Lecture 1: Plasma Physics I

APPH E6101x
Columbia University
Fall, 2023

Syllabus and Class Website

http://sites.apam.columbia.edu/courses/apph6101x/

APPH E6101x Site Information

Plasma Physics 1

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Email: mauel@columbia.edu

General

Welcome to the APPH E6101x class information site.

This is the first semester of a two-semester sequence in plasma physics. Plasma physics is the study of "luminous matter", matter that has been heated sufficiently or prepared specially in order to be ionized. In plasma long-range electromagnetic forces are much more important than short range forces. Plasma dynamics is often dominated by "collective" motion involving the correlated motion of large populations of neighboring particles. Since plasma motion generates electric and magnetic fields, plasma behavior exhibits sometimes very beautiful nonlinear physics.

Plasma is studied in the laboratory and in space. Most of the visible universe is in the plasma state. Laboratory generated plasma are used to studied the fundamental properties of high-temperature matter, and they are employed for many valuable applications like surface processing and lighting. Integrated circuits are manufactured using plasma processing, and plasma displays are status symbols of today's world of entertainment. Controlled fusion energy research reflects the remarkable success of plasma physics. The controlled release of more than 10 MW of fusion power has occurred within the strong confining fields of tokamak devices, and the world is now building the first experimental fusion power source, called ITER.

Topics covered include: Motion of charged particles in space- and time-varying electromagnetic fields. Kinetic description of plasmas. Collisional Boltzmann equation (and collision operators in Fokker-Planck forms.) Classical transport equations and collisional relaxation processes. Linear electrostatic and electromagnetic waves in field-free plasmas. Vlasov equation and Landau damping.

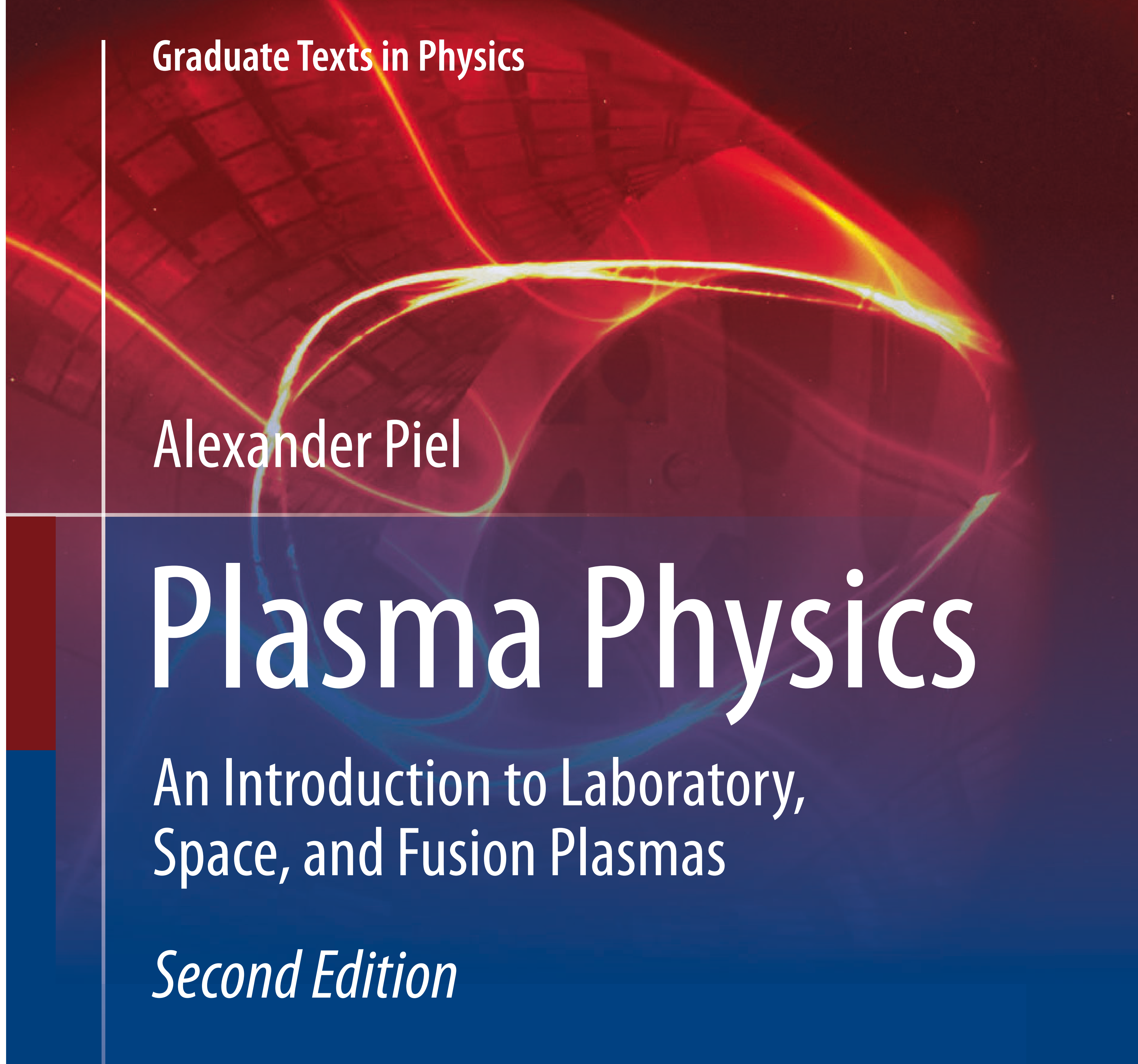
APPH 6101 requires a prior experience with electromagnetics (and some electrodynamics) and partial differential equations. The formal prerequisites are APPH E3300y Applied Electromagnetism and APMA E3102y Applied Mathematics II: Partial Differential Equations. The goal of this course is to provide a solid understanding of both the fundamental aspects of plasma physics and introduce students to research problems in the fields of laboratory and space plasma physics.

Textbook

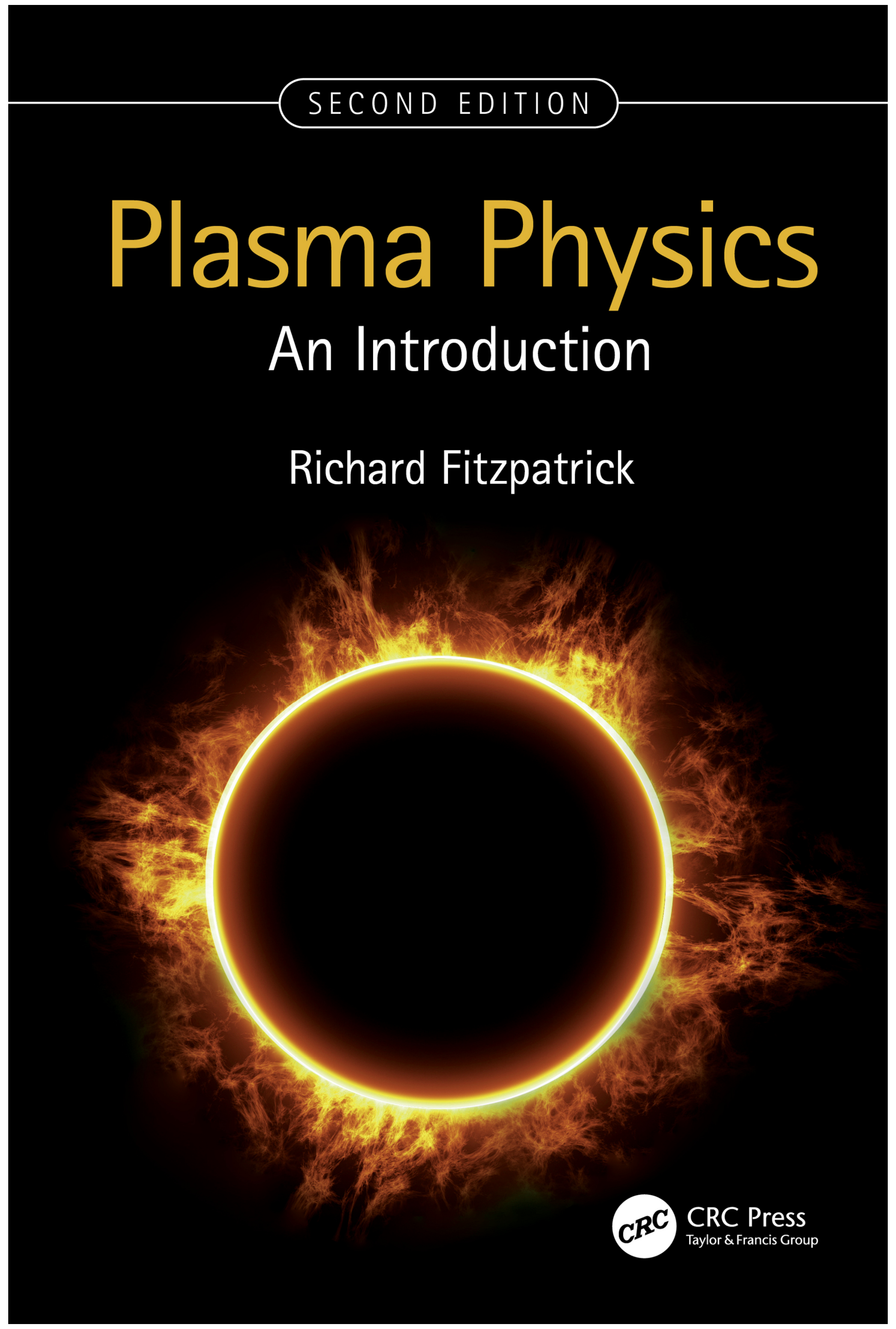
“Plasma Physics offers a broad and modern introduction to the many aspects of plasma science … . A curious student or interested researcher could track down laboratory notes, older monographs, and obscure papers … . with an extensive list of more than 300 references and, in particular, its excellent overview of the various techniques to generate plasma in a laboratory, Plasma Physics is an excellent entree for students into this rapidly growing field. It’s also a useful reference for professional low-temperature plasma researchers.”

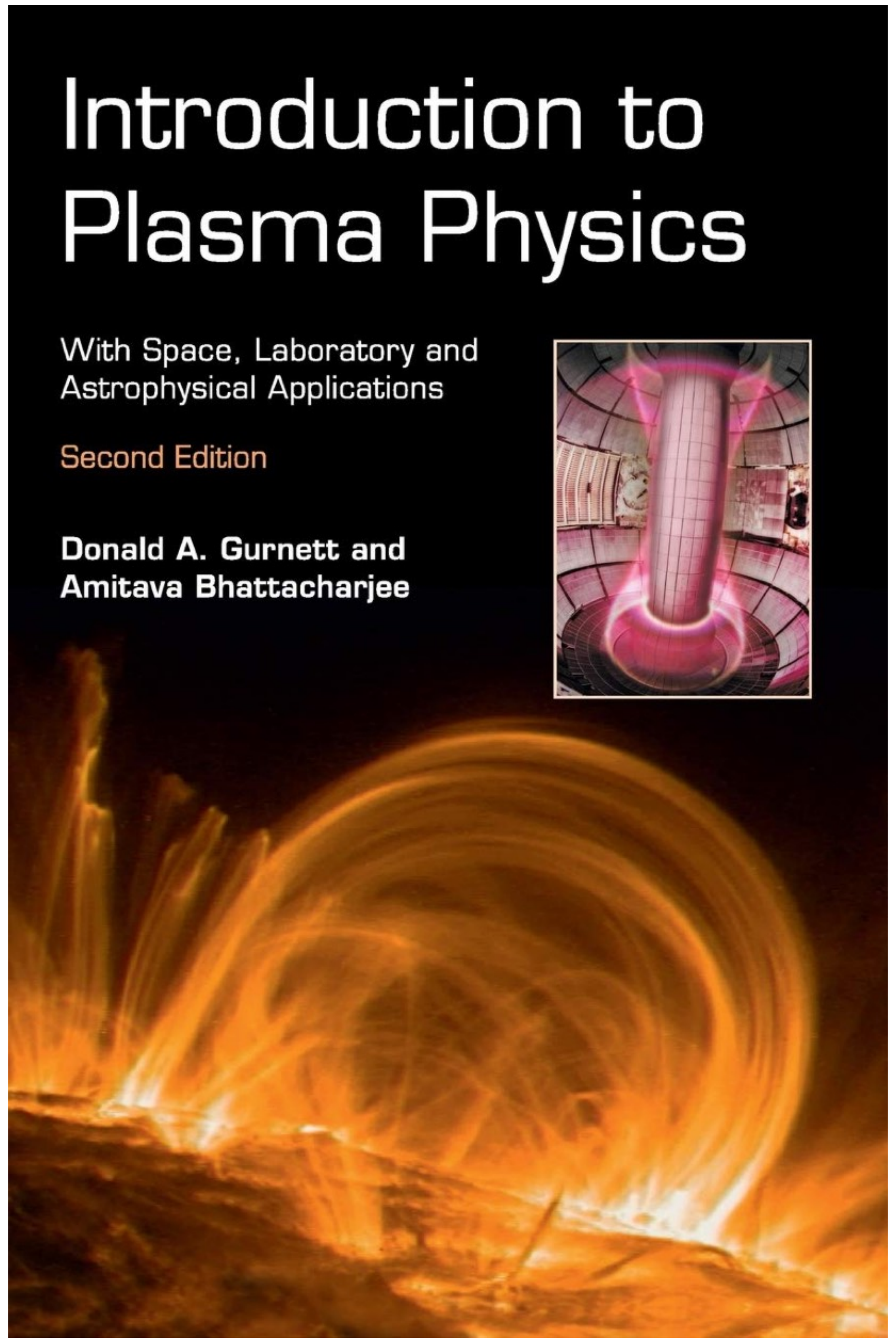
(Michael Brown, *Physics Today*, June, 2011)

