Lecture10-Waves: Plasma Physics 1

APPH E6101x
Columbia University

Last Lecture

- Review of Piel Chapters 1-5 for Quiz

This Lecture

- Introduction to plasma waves
  - Basic review of electromagnetic waves in various media (conducting, dielectric, gyrotropic, …)
  - Basic waves concepts (especially plane waves)
- Electromagnetic waves in *unmagnetized* plasma
- Electrostatic waves in *unmagnetized* plasma

Wave (Helmholtz) Equation

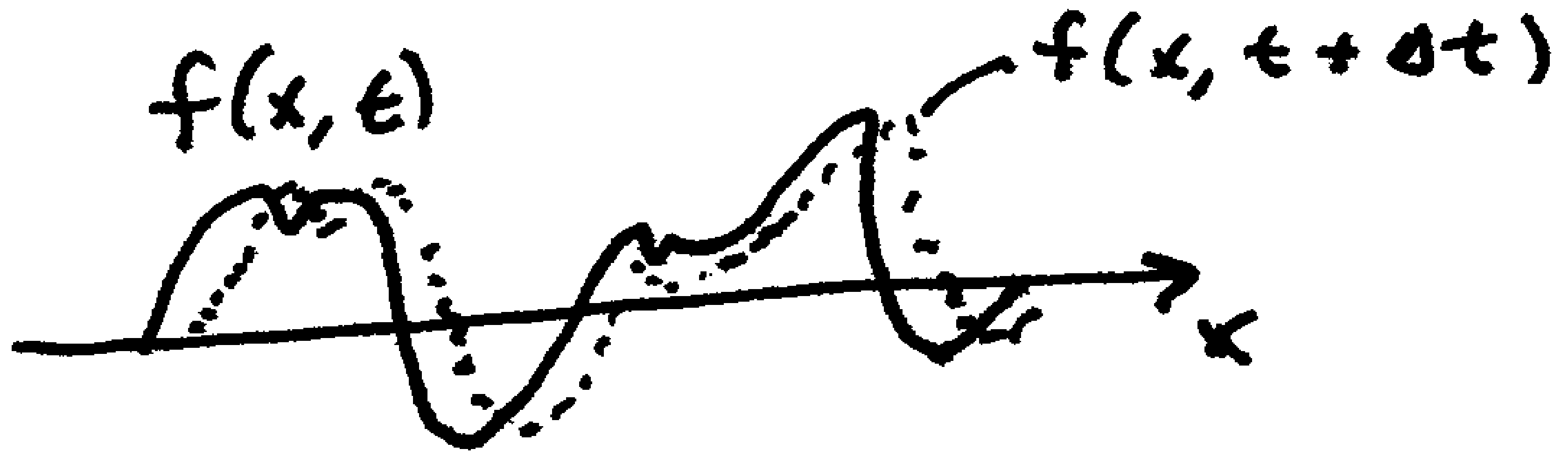
$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}$$

(with c ~ constant)

SOLUTION: $f(x,t) = f(x \pm ct)$

$$\frac{\partial f}{\partial t} = \pm c f' \qquad \frac{\partial f}{\partial x} = f'$$

$$\frac{\partial^2 f}{\partial t^2} = c^2 f'' \qquad \frac{\partial^2 f}{\partial x^2} = f''$$



FOR (SIMPLE) HELMHOLTZ EQ, WAVES MOVE AT CONSTANT SPEED c WITHOUT DISTORTION!

Plane Waves

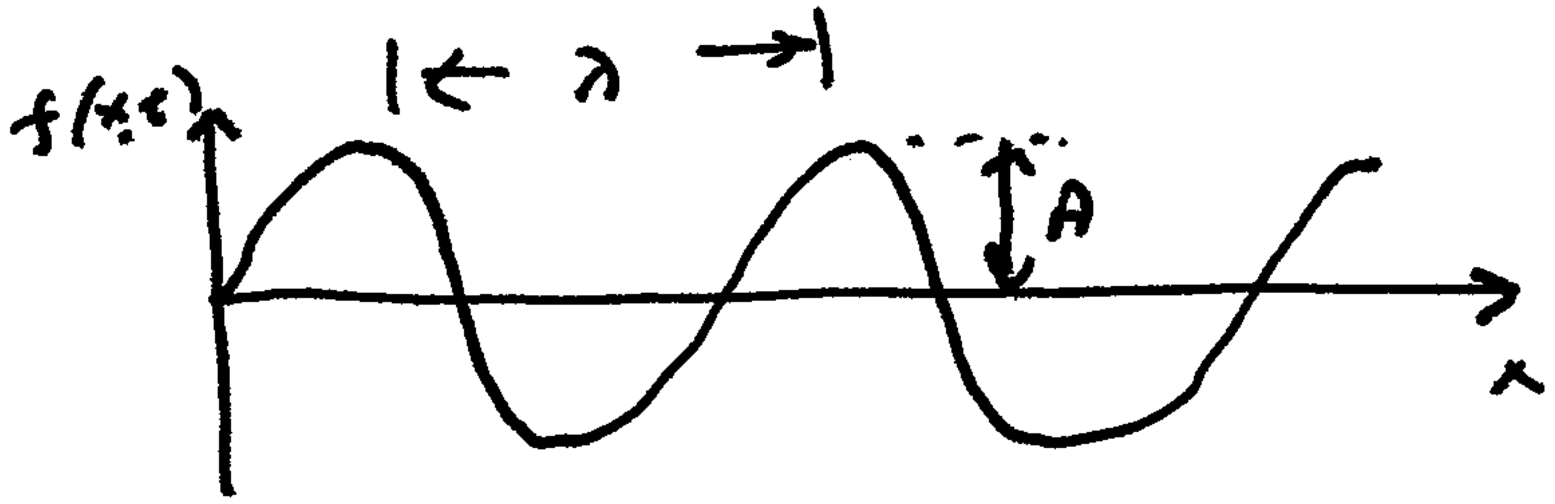
$$\varphi = \mathbf{k} \cdot \mathbf{r} - \omega t$$

