

FITZPATRICK #1

$$\begin{aligned}
 \textcircled{A} \quad \langle \rho \rho \rangle &= \frac{\mu_+^2}{\omega^2} \left\langle \begin{bmatrix} \cos^2 \gamma & -\cos \gamma \sin \gamma \\ -\sin \gamma \cos \gamma & \sin^2 \gamma \end{bmatrix} \right\rangle \\
 &= \frac{\mu_+^2}{\omega^2} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbb{I} = \vec{1} \vec{1}^T} \propto \frac{\mu}{\omega} \quad \left( \mu = \frac{e u_+^2}{\omega} \right)
 \end{aligned}$$

$$\textcircled{B} \quad \text{with } \vec{a}_1 = \lambda \vec{\rho} \times \vec{b}$$

$$\begin{aligned}
 e \langle \vec{a}_1 \times (\vec{\rho} \cdot \vec{\sigma}) \vec{B} \rangle &= e \lambda \langle (\vec{\rho} \times \vec{b}) \times (\vec{\rho} \cdot \vec{\sigma}) \vec{B} \rangle \\
 &= -e \lambda \langle (\vec{b} \times \vec{\rho}) \times (\vec{\rho} \cdot \vec{\sigma}) \vec{B} \rangle \\
 &= -e \lambda \left[ \vec{\rho} (\vec{\rho} \cdot \vec{\sigma}) / B \right] \\
 &= -e \lambda \frac{u_+^2}{\omega^2} \underbrace{\vec{\nabla}_\perp / B}_{\text{since}} \langle \rho \rho \rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
 \end{aligned}$$

$$\textcircled{C} \quad e \langle \vec{u} \cdot (\vec{\rho} \cdot \vec{\sigma}) \vec{E} \rangle = -e \lambda \langle \vec{b} \cdot \vec{\rho} \times (\vec{\rho} \cdot \vec{\sigma}) \vec{E} \rangle$$

$$\text{BUT } \vec{\rho} \vec{\rho} = \frac{\mu}{\omega} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ so}$$

$$\begin{aligned}
 &= -\mu \vec{b} \cdot (\vec{\sigma} \times \vec{E}) \\
 &= -\mu \frac{2B}{\omega}
 \end{aligned}$$

↑  
PERPENDICULAR  
IDENTITY MATRIX

$$\begin{aligned}
 \textcircled{D} \quad e \langle \bar{u} \cdot (\rho \cdot \bar{v}) A \rangle &= e \lambda \langle (\bar{\rho} \times \bar{b}) \cdot (\bar{\rho} \cdot \bar{v}) \bar{A} \rangle \\
 &= -e \lambda \langle \bar{b} \cdot (\bar{\rho} \times (\bar{\rho} \cdot \bar{v}) \bar{A}) \rangle \\
 &= -e \lambda \frac{\mu}{e \lambda} \bar{b} \cdot \underbrace{\nabla \times A}_{\bar{B}} \quad \text{LIKE \#C)} \\
 &= -\mu / B
 \end{aligned}$$

FROM P12C CHAPTER 3

$$\begin{aligned}
 \textcircled{3.1} \quad \text{AMPERE'S LAW} \quad \nabla \times \vec{B} &= \mu_0 \vec{J} \\
 \oint ds \cdot \vec{B} &= \iint dA \cdot \mu_0 \vec{J}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3.2} \quad 2\pi r B_\theta(r) &= \mu_0 J_0 \int_0^r 2\pi r' dr' \left(1 - \frac{r'^2}{a^2}\right) \\
 &= \mu_0 J_0 2\pi \left[ \frac{r^2}{2} - \frac{r^4}{4a^2} \right]
 \end{aligned}$$

3.3 PRACTICE w NUMBERS

$$\begin{aligned}
 \textcircled{3.4} \quad \vec{m} &= -M \hat{z} \quad \text{SO} \quad \hat{r} \cdot \vec{m} = -M \cos \theta \\
 |\vec{B}_r| &\propto \frac{(3r^2 - a^2)(\hat{r} \cdot \vec{m})}{R^5} \\
 |B_\theta| &\propto -\left| \hat{e}_\theta \times \vec{B} \right| = - \left| \frac{-r^2 (\hat{r} \times \vec{m})}{15} \right|
 \end{aligned}$$

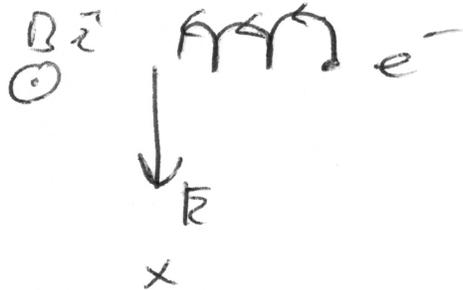
$$\hat{r} \times \hat{z} = \sin \theta \hat{\phi}$$

3.5

$$V_{\text{CONV}} \sim 0.11 \text{ m/sec} \sim \frac{1}{4} \text{ miles/hour}$$

$\leftarrow \phi$

3.6



3.7

EQUATION FOR A FIELD-LINE:

$$\frac{d\bar{s}}{dt} = \frac{\bar{B}}{|\bar{B}|} \quad \therefore \quad \frac{dz}{dt} = \frac{B_z}{|\bar{B}|}$$

$$\frac{dx}{dt} = \frac{B_x}{|\bar{B}|}$$

Ans  $\frac{dz}{dx} = \frac{B_z}{B_x}$

BUT  $B_z \propto 3z(-z) + R^2 = 3z(-z) + z^2 + x^2$

$B_x \propto 3x(-z) \quad \therefore \quad \frac{B_z}{B_x} = \frac{3(-z^2) + z^2 + x^2}{-3zx}$

See:

$$= \frac{2z}{3x} - \frac{1}{3z}$$

```
In[1]:= DSolve[{D[z[x], x] == (2/3) z[x]/x - (1/3) x/z[x], z[1] == 0}, z[x], x]
Plot[z[x] /. Last[%], {x, 0.01, 1}, AspectRatio -> Automatic,
PlotRange -> {{0, 1}, {0, 0.5}}]
```

```
Out[1]= {{z[x] -> -sqrt(-((-1 + x^(2/3)) x^(4/3)))}, {z[x] -> sqrt(-((-1 + x^(2/3)) x^(4/3)))}}
```

