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Recycling in laser-interferometric gravitational-wave detectors

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Laser interferometers may detect gravitational waves by sensing the strain in space produced by their passage. The resultant change in intensity of an interference fringe must be observable against a background noise due to the statistical fluctuations in the number of detected photons. Optimization of the detector sensitivity thus involves devising an optical system which both maximizes the signal and minimizes the noise. This is attempted in the various arrangements known collectively as light recycling. Here, the performance of these systems is quantitatively assessed. Standard or broadband recycling functions essentially by making efficient use of the available light, but it is shown that it may also be made to further enhance the sensitivity within a narrow bandwidth, becoming tuned recycling. This works, as do all the narrow-band variants, by arranging for both the laser light and a gravitational-wave-induced sideband to be resonant in the optical system. The original narrow-band system, resonant recycling, can also be made broadband; the various sensitivity-bandwidth combinations, together with the tuning properties of such a system, are discussed. Furthermore, a new optical arrangement, dual recycling, is proposed. Its optical layout is an extension of standard recycling and its strength lies in its flexibility. It is shown that, relatively simply, it may be made into either a broadband or a narrow-band system, in each case with the same performance as the best of the other schemes. It may be tuned more efficiently and easily over a wide range of frequencies. Uniquely, optimum performance may be obtained with dual recycling without the requirement that the storage time of the optical elements in each arm of the interferometer be comparable with the period of the gravitational wave. This may allow the operation of delay line interferometers down to much lower gravitational-wave frequency and will provide great operational flexibility. Finally, it is shown that dual recycling, together with resonant recycling, is relatively insensitive to imperfections in the geometrical quality of the optical system. When implemented on interferometers with lengths greater than about a kilometer, recycling should allow the attainment of the sensitivity required in order to observe gravitational waves and open up a new window to the Universe.

INTRODUCTION

Gravitational-wave detectors based upon the use of laser interferometry to monitor the separation of widely spaced free masses are under development around the world. The kilometer scale interferometers currently being proposed should open up a new field of astronomy.[1,2] An integral feature of these detectors will be the use of variants of the optical technique known as recycling, an idea first proposed by Drever.[3] Here we quantitatively analyze these variants in a unified way, showing how they improve the sensitivity level of the detector set by photon-counting fluctuations, and propose a new variant, which may have considerable practical and operational advantages.

The basic arrangement of a laser interferometric gravitational-wave detector is shown in Fig. 1 (without mirror M_0). Principles of operation are reviewed in Refs. 1 and 3.

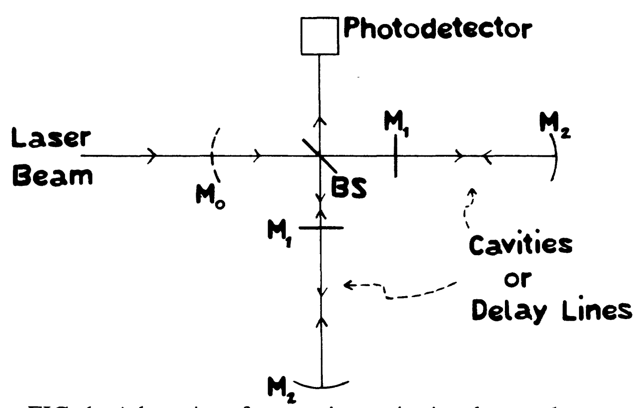


FIG. 1. A laser-interferometric gravitational-wave detector is essentially a Michelson interferometer. A gravitational wave produces opposite phase shifts on the light in the two arms of the interferometer; when interference occurs at the beam-splitter, the result is an intensity change on the light traveling to the photodetector.

