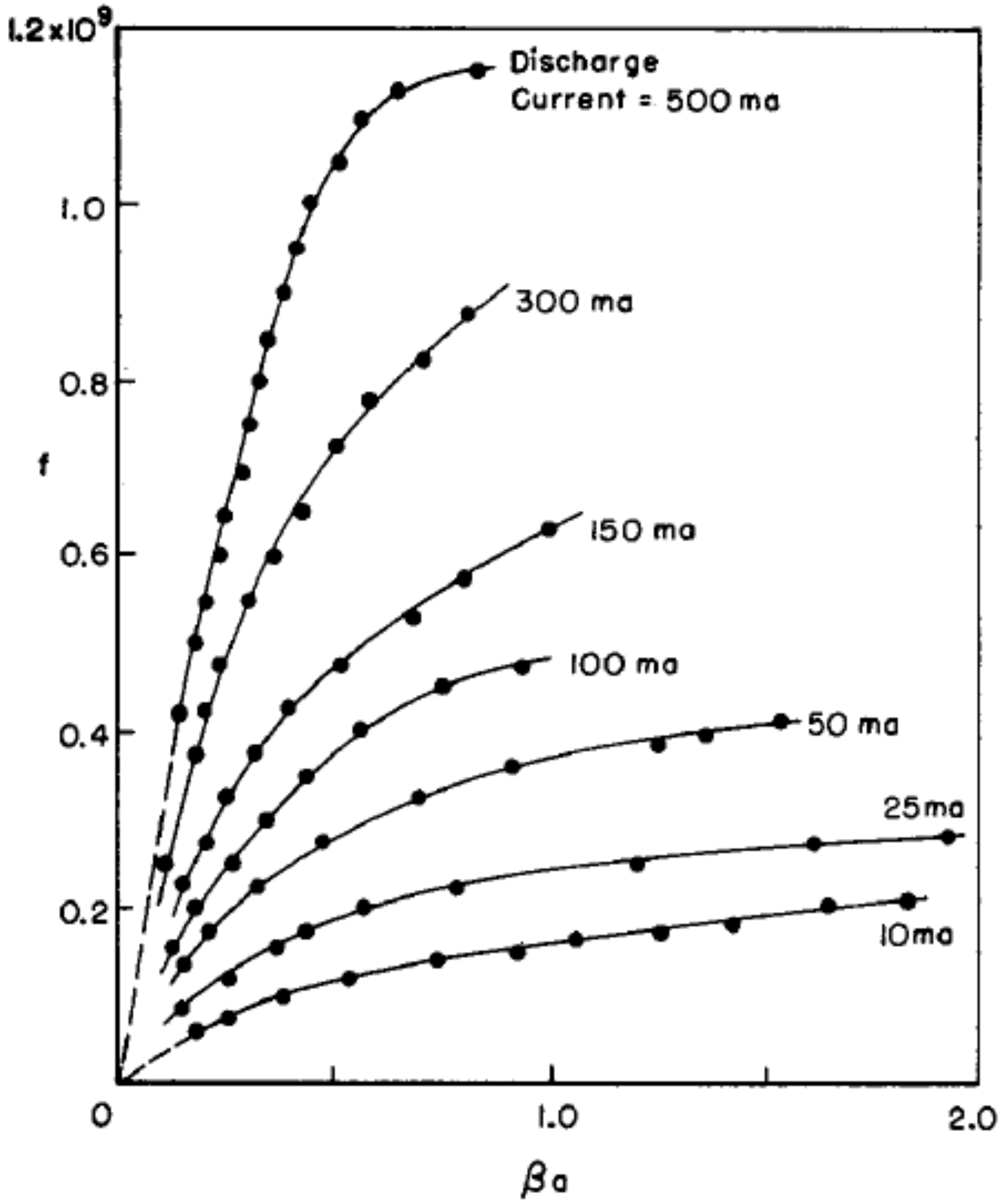


FIG. 13. Measured phase characteristics of plasma space charge waves for no magnetic field for $a=0.52$ cm, $b=0.62$ cm, $K=4.6$.

