Next Week:
In Class Homework

From Fitzpatrick

2. The perturbed electrostatic potential $\delta\Phi$ due to a charge q placed at the origin in a plasma of Debye length λ_D is governed by

$$\left(\nabla^2 - \frac{2}{\lambda_D^2}\right)\delta\Phi = -\frac{q\,\delta(\mathbf{r})}{\epsilon_0}.$$

Show that the nonhomogeneous solution to this equation is

$$\delta\Phi(r) = \frac{q}{4\pi\,\epsilon_0\,r}\,\exp\left(-\frac{\sqrt{2}\,r}{\lambda_D}\right).$$

Demonstrate that the charge density of the shielding cloud is

$$\delta\rho(r) = -\frac{2\,q}{4\pi\,r\,\lambda_D^2}\,\exp\left(-\frac{\sqrt{2}\,r}{\lambda_D}\right),$$

and that the net shielding charge contained within a sphere of radius r, centered on the origin, is

$$Q(r) = -q\left[1 - \left(1 + \frac{\sqrt{2}\,r}{\lambda_D}\right)\exp\left(-\frac{\sqrt{2}\,r}{\lambda_D}\right)\right].$$

Problems *From Piel (answers in back)*

2.1 Prove that the electron Debye length can be written as

$$\lambda_{\mathrm{De}} = 69\,\mathrm{m}\left[\frac{T\,(\mathrm{K})}{n_{\mathrm{e}}(\mathrm{m}^{-3})}\right]^{1/2}$$

2.2 Calculate the electron and ion Debye length
(a) for the ionospheric plasma ($T_{\mathrm{e}} = T_{\mathrm{i}} = 3000\,\mathrm{K}$, $n = 10^{12}\,\mathrm{m}^{-3}$).
(b) for a neon gas discharge ($T_{\mathrm{e}} = 3\,\mathrm{eV}$, $T_{\mathrm{i}} = 300\,\mathrm{K}$, $n = 10^{16}\,\mathrm{m}^{-3}$).

2.3 Consider an infinitely large homogeneous plasma with $n_{\mathrm{e}} = n_{\mathrm{i}} = 10^{16}\,\mathrm{m}^{-3}$. From this plasma, all electrons are removed from a slab of thickness $d = 0.01\,\mathrm{m}$ extending from $x = -d$ to $x = 0$ and redeposited in the neighboring slab from $x = 0$ to $x = d$. (a) Calculate the electric potential in this double slab using Poisson's equation. What are the boundary conditions at $x = \pm d$? (b) Draw a sketch of space charge, electric field and potential for this situation. What is the potential difference between $x = -d$ and $x = d$?

2.4 Show that the equation for the shielding contribution (2.24) results from (2.21) and (2.23).

2.5 Derive the relationship between the coupling parameter for ion-ion interaction Γ Eqs. (2.15) and N_{D} (2.33) under the assumption that $T_{\mathrm{e}} = T_{\mathrm{i}}$.

2.6 Show that the second Lagrange multiplier in Eq. (2.6) is $\lambda = (k_{\mathrm{B}}T)^{-1}$.
Hint: Start from

$$\frac{1}{T} = \frac{\partial S}{\partial \lambda}\frac{\partial \lambda}{\partial U}$$

and use $\sum n_i = 1$.