A gravitational wave (amplitude h) of optimum polarization induces opposite length changes δl in each arm of the Michelson interferometer of magnitude

$$\delta l / l = \tfrac{1}{2} h \cos\omega_g t \;, \tag{1}$$

where l is the length of the interferometer arm and ω_g is the angular gravity-wave frequency. The resultant phase change $\delta\phi$ on a single beam of light spending a time τ_s in the interferometer is

$$\delta\phi = \int_{-\tau_s}^{0} \frac{4\pi}{\lambda}\delta l \, dt = \frac{h\omega}{\omega_g}\sin\frac{\omega_g \tau_s}{2} e^{-i\omega_g \tau_s/2} \;, \tag{2}$$

where λ and ω are the wavelength and angular frequency of the light, respectively. So the signal is a maximum if the storage time τ_s is equal to half the gravitational-wave period—the expected result since the gravitational wave reverses its sign with period $\frac{1}{2}\nu_g$.

In practice, multipass optical delay lines or cavities are used to match the storage time to the gravitational-wave period.[1] The choice of Fabry-Perot cavities makes the analysis more complex, for there are contributions to the signal from light with storage times of all multiples of $2l/c$. However, standard techniques for the analysis of Fabry-Perot interferometers[4] may be used as long as the signal, while arbitrarily fast, is assumed to produce a small phase change on the light and any large offset from resonance is assumed to be slow compared with the cavity storage time.[5] Thus, the field E_R reflected off a cavity with incident field E_0, input mirror of (amplitude) transmission T_1, reflectance R_1, far mirror reflectance R_2, and free spectral range $\nu_0 = c/2l$ is

$$E_R/E_0 = R_1 - \frac{T_1^2 R_2}{(1-R_1R_2)^2}\left[\frac{e^{i\delta}-R_1R_2}{1+F'\sin^2\delta/2}\left[1-\frac{ih\omega}{2\omega_g}\sin\omega_g t\right] + \frac{h\omega e^{i\omega_g t}}{4\omega_g}\frac{e^{i(\delta-\omega_g/\nu_0)}-R_1R_2}{1+F'\sin^2\left[\frac{\delta-\omega_g/\nu_0}{2}\right]} - \frac{h\omega e^{-i\omega_g t}}{4\omega_g}\frac{e^{i(\delta+\omega_g/\nu_0)}-R_1R_2}{1+F'\sin^2\left[\frac{\delta+\omega_g/\nu_0}{2}\right]}\right] \;, \tag{3}$$

where $F' = 4F^2/\pi^2 = 4R_1R_2/(1-R_1R_2)^2$; F is known as the finesse, δ is the phase offset (from resonance) for a single traverse of the interferometer. The different terms can be viewed as embodying the way in which the unmodulated laser light and the sidebands produced by the phase modulation of the gravitational wave resonate in the cavity. The static terms describe the effective reflectivity of the cavity, while the fluctuating terms give the signal. While the general form of (3) is useful (and shall be encountered again), simple interferometers operate with the cavities on resonance, at $\delta = 0$. The phase change of the light is then determined by the size of the fluctuating, imaginary part of the field compared with the static, real part (i.e., relative sideband amplitude). This may be determined by straightforward algebraic manipulation of (3). If the length of the interferometer is small compared with a gravitational wavelength (so that $\omega_g/\nu_0 \ll 1$), the phase shift of the emerging light is

$$\delta\phi = \frac{\alpha_c}{2}\frac{h\omega}{\omega_g}\frac{\omega_g\tau_s}{[1+(\omega_g\tau_s)^2]^{1/2}} \;, \tag{4}$$

where the storage time is

$$\tau_s = \frac{2Fl}{\pi c} = \frac{2l(R_1R_2)^{1/2}}{c(1-R_1R_2)} \;, \tag{5}$$

where c is the speed of light and α_c, the factor by which the field emerging from the cavity is larger than the incident field, is

$$\alpha_c = \frac{T_1^2 R_2}{1-R_1R_2} \sim \frac{2}{1+A^2/T_1^2} \;, \tag{6}$$

where A^2 is the loss coefficient of each mirror ($T^2+R^2+A^2=1$). Note that the field inside the cavity is enhanced by α_c/T_1. From now on it shall be assumed that both R_1 and R_2 are close to 1.