$$f(x,t) = A \sin\left(\frac{2\pi}{\lambda}(x \pm ct)\right)$$

SPEED

WAVELENGTH

AMPLITUDE



FREQUENCY $= \omega = 2\pi \frac{c}{\lambda}$ RAD/SEC

$\left(\frac{c}{\lambda}\right)$ = FREQUENCY IN HERTZ

$\varphi(x,t) \equiv \frac{2\pi}{\lambda}(x \pm ct)$ = PHASE OF WAVE

"PHASE FRONTS" ARE SURFACES OF CONSTANT PHASE

"PHASE FRONTS" MOVE AT SPEED c

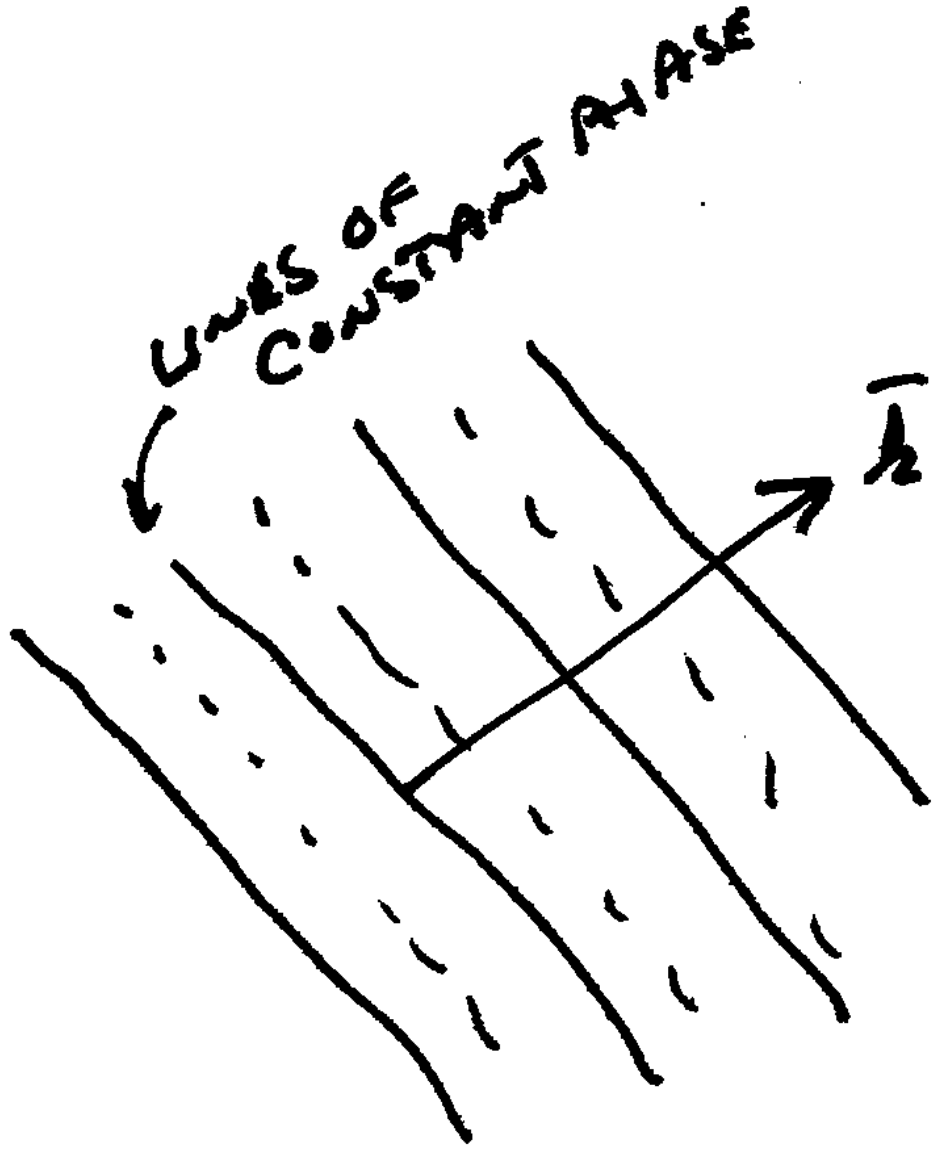
$\underline{c = \text{PHASE VELOCITY}}$

Wave Eq in Multiple Dimensions

$$\frac{\partial^2 f}{\partial t^2} = c^2 \nabla^2 f$$

PLANE WAVES:

$$f(x, y, z, t) = A \sin\Big(\underbrace{\frac{2\pi}{\lambda_x} x + \frac{2\pi}{\lambda_y} y + \frac{2\pi}{\lambda_z} z - \omega t}_{\varphi(x,y,z,t)\ =\ \text{PHASE FRONT}}\Big)$$



$$\bar{k} = \left(\frac{2\pi}{\lambda_x}, \frac{2\pi}{\lambda_y}, \frac{2\pi}{\lambda_z}\right)$$

= WAVE VECTOR

$\bar{k}$ POINTS IN THE DIRECTION OF PHASE VELOCITY

$$f(x, y, z, t) = A \sin(\bar{k} \cdot \bar{x} - \omega t)$$

$$|c| = \frac{\omega}{|\bar{k}|} \qquad \bar{c} = \bar{k}\left(\frac{\omega}{k^2}\right) = \text{PHASE VELOCITY}$$

$$\varphi = \mathbf{k} \cdot \mathbf{r} - \omega t$$

$$\mathbf{E} = \hat{\mathbf{E}} \exp[\mathrm{i}(\mathbf{k} \cdot \mathbf{r} - \omega t)]$$

$$\mathbf{B} = \hat{\mathbf{B}} \exp[\mathrm{i}(\mathbf{k} \cdot \mathbf{r} - \omega t)]$$

$$\mathbf{j} = \hat{\mathbf{j}} \exp[\mathrm{i}(\mathbf{k} \cdot \mathbf{r} - \omega t)]\,.$$

OTHER SOLUTIONS:
- SUM OF PLANE WAVES

Wave Eq in Cylindrical Coordinates

$$\frac{\partial^2 f}{\partial t^2} = c^2 \nabla^2 f$$

CYLINDRICAL COORDINATES:

$$\frac{\partial^2 f}{\partial t^2} = c^2 \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2} \right]$$

CHOOSE SEPARATION OF VARIABLES

$$f(r, \varphi, z, t) \sim g(r)\, e^{\pm i m \theta}\, e^{j k z}\, e^{-j\omega t}$$

$$\frac{\partial^2 f}{\partial \varphi^2} = -m^2 f \qquad \frac{\partial^2 f}{\partial z^2} \sim -k^2 f$$

$$\frac{\partial^2 f}{\partial t^2} = -\omega^2 f$$

SO WAVE EQUATION IS:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial g}{\partial r}\right) + \left(\frac{\omega^2}{c^2} - \frac{m^2}{r^2} - k^2 \right) g(r) = 0$$

CAN BE PUT INTO BESSEL'S EQUATION

$$g(r) = J_m\left(\sqrt{\left(\frac{\omega}{c}\right)^2 - k^2}\; r \right) \;!$$

Wave Packets and Group Velocity

IF $f_1(x,t)$ AND $f_2(x,t)$ ARE SOLUTIONS TO HELMHOLTZ EQUATION, THEN $f_1 + f_2$ IS ALSO A SOLUTION. (THE EQUATION IS LINEAR!)

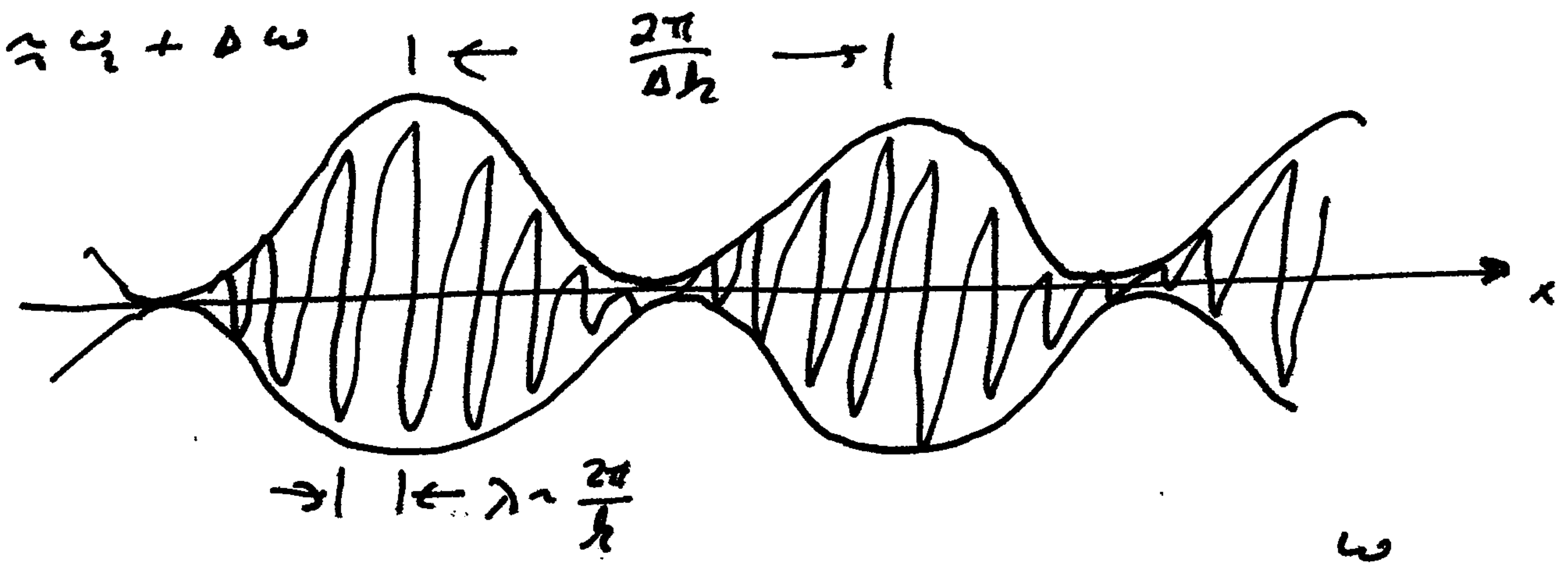
$$f(x,t) = f_1(k_1 x - \omega_1 t) + f_2(k_2 x - \omega_2 t)$$

$$= \sin(k_1 x - \omega_1 t) + \sin(k_2 x - \omega_2 t) \quad \text{(FOR EXAMPLE)}$$

$$= 2\cos\left(\frac{k_1 - k_2}{2} x - \frac{\omega_1 - \omega_2}{2} t\right) \sin\left(\frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t\right)$$

