Example Quantum Program: Notes about gates

AP 4901 & 4903

Qubits: How to Build a Quantum Computer

Outline

- Quantum gates are reversible
- Classical “half-adder”, “full-adder”, and integer adder
- Quantum adder
- How to make classical adder reversible
- Examples

All quantum gates are unitary matrices in the qubit (complex) state-space

$|\psi\rangle = a|1\rangle + b|0\rangle$ (a, b) ARE COMPLEX NUMBERS

$\langle\psi| = \langle 1|a^* + \langle 0|b^*$

$\langle\psi|\psi\rangle = |a|^2 + |b|^2 = 1$ INNER PRODUCT

QUANTUM GATES PRESERVE THE NORM $\langle\psi|\psi\rangle$

$U|\psi\rangle$ = MODIFIED STATE VECTOR

$\langle\psi|U^+ \longrightarrow \langle\psi|U^+U|\psi\rangle = \langle\psi|\psi\rangle$

$U^+U = I$ UNITARY!

Example Single Qubit Gates

$$I \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$Z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$H \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

HADAMARD

$$G_Y \equiv -i \, Y$$

ONE OF THE PAULI MATRICES

$$G_Y^{\dagger} G_Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$Y^{\dagger} Y = I$$

ETC

Quantum Gates are Reversible

$\boxed{H}\,|\psi\rangle$

EXAMPLE LET $|\psi\rangle = |0\rangle$

THEN

$$\boxed{H}\,|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|0\rangle$$

REVERSE

$$\boxed{H^{\dagger}}\,|\psi\rangle = \boxed{H}\,|\psi\rangle$$

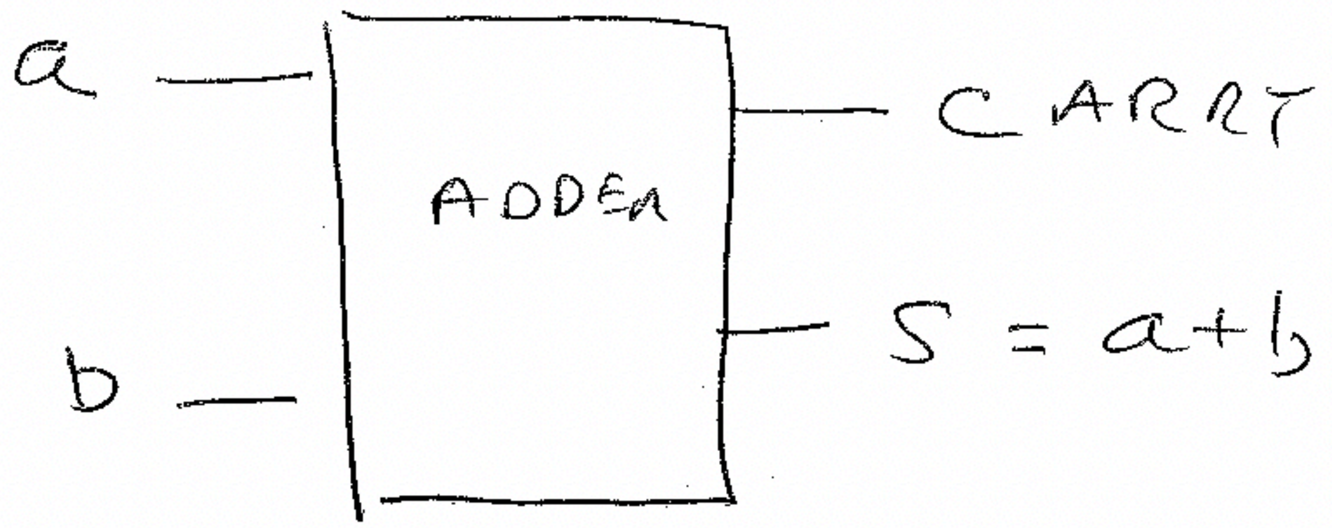
$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

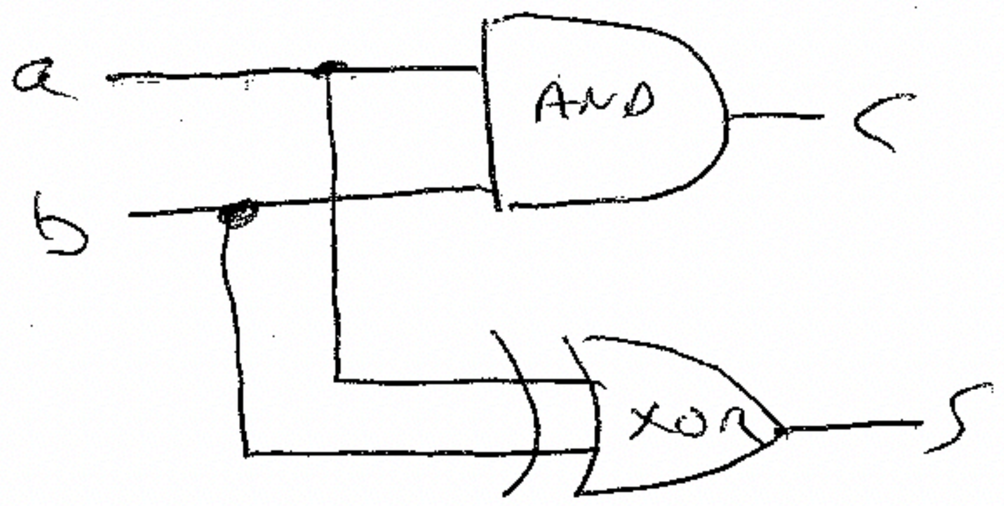
$$= \frac{1}{2}\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |0\rangle$$

How to Make a “Classical” Digital Adder

$a + b = s$

(a, b) ARE DIGITAL BITS (0 OR 1)