So for low loss cavities, as envisaged, the phase change (4) on the light is the same for cavity with $\omega_g\tau_s > 1$ as for a delay line $\tau_s \approx \frac{1}{2}\nu_g$. They will thus have equal potential sensitivities: if the smallest detectable intensity change (produced by the interference of two beams with phase fluctuation $\delta\phi$) is set by photon-counting statistics, then the smallest detectable gravitational-wave amplitude is

$$h_{\rm DL} = \left[\frac{\pi\hbar\lambda\Delta\nu_g}{\epsilon I_0 c}\right]^{1/2}\frac{\nu_g}{\sin(\omega_g\tau_s/2)} \;, \tag{7}$$

$$h_{\rm cav} = \left[\frac{\hbar\lambda[1+(\omega_g\tau_s)^2]\Delta\nu_g}{4\pi\epsilon I_0 c\tau_s^2}\right]^{1/2} \;, \tag{8}$$

where $\Delta\nu_g$ is the measurement bandwidth, ϵ, I_0 is the effective laser power, and $\hbar$ is the reduced Planck's constant.

BROADBAND OR STANDARD RECYCLING

It is important to realize that only a small fraction of the incident optical power is absorbed in the simple detectors considered so far.

A delay line with mirrors of amplitude reflectance R ($A^2 \ll 1$) and N reflections has an overall intensity reflectance of

$$R_{\mathrm{DL}}^2 = 1 - R^{2N} \sim 1 - \frac{c}{l}\tau_s A^2 \tag{9}$$

while that of a cavity is [cf. (3) + algebra]

$$R_c^2 = 1 - \frac{4FA^2/\pi}{1+F'\sin^2\delta/2} . \tag{10}$$

Thus, with $l=1$ km, $A^2=10^{-4}$, and $\tau_s=\frac{1}{2}\tau_g=0.5$ ms, the losses only total about 1.5%. With the interferometer working on a dark fringe the remaining light travels back toward the laser and, in a simple interferometer, is wasted. The simplest version of recycling (sometimes called "broadband" or "standard" recycling) consists of placing another mirror M_0 in the beam (see Fig. 1) with the correct position to coherently send light back to the interferometer.[3] The recycling mirror may also be regarded as an impedance matching device which ensures efficient transfer of power.[6] The resonant enhancement of the laser intensity in this recycling cavity reduces the significance of photon-counting errors: the power increases by the effective number of times the light is recycled, the shot-noise-limited sensitivity is enhanced by the square root of this factor. The increase in power inside the recycling system is limited by the losses in the system, principally the absorption and scattering of the cavity mirrors but with possible contributions from waveform distortions (limiting the efficiency with which the light from the two arms interferes to travel back toward M_0). Specifically, the maximum power gain P is

$$P = \frac{1}{1-R_{\mathrm{eff}}^2} , \tag{11}$$

where $1-R_{\mathrm{eff}}^2$ is the total loss on one round trip from the recycling mirror. If the losses are dominated by the cavity (or delay lines), as will be assumed from now on, then R_{eff}^2 is given by (10) or (9). The choice of storage time is a trade-off between signal and losses; for both delay line and cavity, the optimum choice is a storage time just short of giving the maximum phase shift: $\omega_g\tau_s(\mathrm{opt})=1$ for a cavity, $\tau_s=0.37\tau_g$ for a delay line. The gain in shot-noise-limited sensitivity S compared with a low loss nonrecycled system is then

$$S_c = \left[\frac{\pi\nu_g}{4\nu_0 A^2}\right]^{-1/2} = 10\left[\frac{A^2}{5\times10^{-5}}\right]^{1/2}\left[\frac{\nu_g}{1\text{ kHz}}\right]^{1/2}\left[\frac{l}{1\text{ km}}\right]^{1/2} \tag{12}$$