IF $k_1 \approx k_2 + \Delta k$

$\omega_1 \approx \omega_2 + \Delta\omega$

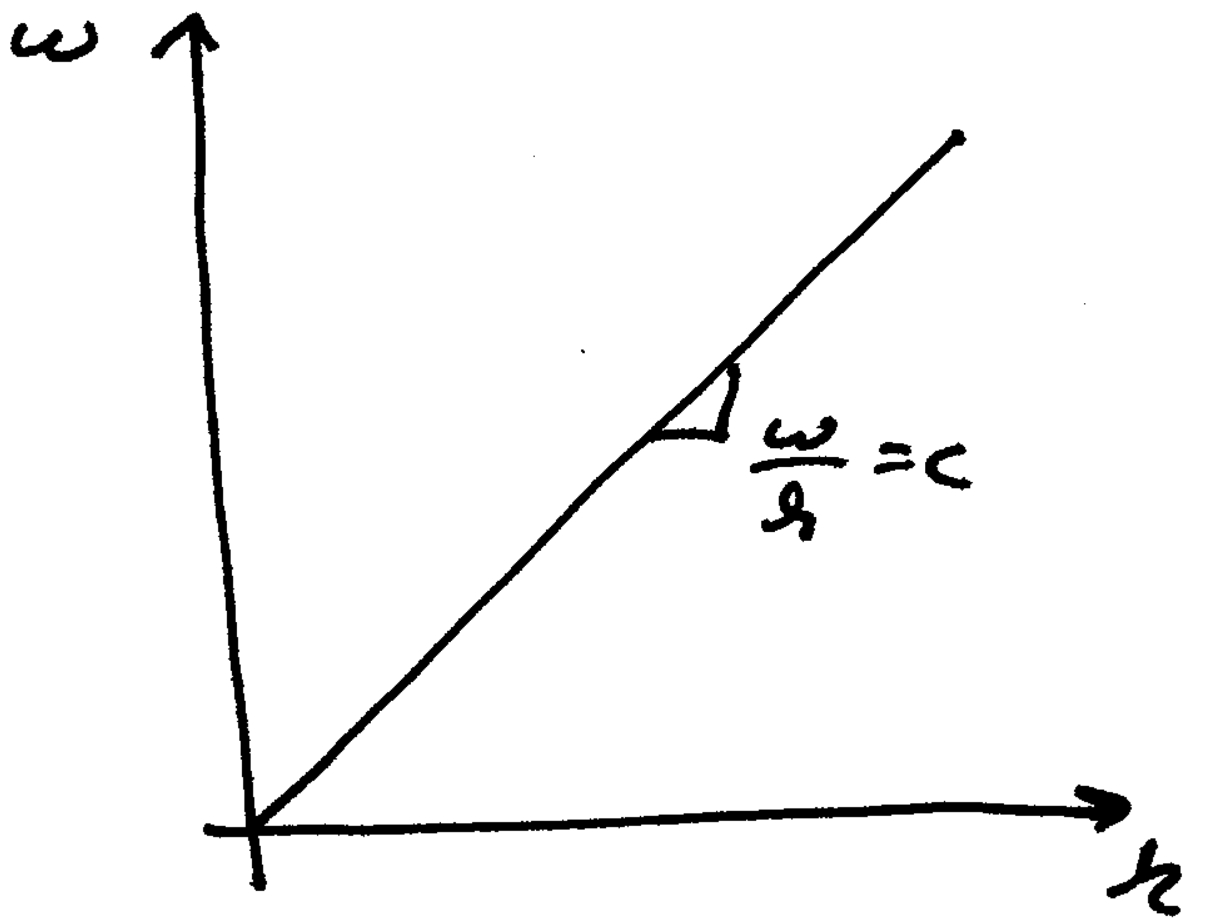


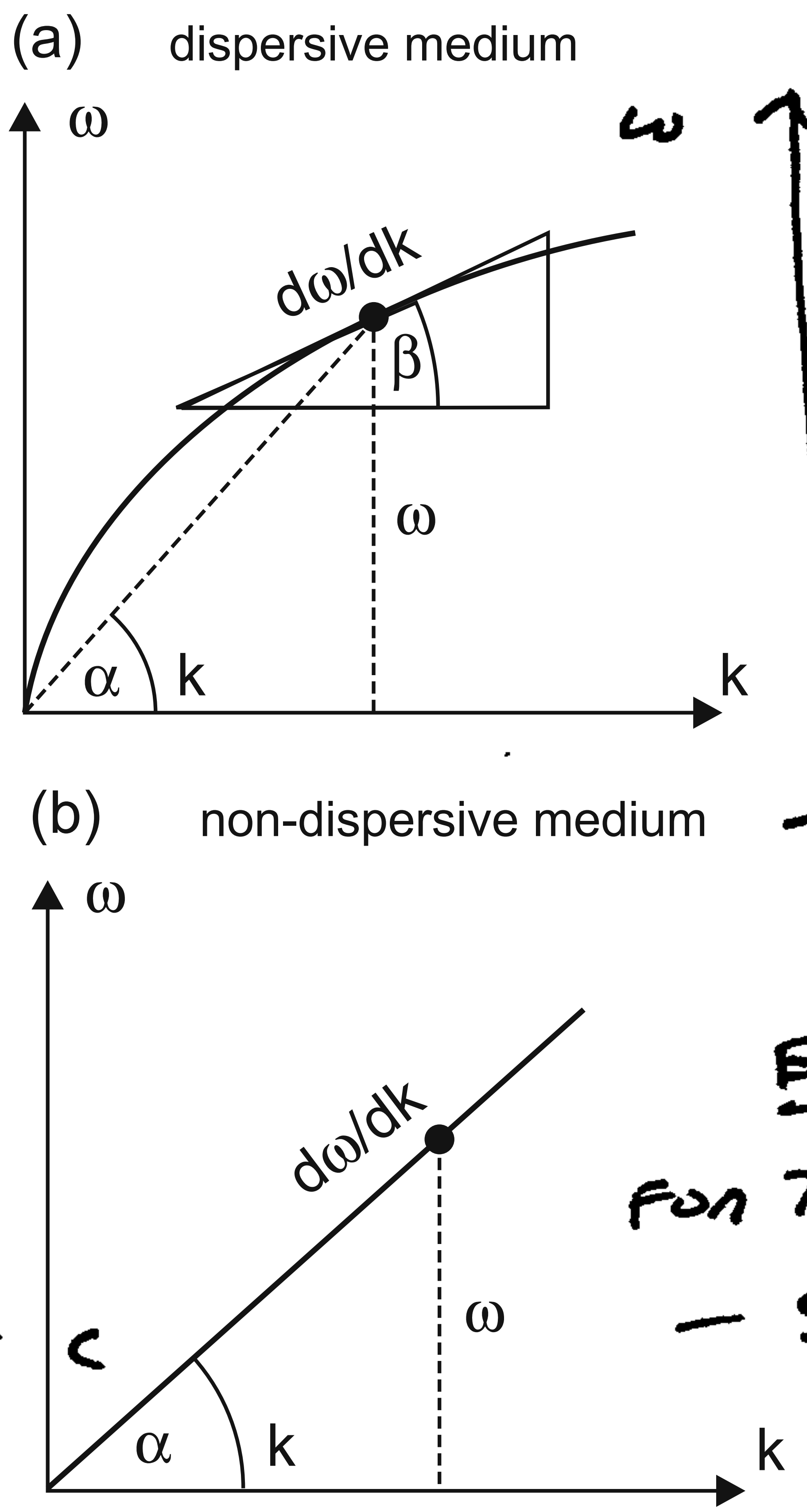
WAVE PHASE FRONTS MOVE AT SPEED $\quad c = \frac{\omega}{|k|}$ = PHASE VELOCITY