INSIDE PLASMA ($r<a$):

$$\bar{E} = -\nabla\Phi \qquad \nabla\cdot(\epsilon_0 K \bar{E}) = 0 \qquad K = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\Rightarrow \nabla^2\Phi = 0$$

OUTSIDE PLASMA ($r>a$):

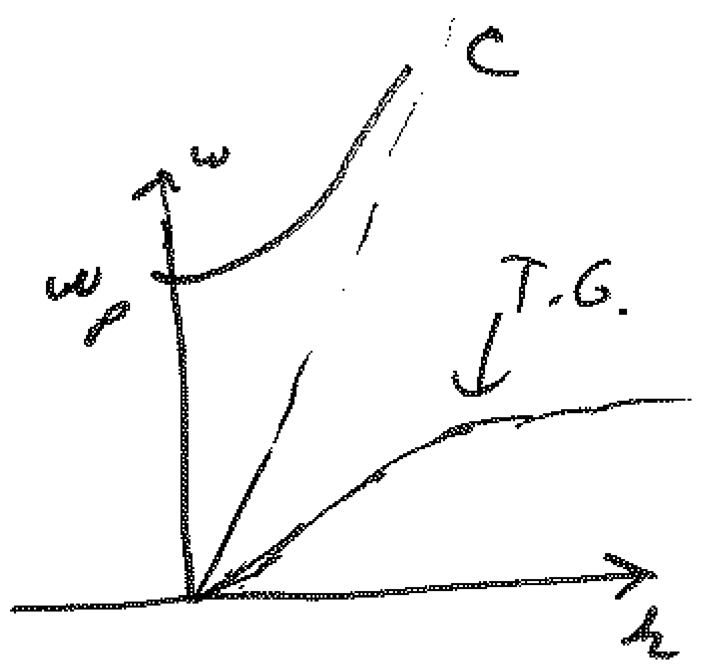
$$\bar{E} = -\nabla\Phi \qquad \nabla\cdot(\epsilon_0 \bar{E}) = 0$$

$$\nabla^2\Phi = 0$$

BOUNDARY CONDITIONS ($r=a$):

$$\hat{n}\cdot(\epsilon_0 K \bar{E}_{IN} - \epsilon_0 \bar{E}_{OUT}) = 0$$

$$\Phi_{IN} = \Phi_{OUT}$$



$$\nabla^2\Phi = 0 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) - k^2\Phi = 0$$

$$\Phi(r,z,t) \propto e^{-j(\omega t - kz)}$$

$$\left.\begin{array}{ll}\Phi(r) \sim I_0(kr) & \text{INSIDE} \\ K_0(kr) & \text{OUTSIDE}\end{array}\right\} r=a$$

$$\left(1-\frac{\omega_p^2}{\omega^2}\right) I_0' C_1 = K_0' C_2$$

$$I_0 C_1 = K_0 C_2$$

$$1 - \frac{\omega_p^2}{\omega^2} = \frac{I_0(ka)K_0'(ka)}{I_0'(ka)K_0(ka)}$$

Raw Data

Two probes, each consisting of a 0.2-mm diameter radial tungsten wire, are placed in the plasma. One probe is connected by coaxial cable to a chopped signal generator. The other probe is connected to a receiver which includes a sharp high-frequency filter, a string of broad-band amplifiers, an rf detector, a video amplifier, and a coherent detector operated at the transmitter chopping frequency. Provision is made to add a reference signal from the transmitter to the receiver rf signal, i.e., we may use the system as an interferometer. The transmitter is set at a series of fixed frequencies, and at each, the receiving probe is moved longitudinally. The position of the receiving probe, which is transduced, is applied to the x axis of an x-y recorder, and the interferometer output or the logarithm of the received power is applied to the y axis.