and

$$S_{\mathrm{DL}} = \left[\frac{1.14\nu_g}{\nu_0 A^2}\right]^{1/2} . \tag{13}$$

While this improvement in sensitivity is optimized for one gravitational-wave frequency, an enhancement is obtained at all frequencies—standard recycling produces a broadband detector. In the case of a cavity, the optimized shot-noise-limited sensitivity will be, combining (12) and (8),

$$h = \left[\frac{2\lambda\hbar A^2\nu_g\Delta\nu_g}{\epsilon I_0 l}\right]^{1/2} = 7\times10^{-24}\left[\frac{\nu_g}{1\text{ kHz}}\right]^{1/2}\left[\frac{\epsilon I_0}{100\text{ W}}\right]^{-1/2}\left[\frac{A^2}{5\times10^{-5}}\right]^{1/2}\left[\frac{l}{1\text{ km}}\right]^{-1/2}\Big/\sqrt{\text{Hz}} \tag{14}$$

assuming $\lambda=5\times10^{-7}$ m.

RESONANT RECYCLING

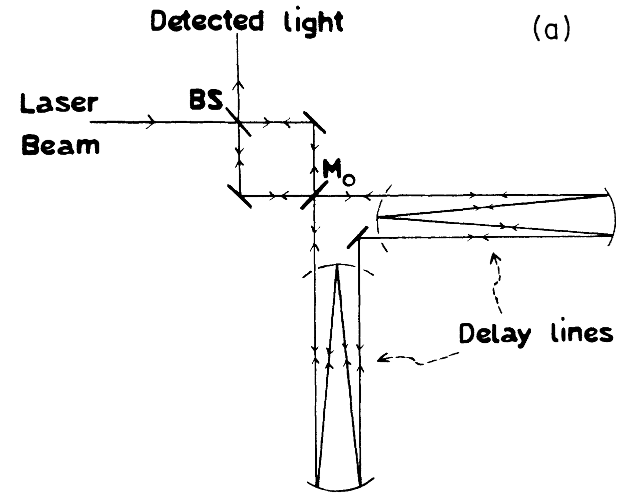
While a broadband detector is desirable when looking for unexpected sources or short bursts of gravitational radiation (GR), there are cases where a narrow-band detector is sufficient and even desirable. Examples include monochromatic sources of GR such as that from pulsars and accreting neutron stars,[1] while the observation of a stochastic background of GR and of the signals from coalescing compact binaries should benefit from an enhanced sensitivity within a restricted bandwidth. A possible way of constructing such a detector was proposed by Drever.[3] This method, known as resonant recycling, is illustrated in Fig. 2(a) (for a delay line) and Fig. 2(b) (for a cavity).

It can be seen that this is a very different optical arrangement from that of standard recycling. In the delay line case, the storage time of the light in each arm of the interferometer is arranged to be half a gravity-wave period, so that the light picks up the maximum phase shift from the gravitational wave; the light then passes directly into the other arm of the interferometer where, because the gravitational wave reverses its sign every half period it sees the *same* sign of phase shift as it did before, with the result that the signal builds up coherently. Roughly, the signal is increased by the number of times the light is cycled round the whole optical system, which is limited by the losses. If the losses are dominated by the cavity mirrors, then use of (9) and an appropriate version of (3) quickly leads to the gain in shot-noise sensitivity S_{RR} compared to a nonrecycled system:

$$S_{RR}(\mathrm{DL}) = \frac{\nu_g}{\nu_0 A^2} . \tag{15}$$

Note that, because signal rather than intensity is recycled, this gain in sensitivity is approximately the square of that obtained by using standard recycling, but it is restricted to a narrow bandwidth $\Delta\nu_g$, since other frequencies become out of step with the gravitational wave:

$$\Delta\nu_g \sim \frac{\nu_g}{S_{RR}} = \nu_0 A^2 = 8\left[\frac{A^2}{5\times10^{-5}}\right]\left[\frac{l}{1\text{ km}}\right]^{-1}\text{ Hz} . \tag{16}$$



