The Basics in a Nutshell

- Plasma instabilities fall into two classes, macroscopic instabilities in real space, like the Rayleigh-Taylor instability, and microinstabilities in velocity space, like the beam-plasma instability.
- The directed flow of a group of fast electrons (beam) can excite electrostatic waves near the electron plasma frequency. This beam-plasma instability has a tremendous growth rate, which depends on \((n_b/n_p)^{1/3}\).
- The instability of the slow wave can be understood from the concept of negative mass or negative energy waves.
- In a system of finite length (Pierce diode) the maximum electron current is limited by the onset of purely-growing, non-oscillating disturbances of the electron beam.

Problems

8.1 For which values of the coefficients \(a\) and \(b\) has the differential equation

\[
\ddot{x} + ax + bx = 0.
\]

stable and unstable solutions? Draw a stability map \(b = f(a)\) and mark regions with damped oscillatory, overdamped, unstable oscillatory and purely growing modes.

8.2 Discuss the instability of a system with counter-streaming electron and positron beams of equal density \(\propto \omega^2\) and equal but opposite velocities \(v\) and \(-v\). Write down the dielectric function for this system in analogy to (8.3). Find the four solutions of \(\varepsilon(\omega, k) = 0\). Show that there is a region \(k < k_{\text{crit}}\) with two real and a pair of conjugate complex solutions. Plot the growth rate of this instability vs. \(kv/\omega_b\).

8.3 Perform the missing steps that lead to (8.23) and (8.24).