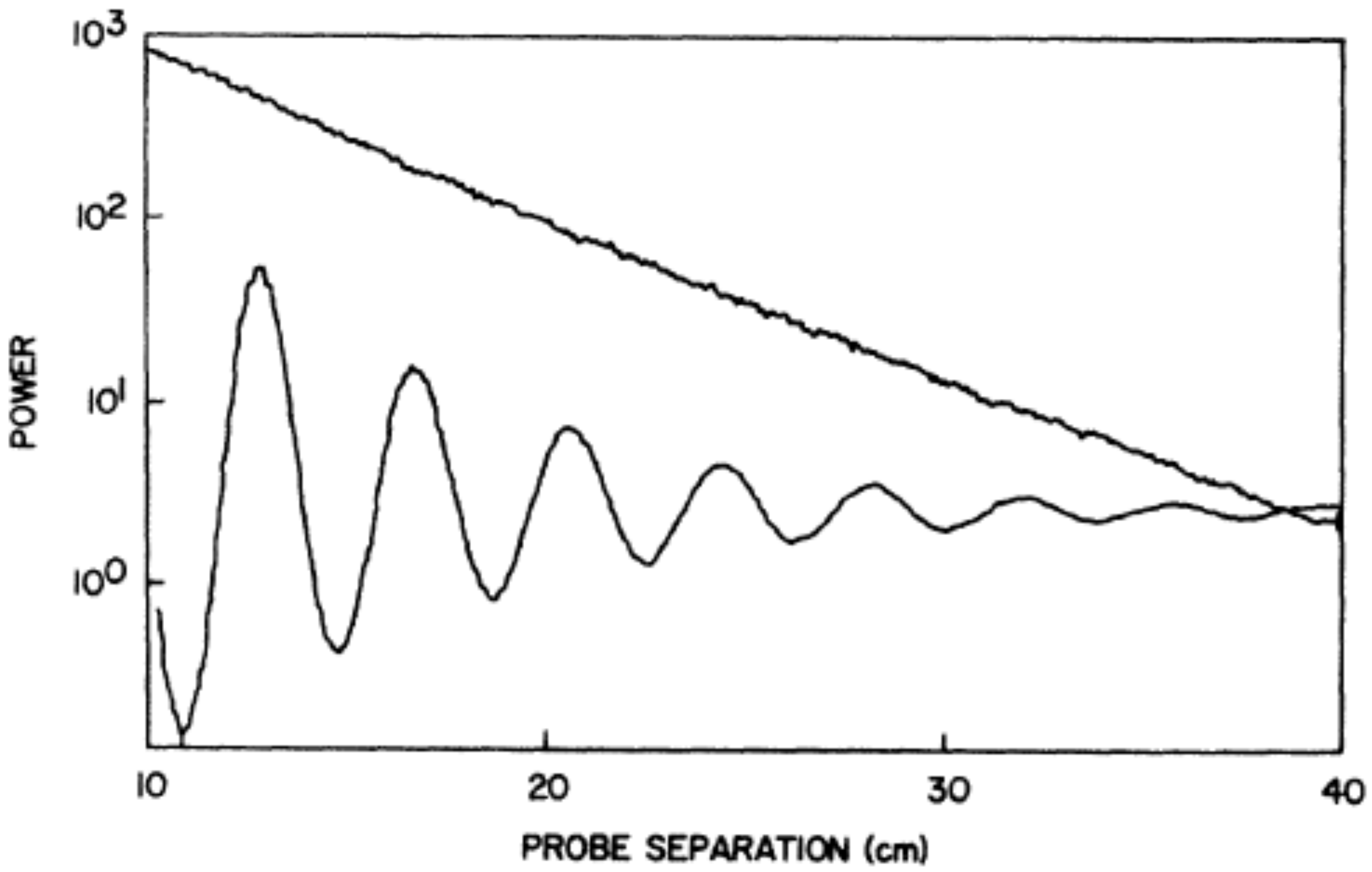


FIG. 1. Raw data. Upper curve is the logarithm of received power. Lower curve is interferometer output. Abscissa is probe separation.

Waves in Magnetized Plasma

$$\frac{\partial \mathbf{v}^{(\alpha)}}{\partial t} = \frac{q_\alpha}{m_\alpha}\left(\mathbf{E}_1 + \mathbf{v}^{(\alpha)} \times \mathbf{B}_0\right) \qquad \alpha = \mathrm{e}, \mathrm{i}$$

$$\hat{v}^{\pm} = \mathrm{i}\frac{q}{\omega m}(\hat{E}^{\pm} \mp \mathrm{i}\hat{v}^{\pm} B_0)$$

$$\hat{v}_x = \mathrm{i}\frac{q}{\omega m}(\hat{E}_x + \hat{v}_y B_0)\,, \qquad \hat{v}_y = \mathrm{i}\frac{q}{\omega m}(\hat{E}_y - \hat{v}_x B_0)$$

$$\boxed{\hat{v}^{\pm} = \mathrm{i}\frac{q}{m}\hat{E}^{\pm}\frac{1}{\omega \mp s\omega_c}}$$

$$\hat{v}^{\pm} = \hat{v}_x \pm \mathrm{i}\hat{v}_y\,, \qquad \hat{E}^{\pm} = \hat{E}_x \pm \mathrm{i}\hat{E}_y$$