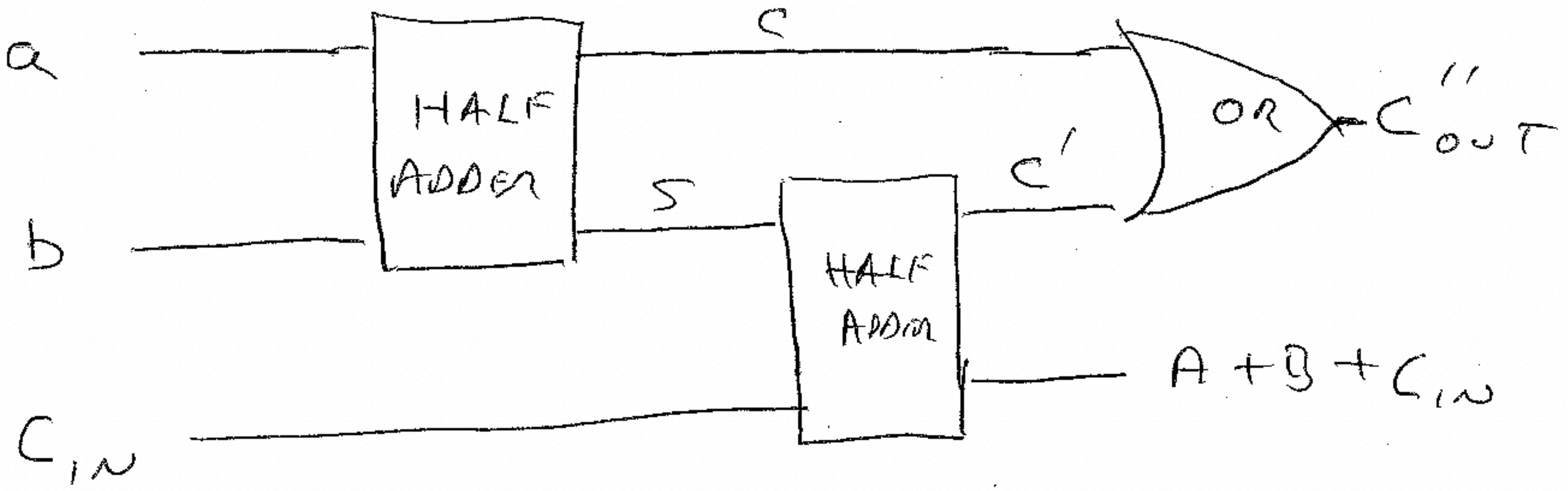
$s = a \oplus b$ (XOR)

