Problems

5.1 (a) Consider the pressure equilibrium in a Z-pinch that has been compressed by its self-generated magnetic field to a radius of $100\,\mu$m. What is the magnetic pressure at the surface of the pinch, when the total current amounts to 10 kA? How compares this to atmospheric pressure?
(b) Assume that the plasma inside the pinch is homogeneous and has $T_e = T_i$ and density $n_e = 10^{24}\,\text{m}^{-3}$. What is the temperature inside this plasma that is necessary to balance the magnetic pressure by gas kinetic pressure?

5.2 Calculate the magnetic field B that is necessary to produce a magnetic pressure at the surface of a magnetically confined fusion that is 4 times the kinetic pressure in the plasma center, when the central density is $n_e = 2\times 10^{20}\,\text{m}^{-3}$ and the temperature $T = 20\,\text{keV}$. This corresponds to $\beta = 25\%$.

5.3 What is the Alfvén speed in a fusion plasma with deuterium ions of $n_i = 10^{20}\,\text{m}^{-3}$ density at a typical magnetic field of $B = 3\,\text{T}$?

5.4 The ionospheric F-layer has a plasma density of $n = 10^{12}\,\text{m}^{-3}$ and consists mainly of O^+-ions.
(a) What is the Alfvén speed at a typical magnetic field of $B = 3 \cdot 10^{-5}\,\text{T}$?
(b) Compare this result with the ion sound speed at a temperature $T_e = T_i = 3000\,\text{K}$.

5.5 For the Parker spiral, draw a log-log plot of the normalized magnetic field $B(r)/B_0$ and its components, B_r/B_0 and B_φ/B_0, vs. the normalized radial position $r/r_\odot$. Assume $u_r = 4 \times 10^5\,\text{m}\,\text{s}^{-1}$ and a solar rotation period of 27 d. Mark the position of the Earth's orbit in this plot.

5.6 A method to determine the temperature of a hot magnetized plasma column is based on measuring the change in magnetic flux when the plasma is switched off. This can be done by a *diamagnetic loop* of N windings, which is wound around the (non-conducting) cylindrical vessel of radius R that is assumed to contain the plasma column. Faraday's induction law gives $\Delta\Phi_{mag} = -N \int U_{ind}\,dt$. Hence, the time integral of the voltage pulse from the diamagnetic loop gives the change in magnetic flux. To derive a relation between plasma temperature and integrated loop voltage, we assume that $T_e = T_i = \text{const}$. The density profile is approximated by a Gaussian $n(r) = n_0 \exp[-(r/a)^2]$ with $a^2 \ll R^2$. Use the pressure equilibrium $p_{kin}(0) + p_{mag}(0) = p_{mag}(R)$ and calculate the total change in magnetic flux $\Delta\Phi_{mag}$ from its vacuum value. Show that $\Delta\Phi_{mag} \approx -\frac{1}{2}\pi a^2 n_0 B_0 \beta$ in the limit $\beta \ll 1$ with β from (5.51).