Waves in Magnetized Plasma

$$\begin{pmatrix} \hat{v}_x \\ \hat{v}_y \\ \hat{v}_z \end{pmatrix} = \mathrm{i}\frac{q}{\omega m} \begin{pmatrix} \dfrac{\omega^2}{\omega^2 - \omega_\mathrm{c}^2} & \mathrm{i}\dfrac{s\omega\omega_\mathrm{c}}{\omega^2 - \omega_\mathrm{c}^2} & 0 \\ -\mathrm{i}\dfrac{s\omega\omega_\mathrm{c}}{\omega^2 - \omega_\mathrm{c}^2} & \dfrac{\omega^2}{\omega^2 - \omega_\mathrm{c}^2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \hat{E}_x \\ \hat{E}_y \\ \hat{E}_z \end{pmatrix}$$

$$\boldsymbol{\sigma}(\omega) = \mathrm{i}\omega\varepsilon_0 \begin{pmatrix} \displaystyle\sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2 - \omega_{\mathrm{c}\alpha}^2} & \displaystyle\mathrm{i}\sum_\alpha s_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2 - \omega_{\mathrm{c}\alpha}^2}\frac{\omega_{\mathrm{c}\alpha}}{\omega} & 0 \\ \displaystyle -\mathrm{i}\sum_\alpha s_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2 - \omega_{\mathrm{c}\alpha}^2}\frac{\omega_{\mathrm{c}\alpha}}{\omega} & \displaystyle\sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2 - \omega_{\mathrm{c}\alpha}^2} & 0 \\ 0 & 0 & \displaystyle\sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2} \end{pmatrix}$$

$$\boxed{\boldsymbol{\varepsilon}_\omega = \mathbf{I} + \frac{\mathrm{i}}{\omega\varepsilon_0}\boldsymbol{\sigma}_\omega}$$

Waves in Magnetized Plasma

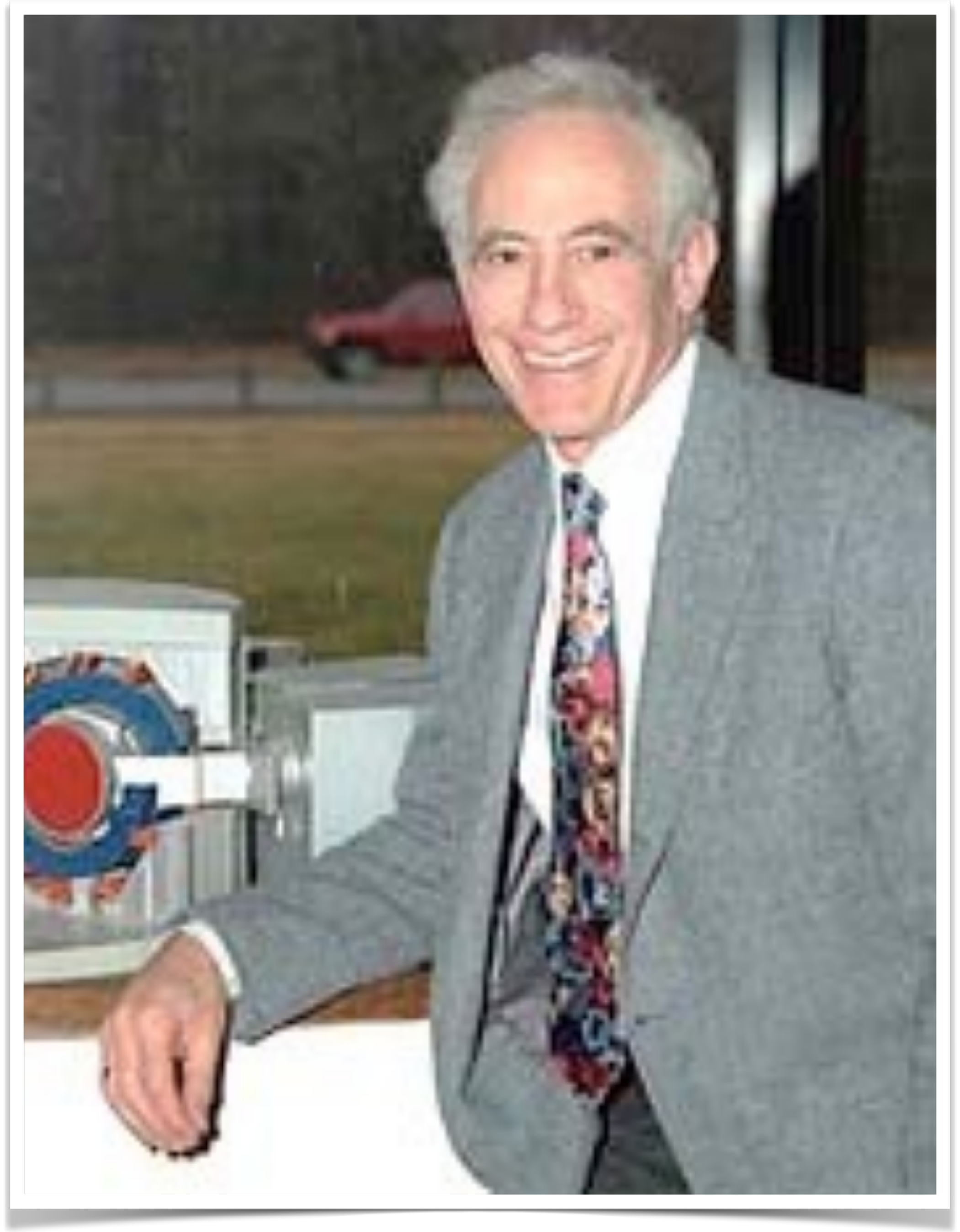
$$\boxed{\boldsymbol{\varepsilon}_\omega = \mathbf{I} + \frac{\mathrm{i}}{\omega\varepsilon_0}\boldsymbol{\sigma}_\omega}$$

