APPH 6101 Plasma Physics I
Homework 7: Due 21 October, 2015.

Questions 1-6

Due problems 6.1 through 6.7 (excluding Prob. 6.6).

Question 7

Don Gurnett was part of the *Voyager I* scientific team that encounter the Jovian magnetosphere in 1980. In the figure below are measurements of whistler waves indicating lighting on the Jovian surface. Note that *Voyager* was near the equator at a radius of about $5.8\,R_J$. If the magnetic field of Jupiter is given by an ideal magnetic dipole, with an equatorial field strength that varies as $B(r) = 0.4(R_j/r)^3$ mT, what information can you learn from the spectrogram about the plasma density in Jupiter's magnetosphere? Keep your answers approximate.

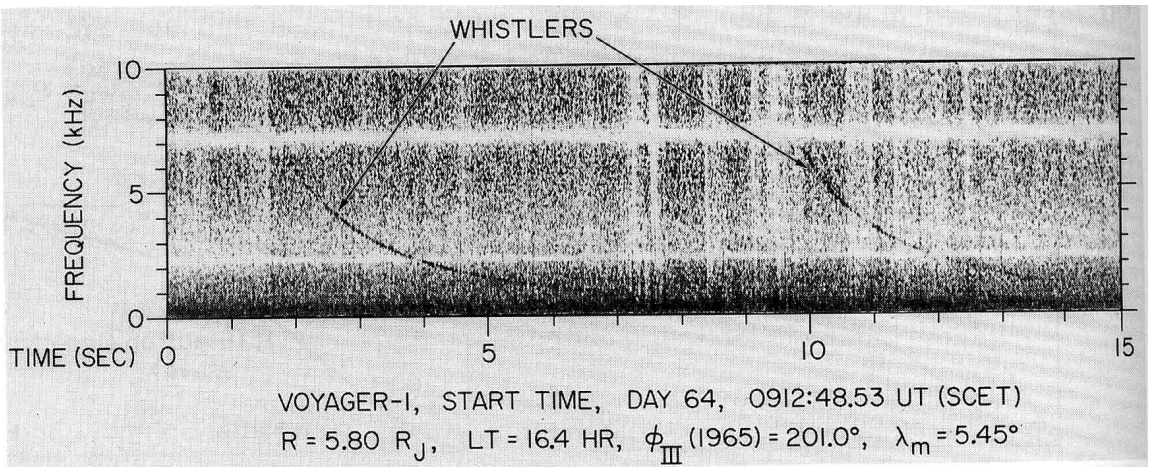

Figure 1: Measurement of whistler waves in Jupiter's magnetosphere.

Problems

6.1 In the limit $T_\mathrm{i} \ll T_\mathrm{e}$ the ion-acoustic wave has the dispersion relation

$$\omega(k) = \frac{\omega_\mathrm{pi}\lambda_\mathrm{De}\,k}{(1+k^2\lambda_\mathrm{De}^2)^{1/2}}$$

(a) Derive an expression for the phase velocity $v_\varphi(k)$ and group velocity $v_\mathrm{g}(k)$ as a function of the wave number k.
(b) Discuss the result with respect to "acoustic behavior" at $k\lambda_\mathrm{De} \ll 1$.

6.2 Assume that in a dielectric medium the relation $v_\varphi \cdot v_\mathrm{g} = c^2$ holds. What is the general shape of the dispersion relation $\omega(k)$ for this case?

6.3 (a) Show that for $\omega_\mathrm{pe}^2 \gg \omega_\mathrm{ce}^2 \gg \omega^2$ the refractive index for Whistler waves takes the limiting form

$$\mathcal{N} = \frac{\omega_\mathrm{pe}}{(\omega\omega_\mathrm{ce})^{1/2}}$$

(b) Calculate phase and group velocity and show that $v_\mathrm{gr} = 2v_\varphi$.

6.4 Determine the minimum plasma density at which a He-Ne Laser at $\lambda = 633\,\mathrm{nm}$ wavelength will be reflected.

6.5 Consider an electron-positron plasma with $n_\mathrm{e} = n_\mathrm{p}$. What is the cut-off frequency for electromagnetic waves in this system?

6.6 The plasma of the ionospheric F-layer has a density $n_\mathrm{e} \approx 2 \times 10^{12}\,\mathrm{m}^{-3}$. The typical magnetic field at mid-latitude is $B = 50\,\mu\mathrm{T}$. Calculate the electron plasma frequency f_pe, electron cyclotron frequency f_ce and the upper hybrid frequency f_uh.

6.7 Prove that $v_\varphi = v_\mathrm{gr}$ requires $\omega = v_\varphi\,k$.