$c = a \wedge b$

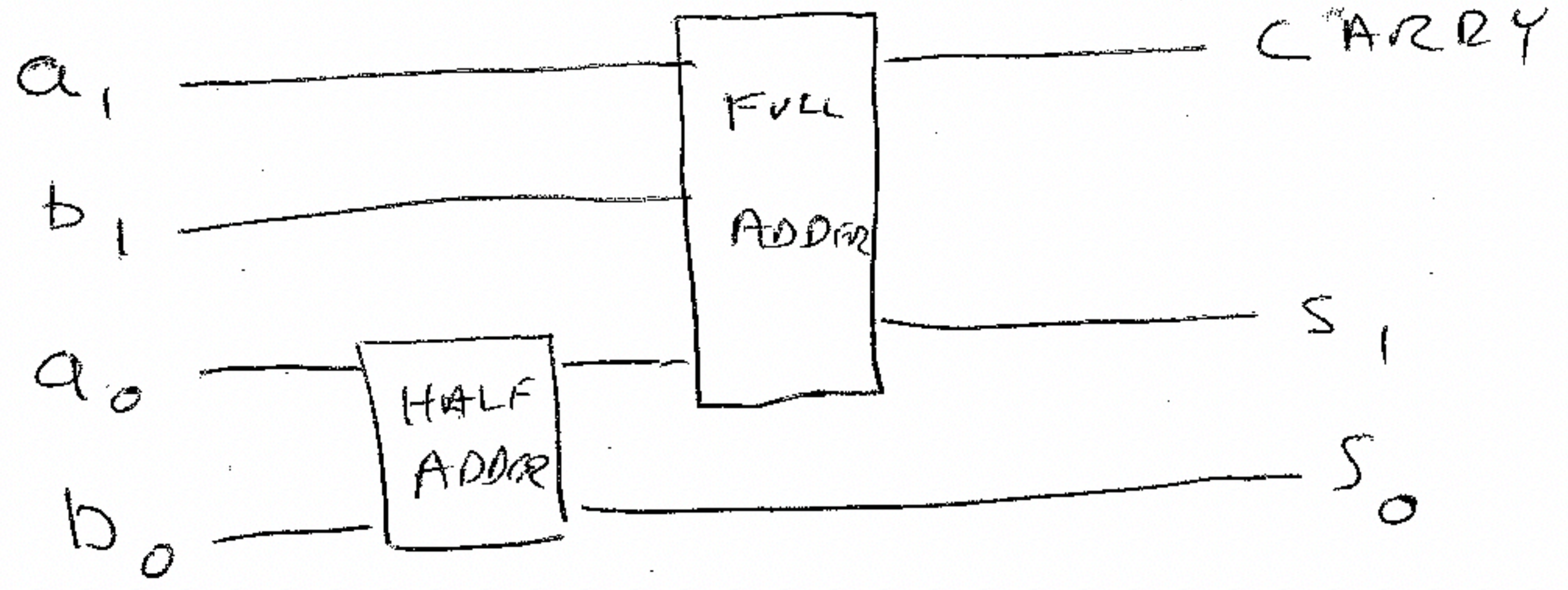
$\oplus$ = XOR BOOLEAN LOGIC

$\wedge$ = AND BOOLEAN LOGIC

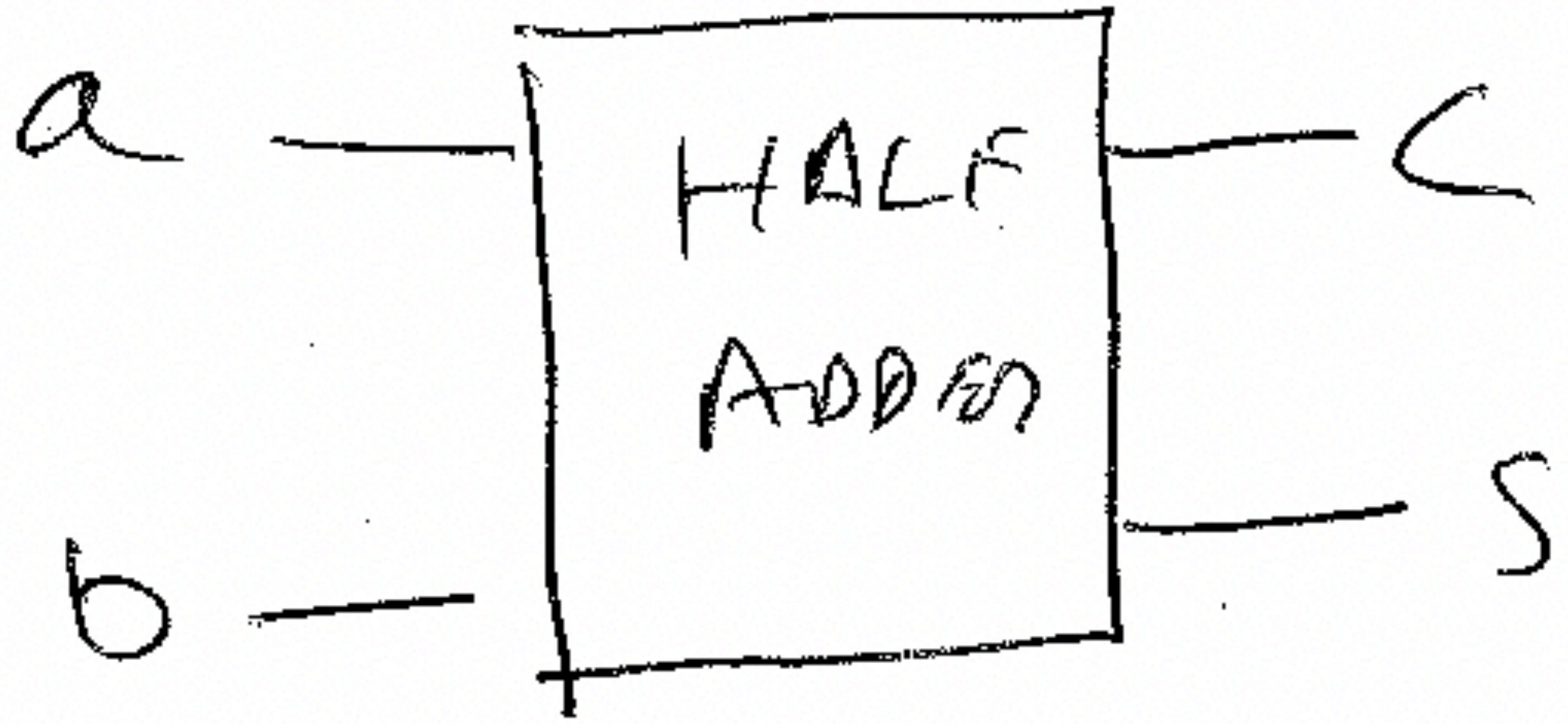
How to Make a Full-Adder



TWO BIT ADDER



