Entanglement in Topological Phases

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Overview of topics

• Topological Phases
• Aspects of Entanglement
• Examples and my work
Topological Phases

• Examples include: fractional quantum Hall, chiral spin states, etc
• Cannot be described by Landau symmetry-breaking
• Characterized by other properties: non-Abelian geometric phase, fractional statistics/charge, topological entanglement entropy
• Where does the name come from?
Why do we care about them?

- It means Landau symmetry-breaking is incomplete
- Topological quantum computing
What is entanglement?

• Concept first discussed by EPR, term coined by Schrödinger
• Basically a result of quantum superposition
• Bell’s inequality – no local hidden variable theories

$$|\psi\rangle = \frac{1}{\sqrt{2}} [|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle]$$
Schmidt Decomposition

- Singular value decomposition for Hilbert spaces
- Write the space explicitly as a product
Entanglement Entropy

- Consider the density matrix of one block
- Calculate the von Neumann entropy
- Scaling law emerges with multiple components

\[ S_A = \alpha L - \gamma + \mathcal{O} \left( \frac{1}{L} \right) \]
Entanglement Spectrum

• Write the density matrix in a suggestive form
• Consider the spectrum of this operator
• Reveals additional information & properties
Recent Developments

• Structure of ES matches spectra for edge
  • Haldane & Li conjecture

• For real-space partition, better matching
Example

• Laughlin 1/3 state

1 particle: \( |m\rangle = \frac{z^m}{\sqrt{2\pi} 2^m m!} \exp\left\{ -\frac{1}{4} |z|^2 \right\} \)

N particles: \( |m, N\rangle \propto \prod\limits_{i<j} (z_i - z_j)^m \exp\left\{ -\frac{1}{4} \sum\limits_{i=1}^{N} |z_i|^2 \right\} \)
Example cont’d

• On a sphere!

\[
\left( a_1^* b_2^* - a_2^* b_1^* \right)^3 = \left[ (a_1^*)^3 (b_2^*)^3 - (a_2^*)^3 (b_1^*)^3 \right] + 3 \left[ (a_1^*)(a_2^*)^2 (b_1^*)^2 (b_2^*) - (a_2^*)(a_1^*)^2 (b_2^*)^2 (b_1^*) \right]
\]

\[
\begin{align*}
| -3, 3 \rangle & \quad | -1, 1 \rangle \\
| 1, 0, 0, 1 \rangle & \quad | 0, 1, 1, 0 \rangle
\end{align*}
\]
My work

- Can handle up to $N=5$, close to EE results in ref. 4
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