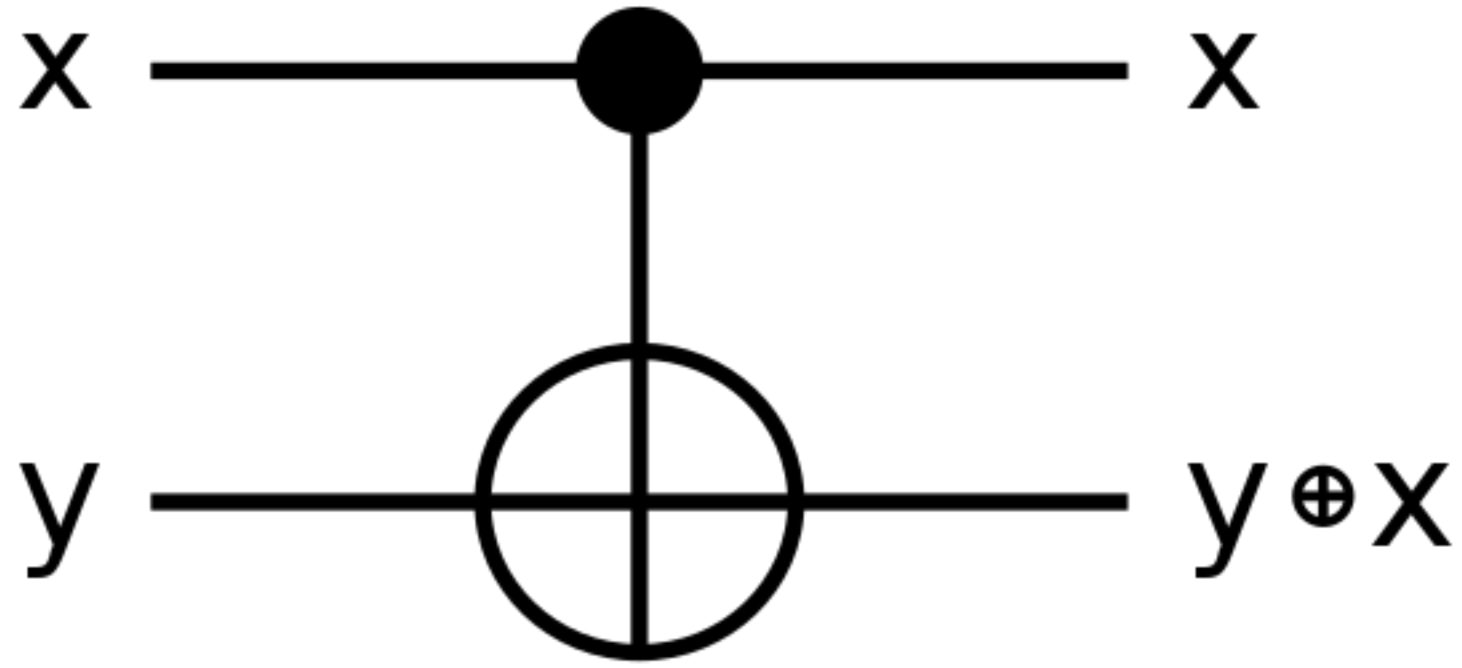
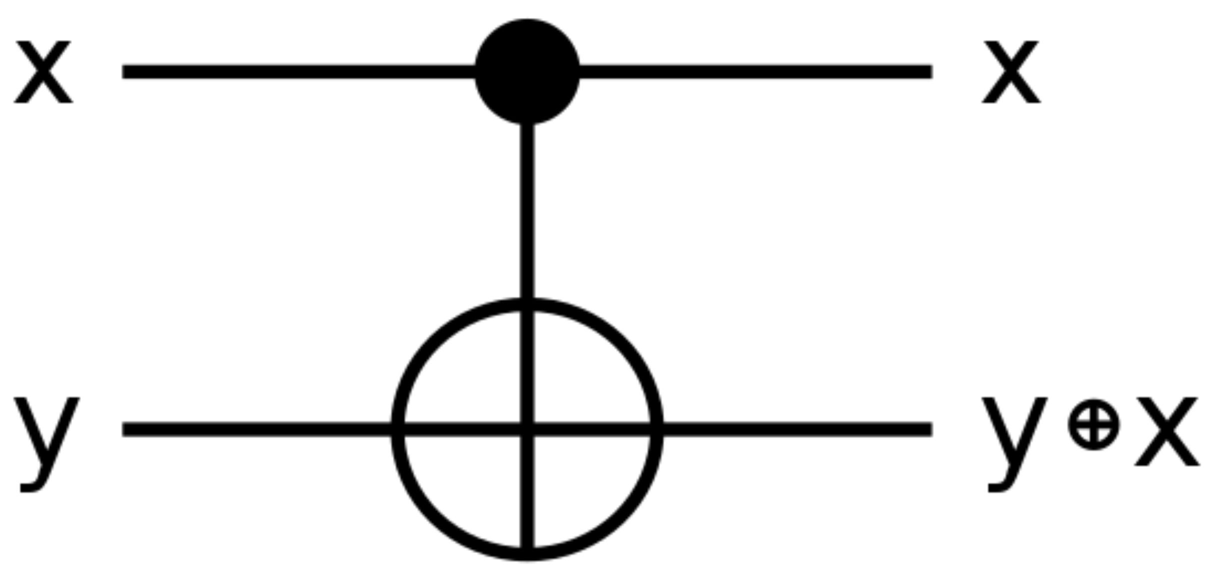
Quantum Computing with Trapped Ions: Building the CNOT Quantum Gate

AP Seminar
Columbia University



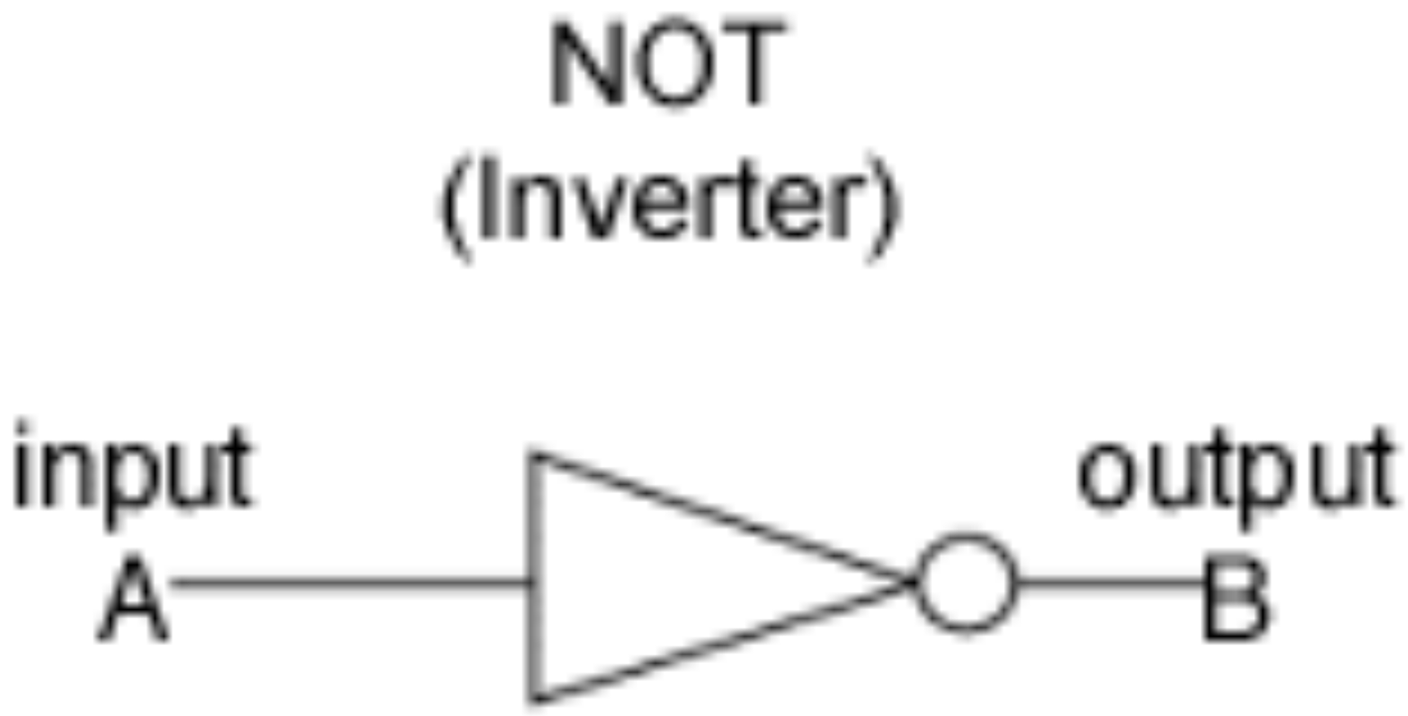
CNOT is not NOT



input		output	
x	y	x	y+x
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

CNOT flips the second qubit (y) if and only if the first qubit (x) is $|1\rangle$