Classical Digital Gates Do Not Need to be Reversible

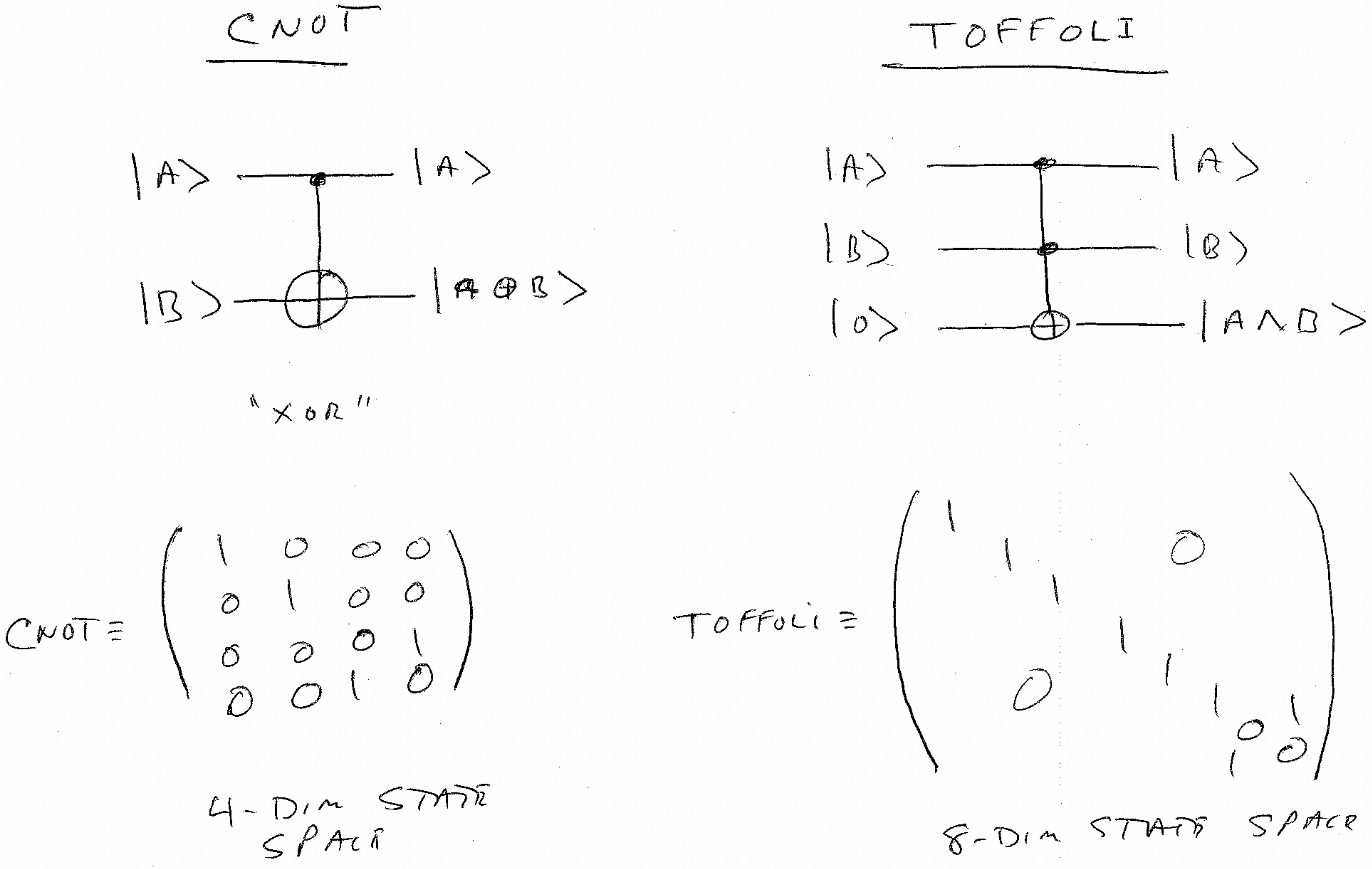


A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

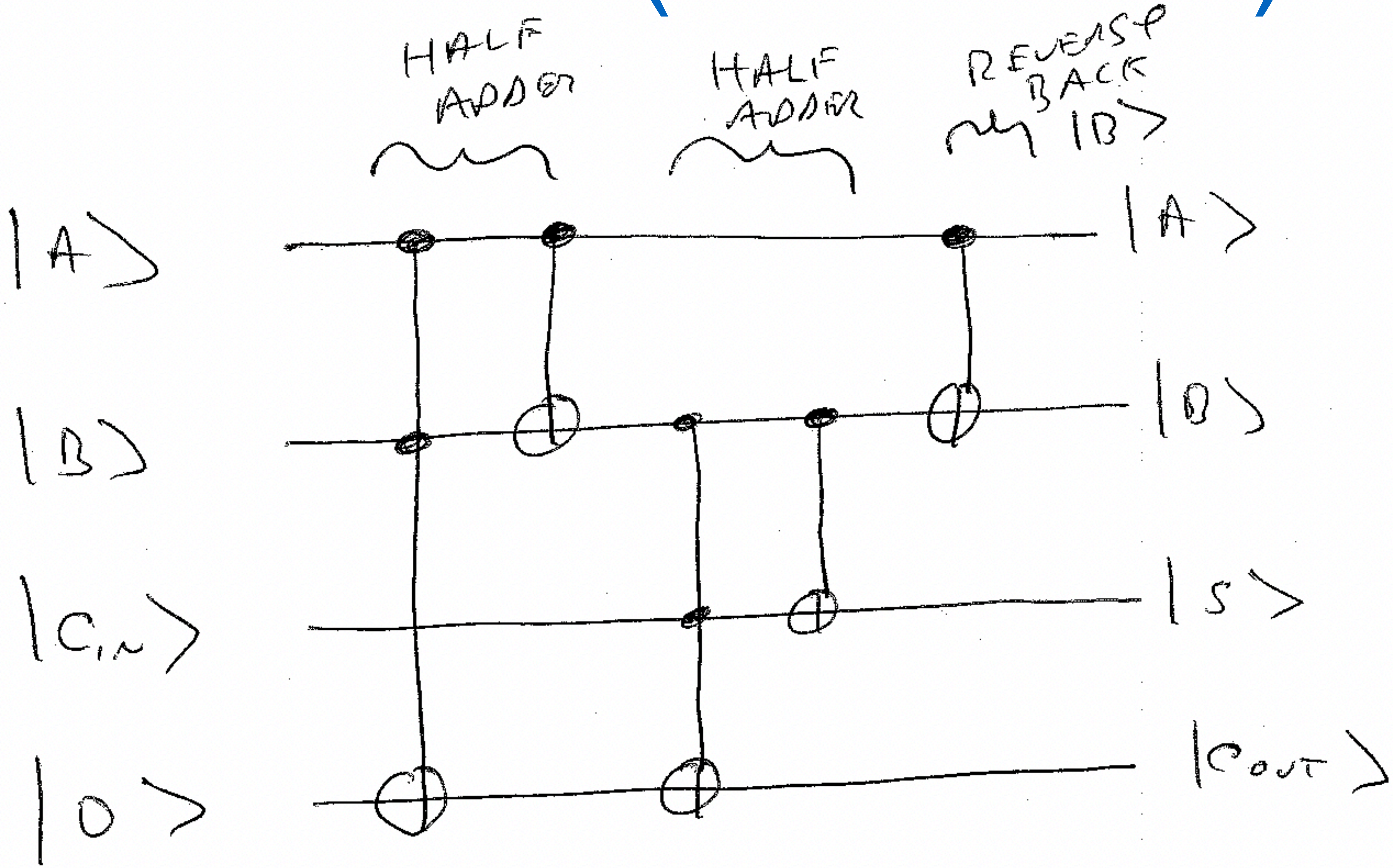