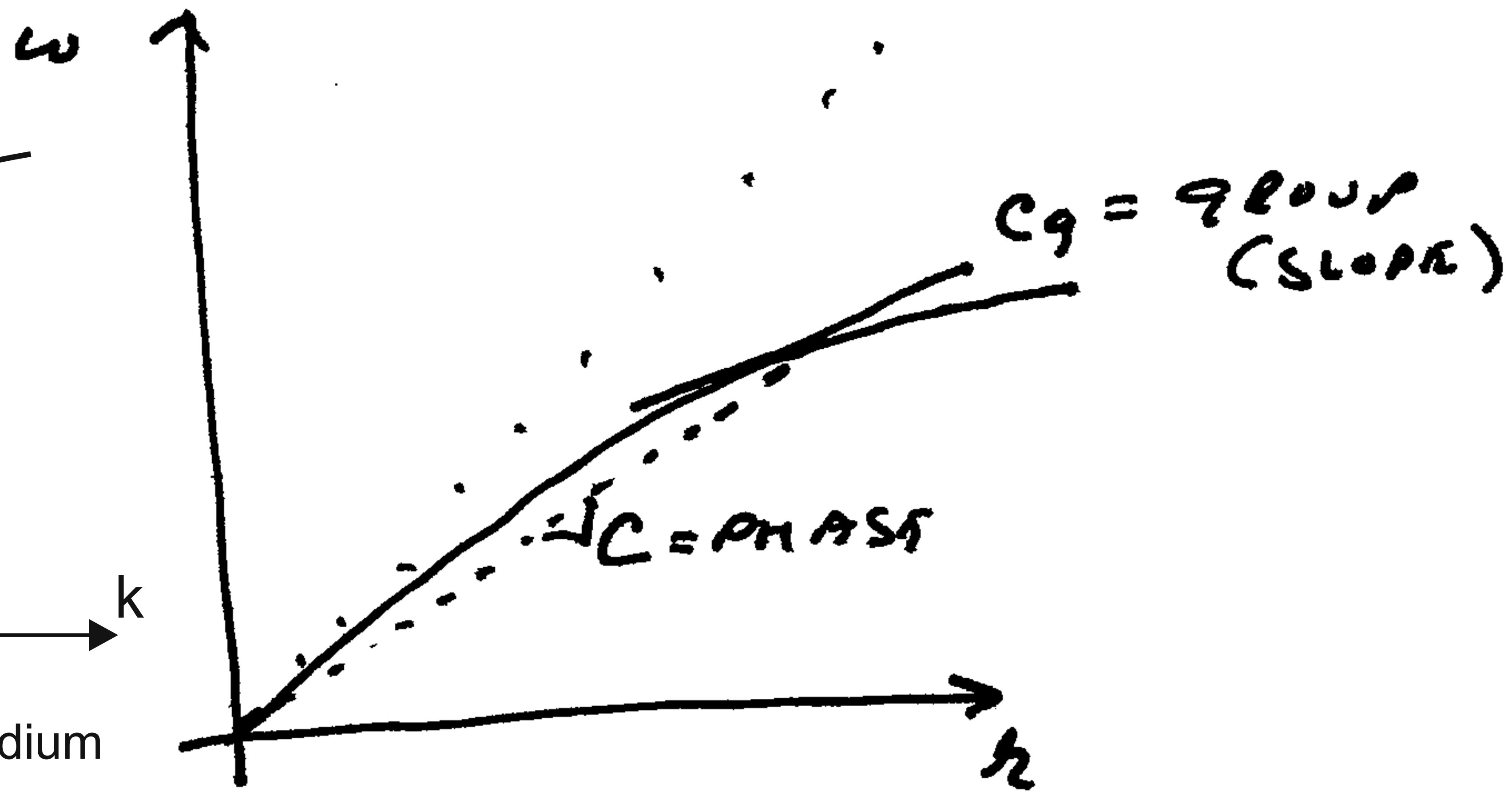
WAVE PACKETS MOVE AT SPEED $\quad c_g = \frac{\Delta\omega}{\Delta k}$ = GROUP VELOCITY

Wave Dispersion

DISPERSIVE WAVES ARE WAVES WHOSE PHASE VELOCITY CHANGES WITH WAVELENGTH (OR FREQUENCY..







NON-DISPERSIVE WAVE

- ALL WAVES MOVE AT c
- GROUP VELOCITY EQUALS PHASE VELOCITY

EXAMPLE DISPERSIVE WAVE

FOR THIS WAVE...

- SHORT WAVELENGTHS MOVE SLOWER THAN LONG WAVELENGTHS
- WAVE PACKETS OF SHORT WAVELENGTH WAVES HAVE VERY SLOW GROUP VELOCITY

Phasors, Sine, Cosine

$$\nabla \times \mathbf{E} \rightarrow \mathbf{ik} \times \hat{\mathbf{E}}\,, \quad \nabla \cdot \mathbf{E} \rightarrow \mathbf{ik} \cdot \hat{\mathbf{E}}\,, \quad \frac{\partial}{\partial t}\mathbf{E} \rightarrow -\mathrm{i}\omega\hat{\mathbf{E}}$$

$$\sin(kx - \omega t) = \mathrm{Re}\left\{ e^{i\frac{\pi}{2}} e^{-j(\omega t - kx)} \right\} \qquad \text{PHASOR} = e^{i\pi/2}$$

$$\cos(kx - \omega t) = \mathrm{Re}\left\{ e^{-j(\omega t - kx)} \right\} \qquad \text{PHASOR} = 1$$

$$\cos(kx - \omega t + \theta) = \mathrm{Re}\left\{ e^{i\theta} e^{-j(\omega t - kx)} \right\} \qquad \text{PHASOR} = e^{i\theta}$$

PDE's become algebraic!

$$\int_0^\lambda \frac{dx}{\lambda} \cos(kx - \omega t) = \int_0^\lambda \frac{dx}{\lambda} \sin(kx - \omega t) = 0$$

$$\int_0^\lambda \frac{dx}{\lambda} \cos^2(kx - \omega t) = \int_0^{2\pi} \frac{d\xi}{2\pi} \cos^2(\xi - \omega t) = \frac{1}{2}$$

$$\int_0^\lambda \frac{dx}{\lambda} \cos(kx - \omega t)\cos(kx - \omega t + \theta) = \frac{1}{2}\cos(\theta)$$

$$\int_0^\lambda \frac{dx}{\lambda} \cos(kx - \omega t)\sin(kx - \omega t) = 0$$

IF A = PHASOR, THEN

$\frac{1}{2}|A|^2 = \frac{1}{2} A^* A$ = SQUARED AVERAGE OF $\mathrm{Re}\left\{ A e^{-j(\omega t - kx)} \right\}$

$|A|/\sqrt{2}$ = RMS OF A

Review of EM Waves

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

$$\begin{aligned} \nabla \times (\nabla \times \mathbf{E}) &= -\nabla \times \frac{\partial \mathbf{B}}{\partial t} \\ &= -\frac{\partial}{\partial t} (\nabla \times \mathbf{B}) \\ &= -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial \mathbf{j}}{\partial t} \end{aligned}$$

Review of EM Waves

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

$$\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j} = \varepsilon_0 \bar{\varepsilon}(\omega) \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{j}(\omega) = \bar{\sigma}(\omega) \cdot \mathbf{E}(\omega)$$

Dielectric tensor

$$\bar{\varepsilon}(\omega) = \bar{1} + \frac{\mathrm{i}}{\omega \varepsilon_0} \bar{\sigma}(\omega)$$

Review of EM Waves

$$\nabla \times (\nabla \times \mathbf{E}) + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu_0 \frac{\partial \mathbf{j}}{\partial t}$$

$$\mathrm{i}\mathbf{k} \times \hat{\mathbf{E}} = \mathrm{i}\omega \hat{\mathbf{B}}$$

$$\mathrm{i}\mathbf{k} \times \hat{\mathbf{B}} = -\mathrm{i}\omega\varepsilon_0\mu_0 \hat{\mathbf{E}} + \mu_0 \hat{\mathbf{j}}_0$$

$$\mathbf{k} \times (\mathbf{k} \times \hat{\mathbf{E}}) = (\mathbf{kk} - k^2 \hat{\mathbf{I}})\hat{\mathbf{E}}$$

All of the plasma physics here

$$\left\{\mathbf{kk} - k^2 \boldsymbol{I} + \frac{\omega^2}{c^2}\boldsymbol{I} + \mathrm{i}\omega\mu_0 \boldsymbol{\sigma}(\omega)\right\} \cdot \hat{\mathbf{E}} = 0$$

$$\left\{\mathbf{kk} - k^2 \boldsymbol{I} + \frac{\omega^2}{c^2} \boldsymbol{\varepsilon}(\omega)\right\} \cdot \hat{\mathbf{E}} = 0$$

Normal Modes ("Dispersion Relation")

$$\begin{pmatrix} k_x k_x - k^2 + \dfrac{\omega^2}{c^2}\varepsilon_{xx} & k_x k_y + \dfrac{\omega^2}{c^2}\varepsilon_{xy} & k_x k_z + \dfrac{\omega^2}{c^2}\varepsilon_{xz} \\ k_y k_x + \dfrac{\omega^2}{c^2}\varepsilon_{yx} & k_y k_y - k^2 + \dfrac{\omega^2}{c^2}\varepsilon_{yy} & k_y k_z + \dfrac{\omega^2}{c^2}\varepsilon_{yz} \\ k_z k_x + \dfrac{\omega^2}{c^2}\varepsilon_{zx} & k_z k_y + \dfrac{\omega^2}{c^2}\varepsilon_{zy} & k_z k_z - k^2 + \dfrac{\omega^2}{c^2}\varepsilon_{zz} \end{pmatrix} \cdot \begin{pmatrix} \hat{E}_x \\ \hat{E}_y \\ \hat{E}_z \end{pmatrix} = 0\,. \tag{6.28}$$

$$0 = D(\omega, \mathbf{k}) = \det\left[\mathbf{kk} - k^2 \boldsymbol{I} + \frac{\omega^2}{c^2}\boldsymbol{\varepsilon}(\omega)\right]. \tag{6.29}$$