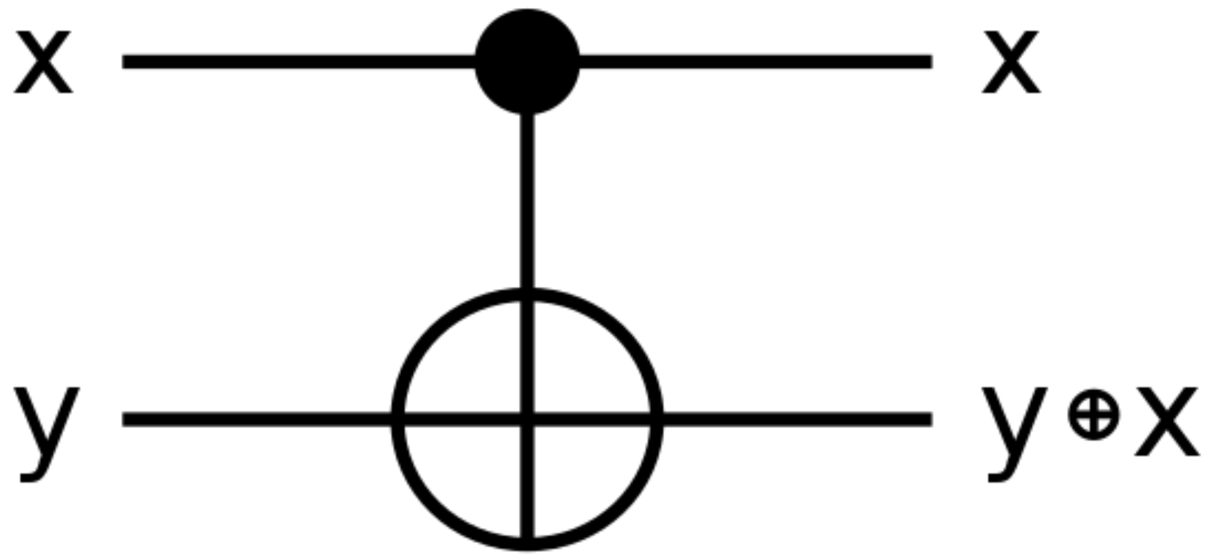
(qubits can be both 0 and 1)



A	B
0	1
1	0

(bits can be either 0 or 1)

CNOT is …



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$\lvert 1\rangle$	$\lvert 0\rangle$	$\lvert 1\rangle$	$\lvert 1\rangle$
$\lvert 1\rangle$	$\lvert 1\rangle$	$\lvert 1\rangle$	$\lvert 0\rangle$

CNOT flips the second qubit (y) if and only if the first qubit (x) is $\lvert 1\rangle$

(qubits can be both 0 and 1)

a “universal” quantum gate

can entangle two qubits

can disentangle two qubits

…

Scientific American (August, 2008)

QUANTUM COMPUTING WITH IONS

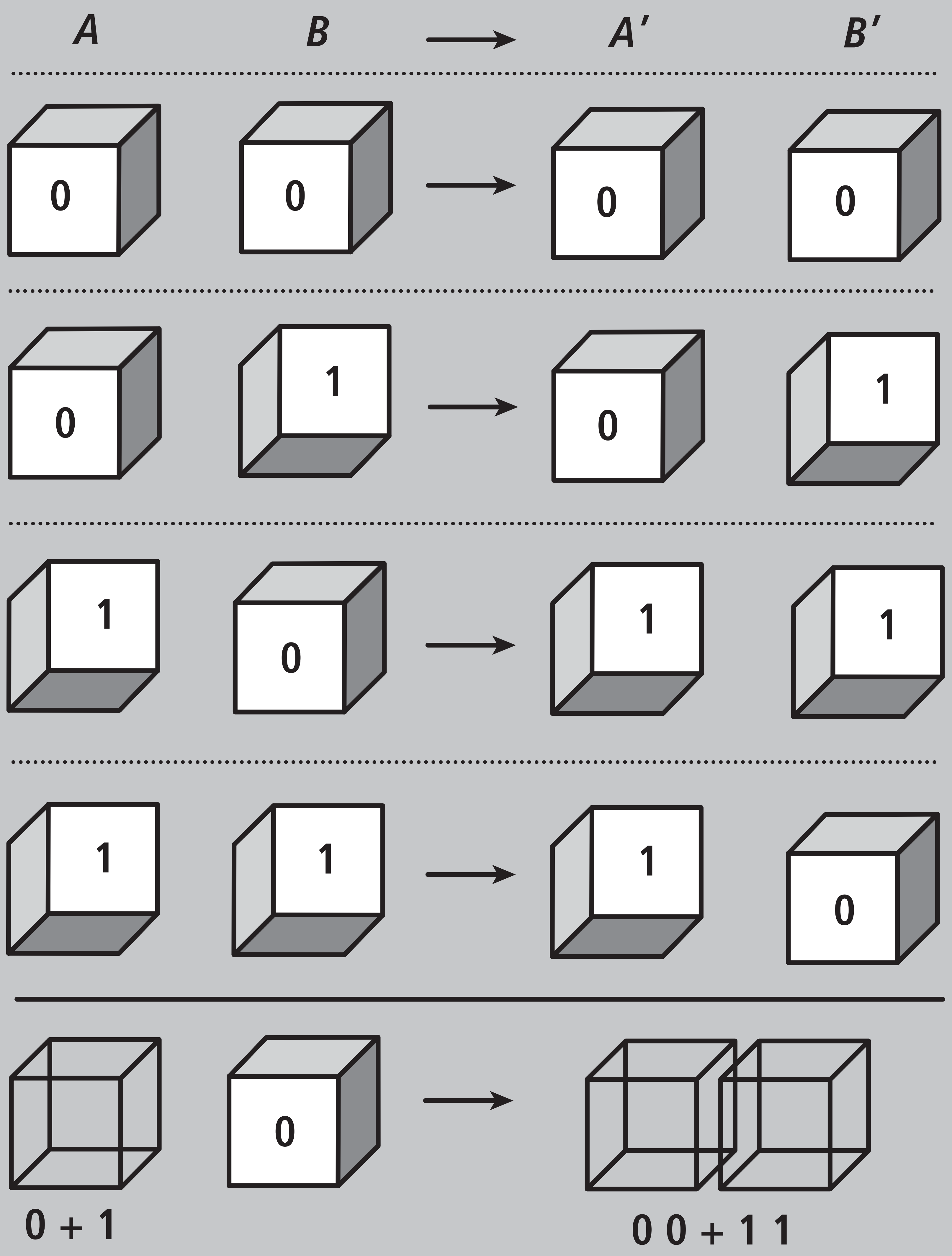
Researchers are taking the first steps toward building ultrapowerful computers that use individual atoms to perform calculations

By Christopher R. Monroe and David J. Wineland

[BASICS]

TRUTH TABLE

A trapped-ion computer would rely on logic gates such as the controlled not (CNOT) gate, which consists of two ions, *A* and *B*. This truth table shows that if *A* (the control bit) has a value of 0, the gate leaves *B* unchanged. But if *A* is 1, the gate flips *B*, changing its value from 0 to 1, and vice versa. And if *A* is in a superposition state (0 and 1 at the same time), the gate puts the two ions in an entangled superposition. (Their state is now identical to the one shown in the box on the bottom of the opposite page.)



POWERS OF TWO

The enormous potential of trapped-ion computers lies in the fact that a system with *N* ions can hold 2^N numbers simultaneously. And as *N* increases, the value of 2^N rises exponentially.