Tommaso Toffoli

https://en.wikipedia.org/wiki/Tommaso_Toffoli

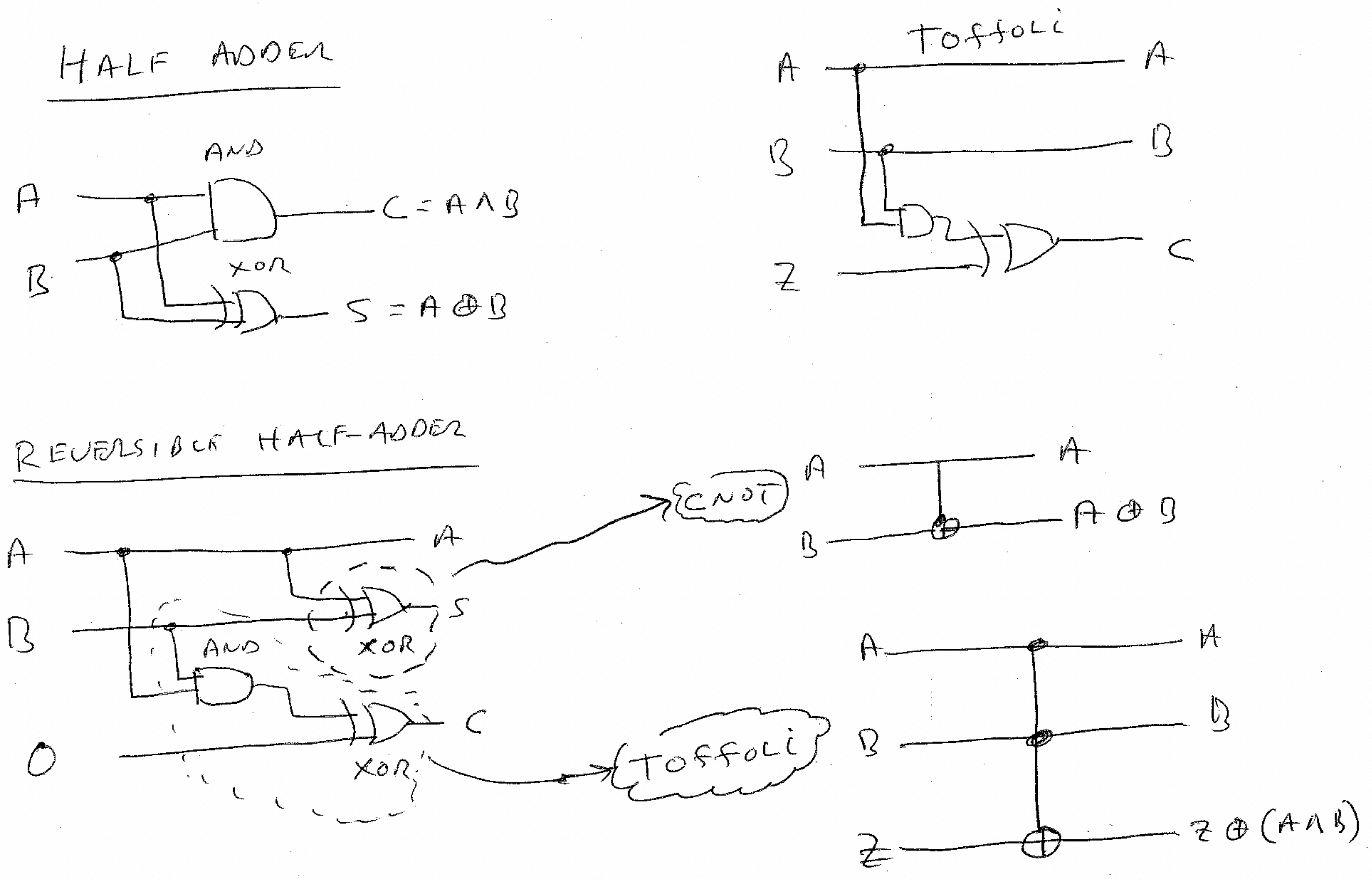
How to make a reversible adder



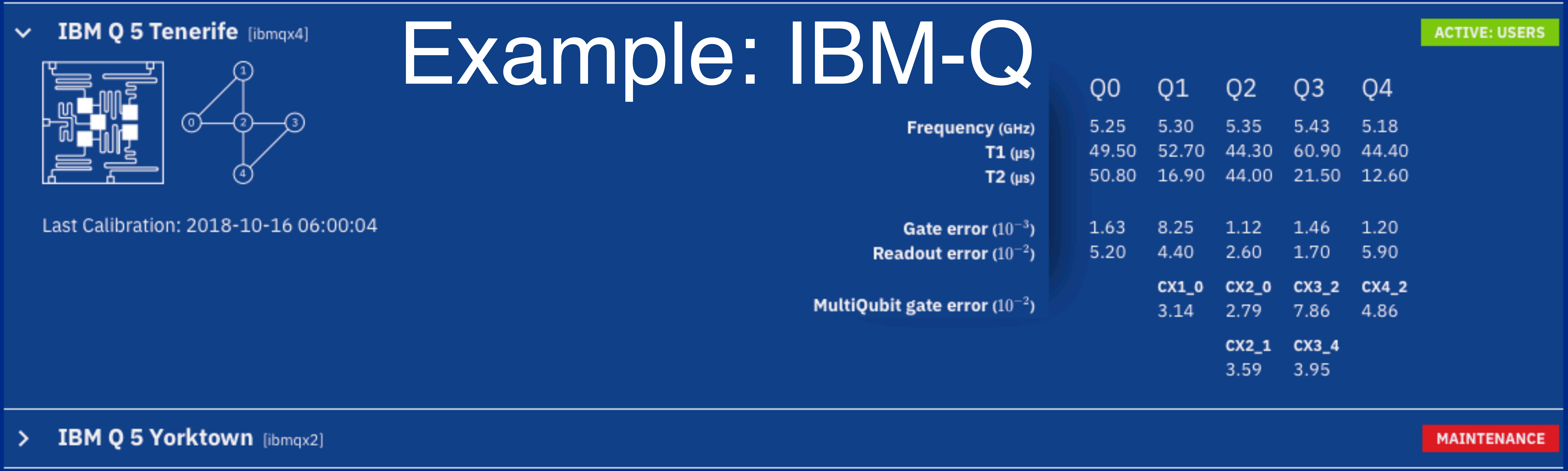
Full Adder (Reversible)

