Problems

9.1 Show that any function $g(\frac{1}{2}mv^2 + q\Phi)$, which only depends on the total energy of a particle, solves the Vlasov equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} - \frac{q}{m}\frac{\partial \Phi}{\partial x}\frac{\partial f}{\partial v} = 0\,.$$

9.2 Verify that the mean velocity of a one-dimensional half-Maxwellian electron distribution is given by

$$v_{\mathrm{mean}} = \frac{1}{n_{\mathrm{e0}}}\int_0^\infty v f_{\mathrm{M}}^{(1)}\,\mathrm{d}v = \left(\frac{k_{\mathrm{B}}T_{\mathrm{e}}}{2\pi m_{\mathrm{e}}}\right)^{1/2}\,.$$

9.3 Start with the velocity distribution (9.25) and prove that the current density in the right half of the diode is the constant (9.28).

9.4 Derive the Landau damping rate for the Bohm-Gross mode (9.55) from (9.44) and (9.54).

9.5 Using (8.8) and (9.86), show that the trapping potential is given by

$$\Phi_{\mathrm{t}} = \frac{m_{\mathrm{e}}}{4e}\,v_0^2\left[\frac{1}{2}\left(\frac{\alpha_{\mathrm{b}}}{2}\right)^{1/3}\right]^2\,.$$

9.6 The mean energy density of the electric wave field is given by $W_E = \frac{1}{2}\varepsilon_0\langle E^2\rangle = \frac{1}{4}\varepsilon_0\hat{E}^2$. Consider the fastest growing mode of the beam-plasma instability (8.24) at the onset of trapping. Use $|\hat{E}| = k\hat{\Phi}$ and $k = \omega_{\mathrm{pe}}/v_0$ and show that the mean field energy is given by

$$W_E = \left(\frac{1}{2}\,n_{\mathrm{b0}}\,m v_0^2\right)\,2^{-31/3}\alpha_{\mathrm{b}}^{1/3}\,.$$