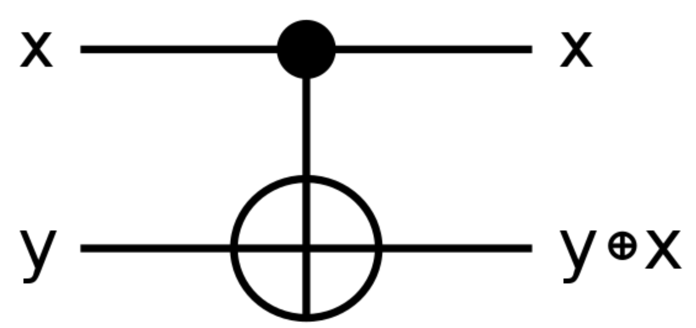
$2^5 = 32$

$2^{10} = 1{,}024$

$2^{50} =$ 1,125,899,906,842,624

$2^{100} =$ 1,267,650,600,228,229,401,496,703,205,376

CNOT can entangle two qubits



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$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

$$\text{CNOT} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|xy\rangle \equiv \begin{bmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{bmatrix}$$

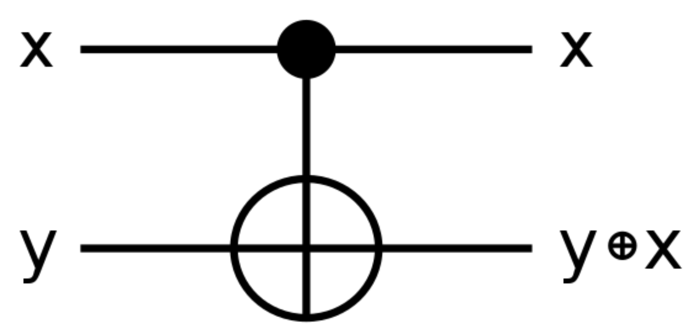
Example: “Classical” Gate

$$x = |1\rangle \text{ and } y = |0\rangle$$

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

(not entangled)

CNOT can entangle two qubits



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$$\text{CNOT} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|xy\rangle \equiv \begin{bmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{bmatrix}$$

Example: Entangled Bell State

$$x = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \text{ and } y = |0\rangle$$

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$\text{CNOT} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

(entangled!)

Demonstration of a Fundamental Quantum Logic Gate

C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland
National Institute of Standards and Technology, Boulder, Colorado 80303
(Received 14 July 1995)

We demonstrate the operation of a two-bit "controlled-NOT" quantum logic gate, which, in conjunction with simple single-bit operations, forms a universal quantum logic gate for quantum computation. The two quantum bits are stored in the internal and external degrees of freedom of a single trapped atom, which is first laser cooled to the zero-point energy. Decoherence effects are identified for the operation, and the possibility of extending the system to more qubits appears promising.

We report the first demonstration of a fundamental quantum logic gate that operates on prepared quantum states. Following the scheme proposed by Cirac and Zoller [1], we demonstrate a controlled-NOT gate on a pair of quantum bits (qubits). The two qubits comprise two internal (hyperfine) states and two external (quantized motional harmonic oscillator) states of a single trapped atom. Although this minimal system consists of only two qubits, it illustrates the basic operations necessary for, and the problems associated with, constructing a large scale quantum computer.

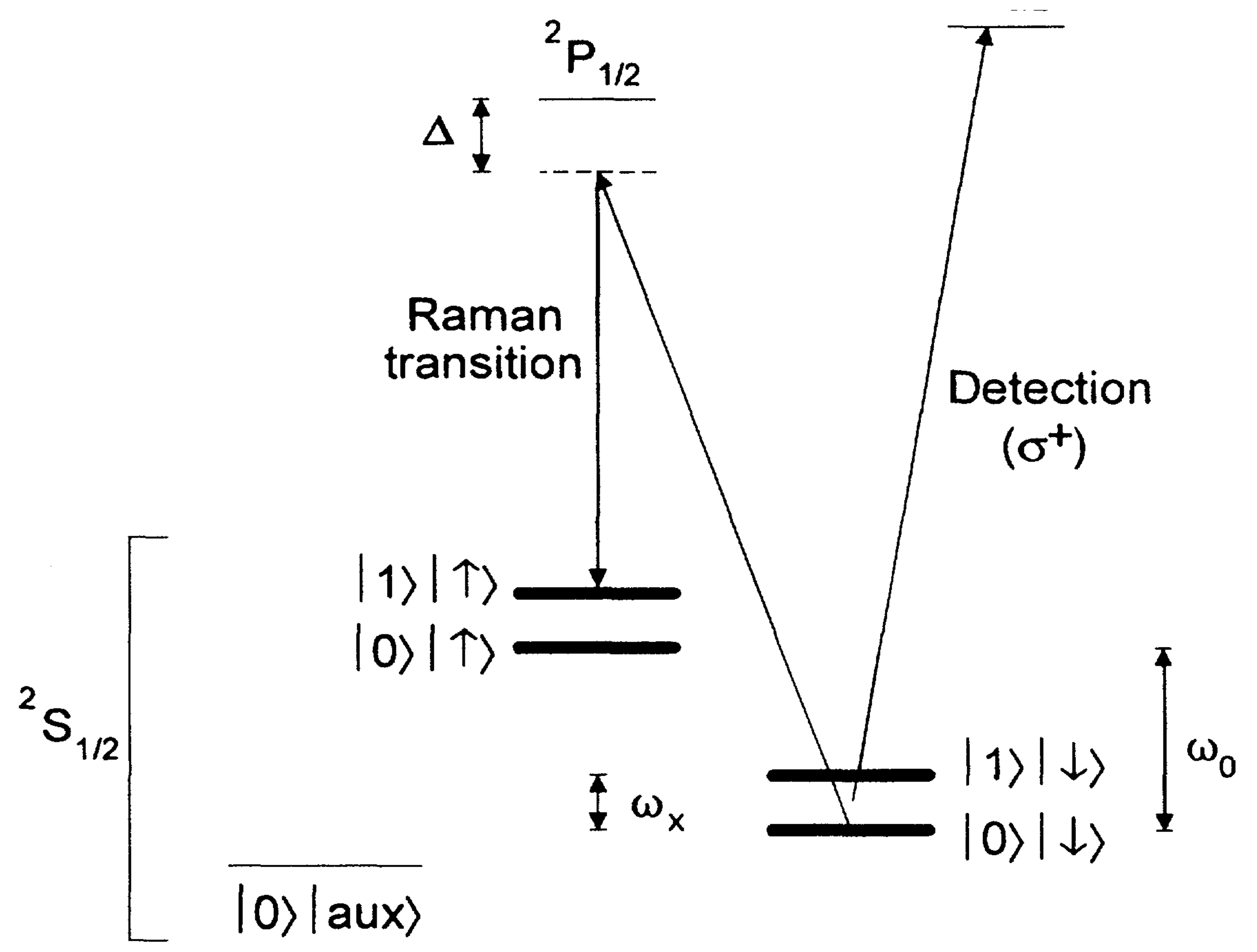

FIG. 1. ^{9}Be$^+$ energy levels. The levels indicated with thick lines form the basis of the quantum register: internal levels are $|S\rangle = |\downarrow\rangle$ and $|\uparrow\rangle$ ($^2S_{1/2}|F = 2, m_F = 2\rangle$ and $^2S_{1/2}|F = 1, m_F = 1\rangle$ levels, respectively, separated by $\omega_0/2\pi \simeq 1.250$ GHz), and $|\mathrm{aux}\rangle = {}^2S_{1/2}|F = 2, m_F = 0\rangle$ (separated from $|\downarrow\rangle$ by $\simeq$2.5 MHz); external vibrational levels are $|n\rangle = |0\rangle$ and $|1\rangle$ (separated by $\omega_x/2\pi \simeq 11.2$ MHz). Stimulated Raman transitions between $^2S_{1/2}$ hyperfine states are driven through the virtual $^2P_{1/2}$ level ($\Delta \simeq 50$ GHz) with a pair of $\simeq$313 nm laser beams. Measurement of S is accomplished by driving the cycling $|\downarrow\rangle \rightarrow {}^2P_{3/2}|F = 3, m_F = 3\rangle$ transition with σ^+-polarized light and detecting the resulting ion fluorescence.