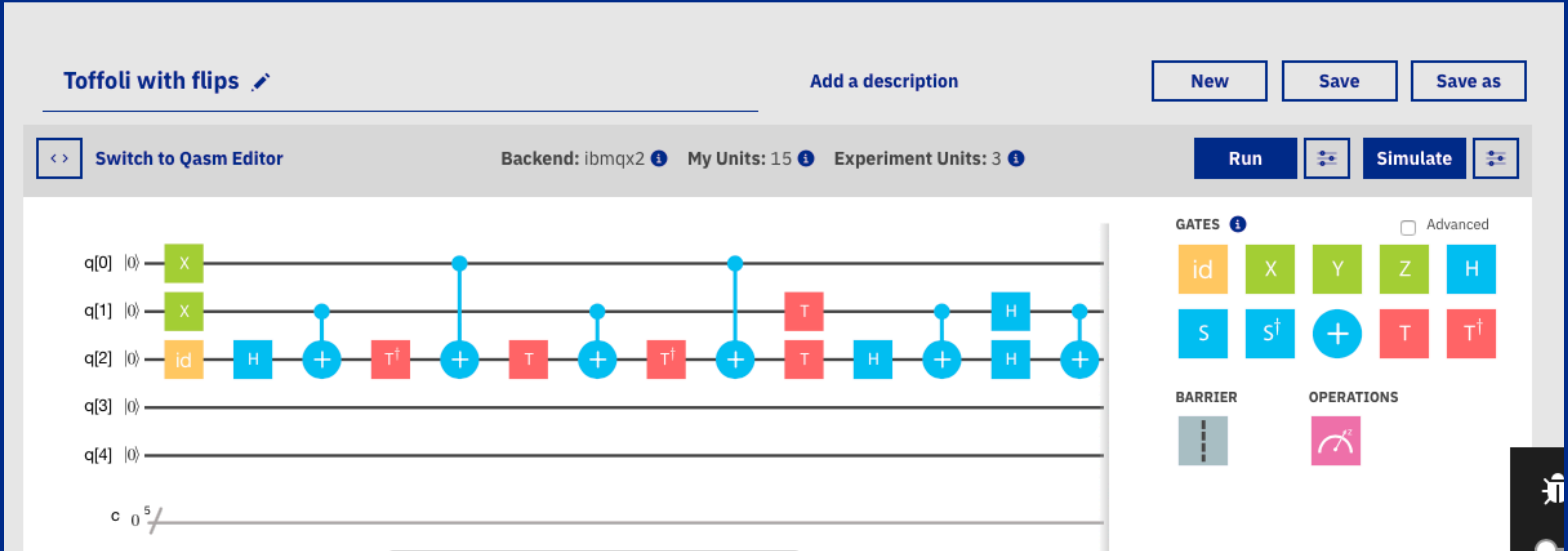


Digital Gates Can be Reversible (How?)



Example: IBM-Q



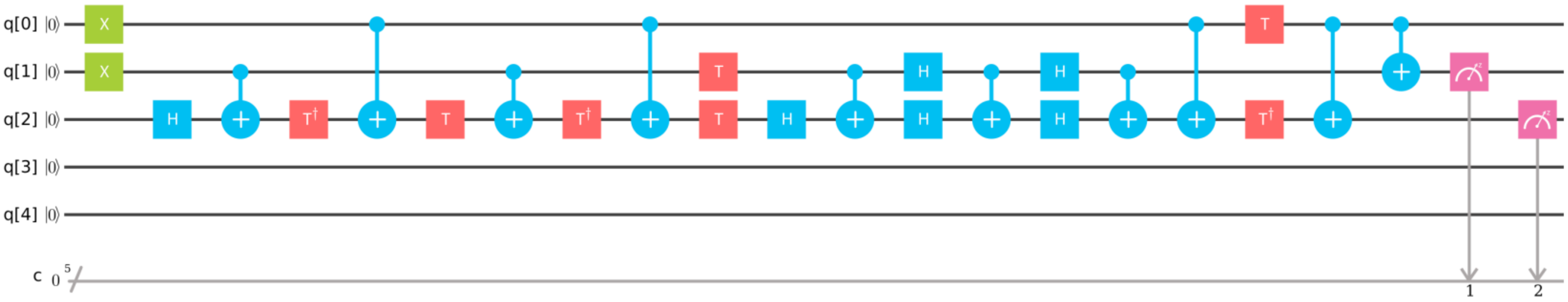


IBM Q: Beginners Guide / Multi-Qubit Gates

https://quantumexperience.ng.bluemix.net/qx/tutorial?sectionId=beginners-guide&page=006-Multi-Qubit_Gates~2F001-Multi-Qubit_Gates

Half-Adder

https://quantumcomputing.stackexchange.com/questions/1654/how-do-i-add-11-using-a-quantum-computer/1661#1661



Realization of the Quantum Toffoli Gate with Trapped Ions

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Gates acting on more than two qubits are appealing as they can substitute complex sequences of two-qubit gates, thus promising faster execution and higher fidelity. One important multiqubit operation is the quantum Toffoli gate that performs a controlled NOT operation on a target qubit depending on the state of two control qubits. Here we present the first experimental realization of the quantum Toffoli gate in an ion trap quantum computer, achieving a mean gate fidelity of 71(3)%. Our implementation is particularly efficient as the relevant logic information is directly encoded in the motion of the ion string.

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Our experimental system consists of a string of $^{40}Ca^+$ ions confined in a linear Paul trap. Each ion represents a qubit, where quantum information is stored in superpositions of the $S_{1/2}(m = -1/2) = |S\rangle \equiv |1\rangle$ ground state and the metastable $D_{5/2}(m = -1/2) = |D\rangle \equiv |0\rangle$ state of the $^{40}Ca^+$ ions [9]. The center-of-mass (COM) vibrational mode of the ion string is used to mediate the interaction between the ion qubits. Each experiment includes: (a) the

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Implementation of a Toffoli gate with superconducting circuits

