Questions 1-6

Due problems 6.1 through 6.7 (excluding Prob. 6.6).

Question 7

Don Gurnett was part of the Voyager I scientific team that encounter the Jovian magnetosphere in 1980. In the figure below are measurements of whistler waves indicating lighting on the Jovian surface. Note that Voyager was near the equator at a radius of about $5.8 R_J$. If the magnetic field of Jupiter is given by an ideal magnetic dipole, with an equatorial field strength that varies as $B(r) = 0.4(R_J/r)^3$ mT, what information can you learn from the spectrogram about the plasma density in Jupiter’s magnetosphere? Keep your answers approximate.

![Figure 1: Measurement of whistler waves in Jupiter’s magnetosphere.](image)

Problems

6.1 In the limit $T_i \ll T_e$ the ion-acoustic wave has the dispersion relation

$$\omega(k) = \frac{\omega_{pi} \lambda_{De} k}{(1 + k^2 \lambda_{De}^2)^{1/2}}$$

(a) Derive an expression for the phase velocity $v_p(k)$ and group velocity $v_g(k)$ as a function of the wave number $k$.
(b) Discuss the result with respect to “acoustic behavior” at $k \lambda_{De} \ll 1$.

6.2 Assume that in a dielectric medium the relation $v_p \cdot v_g = c^2$ holds. What is the general shape of the dispersion relation $\omega(k)$ for this case?

6.3 (a) Show that for $\omega_{pe}^2 \gg \omega_{ce}^2 \gg \omega^2$ the refractive index for Whistler waves takes the limiting form

$$N = \frac{\omega_{pe}}{(\omega \omega_{ce})^{1/2}}$$

(b) Calculate phase and group velocity and show that $v_g = 2v_p$.

6.4 Determine the minimum plasma density at which a He-Ne Laser at $\lambda = 633$ nm wavelength will be reflected.

6.5 Consider an electron-positron plasma with $n_e = n_p$. What is the cut-off frequency for electromagnetic waves in this system?

6.6 The plasma of the ionospheric F-layer has a density $n_e \approx 2 \times 10^{12}$ m$^{-3}$. The typical magnetic field at mid-latitude is $B = 50$ $\mu$T. Calculate the electron plasma frequency $f_{pe}$, electron cyclotron frequency $f_{ce}$ and the upper hybrid frequency $f_{uh}$.

6.7 Prove that $v_p = v_g$ requires $\omega = v_p k$. 