Waves in Unmagnetized Plasma

Collisionless
(reactive)

Collisional
(resistive)

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = q\hat{\mathbf{E}}\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$m\ (-\mathrm{i}\omega + \nu_{\mathrm{m}})\ \hat{v} = q\ \hat{E}$$

90° out of phase

$$\hat{\mathbf{j}} = nq\hat{\mathbf{v}} = \mathrm{i}\frac{ne^2}{\omega m}\hat{\mathbf{E}}$$

$$\hat{v} = \left[\frac{\nu_{\mathrm{m}}}{\omega^2+\nu_{\mathrm{m}}^2} + \frac{\mathrm{i}\omega}{\omega^2+\nu_{\mathrm{m}}^2}\right]\frac{q}{m}\hat{E}$$

In phase
(low-frequency)

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \mathrm{i}\frac{ne^2}{\omega m}$$

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = 1 + \frac{\mathrm{i}}{\omega\varepsilon_0}\sigma_{yy} = 1 - \frac{\omega_{\mathrm{pe}}^2}{\omega^2}$$

$$\omega_{\mathrm{pe}} = \left(\frac{ne^2}{\varepsilon_0 m_{\mathrm{e}}}\right)^{1/2}$$

Waves in Unmagnetized Plasma

$$0 = D(\omega, \mathbf{k}) = \det\left[\mathbf{kk} - k^2\boldsymbol{I} + \frac{\omega^2}{c^2}\boldsymbol{\varepsilon}(\omega)\right] \qquad \mathbf{k} = \{k_x, 0, 0\}$$

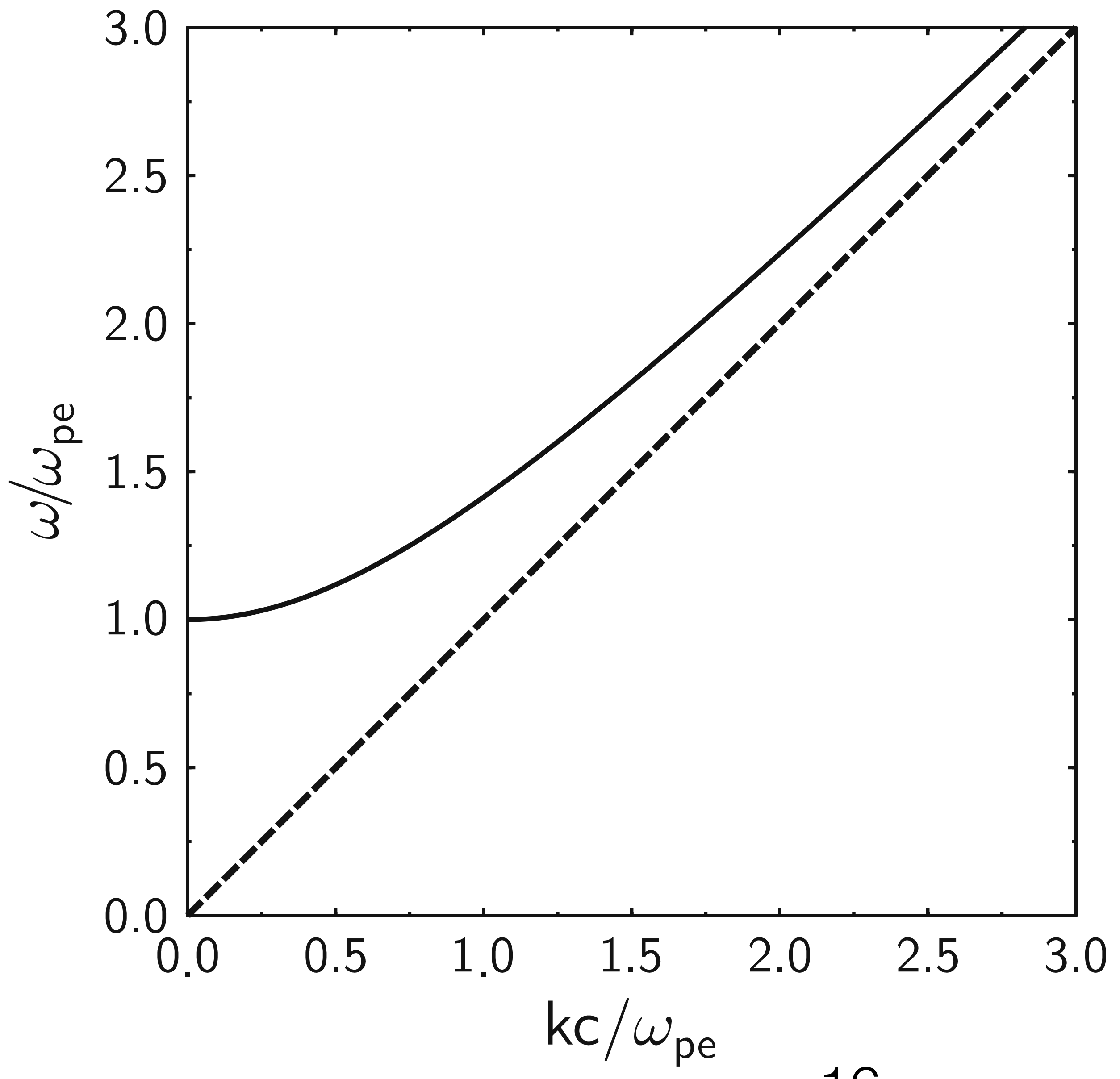
Longitudinal

vs.

Transverse

$$\begin{pmatrix} \frac{\omega^2}{c^2}\left(1-\frac{\omega_{\mathrm{pe}}^2}{\omega^2}\right) & 0 & 0 \\ 0 & -k^2+\frac{\omega^2}{c^2}\left(1-\frac{\omega_{\mathrm{pe}}^2}{\omega^2}\right) & 0 \\ 0 & 0 & -k^2+\frac{\omega^2}{c^2}\left(1-\frac{\omega_{\mathrm{pe}}^2}{\omega^2}\right) \end{pmatrix} \cdot \begin{pmatrix} \hat{E}_x \\ \hat{E}_y \\ \hat{E}_z \end{pmatrix} = 0\,. \tag{6.35}$$