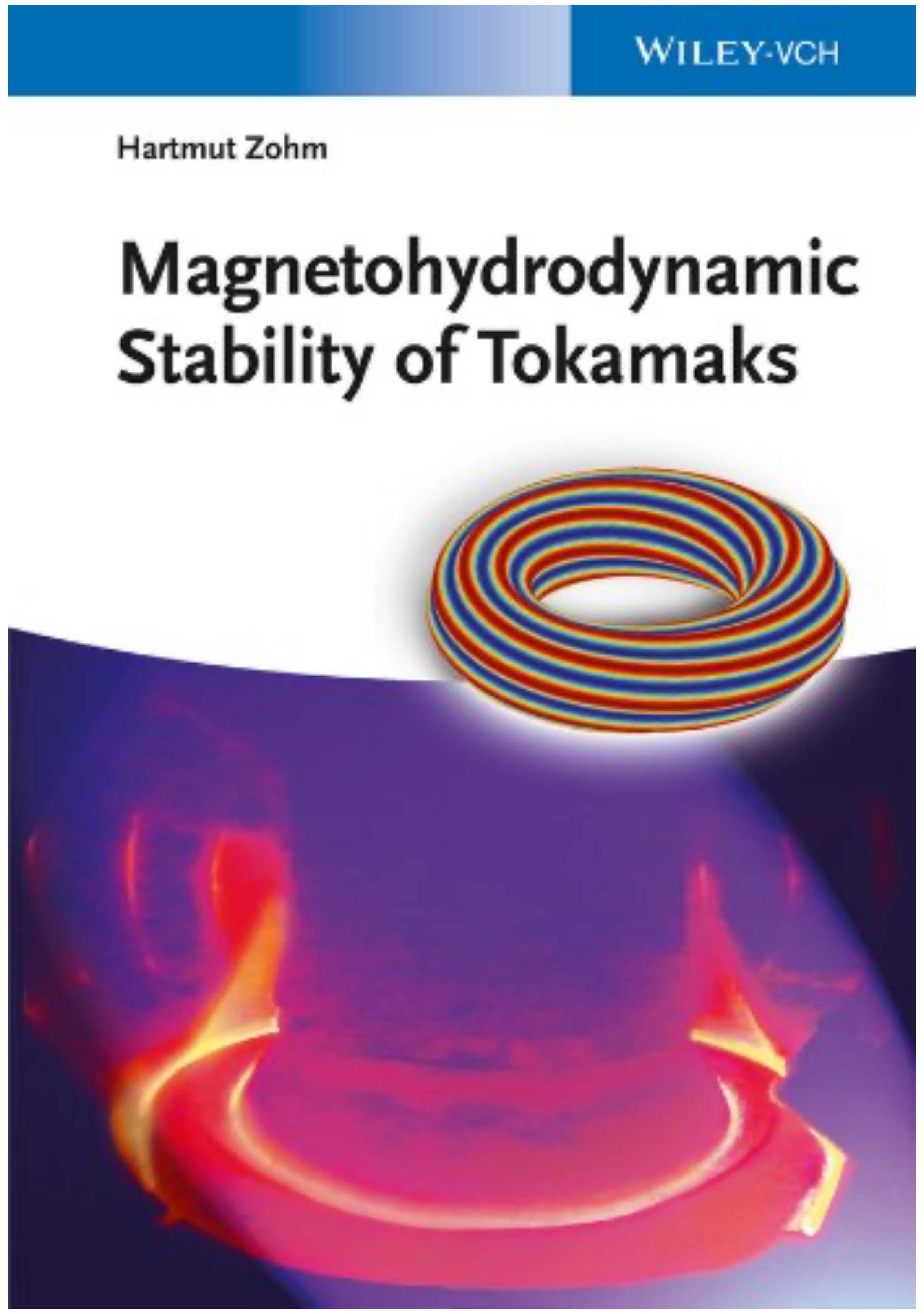
Online here: https://clio.columbia.edu/catalog/13064329

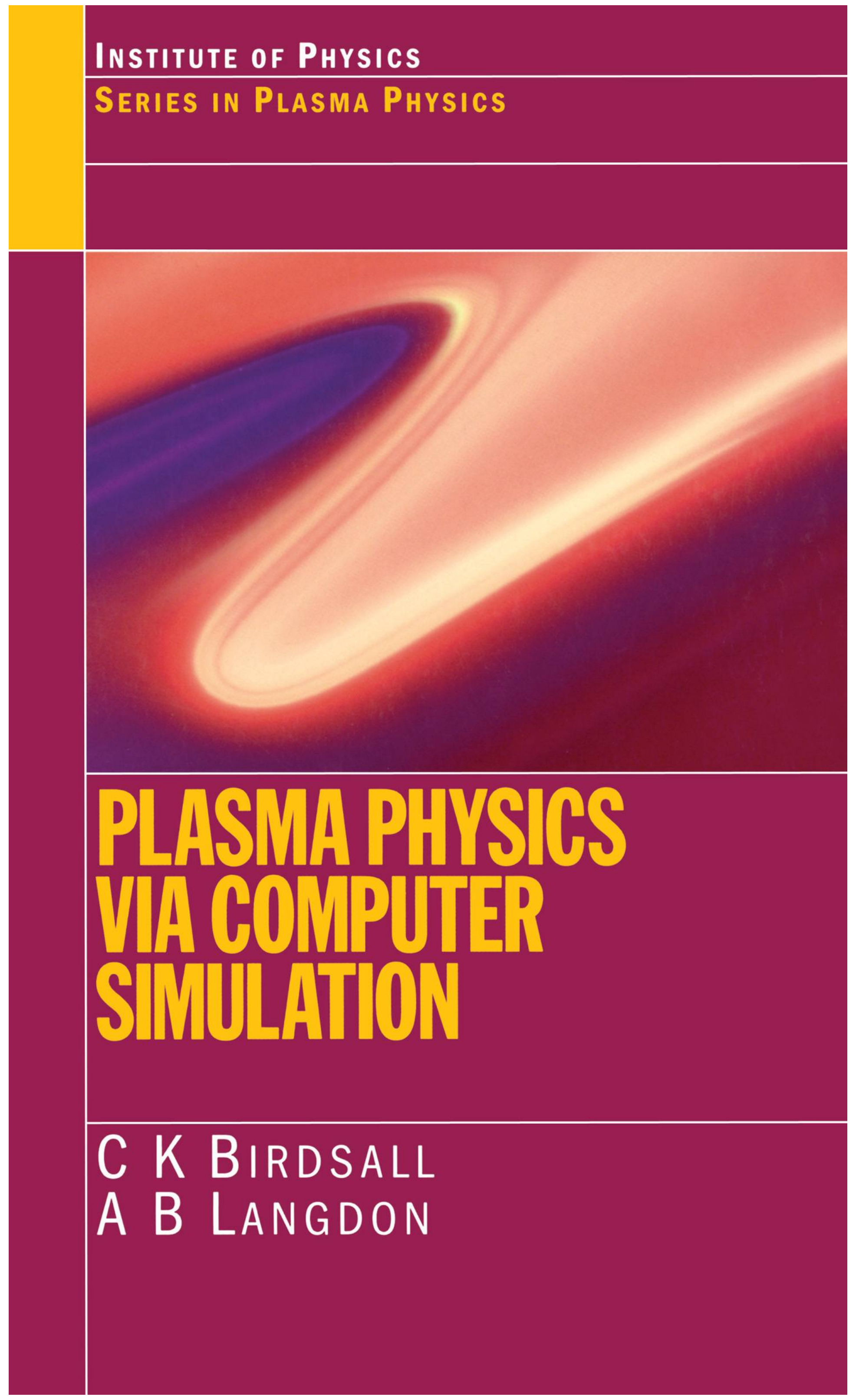


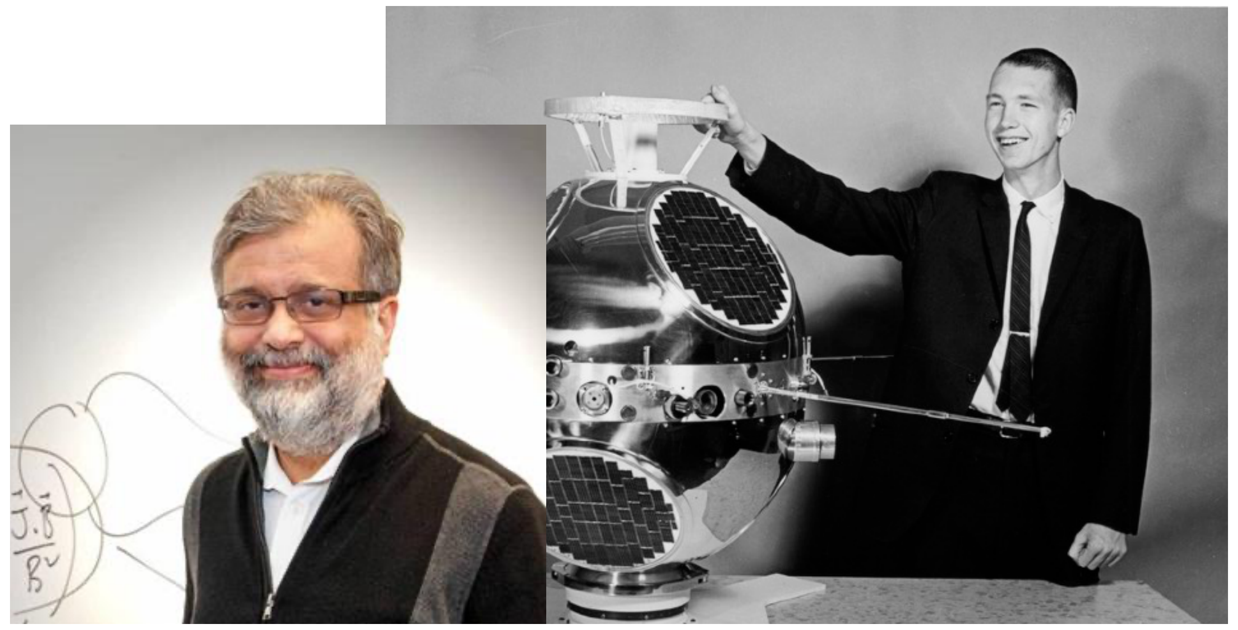
Springer











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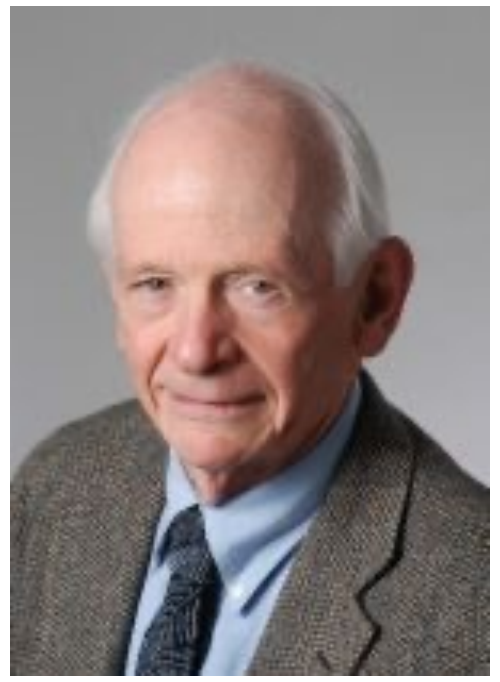


