APPH 6101 Plasma Physics I
Homework 2: Due 18 September, 2023.

Question 1

Irving Langmuir graduated from Columbia University in 1903 and received the Nobel Prize in chemistry for his investigations of surface chemistry. (Langmuir was the first to observe stable films of atoms on tungsten and platinum and formulated the first general theory of adsorbed films.) In plasma physics, he is well known as the individual who characterized ionized matter as "plasma" and for inventing the "Langmuir probe." In one of the most important early papers of plasma physics, Langmuir and his younger colleague, Lewi Tonks, were the first to observe and identify plasma oscillations (also called "Langmuir" oscillations). This was reported in *Physical Review*, **33**, p. 195 (1929). (This paper can be downloaded using Columbia University's subscription to *Physical Review*.)

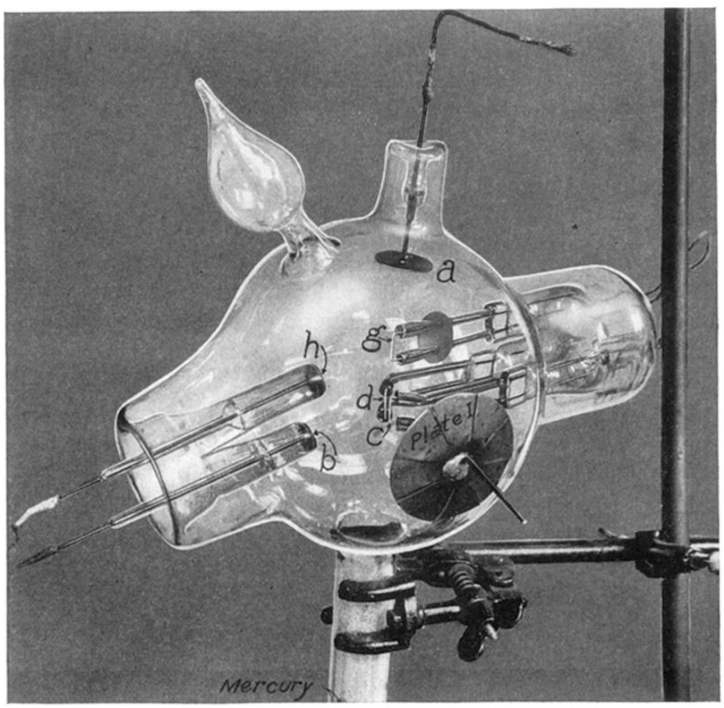

Fig. 1. Experimental tube.

Figure 1: Mercury plasma device (or gas filled lamp) built by Irving Langmuir (1881-1957, top) and Lewi Tonks (1897-1971, bottom) and used to observe plasma oscillations. Probes "b" and "h" are Langmuir probes; "c", "d", and "g" are electron-emitting tungsten filaments; "a" is a positively-biased anode; and "plate 1" is one of three external probes used to measure collective oscillations.

Part A

Tonks and Langmuir wrote:

> ... Taking 10^{10} as a typical low value of electron density, the inter-electron distance is 4.6×10^{-4} cm. If all the electrons throughout a certain region are displaced this distance, the resulting field strength will be 0.028 e.s.u. or 8.3 volts per cm, and the energy density in the field will be 3.0×10^{-5} ergs cm^{-3}.

Confirm these numbers. If the electron temperature was 1 eV (and much warmer than the ions), how does the electrostatic energy density compare with the kinetic energy density of the plasma (*i.e.* the electrons)?

Part B

Tonks and Langmuir wrote:

> ... so that in discharges of the intensity used in our experiments where, roughly, $n = 10^{10}$ and $T_e = 10^4$ [deg K, about 1 eV], $\lambda_D = 0.005$ cm.

How many particles are contained within a Debye sphere (proportional to the "plasma parameter") for this experiment?

Part C

Tonks and Langmuir adjusted the neutral density of mercury by adjusting the temperature of liquid mercury at the bottom of the lamp. They reported adjusting the temperature from 15 deg C to 21 deg C. The tabulated vapor pressure changed from 0.8 mTorr to 1.3 mTorr. What is the neutral density? If the neutral density with the lamp "on" is the same as the tabulated vapor pressure, what is the approximate fraction of ionization (assuming the ions are singlely-charged)? [Info: 1 standard atmosphere is 760 Torr, equivalently, 1 Torr = 1 mm Hg.]

Part D

Fig. 2 shows measurements of the current collected by a "Langmuir probe". When the probe is biased negatively, the current collected in nearly constant and equal to the rate that ions can be accelerated into the probe. This probe current is $I_{probe} \approx enC_e\sqrt{m_e/M_i}\,A_{probe}$, where C_e is the thermal velocity of the electrons, e is the charge of an electron, $\sqrt{m_e/M_i} \sim 1/600$ for mercury, and $A_{probe} \approx 1$ cm^2 was the current collection area for Langmuir's probe. From the figure, estimate the plasma density, n.

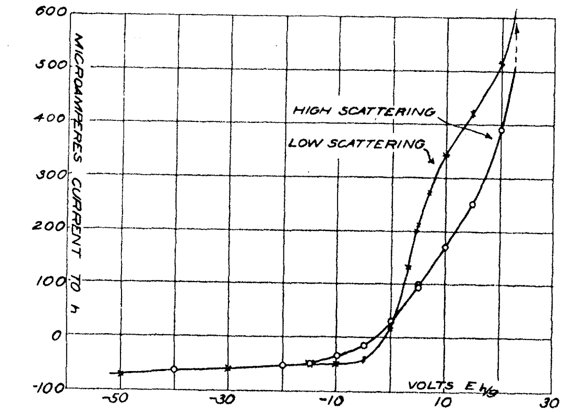

Fig. 8. Volt-ampere characteristics of a collector in the primary electron beam.

Figure 2: Measured current to a "Langmuir probe" as a function of probe bias.

Question 2

A parallel plate capacitor consists of two plates of area, A, separated by a small distance, d. When the region between the plates is a vacuum, the low-frequency capacitance is $C = \epsilon_0 A/d$.

What is the capacitance when the plates are filled by a plasma with a Debye length, λ_D? Show that in the limit that $\lambda_D \ll d$, the capacitance is $C \approx \epsilon_0 A/\sqrt{2}\lambda_D$.

Question 3

Consider a sphere of plasma like Tonks's and Langmuir's of Question 1. The radius of the plasma was 9 cm, and the electron temperature $T_e = 1$ eV, with (infinite) massive ions.

Part A

How much energy is needed to move *all* of the electrons to the surface of the plasma? How does this compare to the kinetic energy of the initially uniform plasma?

Part B

What is the electrostatic potential and electric field as a function of radius?

Part C

How much smaller (in radius) must the plasma be such that the electrostatic energy would equal the the original kinetic energy of the electrons?