$$\boldsymbol{\varepsilon}(\omega) = \begin{pmatrix} S & -\mathrm{i}D & 0 \\ \mathrm{i}D & S & 0 \\ 0 & 0 & P \end{pmatrix}$$

$$S = 1 - \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2 - \omega_{\mathrm{c}\alpha}^2}$$

$$D = \sum_\alpha s_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2 - \omega_{\mathrm{c}\alpha}^2}\frac{\omega_{c\alpha}}{\omega}$$

$$P = 1 - \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2}\,.$$



Working both in the laboratory and with theoretical calculations, he found many ways to put waves to work in fusion research in succeeding decades, and his 1962 book, "The Theory of Plasma Waves," codified the subject in mathematical form for the first time.

http://www.nytimes.com/2001/04/18/nyregion/thomas-h-stix-plasma-physicist-dies-at-76.html

Waves in Magnetized Plasma

$$\begin{pmatrix} S - \mathcal{N}^2 \cos^2\psi & -\mathrm{i}D & \mathcal{N}^2 \cos\psi \sin\psi \\ \mathrm{i}D & S - \mathcal{N}^2 & 0 \\ \mathcal{N}^2 \cos\psi \sin\psi & 0 & P - \mathcal{N}^2 \sin^2\psi \end{pmatrix} \cdot \begin{pmatrix} \hat{E}_x \\ \hat{E}_y \\ \hat{E}_z \end{pmatrix} = 0$$

$$\mathbf{k} \;=\; (k \sin\psi, 0, k\cos\psi)$$

Good places to start:

Propagation along B (ψ = 0)

Propagation ⊥ to B (ψ = π/2)