https://now.uiowa.edu/news/2022/01/legendary-iowa-space-physicist-donald-gurnett-dies

https://ph.utexas.edu/component/cobalt/item/18-physics/429-fitzpatrick-richard

https://www2.eecs.berkeley.edu/Faculty/Homepages/birdsall.html

https://doi.org/10.1063/5.0166683

NRL Plasma Formulary

https://www.nrl.navy.mil/News-Media/Publications/nrl-plasma-formulary/

Publications

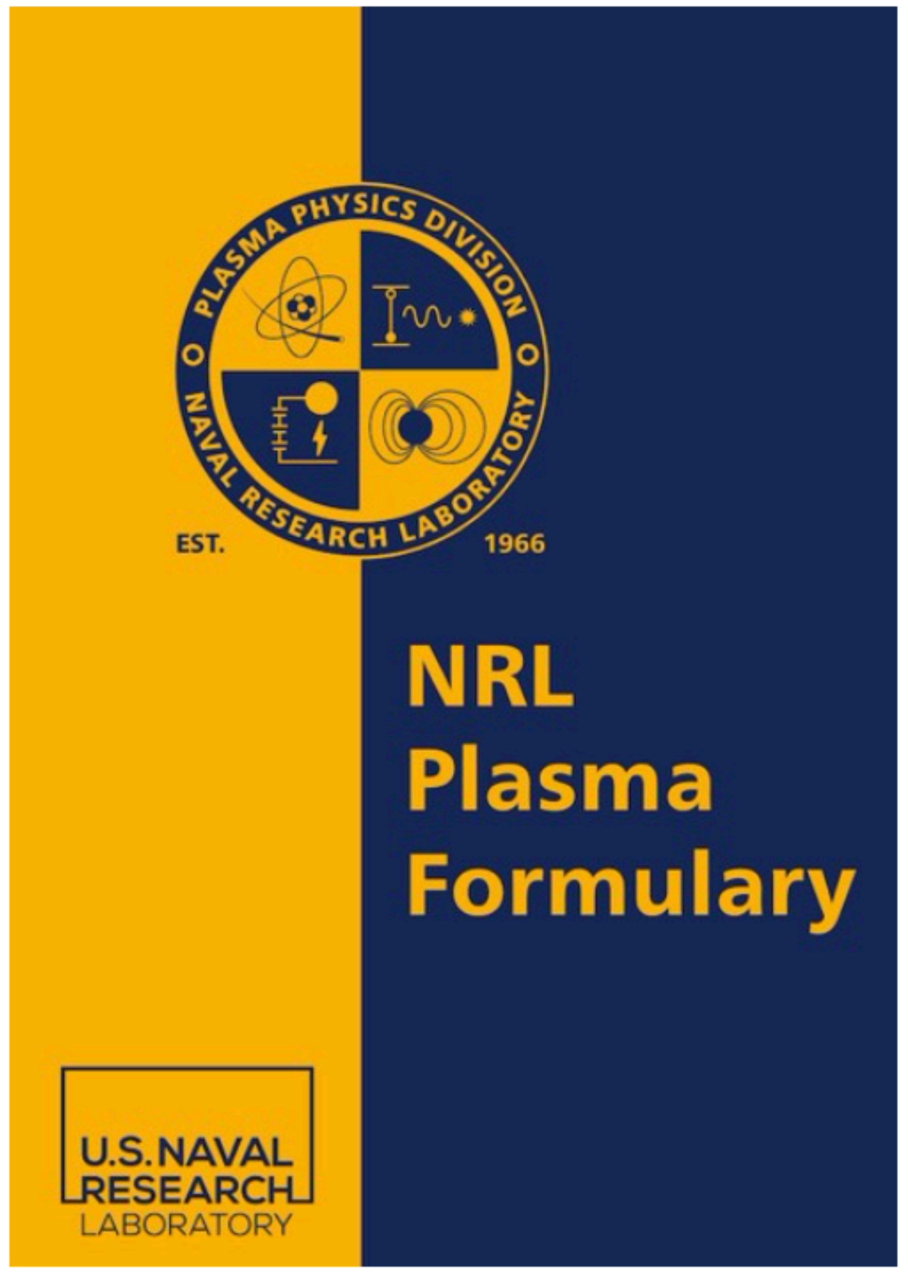
Email: plasma.formulary@nrl.navy.mil

About

The NRL Plasma Formulary has been an invaluable reference for plasma physicists since it was first released in 1975. It is an eclectic compilation of mathematical and scientific formulas, and contains physical parameters pertinent to a variety of plasma regimes, ranging from laboratory devices to astrophysical objects.

To Order Copies

There is no charge for the NRL Plasma Formulary Booklets. To order hard copies of the booklet, please email plasma.formulary@nrl.navy.mil. Please include your name, email, mailing address and number of copies you are requesting.



2019 Plasma Formulary [PDF]

Grading

- Weekly homework (0%)
- Two in-class quizzes (25% *each*)
- Final exam (50%)

Physics of Plasmas EDITORIAL scitation.org/journal/php

Editorial: Preface to the 30th volume of *Physics of Plasmas*

Cite as: Phys. Plasmas **30**, 010401 (2023); doi: 10.1063/5.0141240
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Michael E. Mauel, Editor-in-Chief

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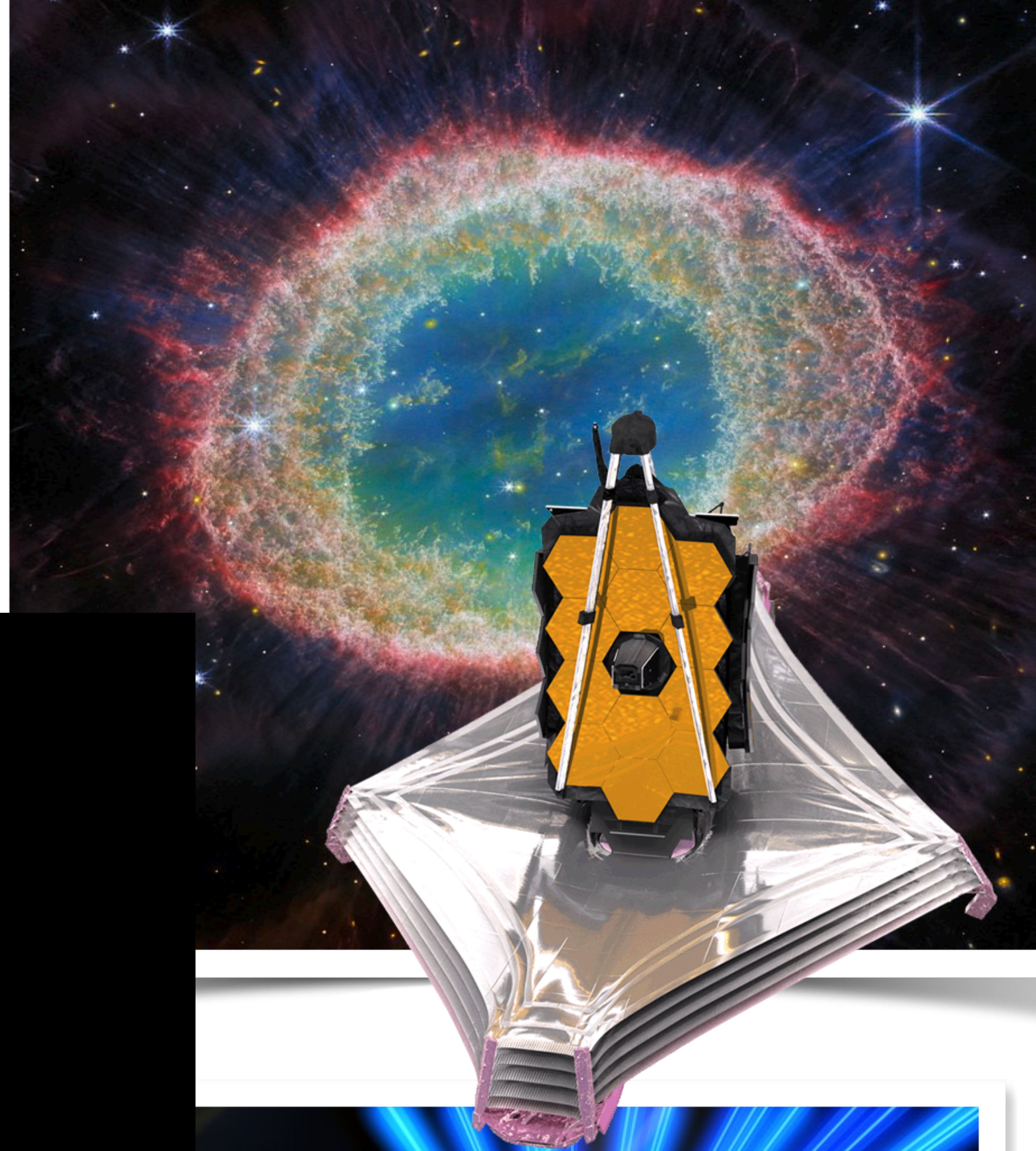