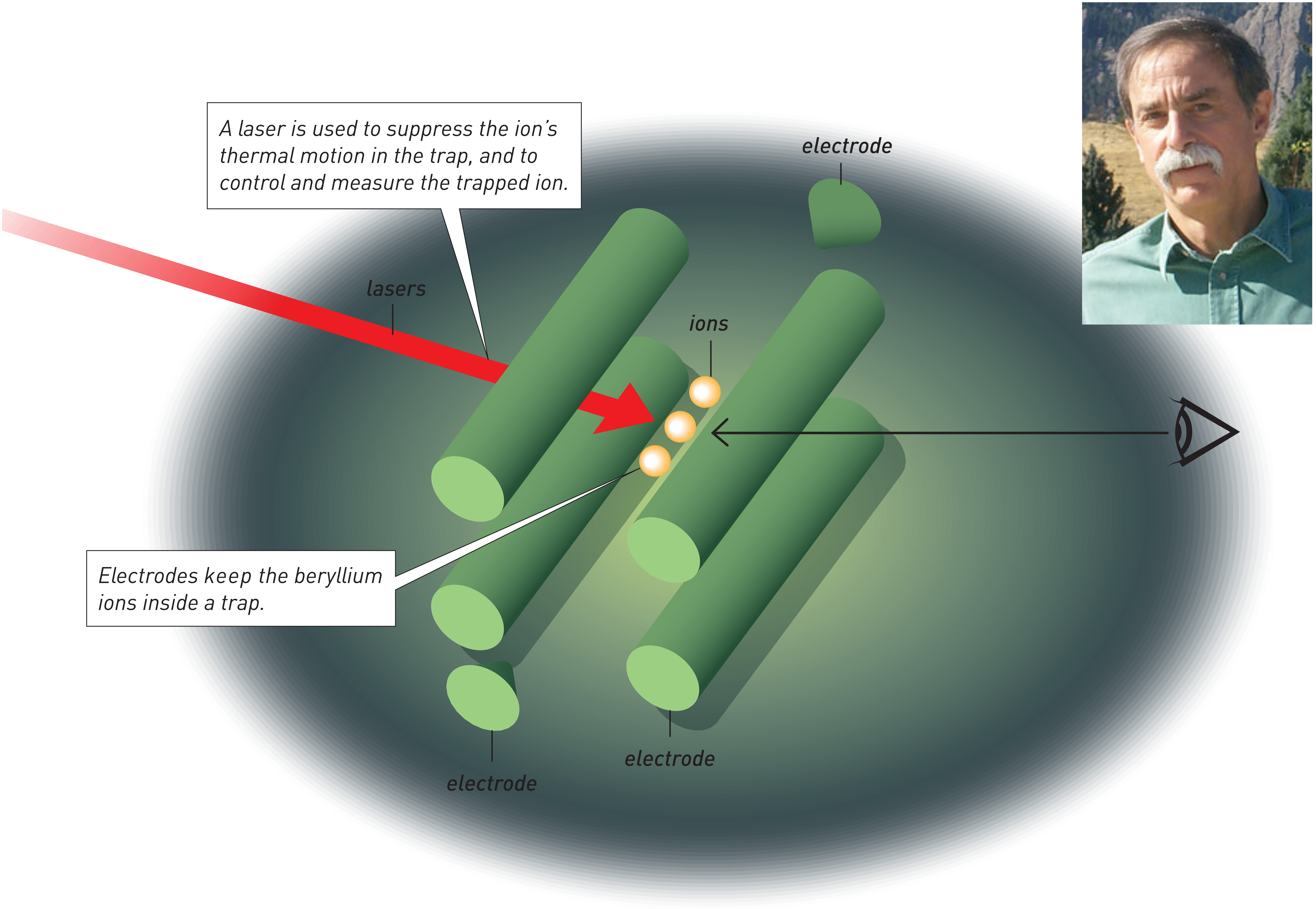

Figure 2. In David Wineland's laboratory in Boulder, Colorado, electrically charged atoms or ions are kept inside a trap by surrounding electric fields. One of the secrets behind Wineland's breakthrough is mastery of the art of using laser beams and creating laser pulses. A laser is used to put the ion in its lowest energy state and thus enabling the study of quantum phenomena with the trapped ion.

References

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The New York Times

Science

The Nobel Prize in Physics 2012

Serge Haroche, David J. Wineland

French and U.S. Physicists Win Nobel Prize

By DENNIS OVERBYE
Published: October 9, 2012

Now scientists are able to direct experiments and catch nature in the act of being quantum and thus explore the boundary between quantum reality and normal life. Their work involves isolating the individual nuggets of nature — atoms and the particles that transmit light, known as photons — and making them play with each other.

Dr. Wineland's work has focused on the material side of where matter meets light. His prize is the fourth Nobel awarded to a scientist associated with the National Institute of Standards and Technology over the past 15 years for work involving the trapping and measuring of atoms. Dr. Wineland and his colleagues trap charged beryllium atoms, or ions, in an electric field and cool them with specially tuned lasers so that they are barely moving, which is another way of saying they are very, very cold.

Quantum dynamics of single trapped ions

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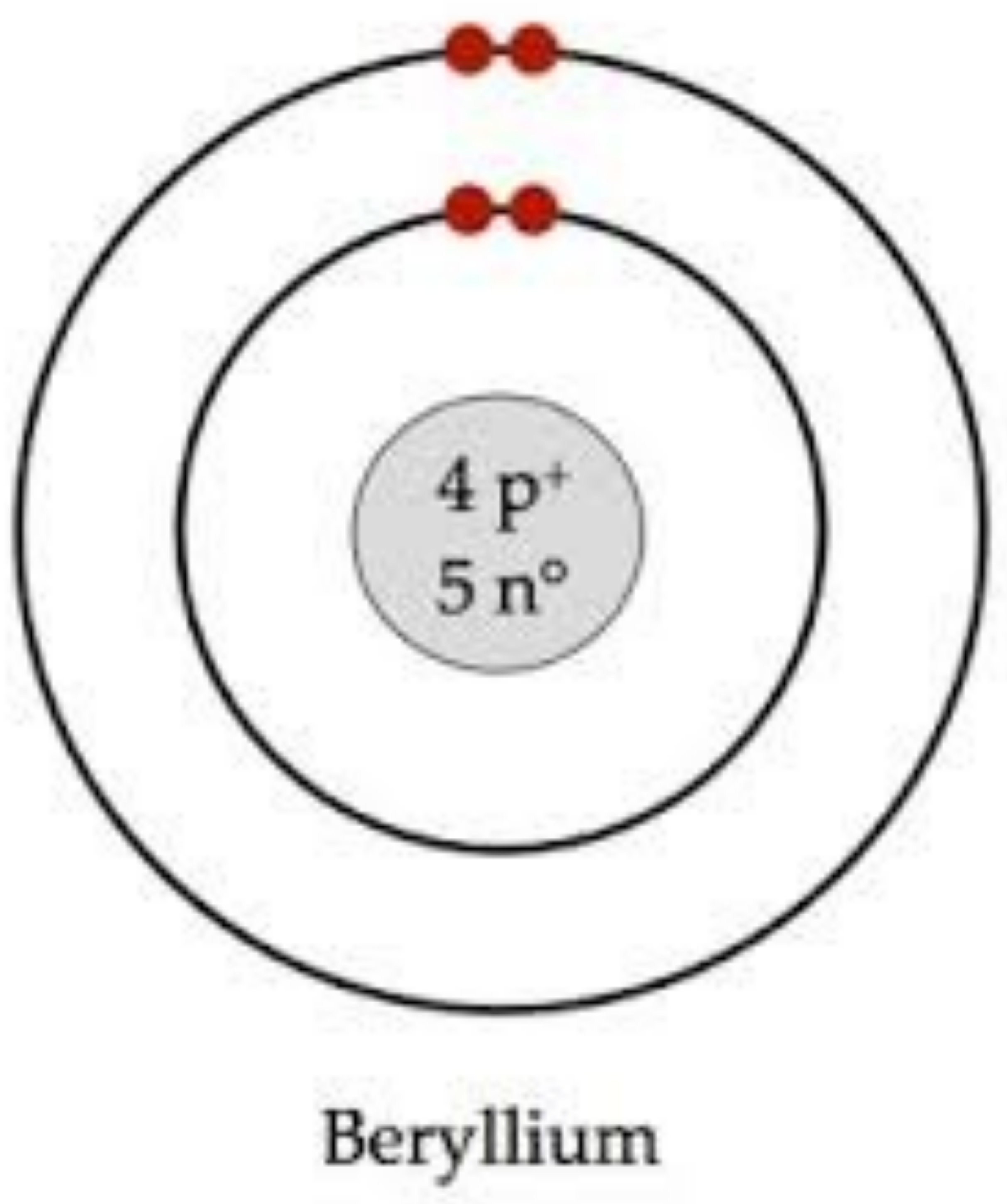
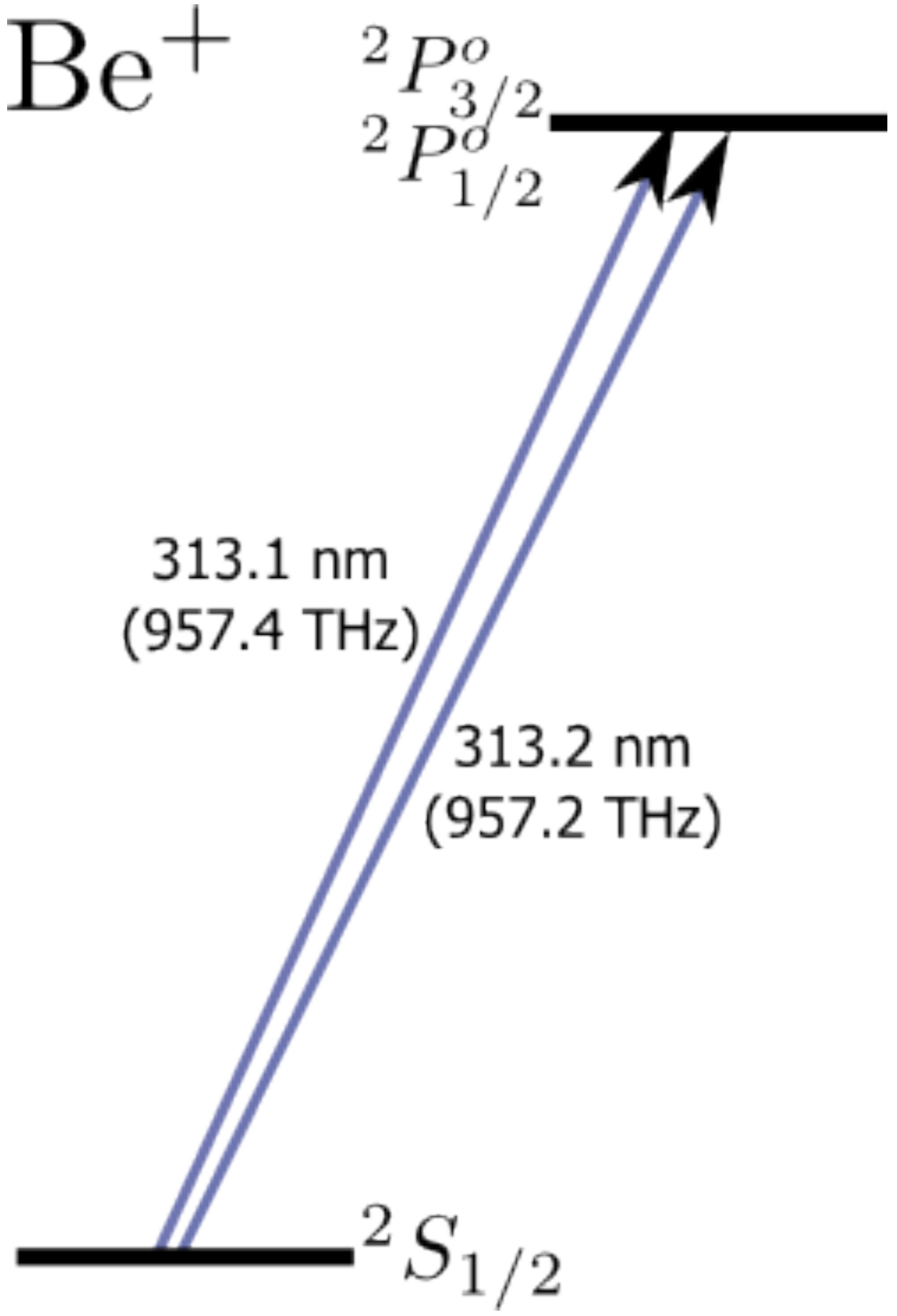
(Published 10 March 2003)