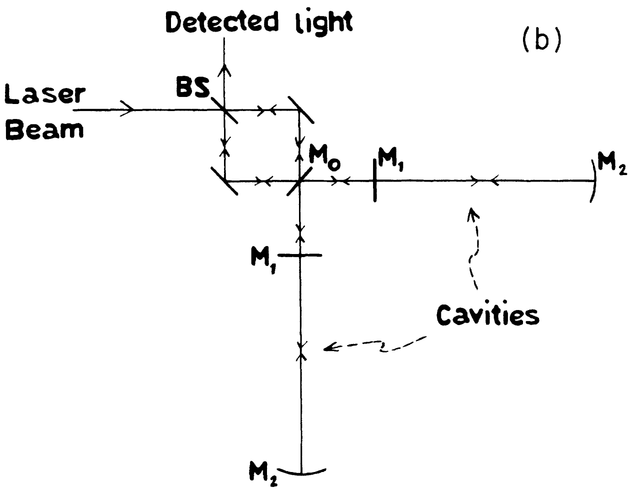


FIG. 2. A schematic diagram of the optical arrangement for resonant recycling (a) for a delay line, (b) for a cavity.

When optical cavities are used for resonant recycling, it is instructive to view the detector as a system of coupled cavities which have two normal modes;[6,7] the laser resonates with one of these, while the action of the gravitational wave is to pump energy into the other mode. In order to understand the reason for this, it is helpful to realize that light reflected off a cavity experiences a frequency-dependent phase shift θ; for the laser frequency (carrier), some manipulation of (3) gives

$$\tan\theta=\frac{(\alpha_c F/\pi)\sin\delta}{F'\sin^2\delta/2-1}\sim\frac{\alpha_c F\delta/\pi}{F^2\delta^2/\pi^2-1}\quad\text{for }\delta\ll 1\ . \tag{17}$$

Remember that δ is a measure of how far off the light is from being perfectly resonant in the cavity. For the sidebands, θ may be found by making the substitution $\delta\to\delta\pm\omega_g/\nu_0$. We see that between being on resonance ($\delta=0$) and far off ($F\delta/\pi\gg 1$) there is a change in the phase of $\sim 180°$ with the phase shift being 90° at $F\delta/\pi=1$. Now when a gravitational wave changes the effective length of a cavity, it imposes two sidebands on the (carrier) light emerging from the cavity, which then travels (via the recycling mirror) to the other cavity. Here it is reflected; if the length of the center cavity is adjusted so that the reflected carrier light has the same phase as that emerging from the other arm (i.e., the laser light is resonant), then, since the signal emerging from the two arms has opposite sign and the corresponding sidebands therefore have opposite phase, at least one of the sidebands must experience a phase shift on reflection which is 180° different from that of the carrier if the signal is to be increased. If the sideband and carrier also experience a relative phase shift of 180° when they are reflected off the original cavity, both carrier and sideband will be resonant, with the sideband always having the correct phase for its amplitude to be increased by the gravitational wave. This resonance condition of an 180° relative phase shift may be produced in practice in several different ways: the most symmetrical is to operate each cavity so that it gives a 90° phase shift of opposite sign to both carrier and sideband. Reference to (17) shows that this requires $F\delta/\pi=1$ or $\delta=\omega_g/2\nu_0$, $\omega_g\tau_s=2$. However, it can be seen from (17) that resonance may also be obtained for $\delta=0$ as long as $\omega_g\tau_s\gg 1$. It must be stressed that even if the isolated cavity is not on resonance, the coupled cavity system is.