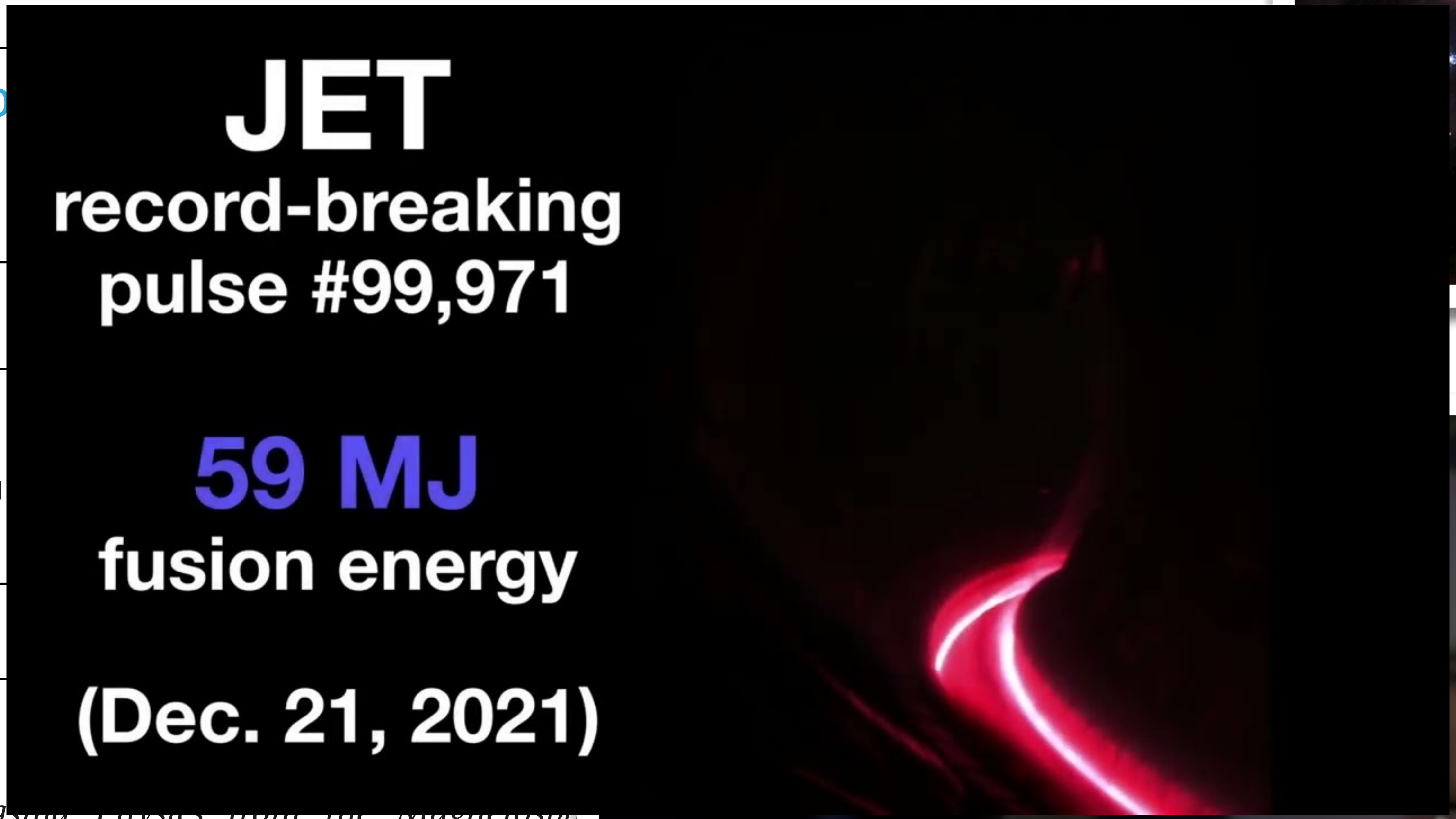

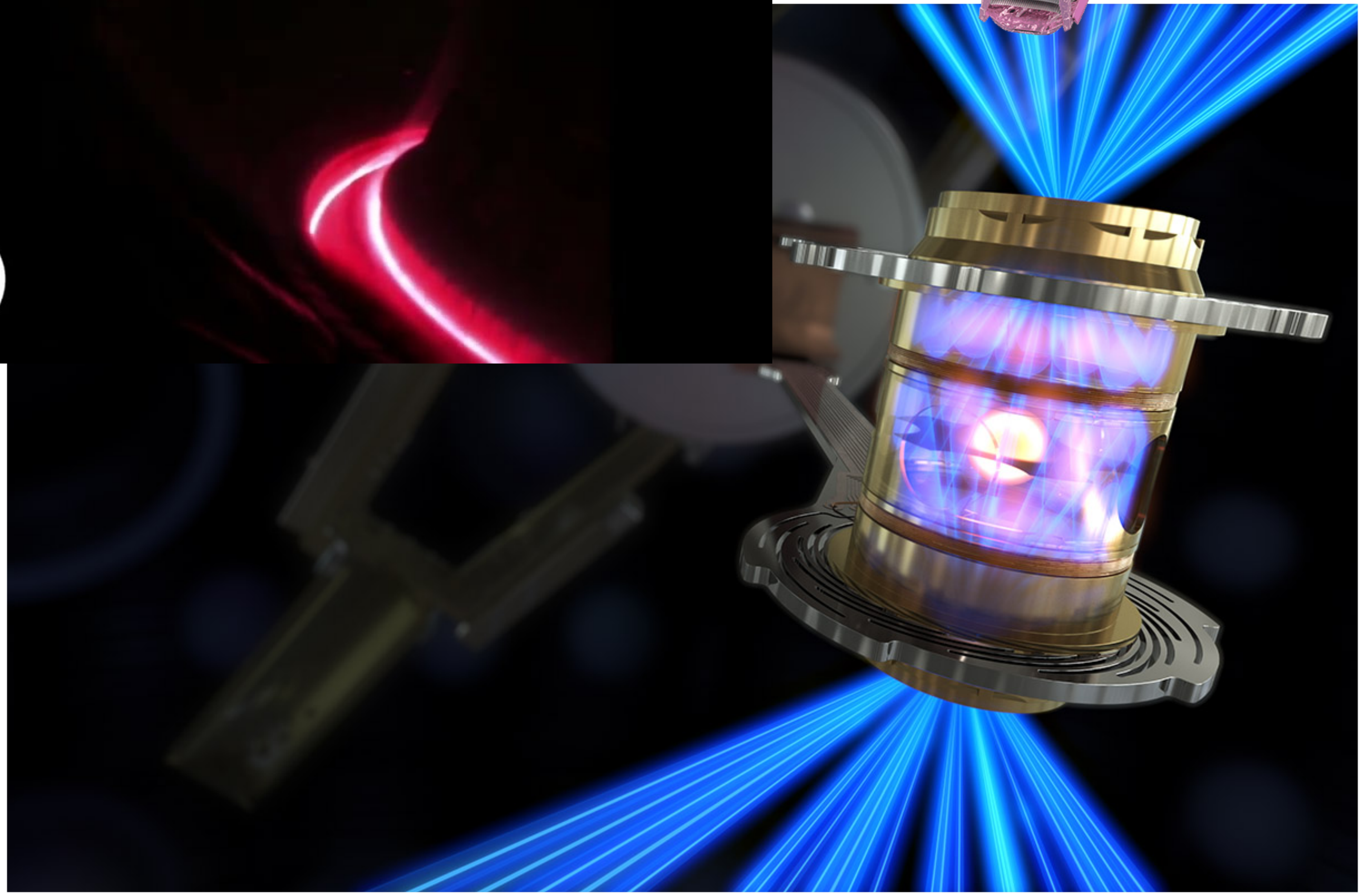

As we begin the 30th volume of *Physics of Plasmas*, I cannot help but remark that 2022 has been a landmark year for plasma physics. During the past few weeks, photos from the James Webb Space Telescope are everywhere and remind us of the beauty of plasma physics.[1] Even more so, 2022 began with the February announcement of the Joint European Torus (JET) record-setting release of 59 MJ of fusion energy during a steady 5 s pulse[2] and ended with the announcement of net fusion energy gain, achieved safely in the National Ignition Facility (NIF).[3] Our instruments view the bright universe with ever greater detail, and our research facilities control high-temperature matter with ever greater precision. Progress in plasma physics is unmistakable.

The Editors at *Physics of Plasmas* are fortunate to be working with the largest number of authors in the field, rapidly reporting their

Plasma Physics from the Magnetosph
Guest Editors Julia Stawarz (Northu
Genestreti (University of New Hampsl
for papers and has become the largest
Plasmas. This collection highlights new
magnetospheric multiscale (MMS) in
broad interest in the plasma physics be
Editors Stuart Bale, Nicola J. Fox, Davi
launched the Special Collection of pape
Sun in Honor of Eugene Parker to highl
and plans for the Parker Solar Probe (PS
and to recognize Eugene Parker's rema
tributions physics.[9] Guest Editors Ste
Wisconsin-Madison), Valerie Izzo, Fiat Lux (General Atomics), and


Physics of Plasmas
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Volume 30, Issue 1, Jan. 2023
Nonlinear interactions of ion acoustic waves explored using fast imaging decompositions
Phys. Plasmas 30, 012109 (2023); doi.org/10.1063/5.0131745
Simon Vincent, Vincent Dolique, and Nicolas Plihon
AIP Publishing
Physics of Plasmas
Volume 30, Issue 3, Mar. 2023
Dispersive and kinetic effects on kinked Alfvén wave packets: A comparative study with fluid and hybrid models
Phys. Plasmas 30, 032101 (2023); doi.org/10.1063/5.0134726
Anna Tenerani, Carlos González, Nikos Sioulas, Chen Shi, and Marco Velli
AIP Publishing
Physics of Plasmas
Volume 30, Issue 5, May 2023
Modeling of carbon and tungsten transient dust influx in tokamak edge plasma
Phys. Plasmas 30, 052506 (2023); doi.org/10.1063/5.0144529
R. D. Smirnov and S. I. Krasheninnikov
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Volume 30, Issue 2, Feb. 2023
The electron diffusion region dominated by electromagnetic turbulence in the reconnection current layer
Phys. Plasmas 30, 022106 (2023); doi.org/10.1063/5.0129591
Keizo Fujimoto and Richard D. Sydora
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Physics of Plasmas
Volume 30, Issue 4, Apr. 2023
Development of the neutral model in the nonlinear MHD code JOREK: Application to E × B drifts in ITER PFPO-1 plasmas
Phys. Plasmas 30, 042509 (2023); doi.org/10.1063/5.0135318
S. Q. Korving, G. T. A. Huijsmans, J.-S. Park, A. Loarte, and JOREK Team
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Vol. 30, Iss. 6, Jun. 2023
How well can VMEC predict the initial saturation of external kink modes in near circular tokamaks and l = 2 stellarators?
R. Ramasamy, M. Hoelzl, S. Henneberg, E. Strumberger, K. Lackner, and S. Günter
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Physics of Plasmas REVIEW scitation.org/journal/php

Quantum computing for fusion energy science applications

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Note: This paper is part of the Special Topic: Papers from the 2022 Sherwood Fusion The

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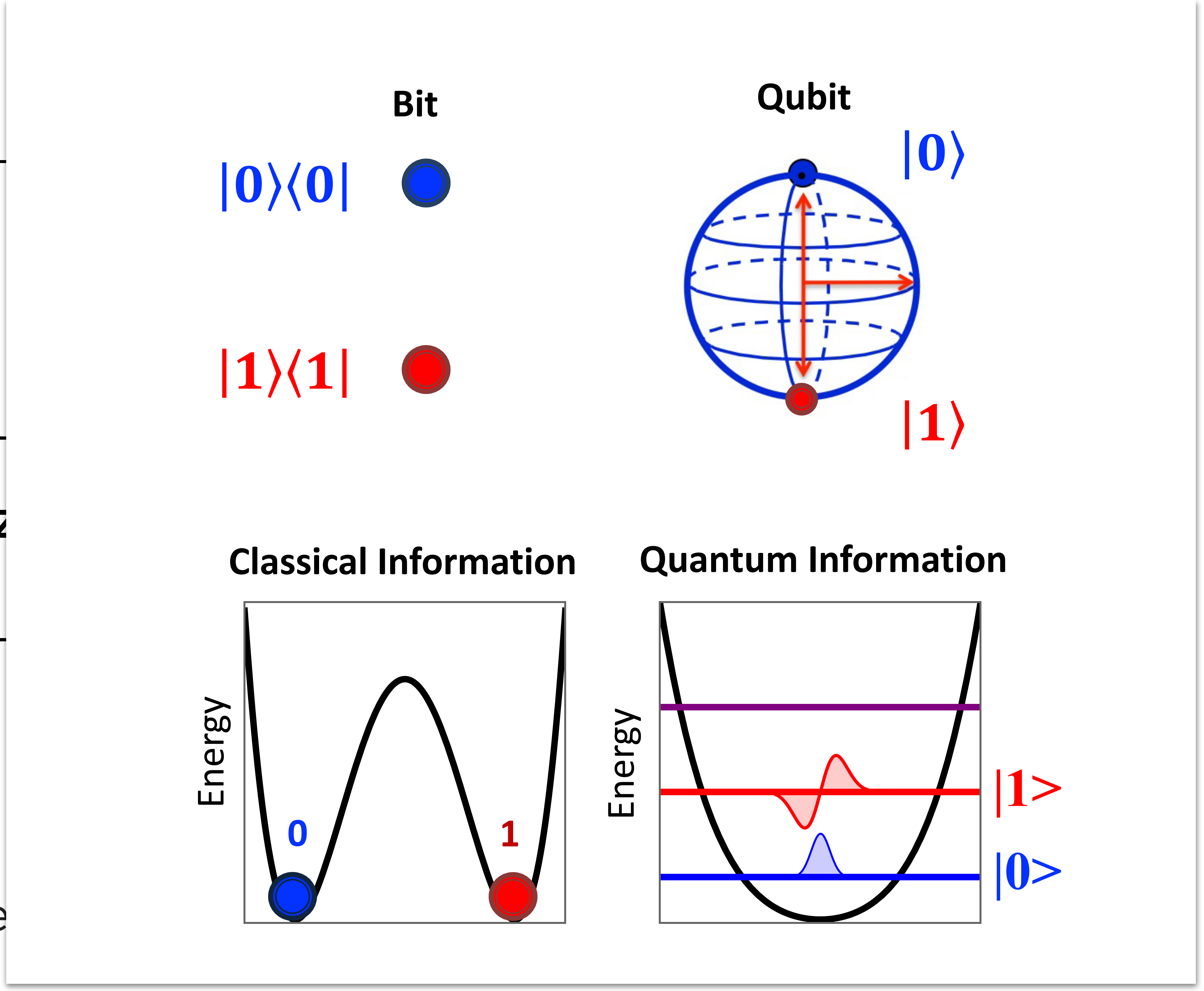