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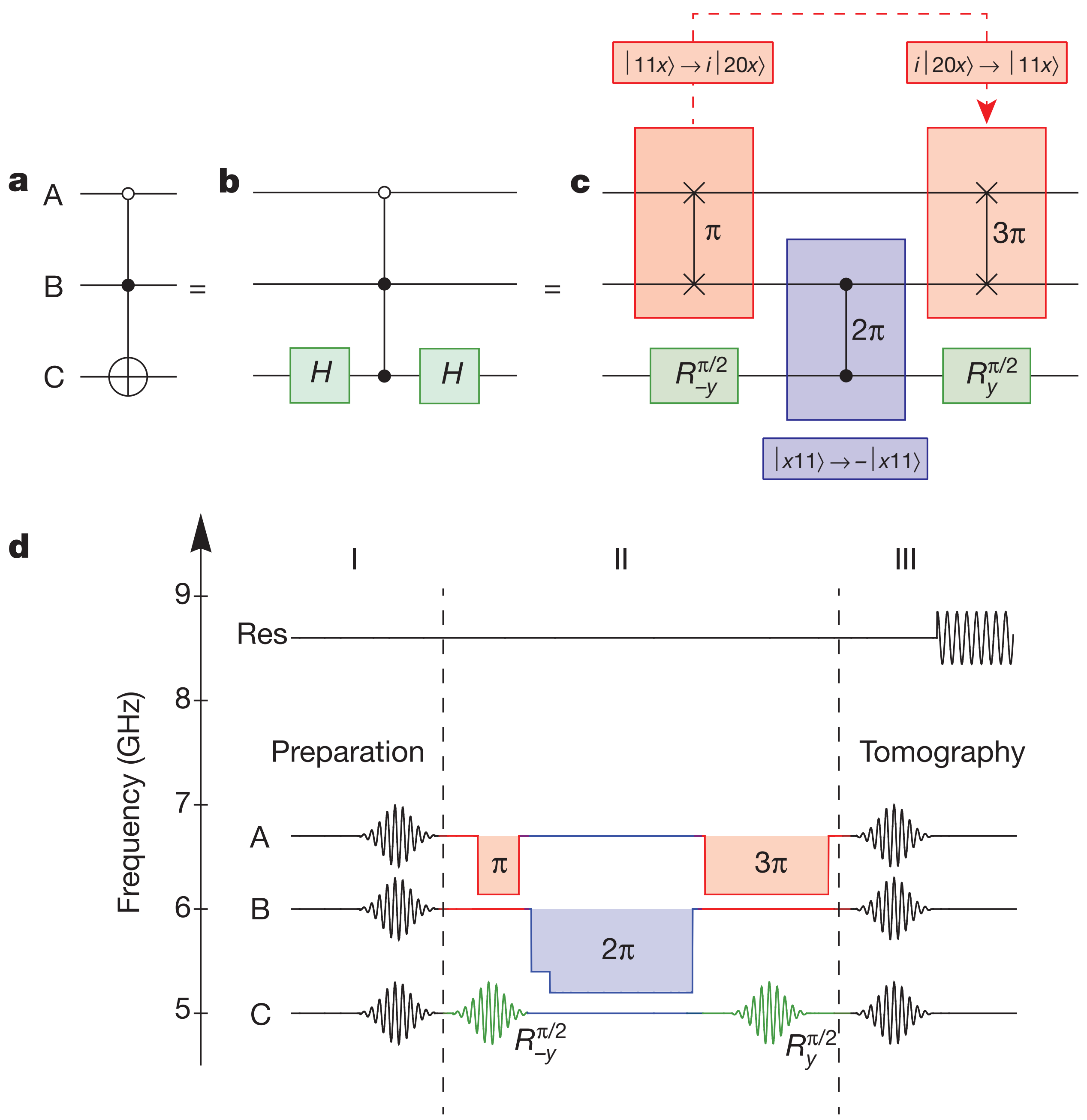
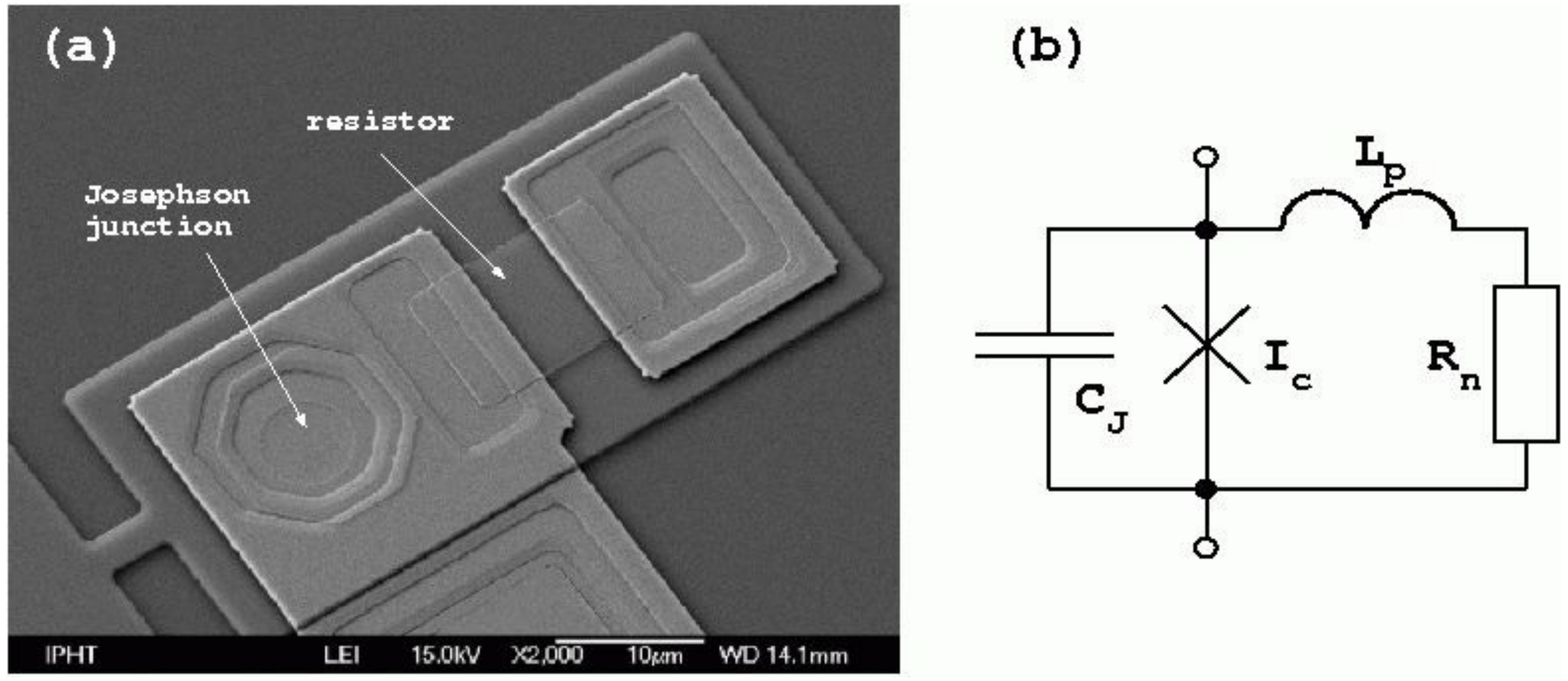
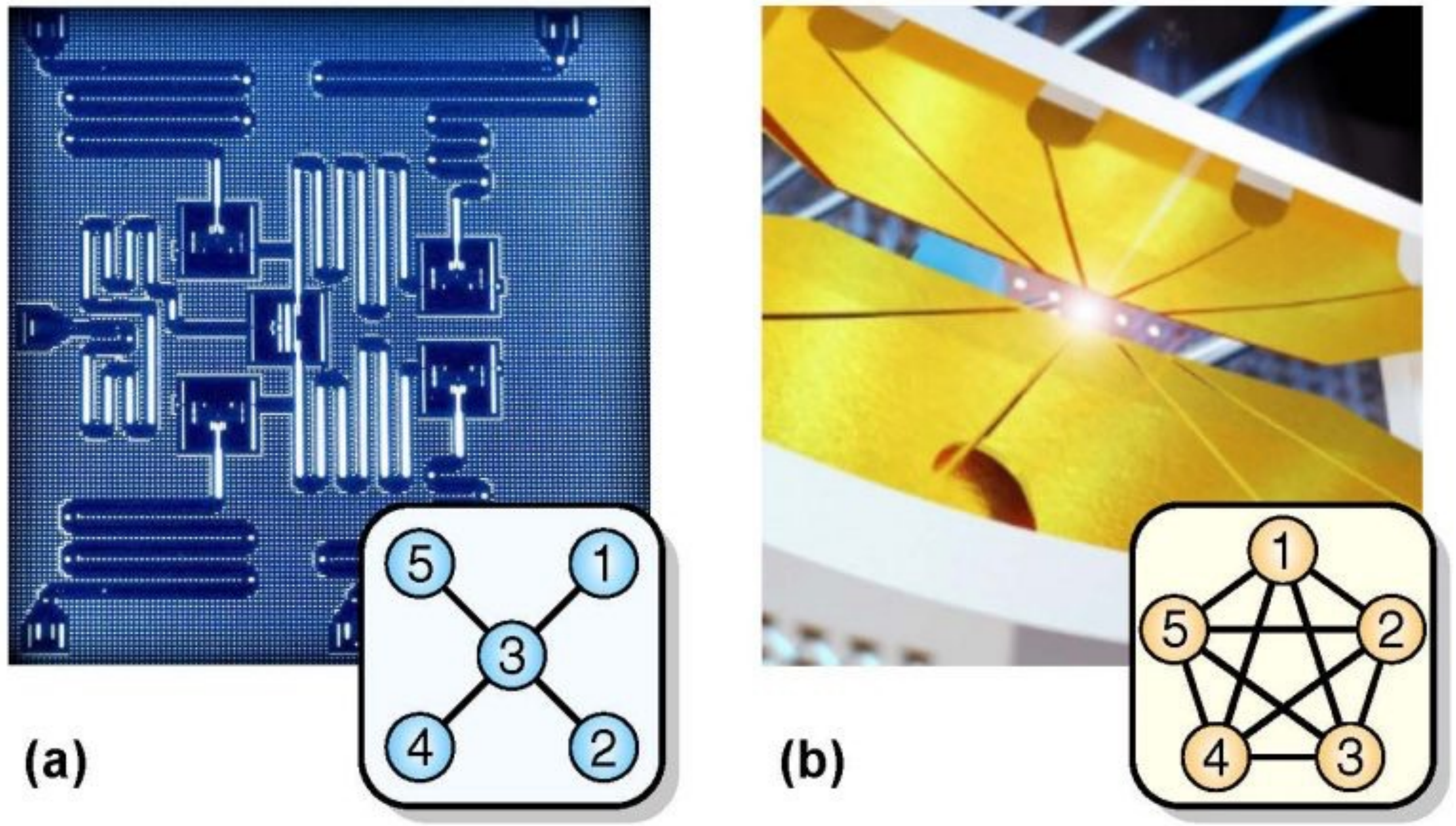