Single trapped ions represent elementary quantum systems that are well isolated from the environment. They can be brought nearly to rest by laser cooling, and both their internal electronic states and external motion can be coupled to and manipulated by light fields. This makes them ideally suited for quantum-optical and quantum-dynamical studies under well-controlled conditions. Theoretical and experimental work on these topics is reviewed in the paper, with a focus on ions trapped in radio-frequency (Paul) traps.

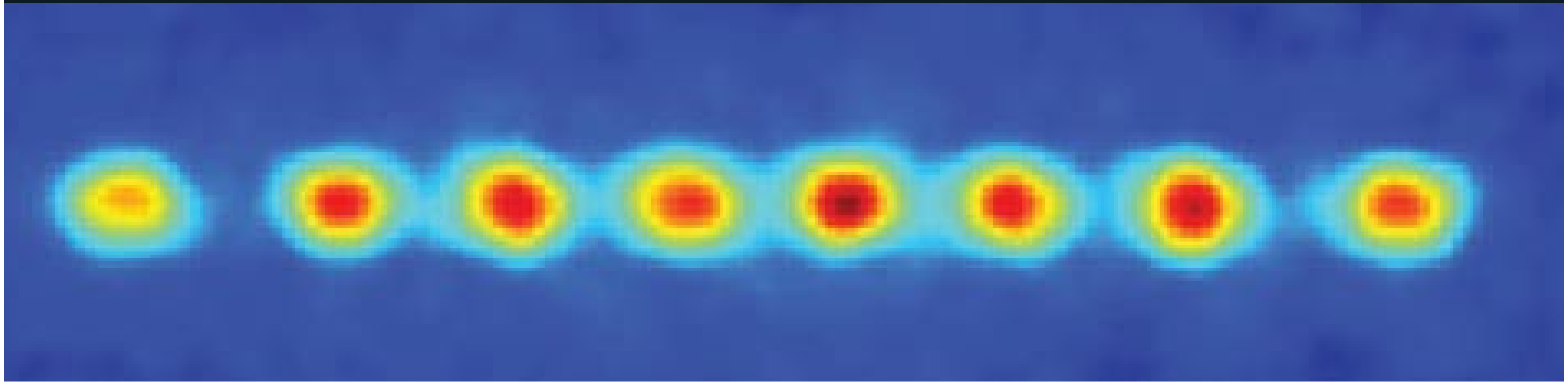
Beryllium+ Ion



Spin-Orbit “Fine-Structure”



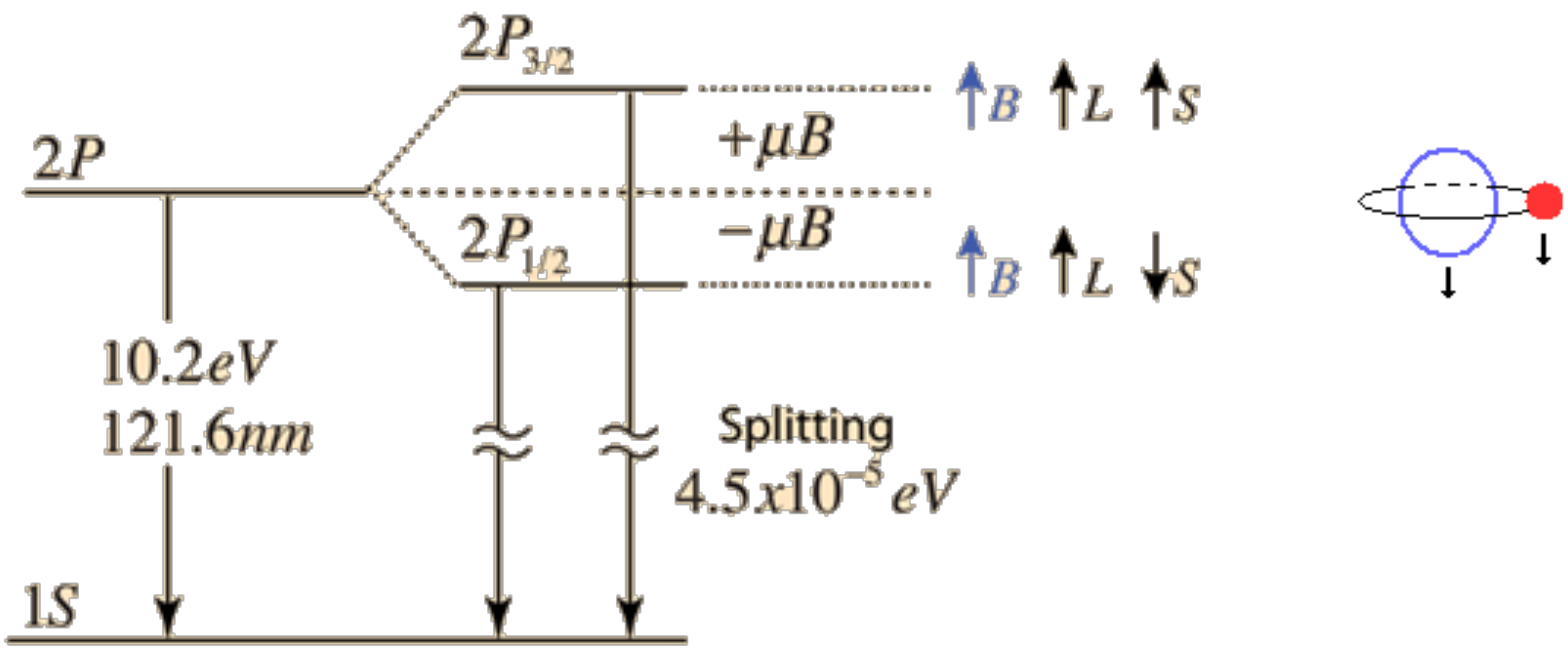
LEVITATED STRING of eight calcium ions are confined in a vacuum chamber and laser-cooled to be nearly at rest. Such a string can perform quantum calculations.