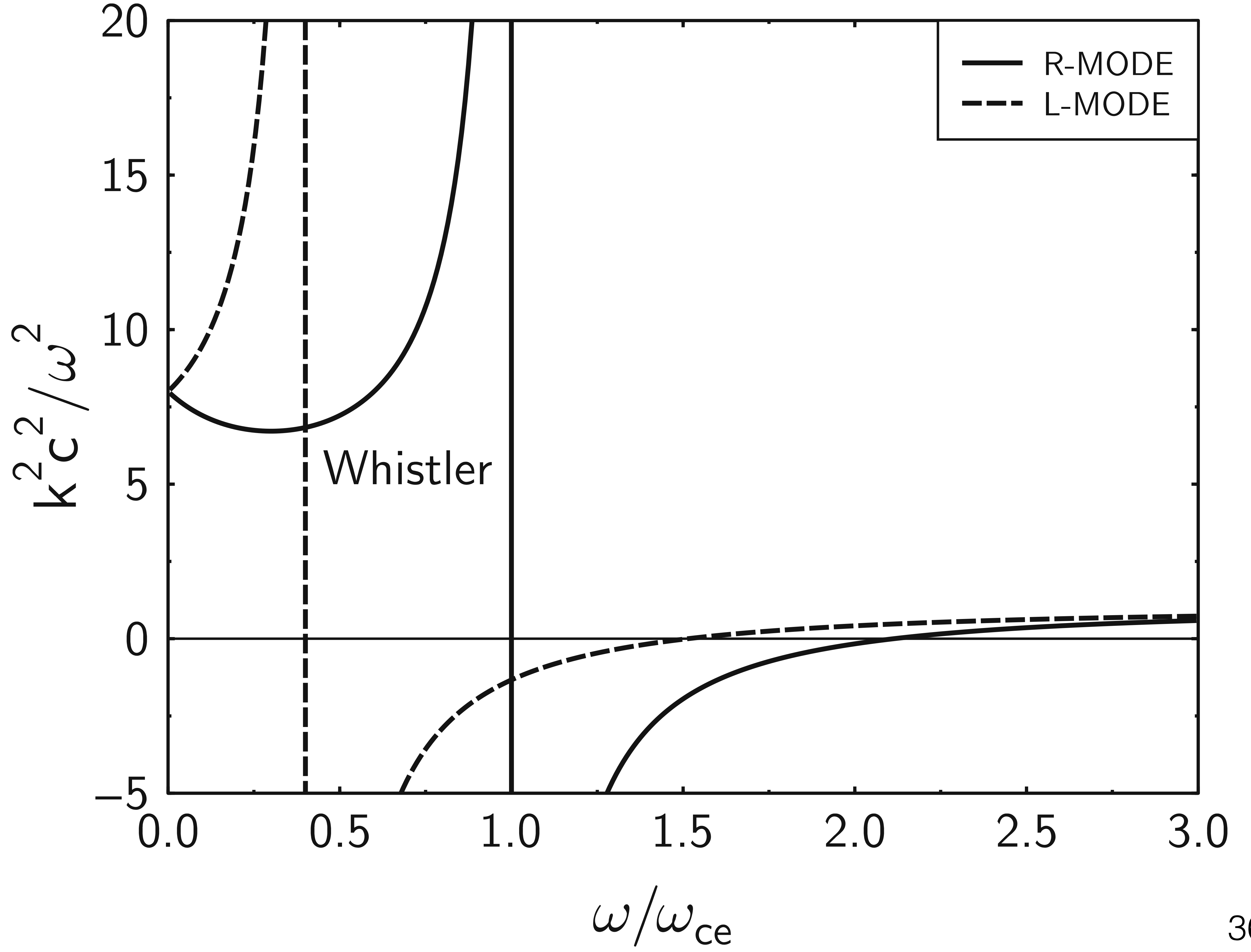
$$S = 1 - \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2 - \omega_{\mathrm{c}\alpha}^2}$$

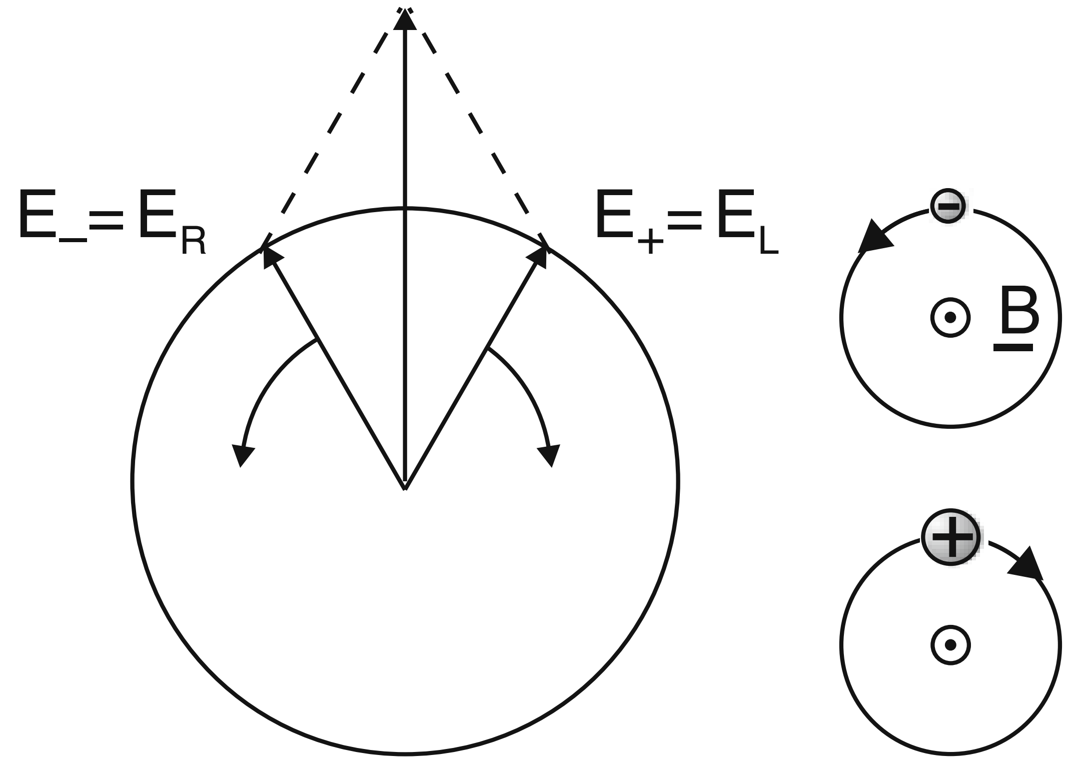
$$D = \sum_\alpha s_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2 - \omega_{\mathrm{c}\alpha}^2} \frac{\omega_{\mathrm{c}\alpha}}{\omega}$$