Figure 1 | Circuit diagram of the Toffoli gate. **a**, A NOT operation ($\oplus$) is applied to qubit C if the control qubits (A and B) are in the ground ($\circ$) and excited states ($\bullet$), respectively. **b**, The Toffoli gate can be decomposed into a CCPHASE gate sandwiched between Hadamard gates (H) applied to qubit C. **c**, The CCPHASE gate is implemented as a sequence of a qubit–qutrit gate, a two-qubit gate and a second qubit–qutrit gate. Each of these gates is realized by tuning the $|11\rangle$ state into resonance with $|20\rangle$ for a $\{\pi, 2\pi, 3\pi\}$ coherent rotation, respectively. For the Toffoli gate, the Hadamard gates are replaced with $\pm\pi/2$ rotations about the y axis (represented by $R^{\pi/2}_{\pm y}$). **d**, Pulse sequence used for the implementation of the Toffoli gate. During the preparation (I), resonant microwave pulses are applied to the qubits on the corresponding gate lines. The Toffoli gate (II) is implemented with three flux pulses and resonant microwave pulses (colour coded as in **c**). The measurement (III) consists of microwave pulses that turn the qubit states to the desired measurement axis, and a subsequent microwave pulse applied to the resonator is used to perform a joint dispersive read-out.

The Toffoli gate is a three-quantum-bit (three-qubit) operation that inverts the state of a target qubit conditioned on the state of two control qubits. It makes universal reversible classical computation[1] possible and, together with a Hadamard gate[2], forms a universal set of gates in quantum computation. It is also a key element in quantum error correction schemes[3–7]. The Toffoli gate has been implemented in nuclear magnetic resonance[3], linear optics[8] and ion trap systems[9]. Experiments with superconducting qubits have also shown significant progress recently: two-qubit algorithms[10] and two-qubit process tomography have been implemented[11], three-qubit entangled states have been prepared[12,13], first steps towards quantum teleportation have been taken[14] and work on quantum computing architectures has been done[15]. Implementation of the Toffoli gate with only single- and two-qubit gates requires six controlled-NOT gates and ten single-qubit operations[16], and has not been realized in any system owing to current limits on coherence. Here we implement a Toffoli gate with three superconducting transmon qubits coupled to a microwave resonator. By exploiting the third energy level of the transmon qubits, we have significantly reduced the number of elementary gates needed for the implementation of the Toffoli gate, relative to that required in theoretical proposals using only two-level systems. Using full process tomography and Monte Carlo process certification, we completely characterize the Toffoli gate acting on three independent qubits, measuring a fidelity of 68.5 ± 0.5 per cent. A similar approach[15] to realizing characteristic features of a Toffoli-class gate has been demonstrated with two qubits and a resonator and achieved a limited characterization considering only the phase fidelity. Our results reinforce the potential of macroscopic superconducting qubits for the implementation of complex quantum operations with the possibility of quantum error correction[17].

Transmon vs Trapped Ions





Christopher Monroe (University of Maryland)

http://iontrap.umd.edu/2017/05/13/ions-vs-superconductors-quantum-connections/

Next Week:
How to make a technical presentation