Energy transfer of the solar wind turbulence based on Parker solar probe and other spacecraft observations

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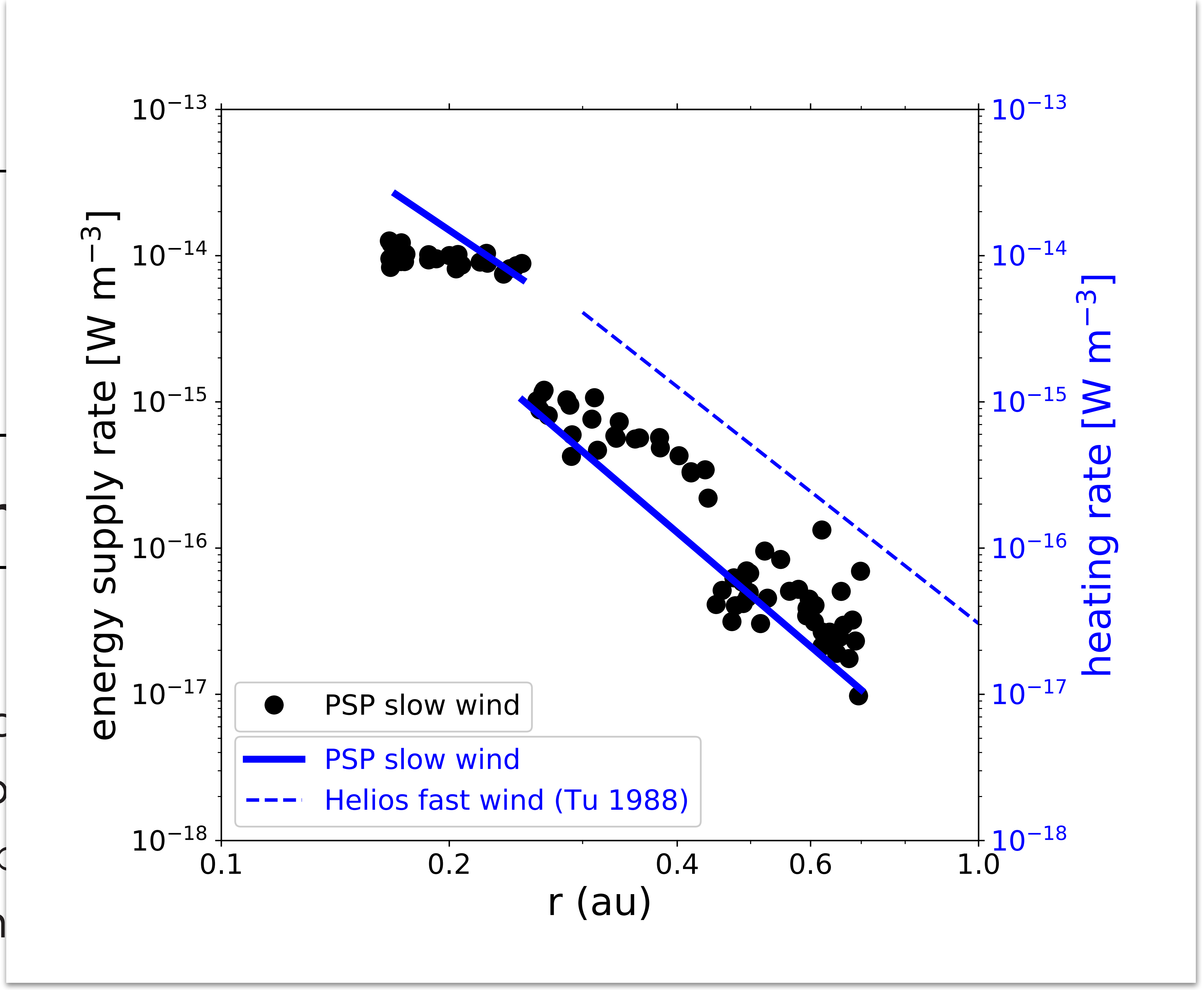

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Note: This paper is part of the Special Topic: Plasma Physics of the Sun in Honor of Eugene Parker.
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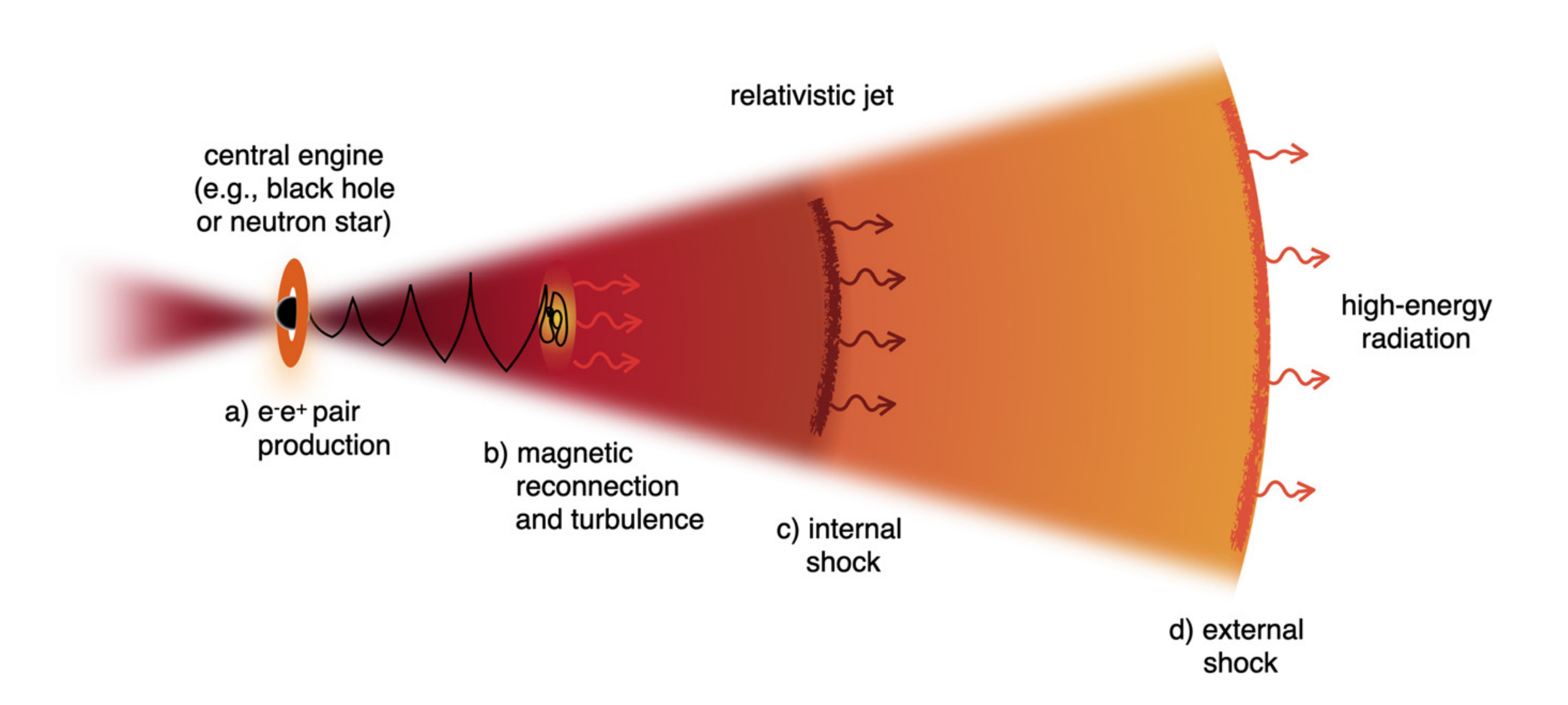
Perspectives on relativistic electron–positron pair plasma experiments of astrophysical relevance using high-power lasers



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Physics of Plasmas REVIEW scitation.org/journal/php

Formation and evolution of coherent structures in 3D strongly turbulent magnetized plasmas

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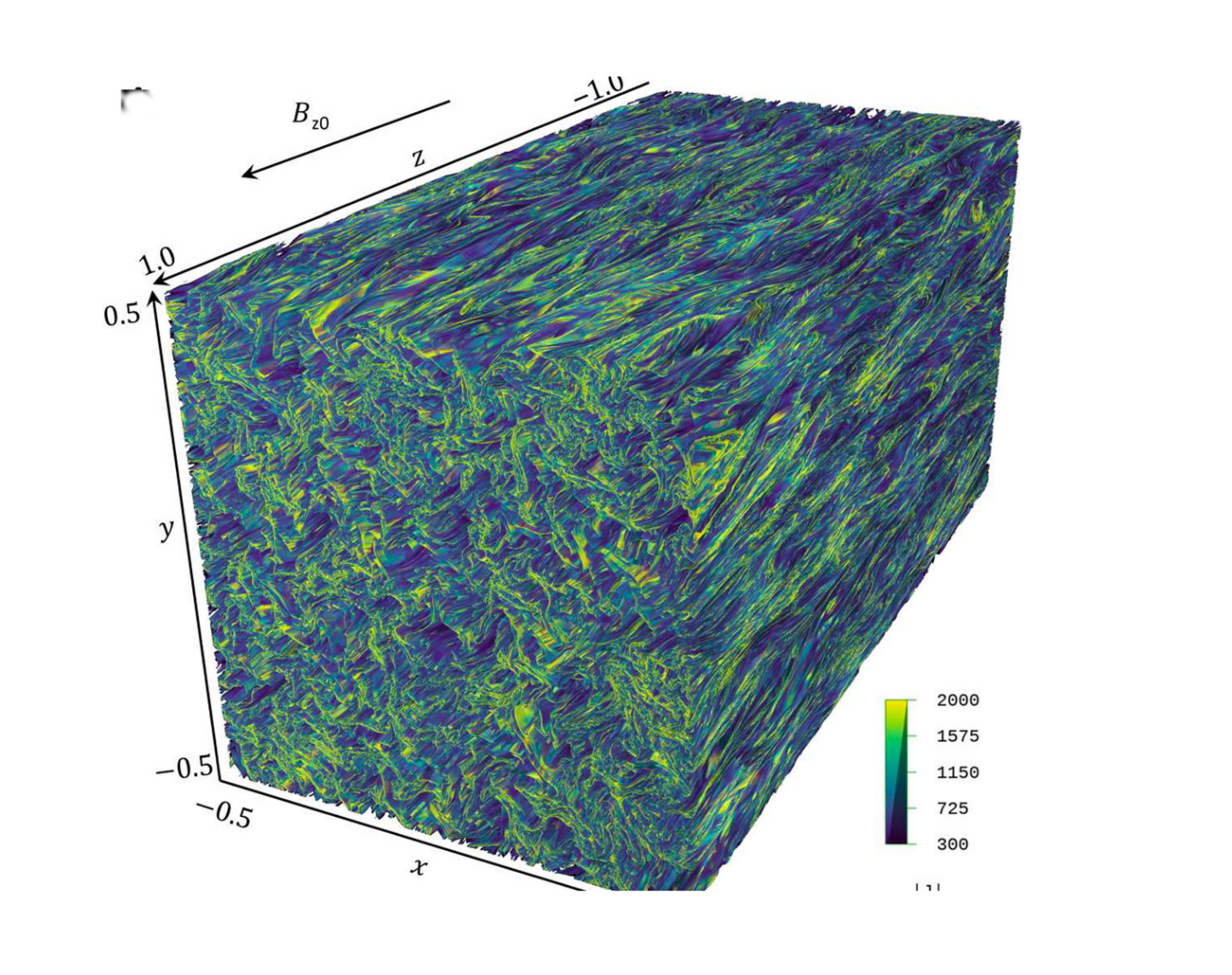

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Physics of Plasmas REVIEW pubs.aip.org/aip/php

Unexpected energetic particle observations near the Sun by Parker Solar Probe and Solar Orbiter