$$P = 1 - \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2}{\omega^2}\,.$$

Waves k || B

$$\begin{pmatrix} S-\mathcal{N}^2 & -\mathrm{i}D & 0 \\ \mathrm{i}D & S-\mathcal{N}^2 & 0 \\ 0 & 0 & P \end{pmatrix} \cdot \begin{pmatrix} \hat{E}_x \\ \hat{E}_y \\ \hat{E}_z \end{pmatrix} = 0$$





Waves k ⊥ B

Extra-Ordinary Mode

$$\begin{pmatrix} S & -\mathrm{i}D \\ \mathrm{i}D & (S - \mathcal{N}^2) \end{pmatrix} \cdot \begin{pmatrix} \hat{E}_x \\ \hat{E}_y \end{pmatrix} = 0$$

$$\mathcal{N}_{\mathrm{X}} = \left(\frac{S^2 - D^2}{S} \right)^{1/2}$$

$$\omega_{\mathrm{uh}} = (\omega_{\mathrm{ce}}^2 + \omega_{\mathrm{pe}}^2)^{1/2}$$

$$\omega_{\mathrm{lh}} = \left(\omega_{\mathrm{ci}}^2 + \frac{\omega_{\mathrm{pi}}^2 \omega_{\mathrm{ce}}^2}{\omega_{\mathrm{pe}}^2 + \omega_{\mathrm{ce}}^2} \right)^{1/2}$$