$$\omega^2 = \omega_{\mathrm{pe}}^2 + k^2c^2$$



$$v_\varphi = \left(\frac{\omega_{\mathrm{pe}}^2}{k^2} + c^2\right)^{1/2}$$

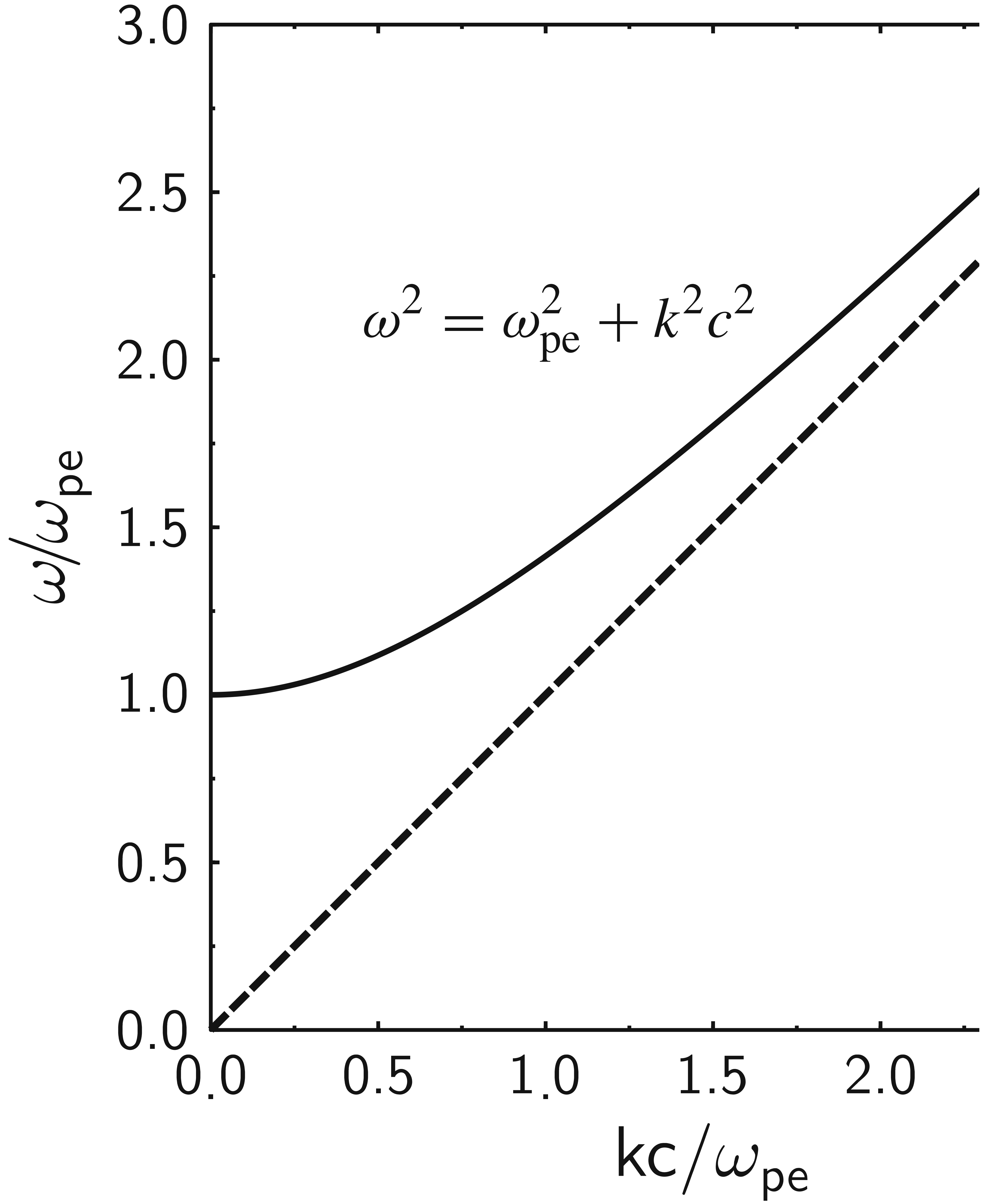
$$v_{\mathrm{gr}} = \frac{kc^2}{\left(\omega_{\mathrm{pe}}^2 + k^2c^2\right)^{1/2}}$$

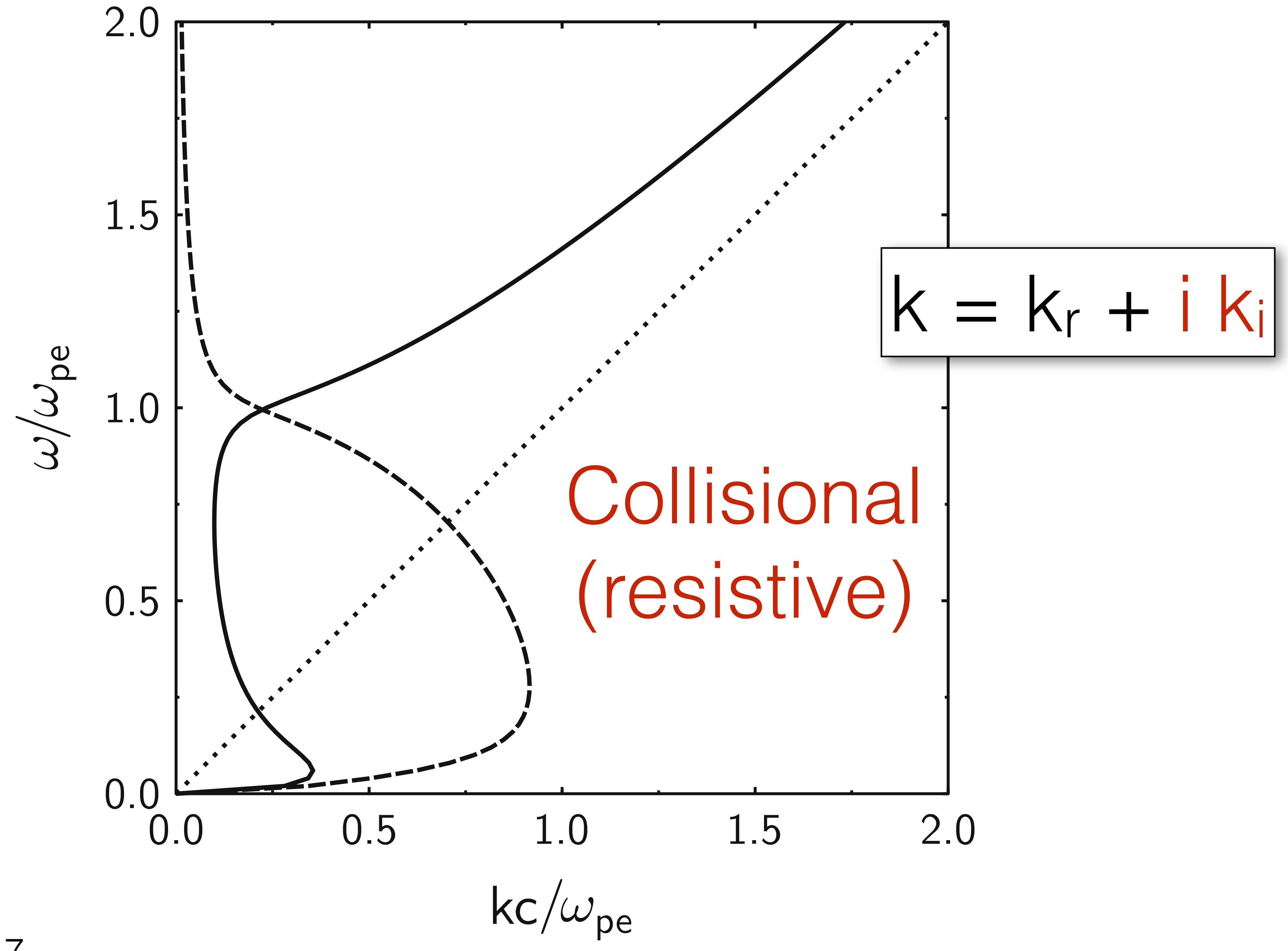
Waves in Unmagnetized Plasma

$$m\ (-\mathrm{i}\omega + \nu_\mathrm{m})\ \hat{v} = q\ \hat{E}$$

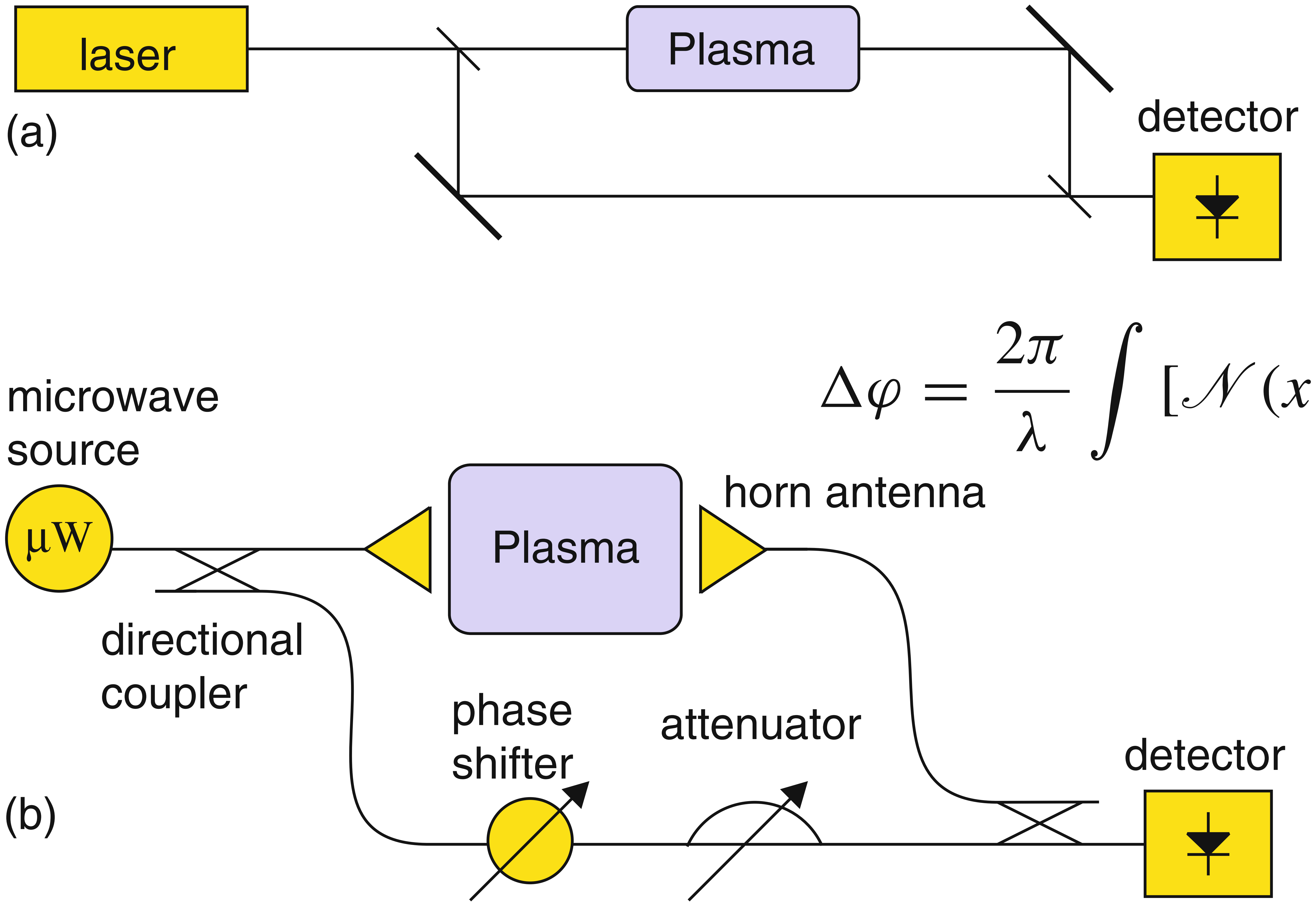
$$\hat{v} = \left[\frac{\nu_\mathrm{m}}{\omega^2 + \nu_\mathrm{m}^2} + \frac{\mathrm{i}\omega}{\omega^2 + \nu_\mathrm{m}^2}\right]\frac{q}{m}\hat{E}$$