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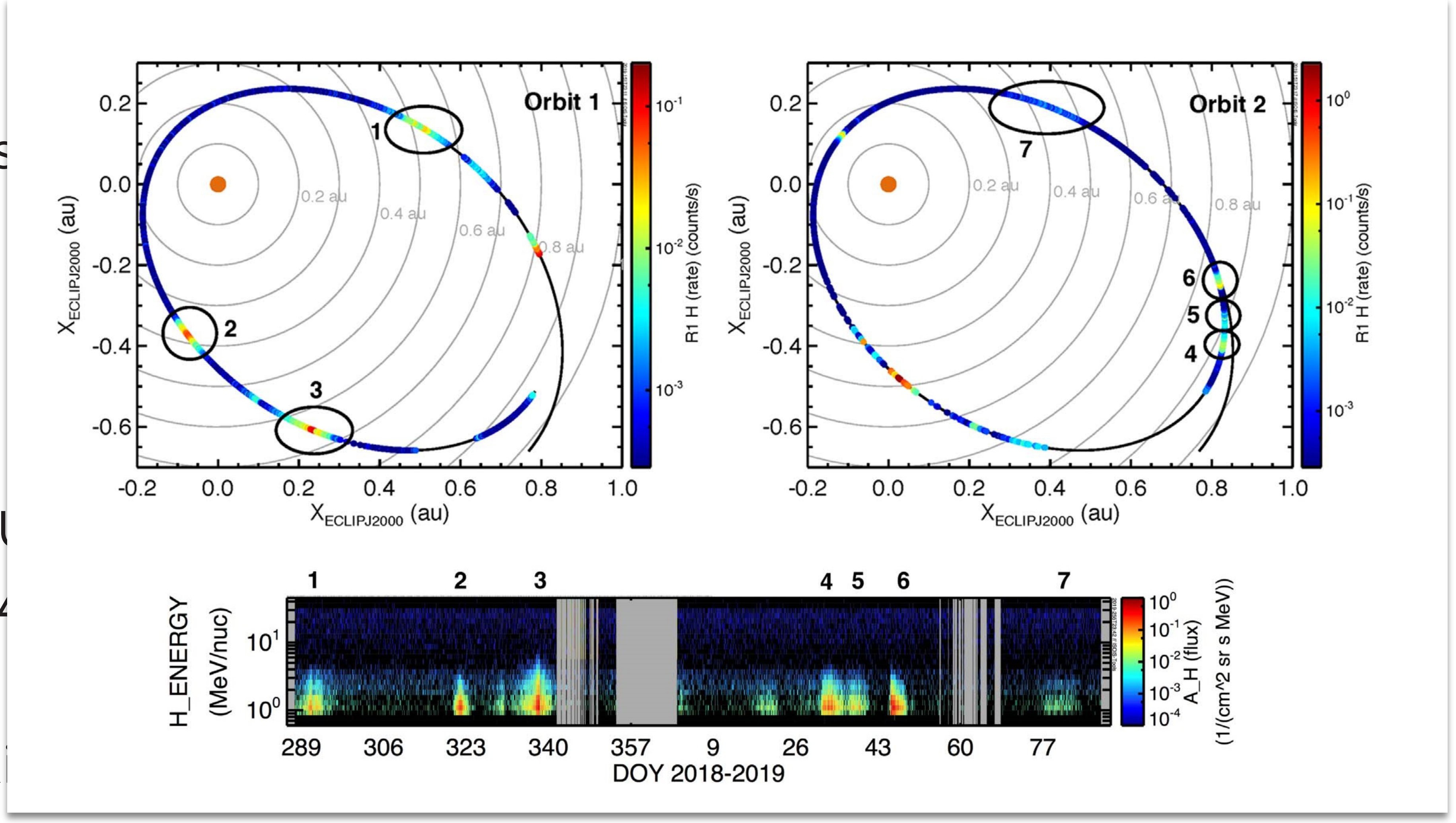

Plasma application in atomic layer etching

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Note: This paper is part of the Special Topic: Plasma Sources for Advanced Semiconductor /
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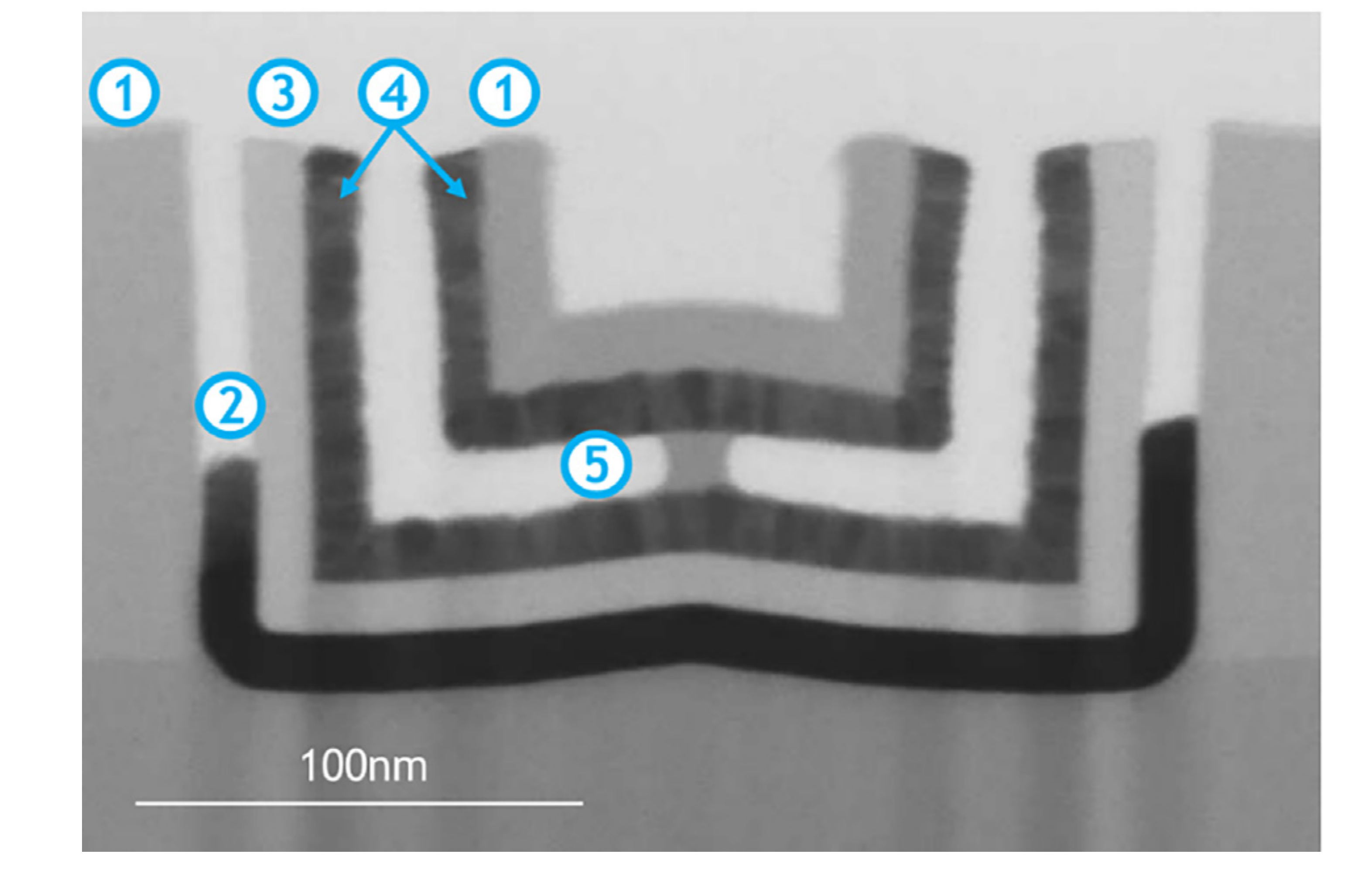


Key Concepts from Plasma 1

- Debye length and the plasma sheath
- Describing a plasma (part 1): how do charged particles move?
- Describing a plasma (part 2): distribution of charged particles and collisions
- Moments of the distribution function: plasma “fluid” theory and “collisionless closure”
- Linear waves in “cold” plasma
- Linear waves in bounded plasma
- Waves in “warm” plasma: the wave-particle resonance
- Magnetohydrodynamics (MHD) and examples (solar wind and plasma dynamo)
- Equilibria of magnetized plasmas (Straight pinches and the Grad-Shafranov Equation)
- Ideal and resistive instabilities in the large-aspect ratio (“straight”) tokamak

Plasma Parameters

- $n(r,t)$ – density - plasma frequency, ω_p
- $T(r,t)$ – temperature, $v_{th} = (kT/m)^{1/2}$
- λ_D – Debye length, v_{th}/ω_p
- N_D - plasma parameter, $(4\ \lambda_D{}^3/3)\ n >> 1$

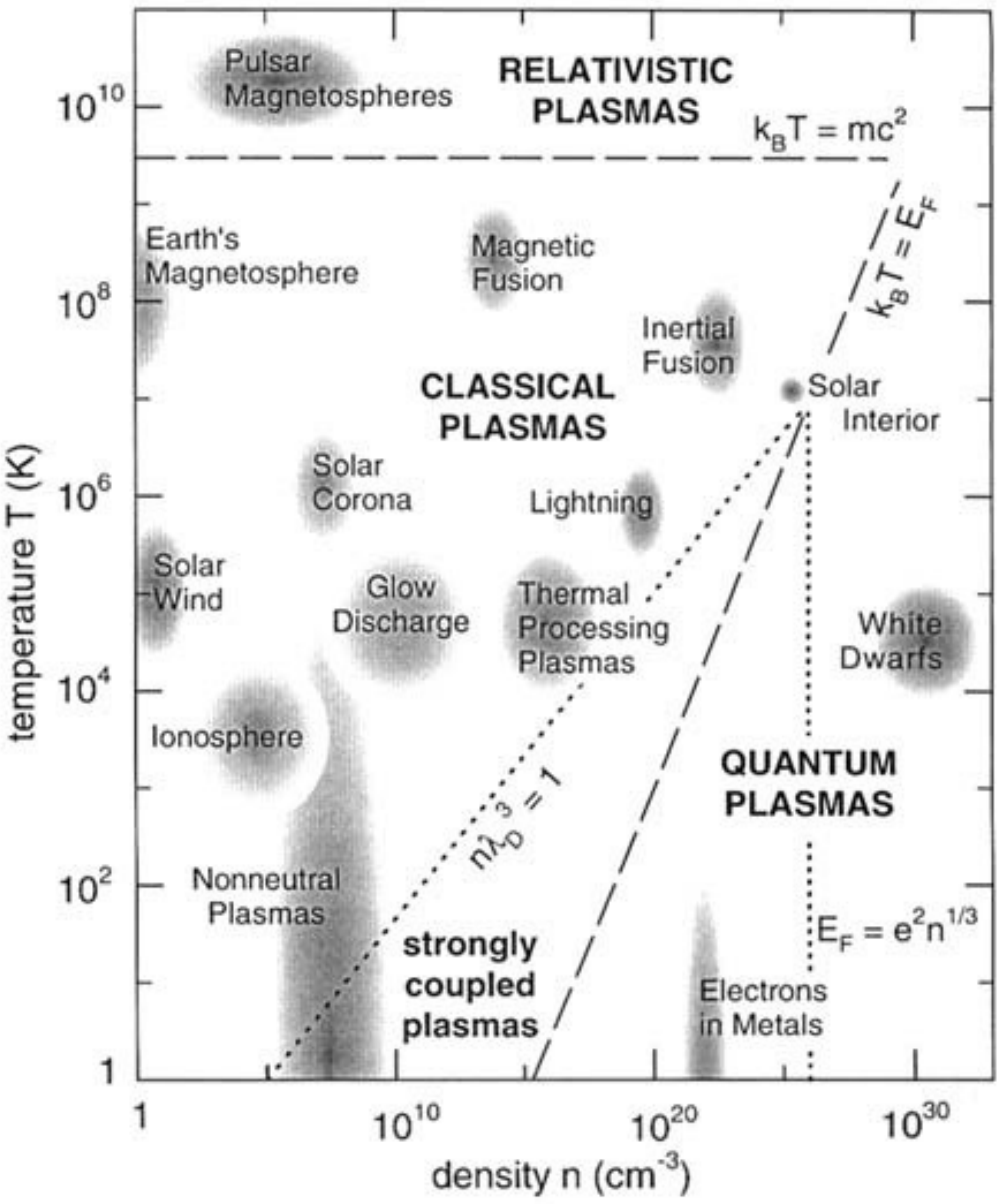
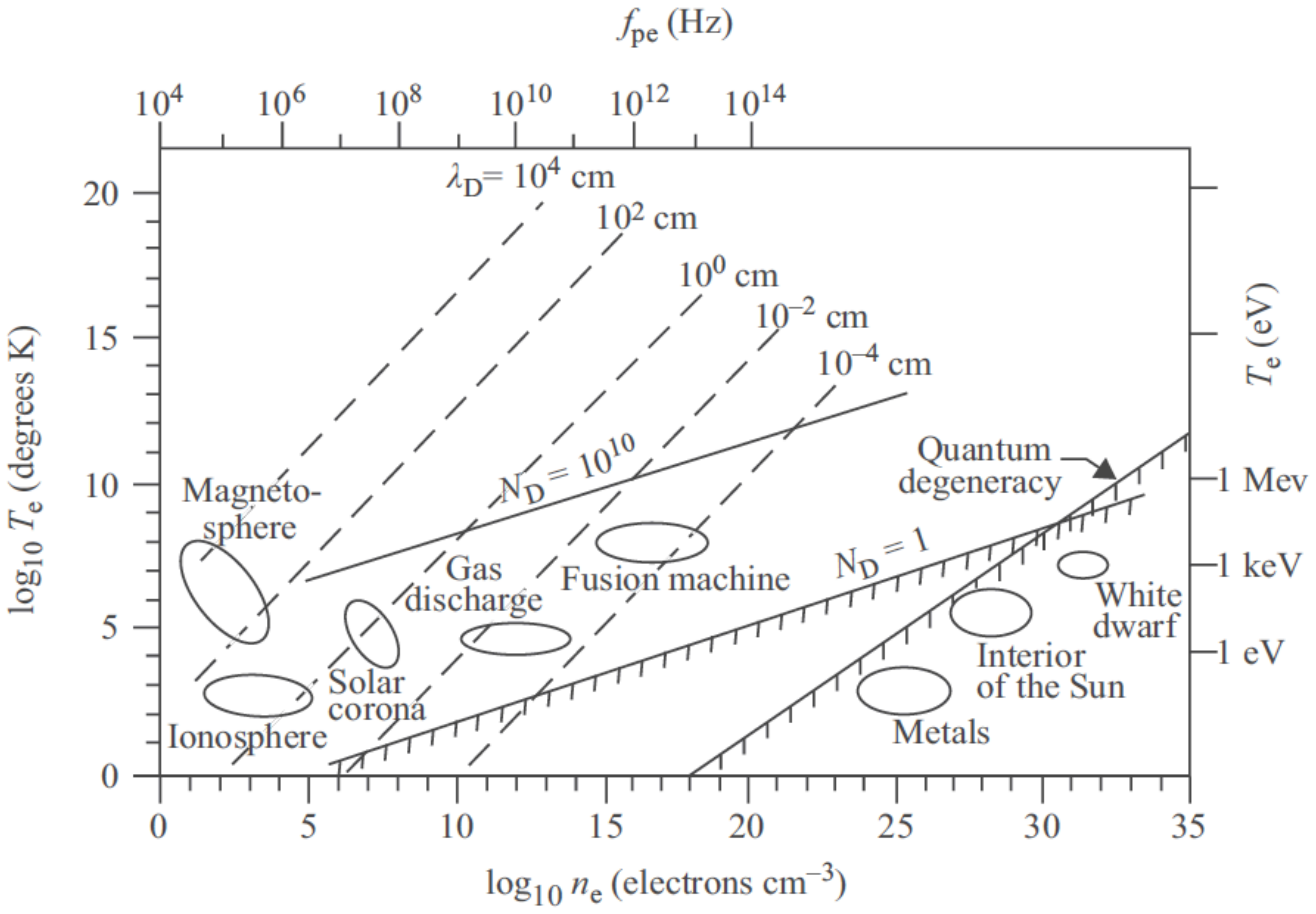


