Homework #4 (Ch. 4)

APPH E6101x
Plasma Physics 1

- Fitzpatrick: Read Chapter 3 about collisions, and discuss the meaning of “Rosenbluth Potentials” and the collision operator in Eq. 3.112
- Piel: All nine problems in Ch. 4 (answers in back of text)

Fitzpatrick: Read Chapter 3 about collisions, and discuss the meaning of "Rosenbluth Potentials" and the collision operator in Eq. 3.112

The quantities $H_{s'}(\mathbf{v})$ and $G_{s'}(\mathbf{v})$ are known as *Rosenbluth potentials* (Rosenbluth, MacDonald, and Judd 1957), and can easily be seen to satisfy

$$\nabla_v^2 H_{s'} = -4\pi\, f_{s'}(\mathbf{v}), \tag{3.110}$$

$$\nabla_v^2 G_{s'} = 2\, H_{s'}(\mathbf{v}), \tag{3.111}$$

where ∇_v^2 denotes a velocity-space Laplacian operator. The former result follows because $\nabla_v^2(1/v) = -4\pi\,\delta(\mathbf{v})$, and the latter because $\nabla_v^2(v) = 2/v$.

When expressed in terms of the Rosenbluth potentials, the Landau collision operator, (3.93), takes the form

$$\begin{aligned} C_{ss'} &= -\Gamma_{ss'}\,\frac{\partial}{\partial v_\alpha}\left(\frac{m_s}{m_{s'}}\,\frac{\partial H_{s'}}{\partial v_\alpha}\,f_s - \frac{1}{2}\,\frac{\partial^2 G_{s'}}{\partial v_\alpha\,\partial v_\beta}\,\frac{\partial f_s}{\partial v_\beta}\right) \\ &= -\Gamma_{ss'}\left[\left(1+\frac{m_s}{m_{s'}}\right)\frac{\partial}{\partial v_\alpha}\left(\frac{\partial H_{s'}}{\partial v_\alpha}\,f_s\right) - \frac{1}{2}\,\frac{\partial^2}{\partial v_\alpha\,\partial v_\beta}\left(\frac{\partial^2 G_{s'}}{\partial v_\alpha\,\partial v_\beta}\,f_s\right)\right], \end{aligned} \tag{3.112}$$

where

$$\Gamma_{ss'} = \frac{2\,\gamma_{ss'}}{m_s^2} = \left(\frac{e_s\,e_{s'}}{4\pi\,\epsilon_0\,m_s}\right)^2 4\pi\,\ln\Lambda_c. \tag{3.113}$$

Problems

4.1 Show that the maximum of the Maxwell distribution function $f_M(|v|)$ is found at v_T.

4.2 Prove that the mean thermal velocity in (4.7) is $v_{\text{th}} = [(8k_{\text{B}}T)/(\pi m)]^{1/2}$.

4.3 Prove that the mean kinetic energy in (4.8) is $(\text{m}/2)\langle v^2\rangle = (3/2)k_{\text{B}}T$.

4.4 Derive (4.66) from conservation of energy and momentum by considering the scattering of a light particle on a heavy particle. Hint: In this limit the modulus of momentum of the scattered electron is the same as the momentum before the collision.

4.5 Solve the integral in (4.67).

4.6 Solve the integrals in (4.62) and derive (4.63).

4.7 Show that the velocities in (4.63) can also be derived from the force balance between friction and total Lorentz force

$$m_{\text{i}}\nu_{\text{m}}\mathbf{v} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \ .$$

4.8 Assume that the radial electron density profile in a long cylindrical discharge tube of radius a is parabolic

$$n_{\text{e}}(r) = n(0)\left[1 - \frac{r^2}{a^2}\right].$$

Determine the equivalent electron density of a homogeneous density distribution that would give the same current.

4.9 Perform the intermediate steps for proving the statement that the ignition line for fusion is equivalent to $\eta = 0.154$ in the Lawson curves.