$$k = \frac{1}{c}\left(\omega^2 - \frac{\omega_\mathrm{pe}^2}{1 + \mathrm{i}(\nu_\mathrm{m}/\omega)}\right)^{1/2}$$





Interferometry



$$\Delta\varphi = \frac{2\pi}{\lambda}\int [\mathcal{N}(x) - 1]\,\mathrm{d}x$$

Fig. 6.5 (**a**) Laser interferometer in Mach-Zehnder arrangement, (**b**) microwave interferometer. The optical arrangement uses partially-reflecting and fully-reflecting mirrors. The analog to a partially reflecting mirror is the directional coupler for microwaves

Interferometry

$$\Delta\varphi = \frac{2\pi}{\lambda} \int [\mathcal{N}(x) - 1]\, \mathrm{d}x$$

$$\mathcal{N} = \sqrt{1 - \omega_{\mathrm{pe}}^2/\omega^2} \approx 1 - \frac{1}{2}\frac{\omega_{\mathrm{pe}}^2}{\omega^2} = 1 - \frac{1}{2}\frac{n}{n_{\mathrm{co}}}$$

Table 6.1 Cut-off densities for microwave and laser interferometers

Source	Wavelength λ	Frequency f	Cut-off-density $n_{\mathrm{co}}\,(\mathrm{m}^{-3})$
Microwave	3 cm	10 GHz	1.2×10^{18}
	8 mm	37 GHz	1.7×10^{19}
	4 mm	75 GHz	7.0×10^{19}
HCN-laser	337 μm	890 GHz	9.8×10^{21}
CO_2 laser	10.6 μm	28 THz	9.9×10^{24}
He-Ne laser	3.39 μm	88 THz	9.7×10^{25}
	0.633 μm	474 THz	2.8×10^{27}

Interferometry

$$\Delta\varphi = \frac{2\pi}{\lambda}\int [\mathcal{N}(x) - 1]\,\mathrm{d}x$$

$$\mathcal{N} = \sqrt{1 - \omega_{pe}^2/\omega^2} \approx 1 - \frac{1}{2}\frac{\omega_{pe}^2}{\omega^2} = 1 - \frac{1}{2}\frac{n}{n_{co}}$$

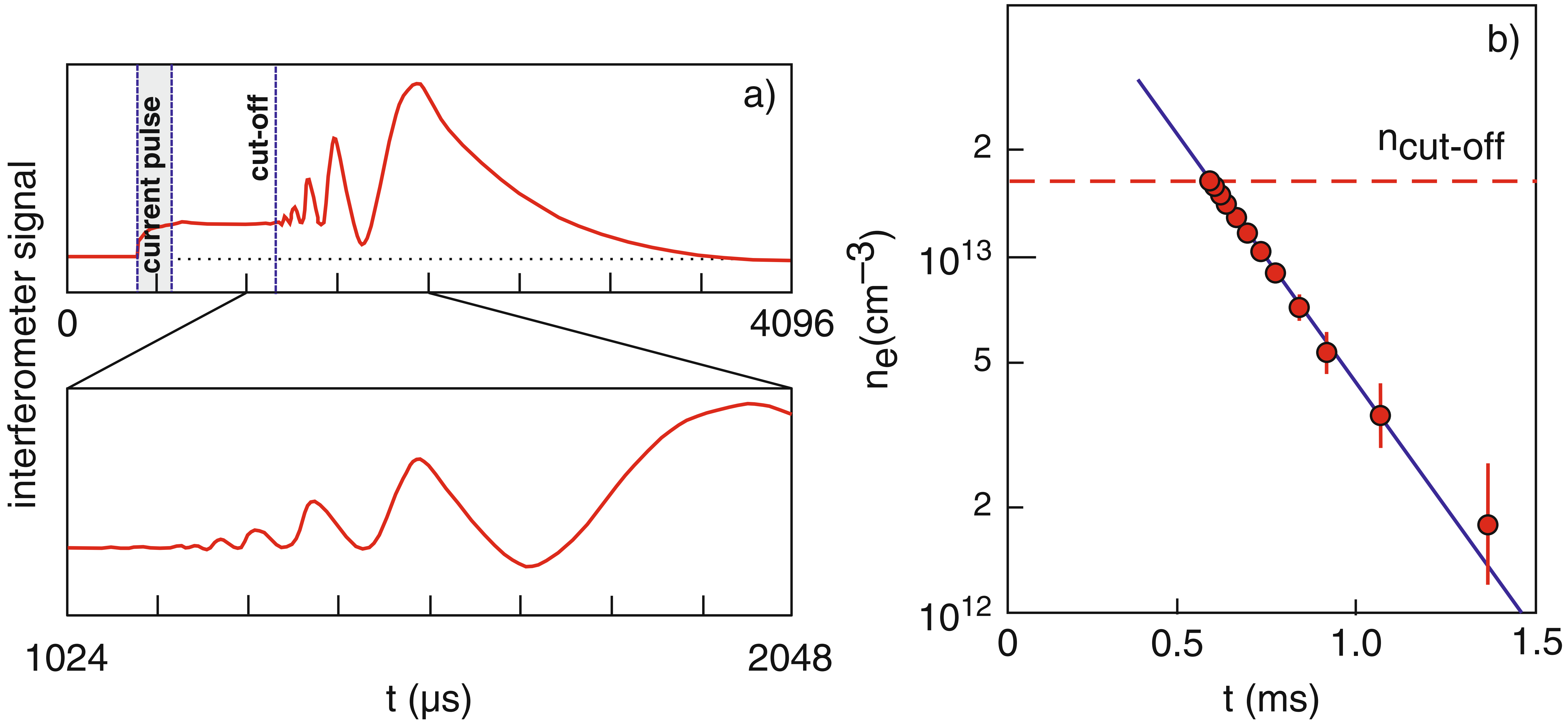


Fig. 6.6 (**a**) Interferogram in a pulsed gas discharge. (**b**) Reconstruction of the decaying electron density by counting interferometer *fringes*

Electrostatic Waves

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$-\nabla \cdot \nabla \Phi = \frac{\rho}{\epsilon_0}$$

$$k^2 \Phi = \frac{\rho}{\epsilon_0} \approx -\frac{e n_e}{\epsilon_0}$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0$$

$$m\dot{v} = -q\frac{\mathrm{d}\phi}{\mathrm{d}x} - \frac{\gamma}{n}\frac{\mathrm{d}(nk_\mathrm{B}T)}{\mathrm{d}x}$$

Electron Pressure Force

$$nm\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = nq(\mathbf{E} + \mathbf{u}\times\mathbf{B}) - \nabla p\,. \qquad (5.28)$$

