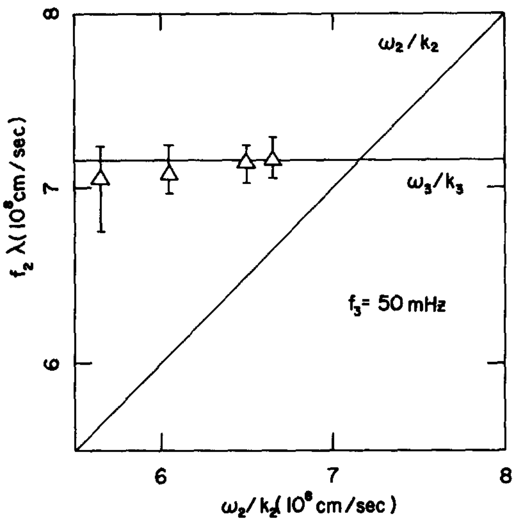

FIG. 3. Echo phase velocity vs ω_2/k_2. The triangles refer to the echo decay.

taken by varying both f_1 and f_2, but keeping the echo frequency f_3 constant.

Data for the echo rise, $L > (f_3/f_2)x$, are plotted as circles in Fig. 2. The rise phase velocity follows the phase velocity of the first wave, as predicted, and it is clearly not characteristic of ω_3/k_3, which is a constant for all points.

In the echo decay, the observed structure is dominated by the A term in Eq. (1). Decay data are presented as triangles in Fig. 2, and the phase velocity is constant, characteristic of f_3, and independent of the phase velocity of the first wave. This contrasts sharply with the data for the echo rise. These points are replotted in Fig. 3 as a function of the phase velocity of the second wave to demonstrate that the phase velocity is constant and distinguishable from that of the second wave. These results are not sensitive to the rf power applied to the transmitter probes for the small powers used.

Data such as that exhibited in Figs. 2 and 3 cannot be interpreted as a ballistic effect resulting from electron velocity perturbations by the antenna fields. A ballistic echo would not exhibit a wavelength dependent on ω_1/k_1 for fixed ω_3 or a sudden change in wavelength at $x = L'$. The observed echo contains information about the primary wave of phase velocity ω_1/k_1 and therefore implies the reversibility of its damping.

In summary, we have observed second-order echoes demonstrably resulting from collective plasma oscillations. The phase velocity of the echo rise is that of the first primary wave and the echo decay is characteristic of the local resonant plasma response at the echo frequency.

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Observations of the Beam-Plasma Instability

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The nonlinear limit of the instability driven by a low density cold electron beam in a collisionless plasma is experimentally found to be determined by the trapping of the beam by the most rapidly growing wave.

Recently, Drummond *et al.*[1] proposed a model for the nonlinear limiting of the instability driven by a cold electron beam in a collisionless plasma. The essential feature of their theory is that the spectrum is dominated by a single wave, the most unstable mode. Wave growth is halted when this mode traps the beam and turns the electrons around in the wave frame. The model makes a unique prediction for the maximum wave energy and further predicts that the wave energy will oscillate below this maximum with the oscillation frequency of the trapped electrons. The time scale for this oscillation and any subsequent evolution is much slower than the linear growth rates, for the wave with trapped particles is a nearly stable Bernstein–Greene–Kruskal-like mode. Similar results have been obtained in computer simulations.[2–4] Here, we report experimental observations of the phenomena in a one-dimensional column of collisionless plasma into which a weak, cold electron beam is injected.

The apparatus has previously been described,[5] where it was used for a similar experiment to test quasilinear theory. The apparatus and techniques are also very similar to those used by Malmberg and Wharton[6] for the problem. The present work complements and extends the results of that paper in the nonlinear regime. For these experiments, the two-meter column contained a 3-cm diam plasma with a density near 10^9 and a temperature of 20 eV. The magnetic field of 1 kG was suf-