FIGURE S.1 Plasmas that occur naturally or can be created in the laboratory are shown as a function of density (in particles per cubic centimeter) and temperature (in kelvin). The boundaries are approximate and indicate typical ranges of plasma parameters.

Distinct plasma regimes are indicated:

- For thermal energies greater than that of the rest mass of the electron ($k_BT>mc^2$), relativistic effects are important.
- At high densities, where the Fermi energy is greater than the thermal energy ($E_F>k_BT$), quantum effects are dominant.
- In strongly coupled plasmas (i.e., $n\lambda_D^3 < 1$, where λ_D is the Debye screening length), the effects of the Coulomb interaction dominate thermal effects; and
- When $E_f>e^2n^{1/3}$, quantum effects dominate those due to the Coulomb interaction, resulting in nearly ideal quantum plasmas.
- At temperatures less than about 105 K, recombination of electrons and ions can be significant, and the plasmas are often only partially ionized.

http://www.nap.edu/catalog/4936/plasma-science-from-fundamental-research-to-technological-applications



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\

The Equations of Plasma Physics

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \tag{5.1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{5.2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{5.3}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) . \tag{5.4}$$

Distribution Function

$$f^{(\alpha)}(\mathbf{r}.\mathbf{v}, t) = \sum_k \delta(\mathbf{r} - \mathbf{r}_k(t))\delta(\mathbf{v} - \mathbf{v}_k(t)) , \tag{9.4}$$

$$N^{(\alpha)} = \iint f^{(\alpha)}(\mathbf{r}, \mathbf{v}, t)\, \mathrm{d}^3 r\, \mathrm{d}^3 v , \tag{9.5}$$

$$\rho_m(\mathbf{r}, t) = \sum_\alpha m^{(\alpha)} n^{(\alpha)}(\mathbf{r}, t) \tag{9.7}$$

$$\rho(\mathbf{r}, t) = \sum_\alpha q^{(\alpha)} n^{(\alpha)}(\mathbf{r}, t) . \tag{9.8}$$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_r f + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v f = 0 . \tag{9.13}$$

Particle Motion

$$m\dot{\mathbf{v}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) , \tag{3.1}$$

Debye Length:
The small scale of electric fluctuations

$$f(W = \text{ENERGY}) \propto e^{-W/kT}$$

THERMODYNAMIC EQUILIBRIUM

$$kT = \tfrac{1}{2} m V_{TH}^2$$

$$W = \tfrac{1}{2} m v^2 + q\Phi$$

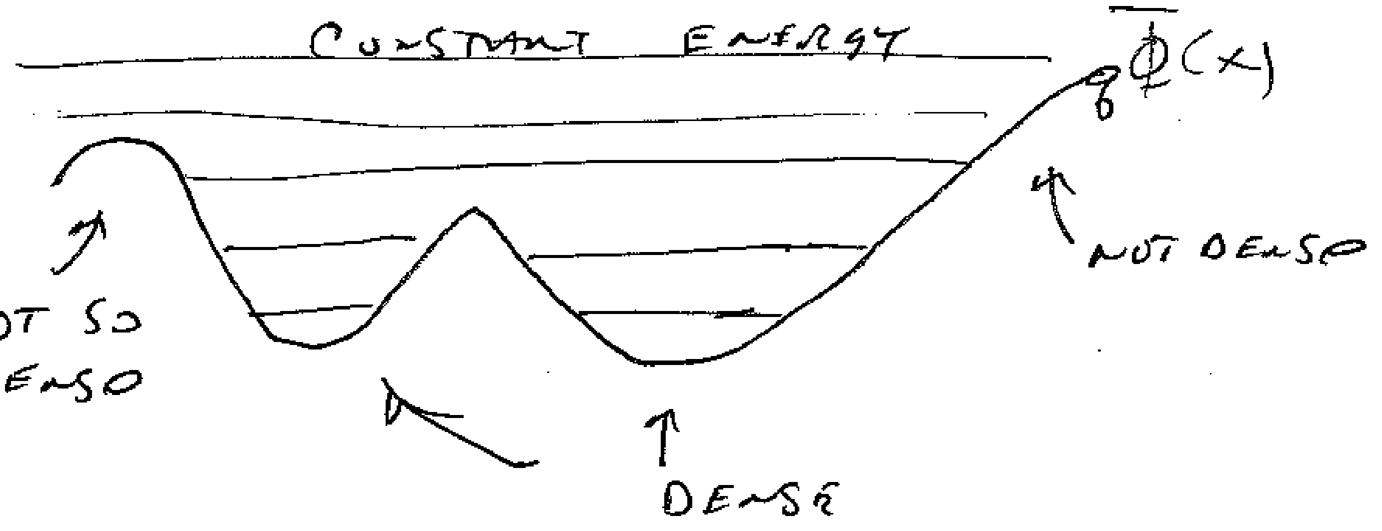
KINETIC

POTENTIAL

$$f(\bar{v}, \bar{x})$$

VELOCITY

SPACE



$$n_e(\bar{x}) \propto e^{-q_e \Phi(x)/kT_e}$$

$$n_i(\bar{x}) \propto e^{-q_i \Phi(x)/kT_i}$$

$$n_e(\bar{x}) - n_i(\bar{x}) \approx -e\,\Phi(x)\, n_0 \left(\frac{1}{kT_i} + \frac{1}{kT_e}\right)$$

$$\Phi(r) = ?$$

$$\nabla \cdot E = \rho/\epsilon_0$$

$$= \frac{Q}{\epsilon_0}\delta(\bar{r}) - \frac{e\,n_e(x)}{\epsilon_0} + \frac{e\,n_i(x)}{\epsilon_0}$$

$$\bar{E} = -\nabla\Phi$$

$$\nabla^2\Phi = -\rho/\epsilon_0 = -\frac{Q}{\epsilon_0}\delta(r) + \Phi(r)\underbrace{\left[\frac{e^2 n_0}{\epsilon_0 kT_e} + \frac{e^2 n_0}{\epsilon_0 kT_i}\right]}_{\lambda_D^{-2}}$$

NOTE: λ_D IS INDEPENDENT OF MASS!

Debye Length:
Potential near a change within a plasma

$$\nabla^2 \Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) = \frac{\partial^2 \Phi}{\partial r^2} + \frac{2}{r}\frac{\partial \Phi}{\partial r}$$

$$\iiint d^3r \, \nabla \cdot \nabla \Phi = -\frac{Q}{\epsilon_0}$$

$$\iint d\bar{A} \cdot \nabla \Phi \Rightarrow \Phi = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$$

BUT WITH PLASMA SHIELDING, WE CAN FIND A SOLUTION

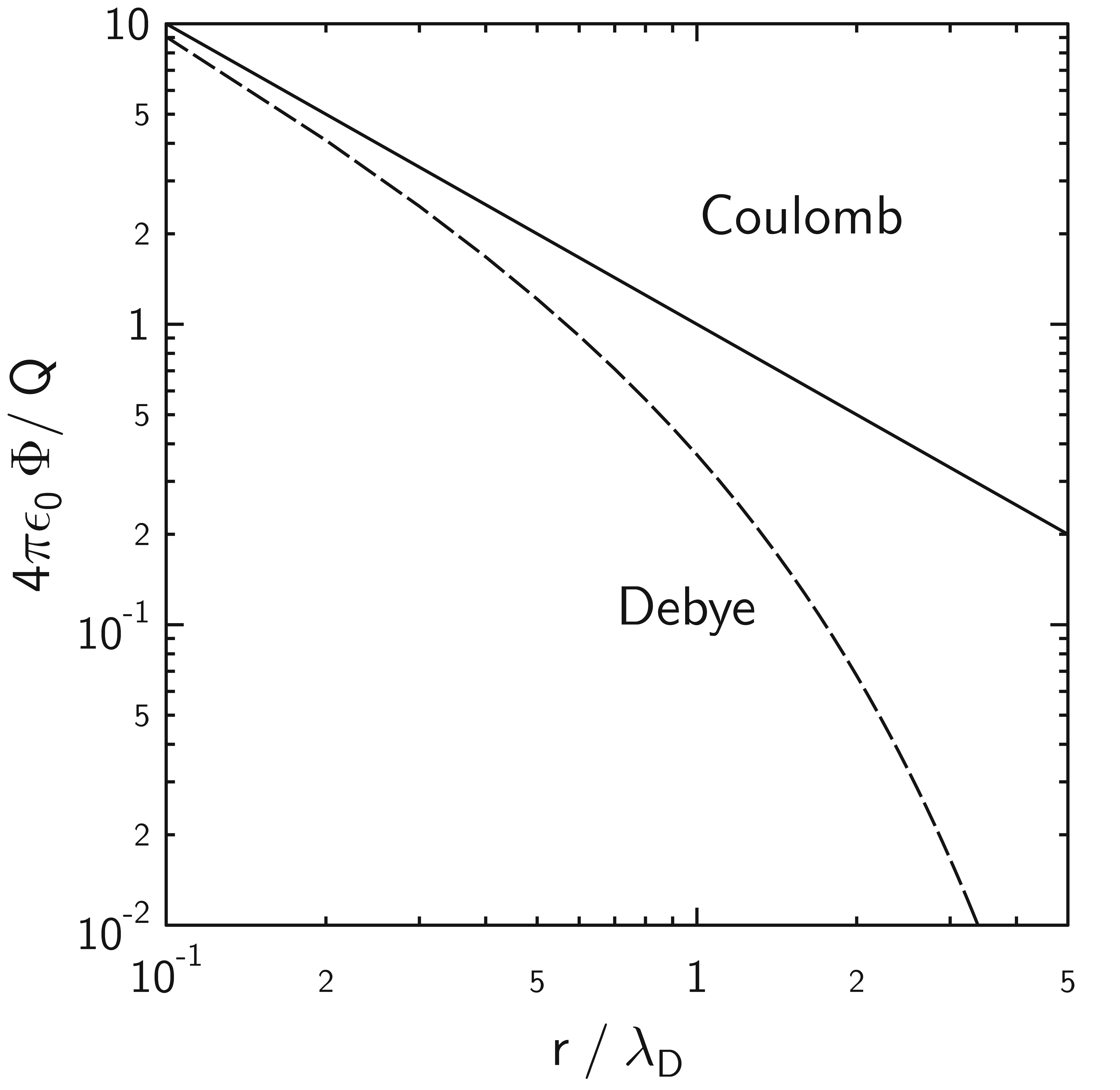
$$\Phi(r) \sim \frac{Q}{4\pi\epsilon_0}\frac{1}{r} f(r)$$