Electrostatic Plasma Waves

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$-\nabla \cdot \nabla \Phi = \frac{\rho}{\epsilon_0}$$

$$k^2 \Phi = \frac{\rho}{\epsilon_0} \approx -\frac{e n_e}{\epsilon_0}$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0$$

$$-\,\mathrm{i}\omega \hat{n} + \mathrm{i}k n_0 \hat{v} = 0$$

$$m\dot{v} = -q\frac{\mathrm{d}\phi}{\mathrm{d}x} - \frac{\gamma}{n}\frac{\mathrm{d}(n k_\mathrm{B} T)}{\mathrm{d}x}$$

$$-\,\mathrm{i}\omega m \hat{v} = -\mathrm{i}kq\hat{\phi} - \mathrm{i}k\gamma k_\mathrm{B} T \hat{n}$$

Linearized Electron Pressure Force in “isothermal” plasma

Electrostatic Plasma Waves

$$\omega = \left(\omega_{\mathrm{pe}}^2 + \frac{3}{2}k^2 v_{\mathrm{Te}}^2\right)^{1/2} = \omega_{\mathrm{pe}}\left(1 + 3k^2\lambda_{\mathrm{De}}^2\right)^{1/2}$$

Electron Pressure Force

Electron Pressure Force

Electrostatic Ion Sound Waves

Ion Pressure Force

$$-\,\mathrm{i}\omega m_\mathrm{i}\hat{v}_\mathrm{i} = e\hat{E} - \frac{\mathrm{i}k}{n_{\mathrm{i}0}}(\gamma_\mathrm{i} k_\mathrm{B} T_\mathrm{i})\hat{n}_\mathrm{i} \qquad \hat{n}_\mathrm{i} = \frac{ek}{-\mathrm{i}\omega^2 m_\mathrm{i} + \mathrm{i}k^2\gamma_\mathrm{i} k_\mathrm{B} T_\mathrm{i}}\hat{E}$$

$$0 = -e\hat{E} - \frac{\mathrm{i}k}{n_{\mathrm{e}0}}(k_\mathrm{B} T_\mathrm{e})\hat{n}_\mathrm{e} \qquad \hat{n}_\mathrm{e} = \frac{-e}{\mathrm{i}k k_\mathrm{B} T_\mathrm{e}}\hat{E}\,,$$

Electron Pressure Force

Look!

No electron acceleration

$$k^2\Phi = \frac{\rho}{\epsilon_0} \approx \frac{e}{\epsilon_0}\left(n_i - n_e\right)$$

$$\mathrm{i}k\hat{E} = \left(\frac{n_{\mathrm{i}0}e^2}{\varepsilon_0 m_\mathrm{i}}\right)\frac{k}{-\mathrm{i}\omega^2 + \mathrm{i}k^2\gamma_\mathrm{i} k_\mathrm{B} T_\mathrm{i}/m_\mathrm{i}}\hat{E} + \left(\frac{n_{\mathrm{e}0}e^2}{\varepsilon_0 k_\mathrm{B} T_\mathrm{e}}\right)\frac{1}{\mathrm{i}k}\hat{E}$$

Electrostatic Ion Sound Waves

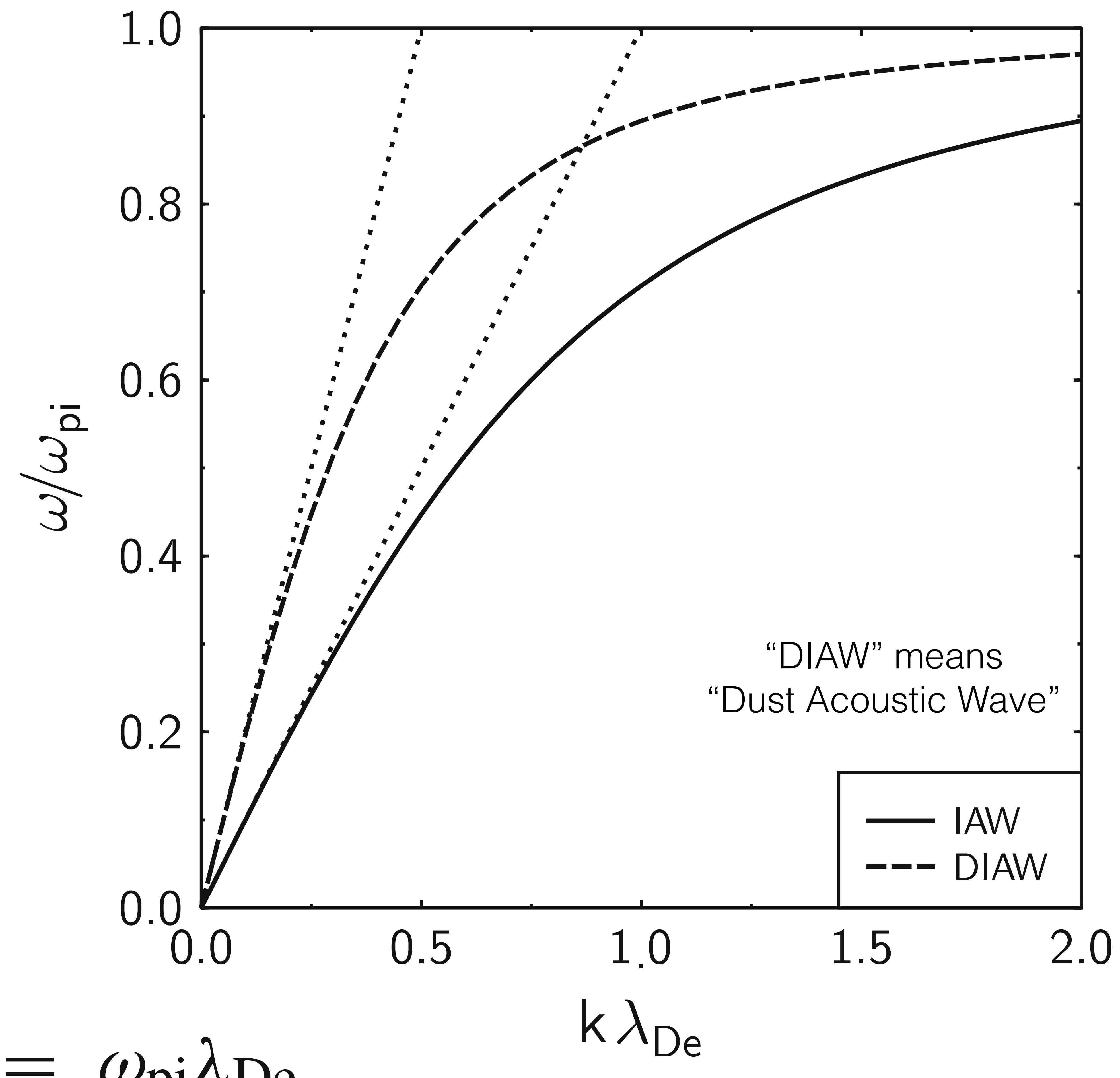
$$\varepsilon(k,\omega) = 1 - \frac{\omega_{\mathrm{pi}}^2}{\omega^2 - k^2\gamma_{\mathrm{i}} k_{\mathrm{B}} T_{\mathrm{i}}/m_{\mathrm{i}}} + \frac{1}{k^2\lambda_{\mathrm{De}}^2}$$

$$\omega^2 = k^2\left(\frac{\gamma_{\mathrm{i}} k_{\mathrm{B}} T_{\mathrm{i}}}{m_{\mathrm{i}}} + \frac{\omega_{\mathrm{pi}}^2\lambda_{\mathrm{De}}^2}{1+k^2\lambda_{\mathrm{De}}^2}\right)$$

Why is $T_e/T_i >> 1$ important?

$$\omega \approx \frac{k\,C_{\mathrm{s}}}{\sqrt{1+k^2\lambda_{\mathrm{De}}^2}}$$

$T_{\mathrm{e}} \gg T_{\mathrm{i}}$ $\quad C_{\mathrm{s}} = \omega_{\mathrm{pi}}\lambda_{\mathrm{De}}$



Important Wave Concepts

- Linear vs. nonlinear
- Dispersion
- Phase and group velocity
- "Polarization" and wave structure

➡ **Energy & intensity (Poynting's Theorem)**

- ***Inhomogeneity***

$$W = \frac{1}{2} \int_V dV \, (\mathbf{H} \cdot \mathbf{B} + \mathbf{E} \cdot \mathbf{D})$$

$$\frac{\partial W}{\partial t} + \int_S \mathbf{N} \cdot d\mathbf{S} = - \int_V dV \, \mathbf{J} \cdot \mathbf{E},$$

Next Lecture

- Chapter 6: “Plasma Waves”
    - Waves in magnetized plasma
    - Inhomogenous plasma (part 1)