Plus: Ordinary Mode

$$\omega^2 = \omega_{\mathrm{pe}}^2 + k^2 c^2$$

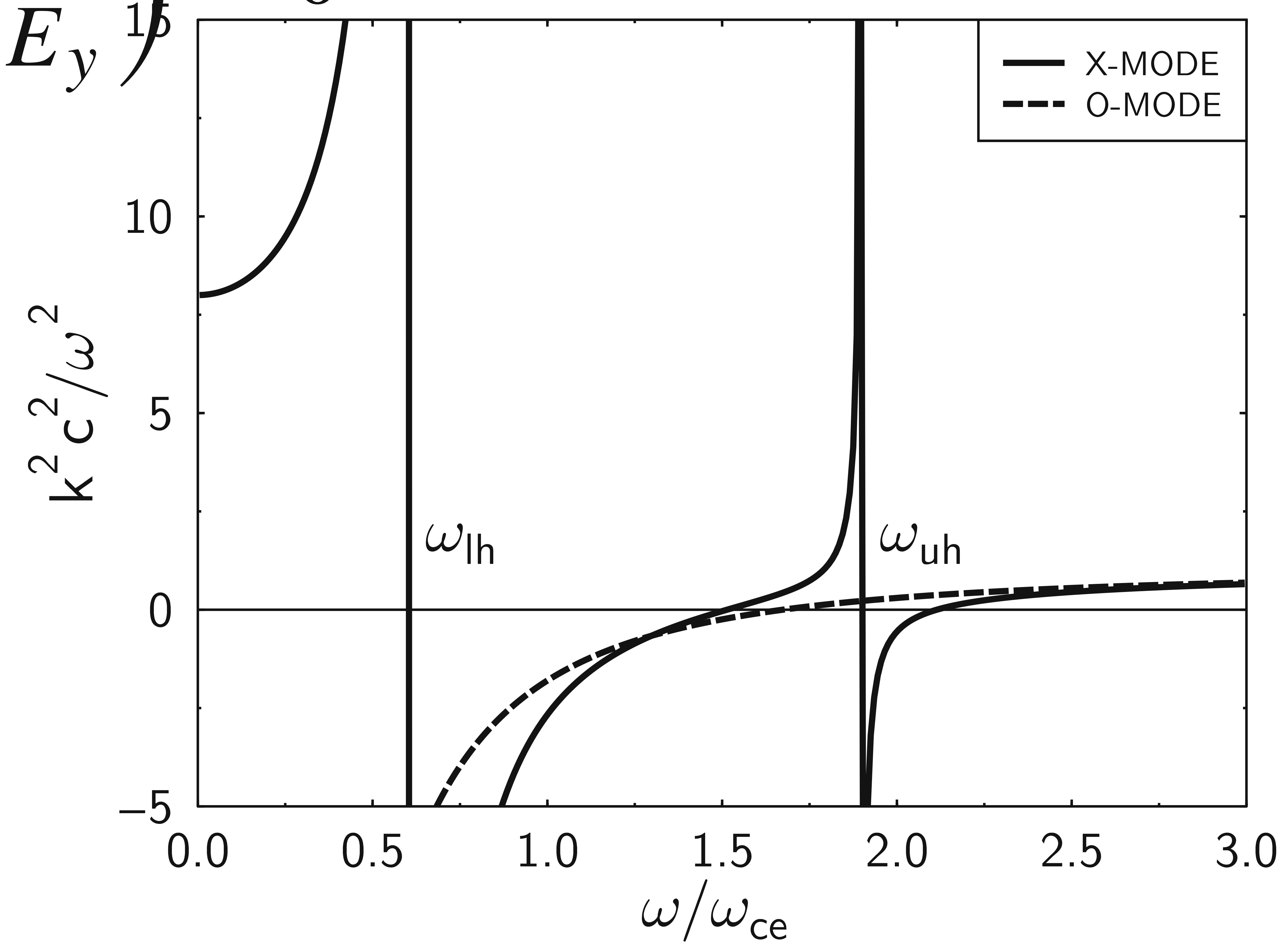
Waves k ⊥ B

Extra-Ordinary Mode

$$\begin{pmatrix} S & -\mathrm{i}D \\ \mathrm{i}D & (S-\mathcal{N}^2) \end{pmatrix} \cdot \begin{pmatrix} \hat{E}_x \\ \hat{E}_y \end{pmatrix} = 0$$

Plus: Ordinary Mode

$$\omega^2 = \omega_{\mathrm{pe}}^2 + k^2 c^2$$



Next Lectures

- Chapter 6: Problems
  - Seeking volunteers…
- Chapter 7: “Plasma Boundaries”
  - Probes (!)

Ch 6 Wave Problems

—— —— 6.1

6.1 In the limit $T_i \ll T_e$ the ion-acoustic wave has the dispersion relation

$$\omega(k) = \frac{\omega_{pi}\lambda_{De}\, k}{(1 + k^2\lambda_{De}^2)^{1/2}}$$

(a) Derive an expression for the phase velocity $v_\varphi(k)$ and group velocity $v_g(k)$ as a function of the wave number k.
(b) Discuss the result with respect to "acoustic behavior" at $k\lambda_{De} \ll 1$.

—— —— 6.2

6.2 Assume that in a dielectric medium the relation $v_\varphi \cdot v_g = c^2$ holds. What is the general shape of the dispersion relation $\omega(k)$ for this case?

—— —— 6.3

6.3 (a) Show that for $\omega_{pe}^2 \gg \omega_{ce}^2 \gg \omega^2$ the refractive index for Whistler waves takes the limiting form

$$\mathcal{N} = \frac{\omega_{pe}}{(\omega\omega_{ce})^{1/2}}$$

(b) Calculate phase and group velocity and show that $v_{gr} = 2v_\varphi$.

Ch 6 Wave Problems

—— —— 6.4

—— —— 6.5

—— —— 6.6

—— —— 6.7

6.4 Determine the minimum plasma density at which a He-Ne Laser at $\lambda = 633\,\mathrm{nm}$ wavelength will be reflected.

6.5 Consider an electron-positron plasma with $n_\mathrm{e} = n_\mathrm{p}$. What is the cut-off frequency for electromagnetic waves in this system?

6.6 The plasma of the ionospheric F-layer has a density $n_\mathrm{e} \approx 2 \times 10^{12}\,\mathrm{m}^{-3}$. The typical magnetic field at mid-latitude is $B = 50\,\mu\mathrm{T}$. Calculate the electron plasma frequency f_pe, electron cyclotron frequency f_ce and the upper hybrid frequency f_uh.

6.7 Prove that $v_\varphi = v_\mathrm{gr}$ requires $\omega = v_\varphi\, k$.