Ion Highways

But can researchers really make a full-fledged quantum computer out of trapped ions? Unfortunately, it appears that longer strings of ions—those containing more than about 20 qubits—would be nearly impossible to control because their many collective modes of common motion would interfere with one another. So scientists have begun to explore the idea of dividing the quantum hardware into manageable chunks, performing calculations with short chains of ions that could be shuttled from place to place on the quantum computer chip. Electric forces can move the ion strings without disturbing their internal states, hence preserving the data they carry. And researchers could entangle one string with another to transfer data and perform processing tasks that require the action of many logic gates. The resulting architecture would somewhat resemble the familiar charge-coupled device (CCD) used in digital cameras; just as a CCD can move electric charge across an array of capacitors, a quantum chip could propel strings of individual ions through a grid of linear traps.

Hydrogen Hyperfine Structure



Be+ Hyperfine Structure

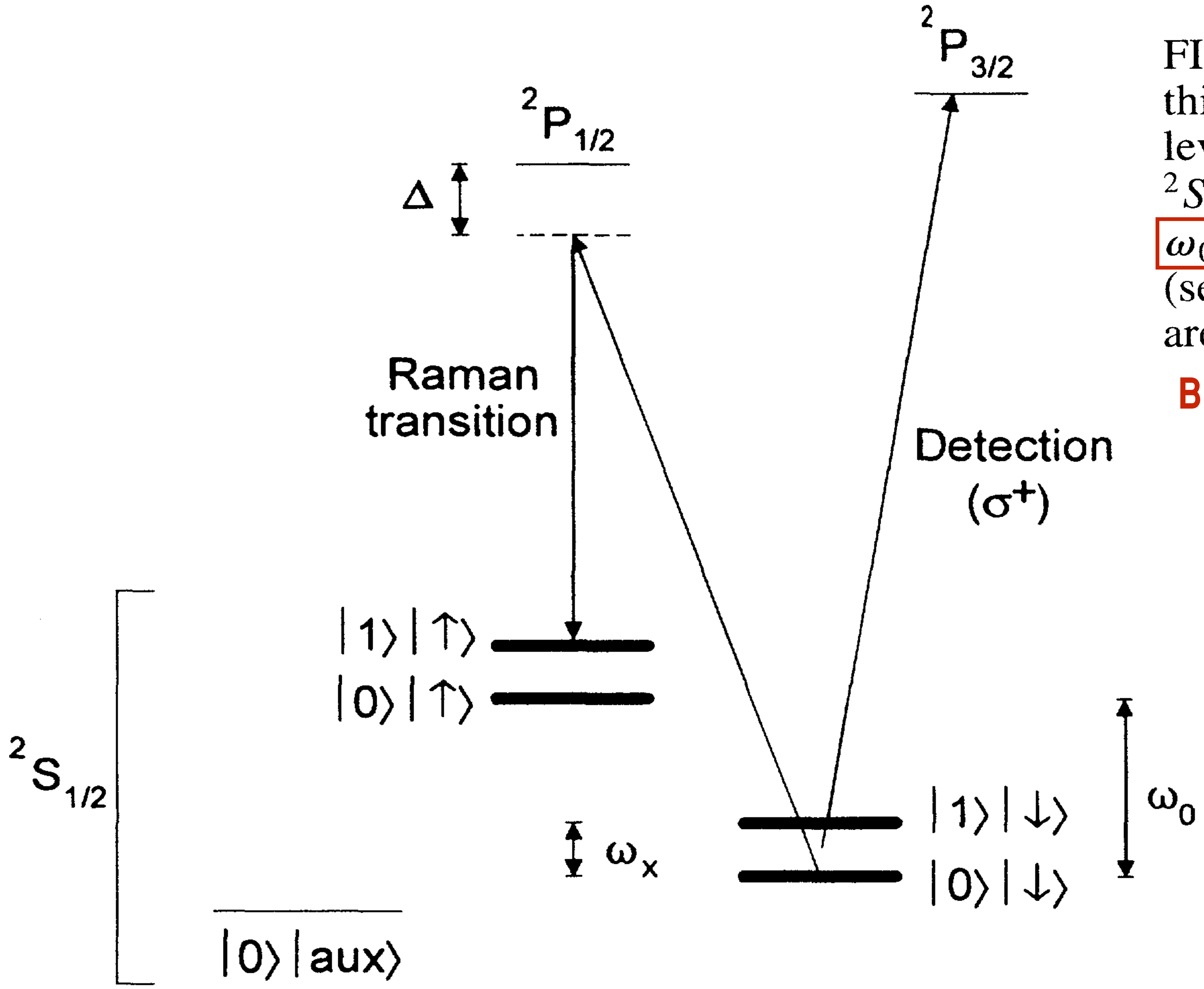


FIG. 1. $^9Be^+$ energy levels. The levels indicated with thick lines form the basis of the quantum register: internal levels are $|S\rangle = |\downarrow\rangle$ and $|\uparrow\rangle$ ($^2S_{1/2}|F=2, m_F=2\rangle$ and $^2S_{1/2}|F=1, m_F=1\rangle$ levels, respectively, separated by $\omega_0/2\pi \simeq 1.250$ GHz), and $|aux\rangle = {}^2S_{1/2}|F=2, m_F=0\rangle$ (separated from $|\downarrow\rangle$ by $\simeq$2.5 MHz); external vibrational levels are $|n\rangle = |0\rangle$ and $|1\rangle$ (separated by $\omega_x/2\pi \simeq 11.2$ MHz).