$$\frac{\partial \Phi}{\partial r} \sim -\frac{Q}{4\pi\epsilon_0 r^2} f(r) + \frac{Q}{4\pi\epsilon_0}\frac{1}{r} f'$$

$$\frac{\partial^2 \Phi}{\partial r^2} = \frac{2Q}{4\pi\epsilon_0 r^3} f(r) - \frac{Q}{4\pi\epsilon_0 r^2} f' - \frac{Q}{4\pi\epsilon_0 r^2} f' + \frac{Q}{4\pi\epsilon_0 r} f''$$

$$\boxed{f'' = \frac{1}{\lambda_D^2} f}$$

$$\Phi = \frac{Q}{4\pi\epsilon_0}\frac{1}{r} e^{-r/\lambda_D}$$



Plasma Parameter

$N_D = \frac{4}{3}\pi\lambda_D^3 n \gg 1$ For a "usual" plasma

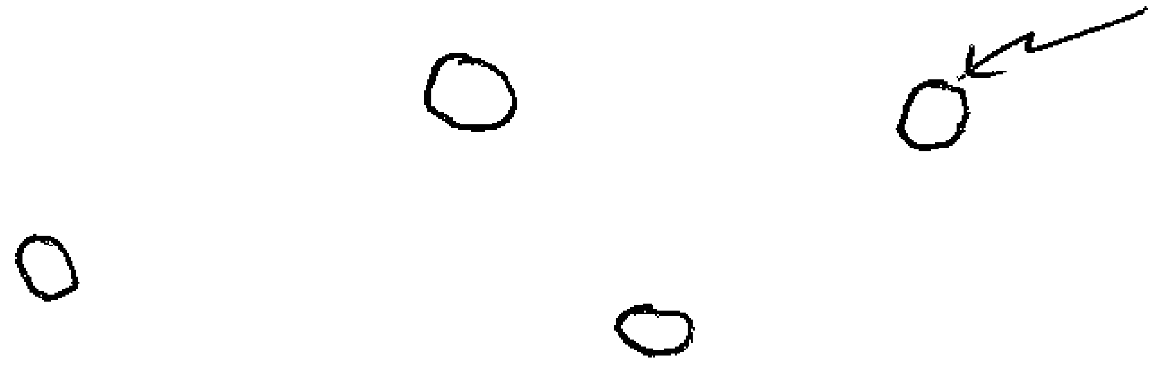
$N_D = \left(\frac{1}{3\Gamma_i}\right)^{3/2}$ $\Gamma_i \sim \frac{q^2/4\pi\epsilon_0\langle d\rangle}{kT_i}$ $\left(\frac{4\pi}{3}\langle d\rangle^3 \sim \frac{1}{n_i}\right)$

When $\Gamma_i \sim$ large, then "plasma" is strongly coupled

Usually only for case where there exists large q & small T_i

Components within plasma

Like dusty plasma



Dust 10-1000 nm dia

$Q \sim 10^4 - 10^6 e$

$T_i \sim 0.01$ room temperature

N_D or Λ: Plasma Parameter

Table 1.1 Key parameters for some typical weakly coupled plasmas.

Plasma	$n(\mathrm{m}^{-3})$	T(eV)	$\Pi(\sec^{-1})$	λ_D(m)	Λ
Solar wind (1AU)	10^7	10	2×10^5	7×10^0	5×10^{10}
Tokamak	10^{20}	10^4	6×10^{11}	7×10^{-5}	4×10^8
Interstellar medium	10^6	10^{-2}	6×10^4	7×10^{-1}	4×10^6
Ionosphere	10^{12}	10^{-1}	6×10^7	2×10^{-3}	1×10^5
Inertial confinement	10^{28}	10^4	6×10^{15}	7×10^{-9}	5×10^4
Solar chromosphere	10^{18}	2	6×10^{10}	5×10^{-6}	2×10^3
Arc discharge	10^{20}	1	6×10^{11}	7×10^{-7}	5×10^2

Plasma Frequency:
"Fast" Electron Motion of Plasma

$$\nabla \cdot \bar{E} = \rho/\epsilon_0$$

$$\rho = \rho_i + \rho_e \approx -e\tilde{n}_e$$ (IONS DON'T MOVE)

$$\frac{\partial f}{\partial t} + \bar{v}\cdot\bar{\nabla} f + \frac{q}{m}\bar{E}\cdot\nabla_v f = 0$$

REMEMBER $(\bar{x}, \bar{v}, t)$ IS A 6+1 DIMENSIONAL SPACE

$$\iiint d^3v\,(\,``\,) \Rightarrow \qquad \frac{\partial n}{\partial t} + \bar{\nabla}\cdot\bar{v}n = 0$$

NO PRESSURE

$$mn\frac{d\bar{v}}{dt} = qn\bar{E}$$

$$\bar{v}n = \iiint d^3v\,\bar{v}f$$

$\Rightarrow$ LINEARIZE

$$n = n_0 + \tilde{n} \qquad (\tilde{n}/n \lll 1)$$

$$\frac{\partial \tilde{n}}{\partial t} + n_0\bar{\nabla}\cdot\tilde{v} \approx 0$$

BUT $\nabla\cdot\frac{\partial \bar{v}}{\partial t} = \frac{q}{m}\nabla\cdot\bar{E}$

$$\frac{\partial^2 \tilde{n}}{\partial t^2} + n_0\bar{\nabla}\cdot\frac{\partial \bar{v}}{\partial t} \approx 0$$

$$\boxed{\frac{\partial^2 \tilde{n}}{\partial t^2} + \frac{q^2 n_0}{m_0\epsilon_0}\tilde{n} = 0}$$

From *Piel:* Current questions in plasma physics?

- **Space and astrophysical plasmas**
  - What are the origins and the evolution of plasma structures throughout the magnetized universe?
  - How are particles accelerated throughout the universe?
  - How do plasmas interact with non-plasmas?
- **Low temperature plasmas**
  - How can plasmas be used in the next generation of energy-efficient light sources?
  - How can plasma methods be optimized for purifying drinking water and for other environmental problems?
  - To which extent can new materials or advanced nanoparticles and nanowires be tailored by plasma processes?
- **Plasma physics at high energy densities**
  - Can we achieve fusion ignition and, eventually, useful fusion energy from compressed and heated fusion plasma?
  - Can we generate, using intense short-pulse lasers, electric fields in the multi-GeV/cm range for accelerating charged particles to energies far beyond the present limits of standard accelerators?
  - Can we better understand some aspects of observed high-energy astrophysical phenomena, such as supernova explosions or galactic jets, by carrying out appropriately scaled experiments?
- **Basic plasma science**
  The fields of basic research at the present forefront of plasma science are:
  - Non-neutral plasmas and single-component plasmas
  - Ultracold neutral plasmas
  - Dusty plasmas
  - Laser produced and high energy density plasmas
  - Microplasmas at atmospheric pressure
  - Plasma turbulence and turbulent transport
  - Magnetic fields in plasmas
  - Plasma waves, structures and flows

Next Week:
In Class Homework

From Fitzpatrick

2. The perturbed electrostatic potential $\delta\Phi$ due to a charge q placed at the origin in a plasma of Debye length λ_D is governed by

$$\left(\nabla^2 - \frac{2}{\lambda_D^2}\right)\delta\Phi = -\frac{q\,\delta(\mathbf{r})}{\epsilon_0}.$$

Show that the nonhomogeneous solution to this equation is

$$\delta\Phi(r) = \frac{q}{4\pi\,\epsilon_0\,r}\,\exp\left(-\frac{\sqrt{2}\,r}{\lambda_D}\right).$$

Demonstrate that the charge density of the shielding cloud is

$$\delta\rho(r) = -\frac{2\,q}{4\pi\,r\,\lambda_D^2}\,\exp\left(-\frac{\sqrt{2}\,r}{\lambda_D}\right),$$

and that the net shielding charge contained within a sphere of radius r, centered on the origin, is

$$Q(r) = -q\left[1 - \left(1 + \frac{\sqrt{2}\,r}{\lambda_D}\right)\exp\left(-\frac{\sqrt{2}\,r}{\lambda_D}\right)\right].$$

Problems *From Piel (answers in back)*

2.1 Prove that the electron Debye length can be written as

$$\lambda_{\mathrm{De}} = 69\,\mathrm{m}\left[\frac{T\,(\mathrm{K})}{n_{\mathrm{e}}(\mathrm{m}^{-3})}\right]^{1/2}$$

2.2 Calculate the electron and ion Debye length
(a) for the ionospheric plasma ($T_{\mathrm{e}} = T_{\mathrm{i}} = 3000\,\mathrm{K}$, $n = 10^{12}\,\mathrm{m}^{-3}$).
(b) for a neon gas discharge ($T_{\mathrm{e}} = 3\,\mathrm{eV}$, $T_{\mathrm{i}} = 300\,\mathrm{K}$, $n = 10^{16}\,\mathrm{m}^{-3}$).

2.3 Consider an infinitely large homogeneous plasma with $n_{\mathrm{e}} = n_{\mathrm{i}} = 10^{16}\,\mathrm{m}^{-3}$. From this plasma, all electrons are removed from a slab of thickness $d = 0.01\,\mathrm{m}$ extending from $x = -d$ to $x = 0$ and redeposited in the neighboring slab from $x = 0$ to $x = d$. (a) Calculate the electric potential in this double slab using Poisson's equation. What are the boundary conditions at $x = \pm d$? (b) Draw a sketch of space charge, electric field and potential for this situation. What is the potential difference between $x = -d$ and $x = d$?

2.4 Show that the equation for the shielding contribution (2.24) results from (2.21) and (2.23).

2.5 Derive the relationship between the coupling parameter for ion-ion interaction Γ Eqs. (2.15) and N_{D} (2.33) under the assumption that $T_{\mathrm{e}} = T_{\mathrm{i}}$.

2.6 Show that the second Lagrange multiplier in Eq. (2.6) is $\lambda = (k_{\mathrm{B}}T)^{-1}$.
Hint: Start from

$$\frac{1}{T} = \frac{\partial S}{\partial \lambda}\frac{\partial \lambda}{\partial U}$$

and use $\sum n_i = 1$.

Next Week: Mechanics of Charged Particles

- Tonks and Langmuir, “Oscillations in Ionized Gases,” *PR*, 1929.
- Charged particle motion in inhomogeneous, static and slowly-varying electric and magnetic fields

$$m\dot{\mathbf{v}} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \,, \tag{3.1}$$