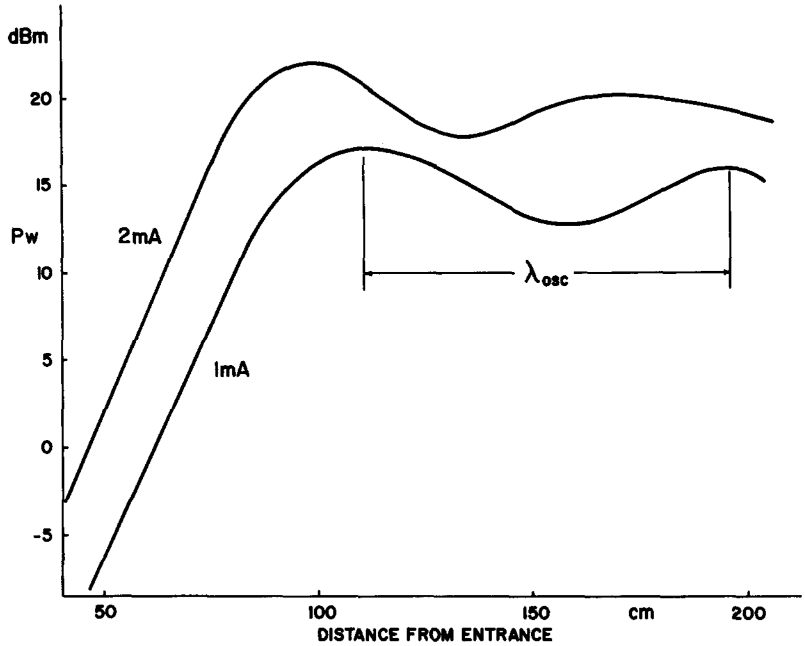

FIG. 1. Total wave power as a function of distance from the point of injection of the beam into the plasma column. The background plasma had a density of 7×10^8 and a temperature of 18 eV.

ficient to render the dynamics one dimensional and the background pressure of 8×10^{-6} Torr precluded collisional effects. The plasma is quiet, with low-frequency density fluctuations of only a few percent. The axial density uniformity is excellent. Although the density drops approximately 20% over the 40 cm near the gun, no measurable gradient exists over the remainder of the column. A gradient of more than a few percent would be readily detectable. The density gradient near the gun is not significant. The waves are still far from saturation when they enter the uniform region, and all the important physics occurs in that region.

Typical results for the development of the instability are shown in Fig. 1. The wave power is measured with a loosely coupled, calibrated (-32 dB) coaxial probe connected to a broad band amplifier and rf voltmeter. The probe may be moved the length of the machine, and the figure shows the result for the absolute power in the waves. The qualitative behavior is precisely that predicted: linear growth to a peak, followed by slow oscillation. Although the theory was presented for infinite geometry and an initial value problem, the argument can easily be applied to finite geometry and growth in space. The peak wave power should be

$$P_w = 2^{2/3}\eta^{1/3}P_B, \tag{1}$$

where $P_b = I_b V_b$, the input beam power, and

$$\eta = \int_0^a n_b(r)r\,dr \left(\int_0^a n_p(r)r\,dr\right)^{-1}. \tag{2}$$

These integrals of the beam and plasma densities over the cross section give the finite geometry equivalent of the η in one dimension. Since η is proportional to the beam current, Eq. (1) implies that the peak wave power should scale as $I_b^{4/3}$ if all other parameters are held fixed. A test of this prediction is presented in Fig. 2 This log plot confirms that the peak power is proportional to I^n with an $n = 1.4\pm0.1$, quite consistent with the $n = 1.33$ predicted. This covers the range of currents from those just large enough to reach a clear peak within the system to those so large that they affect the background plasma by modifying the flow of plasma from the source into the column.

Equation (1) was also tested for absolute magnitude. For example, the 2-mA beam in Fig. 1 produced a peak wave power of $+22$ dBm (160 mW). Evaluating Eq. (1) for this beam and plasma gives a prediction of $+23$ dBm (200 mW) for wave power. This is excellent quantitative agreement, well within the 2-dB uncertainties in the calibration of the probe coupling constants. The observed nonlinear limit of the instability agrees well with that predicted by the model of beam trapping by a single wave.

Additional evidence for the dominance of the single wave may be adduced by examining the wave signal at the peak. Depending upon experimental conditions, the spectrum has a full width at half-maximum of 3%–10%. However, examination of the signal with a fast oscilloscope in real time shows that this is caused by a weak amplitude modulation of a single frequency. The frequency is always found to be constant within 2% over times of 0.3 μsec. The frequency is coherent for 20–30 wave periods, which exceeds the transit time of an electron through the machine. The frequency is equally constant in space over the region of uniform density. The signal is unquestionably of sufficiently constant frequency to appear as a single wave to the particles during growth and trapping.