B = 1.8 Gauss

$$|\uparrow\rangle = {}^2S_{1/2}(F=2, m_F=2)$$

$$|\downarrow\rangle = {}^2S_{1/2}(F=1, m_F=1)$$

Be^+ ($I = 3/2$) + e^-($S = 1/2$) → {2, 1, 0, -1, -2}

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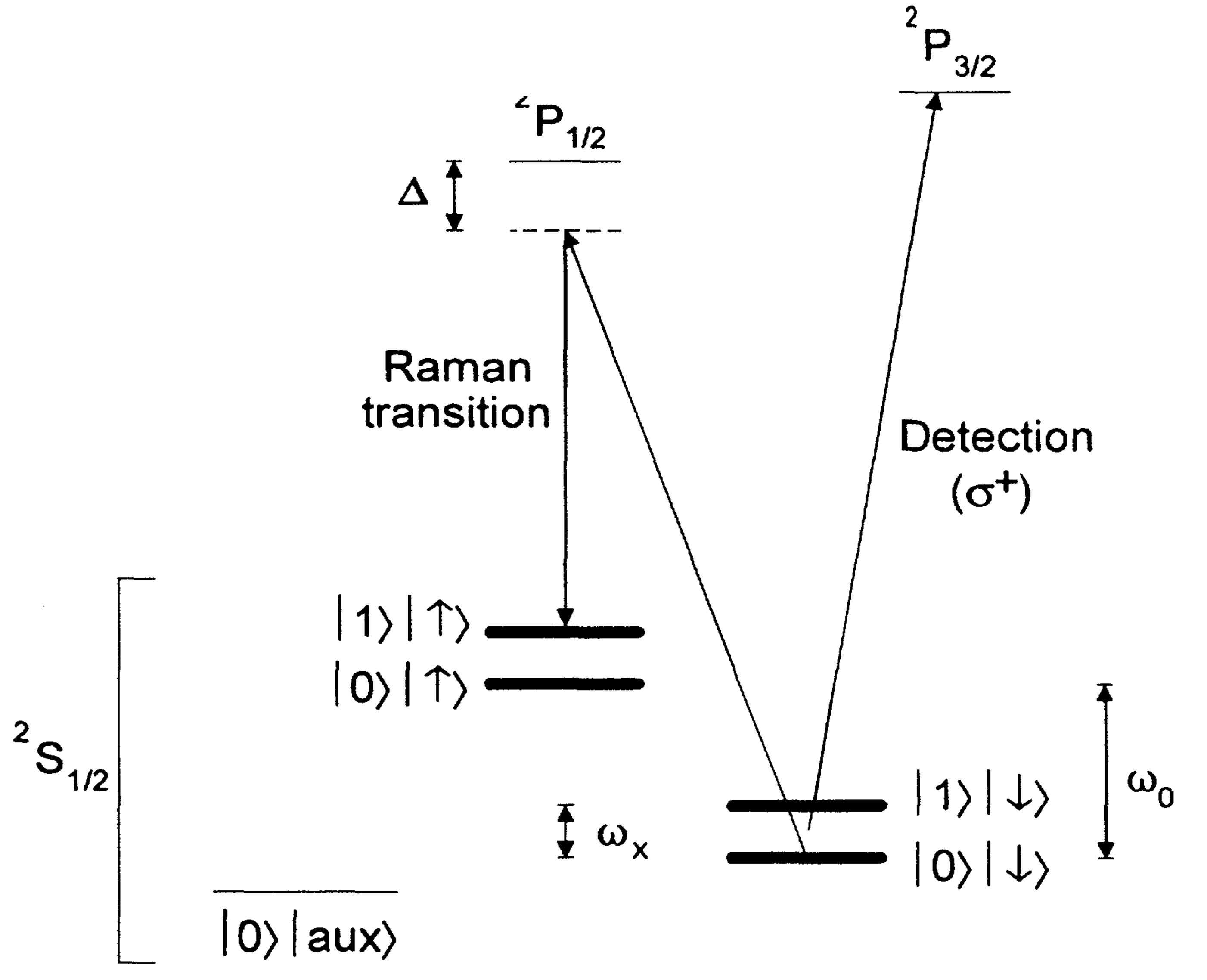
We realize the controlled-NOT gate by sequentially applying three pulses of the Raman beams to the ion according to the following format:

(a) A $\pi/2$ pulse is applied on the carrier transition. The effect is described by the operator $V^{1/2}(\pi/2)$ in the notation of Ref. [1].

(b) A 2π pulse is applied on the blue sideband transition between $|\uparrow\rangle$ and an auxiliary atomic level $|\text{aux}\rangle$ (see Fig. 1). (1)

(c) A $\pi/2$ pulse is applied on the carrier transition, with a π phase shift relative to (a), leading to the operator $V^{1/2}(-\pi/2)$ of Ref. [1].

The $\pi/2$ pulses in steps (a) and (c) cause the spin $|S\rangle$ to undergo $+1/4$ and $-1/4$ of a complete Rabi cycle, respectively, while leaving $|n\rangle$ unchanged. The auxiliary transition in step (b) simply reverses the sign of any component of the register in the $|1\rangle|\uparrow\rangle$ state by inducing a complete Rabi cycle from $|1\rangle|\uparrow\rangle \rightarrow |0\rangle|\text{aux}\rangle \rightarrow -|1\rangle|\uparrow\rangle$. The auxiliary level $|\text{aux}\rangle$ is the $^2S_{1/2}\ |F=2, m_F=0\rangle$ ground state, split from the $|\downarrow\rangle$ state by virtue of a Zeeman shift of $\simeq$2.5 MHz resulting from a 0.18 mT applied magnetic field (see Fig. 1). Any component of the quantum register in the $|n\rangle = |0\rangle$ state is unaffected by the blue sideband transition of step (b), and the effects of the two Ramsey $\pi/2$ pulses cancel. On the other hand, any component of the quantum register in the $|1\rangle|\uparrow\rangle$ state acquires a sign change in step (b), and the two Ramsey pulses add constructively, effectively "flipping" the target qubit by π radians. The truth table of the CN operation



Input state $\rightarrow$ Output state

$$|0\rangle|\downarrow\rangle \rightarrow |0\rangle|\downarrow\rangle$$
$$|0\rangle|\uparrow\rangle \rightarrow |0\rangle|\uparrow\rangle$$
$$|1\rangle|\downarrow\rangle \rightarrow |1\rangle|\uparrow\rangle$$
$$|1\rangle|\uparrow\rangle \rightarrow |1\rangle|\downarrow\rangle.$$

Varying Spin-Down Probability

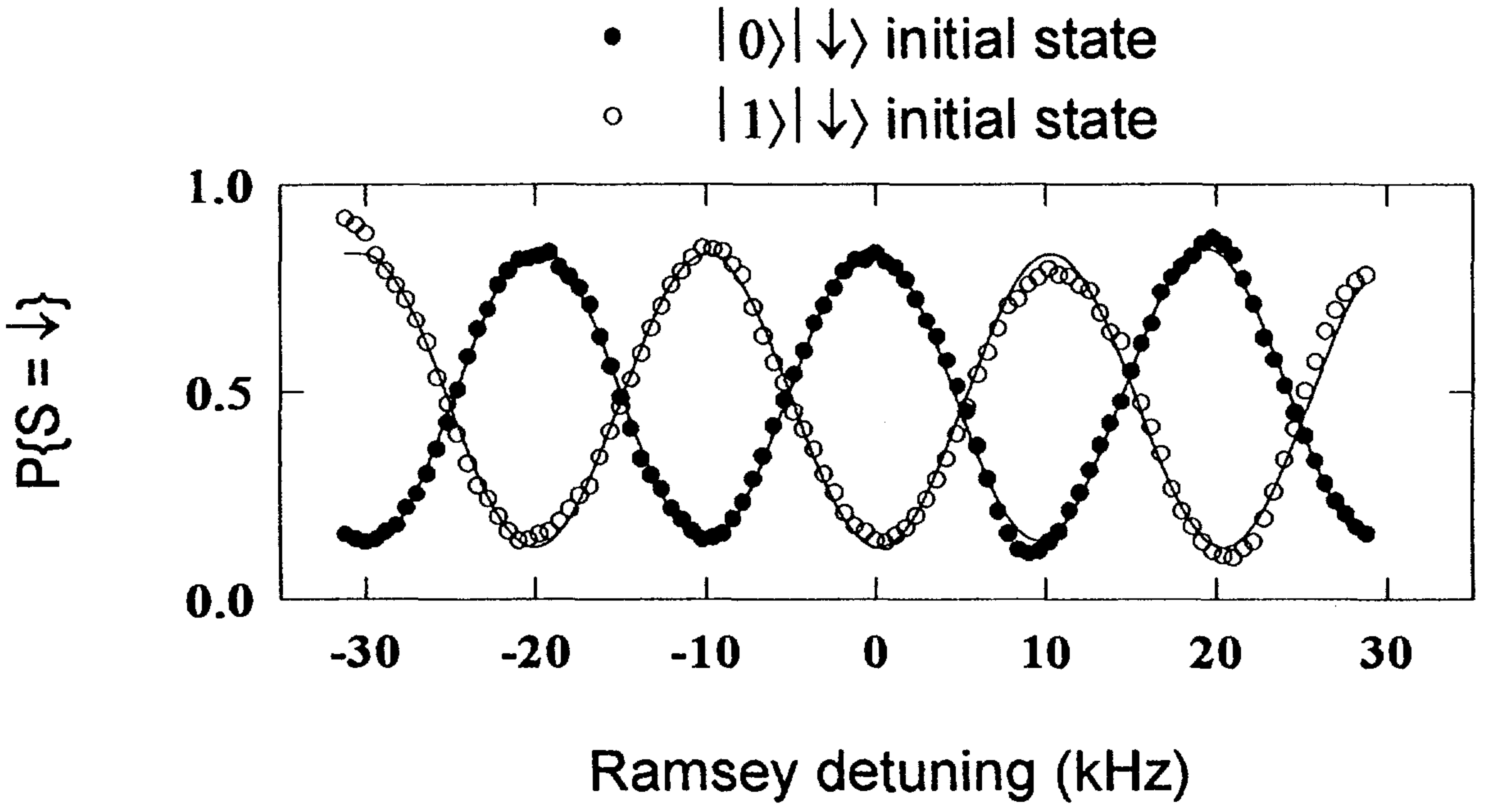
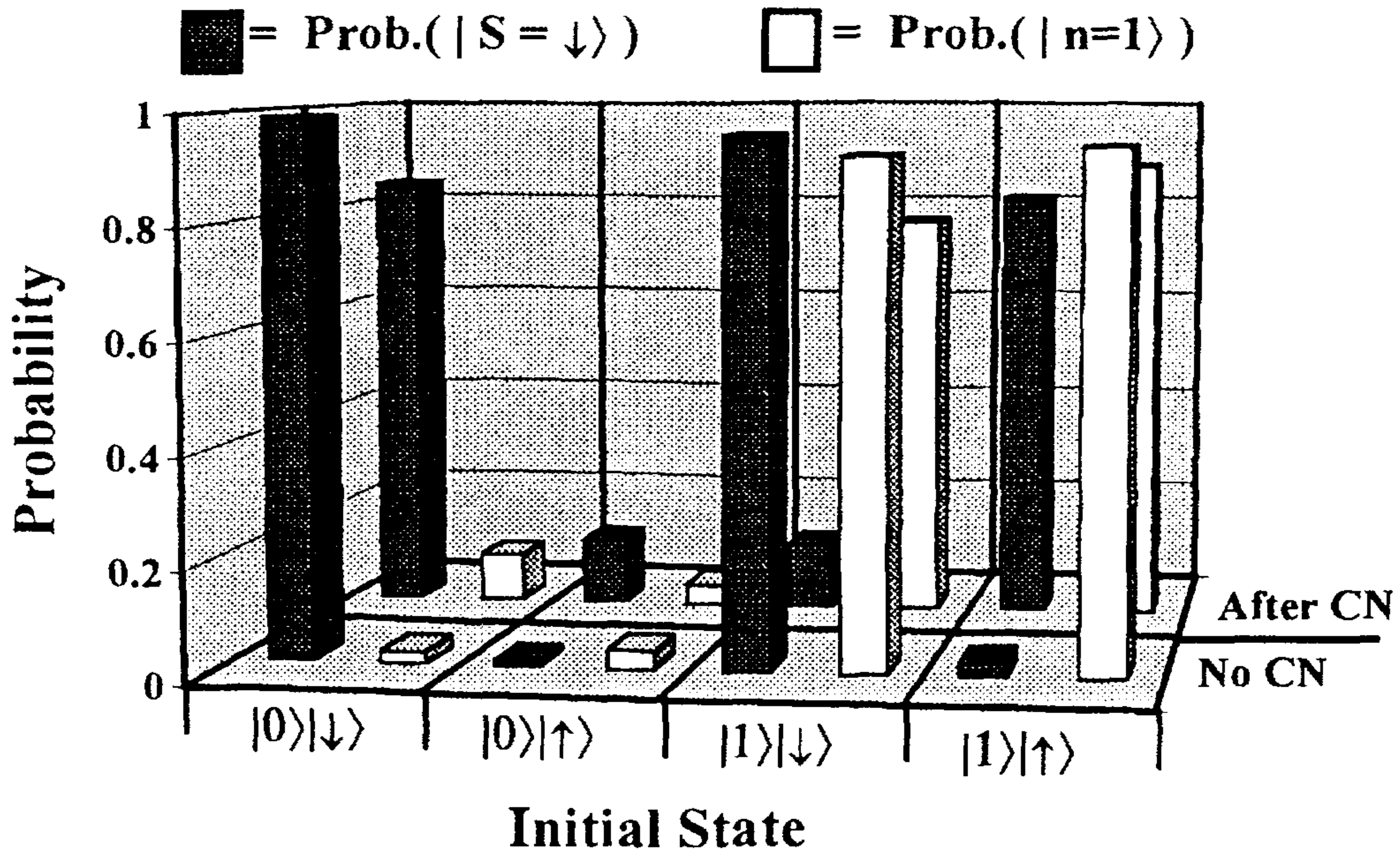


FIG. 3. Ramsey spectra of the controlled-NOT (CN) gate. The detuning of the Ramsey $\pi/2$ pulses of the CN gate [steps (a) and (c)] is swept, and S is measured, expressed in terms of the probability $P\{S=\downarrow\}$. The solid points correspond to initial preparation in the $|n\rangle|S\rangle = |0\rangle|\downarrow\rangle$ state, and the hollow points correspond to preparation in the $|n\rangle|S\rangle = |1\rangle|\downarrow\rangle$ state. The resulting patterns are shifted in phase by π rad. This flipping of $|S\rangle$ depending on the state of the control qubit indicates the conditional dynamics of the gate. Similar curves are obtained when the $|n\rangle|S\rangle = |0\rangle|\uparrow\rangle$ and $|1\rangle|\uparrow\rangle$ states are prepared. The lines are fits by a sinusoid, and the width of the Ramsey fringes is consistent with the $\simeq$50 μsec duration of the CN operation.

CNOT Performance



Input state → Output state

$$|0\rangle|\downarrow\rangle \rightarrow |0\rangle|\downarrow\rangle$$
$$|0\rangle|\uparrow\rangle \rightarrow |0\rangle|\uparrow\rangle$$
$$|1\rangle|\downarrow\rangle \rightarrow |1\rangle|\uparrow\rangle$$
$$|1\rangle|\uparrow\rangle \rightarrow |1\rangle|\downarrow\rangle .$$

FIG. 2. Controlled-NOT (CN) truth table measurements for eigenstates. The two horizontal rows give measured final values of n and S with and without operation of the CN gate, expressed in terms of the probabilities $P\{n = 1\}$ and $P\{S = \downarrow\}$. The measurements are grouped according to the initial prepared eigenstate of the quantum register ($|0\rangle|\downarrow\rangle$, $|0\rangle|\uparrow\rangle$, $|1\rangle|\downarrow\rangle$, or $|1\rangle|\uparrow\rangle$). Even without CN operations, the probabilities are not exactly 0 or 1 due to imperfect laser-cooling, state preparation and detection preparation, and decoherence effects. However, with high probability, the CN operation preserves the value of the control qubit n, and flips the value of the target qubit S only if $n = 1$.

Summary (1)

We report the first demonstration of a fundamental quantum logic gate that operates on prepared quantum states. Following the scheme proposed by Cirac and Zoller [1], we demonstrate a controlled-NOT gate on a pair of quantum bits (qubits). The two qubits comprise two internal (hyperfine) states and two external (quantized motional harmonic oscillator) states of a single trapped atom. Although this minimal system consists of only two qubits, it illustrates the basic operations necessary for, and the problems associated with, constructing a large scale quantum computer.

The distinctive feature of a quantum computer is its ability to store and process superpositions of numbers [2]. This potential for parallel computing has led to the discovery that certain problems are more efficiently solved on a quantum computer than on a classical computer [3]. The most dramatic example is an algorithm presented by Shor [4] showing that a quantum computer should be able to factor large numbers very efficiently. This appears to be of considerable interest, since the security of many data encryption schemes [5] relies on the inability of classical computers to factor large numbers.

Summary (2)

Experimental realization of a quantum computer requires isolated quantum systems that act as the qubits, and the presence of controlled unitary interactions between the qubits that allow construction of the CN gate. As pointed out by many authors [6,9,10], if the qubits are not sufficiently isolated from outside influences, decoherences can destroy the quantum interferences that form the computation. Several proposed experimental schemes for quantum

In our implementation of a quantum CN logic gate, the target qubit $|S\rangle$ is spanned by two $^2S_{1/2}$ hyperfine ground states of a single $^9Be^+$ ion (the $|f = 2, m_F = 2\rangle$ and $|F = 1, m_F = 1\rangle$ states, abbreviated by the equivalent spin-1/2 states $|\downarrow\rangle$ and $|\uparrow\rangle$) separated in frequency by $\omega_0/2\pi \simeq 1.250$ GHz. The control qubit $|n\rangle$ is spanned by the first two quantized harmonic oscillator states of the trapped in ($|0\rangle$ and $|1\rangle$), separated in frequency by the vibrational frequency $\omega_x/2\pi \simeq 11$ MHz of the harmonically trapped ion. Figure 1 displays the relevant $^9Be^+$ energy levels. Manipulation between the four basis

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