A prediction unique to the single wave theory is that the wave energy will not only decay after the first

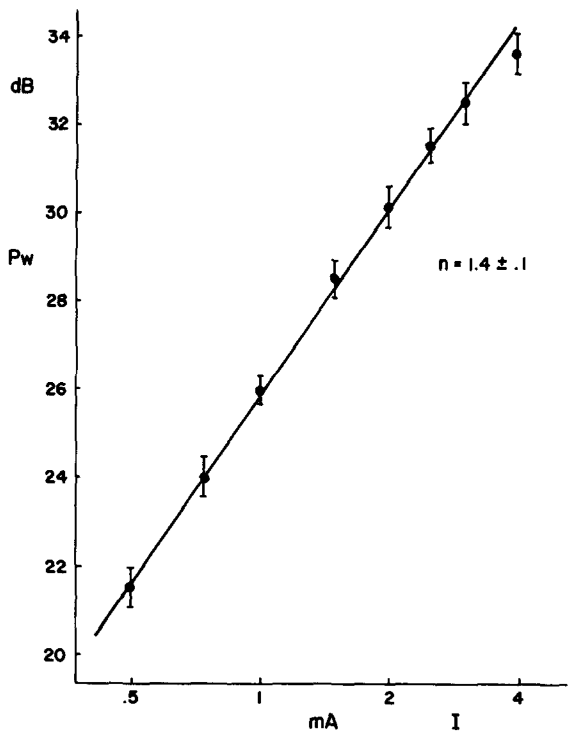

FIG. 2. Total wave power measured at the spatial maximum as a function of beam current. The wave power is in relative units.

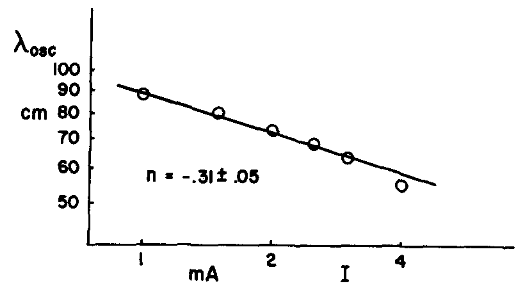

FIG. 3. Oscillation wavelength of the wave energy as a function of beam current.

maximum but will oscillate as the trapped particles exchange energy with the wave in their periodic oscillations in the potential well. In space, the oscillations will appear with a wavelength determined by the particle oscillation frequency in the well and the propagation velocity, the beam velocity u. If the potential is sinusoidal and the particles are trapped near the bottom of the well, the oscillation length is given by

$$\lambda_{osc} = 2\pi u (m/ekE)^{1/2}. \qquad (3)$$

Making use of Eq. (1), we note that this implies that

$$\lambda_{osc} \propto I_b^{-1/3}. \qquad (4)$$

A test of this scaling law is presented in Fig. 3. The oscillation length definitely scales with current as predicted by the trapped beam model. Equation (3) may also be examined for absolute magnitude. For the 2-mA beam of Fig. 1, the oscillation length observed is 70 cm, whereas the prediction of Eq. (3) is 50 cm, using the electric field at the peak. This is somewhat outside the experimental uncertainties, but Eq. (3) is not strictly correct. It assumes the particles are trapped near the bottom of the sinusoidal well, whereas the theory and computer simulations both indicate that the electrons are trapped near the top. Accepting the experimental oscillation length, one would conclude that the electrons are trapped approximately one-quarter of the way down the well. This is quite consistent with the computer results.[2] Regardless of the interpretations of the exact magnitude, the observations clearly support the identification of the energy oscillations with trapping. The assumption of a sinusoidal wave shape is also justified by the experiment. Oscilloscope pictures of the waveform show a pure sine wave, and direct measurements of harmonic content generally show powers at least 20 dB below the fundamental.

These experiments demonstrate that the beam-plasma instability grows until stopped by the trapping of the beam in a single large-amplitude wave. The oscillation in energy after the first maximum further confirms the importance of trapped particles. One should not infer from this that the simple trapped particle model is sufficient to explain the subsequent evolution of the system, however. The amplitude of the wave energy oscillations is rather less than suggested by the model, and the oscillations decay rapidly. The spectrum spreads in frequency after the first minimum, and no more than one subsequent maximum has ever been observed. Additional processes are occurring on the time scale of the trapped particle oscillations. Although these Bernstein–Greene–Kruskal modes are known to be unstable to sidebands at the bounce frequency,[7–9] Wong has estimated the growth rates to be less than those required to produce the observed spreading. The long-term evolution of the trapped particle state remains to be explained.

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