In order to make this analysis quantitative, it is necessary to calculate the phase change produced by a gravitational wave on the light emerging from a cavity which (by itself) is not perfectly resonant. Reference to (3) plus, once more, some straightforward algebra, gives an amplitude E_s for sideband emerging from such a cavity of

$$|E_s/E_0|=\left[\frac{\alpha_c}{2}\right]\frac{h\omega}{2\omega_g}\frac{\omega_g\tau_s}{\left[1+F'\sin^2\delta/2\right]^{1/2}\left[1+F'\sin^2\dfrac{\delta-\omega_g/\nu_0}{2}\right]^{1/2}}\ . \tag{18}$$

This is the contribution, due to a single sideband, to the phase change on the reflected light from an isolated cavity. If this sideband is resonant, it will emerge from the recycling optics and be detectable with amplitude

$$|E_s/E_0|_{\text{tot}}=\frac{h\omega}{\omega_g}S_{RR}=\frac{\frac{1}{2}\alpha_c T_0^2}{(1-R_0R_c^2)(1-R_0R_{cs}^2)}\frac{h\omega}{\omega_g}\frac{\omega_g\tau_s}{(1+F'\sin\delta/2)^{1/2}\left[1+F'\sin^2\dfrac{\delta-\omega_g\nu_0}{2}\right]^{1/2}}\ , \tag{19}$$

where R_c and R_{cs} are the reflectivity of the cavity for the carrier and sideband, respectively [cf. (10)]. If we want to maximize the sensitivity at one frequency, then the best choice of recycling mirror transmission is

$$T_0^2=\frac{\frac{8F}{\pi}A^2}{(1+F'\sin^2\delta/2)^{1/2}\left[1+F'\sin^2\frac{\delta-\omega_g/\nu_0}{2}\right]^{1/2}} \tag{20}$$

which gives for the gain S_{RR} in shot-noise-limited sensitivity over a nonrecycled system,

$$S_{RR}(\text{cav})=\frac{\pi\nu_g}{4\nu_0A^2}=100\left[\frac{\nu_g}{1\text{ kHz}}\right]\left[\frac{l}{1\text{ km}}\right]\left[\frac{A^2}{5\times10^{-5}}\right]. \tag{21}$$

Note that the gain from resonant recycling is, within its bandwidth, just the square of that obtainable with broadband recycling.

So the shot-noise-limited sensitivity is

$$h_{RR}(\text{cav})=\left[\frac{2\lambda\hbar c}{\pi\epsilon I_0\tau_{\text{int}}}\right]^{1/2}\frac{A^2}{l}=5\times10^{-28}\left[\frac{\epsilon I_0}{100\text{ W}}\right]^{-1/2}\left[\frac{A^2}{5\times10^{-5}}\right]\left[\frac{l}{1\text{ km}}\right]^{-1}\left[\frac{\tau_{\text{int}}}{10^6\text{s}}\right]^{-1/2}, \tag{22}$$

where τ_{int} is the integration time.

Note that delay line and cavity detectors give virtually the same performance even though the latter only use one sideband: this is because cavity detectors can have a higher Q, resulting from the lower loss of a cavity "off resonance," and therefore the signal sideband has a higher coupling to the gravitational wave. One consequence is that the bandwidth of a cavity detector (at maximum sensitivity) is somewhat smaller than for a delay line detector: the different reflection phase shifts $\Delta\phi$ for different frequencies and the requirement that $S\Delta\theta<1$ gives

$$\Delta\nu_g(\text{cav})=\frac{2\nu_0}{\pi}A^2, \tag{23}$$

a factor of $\frac{1}{2}\pi$ smaller bandwidth than that of a corresponding delay line system.

It is possible to tune the resonant frequency of a cavity detector by altering the length of the long cavities (i.e., δ), while adjusting the length of the center cavity to keep the intensity there a maximum. With cavities of storage time τ_s one can obtain resonance at frequency ω_g with an offset δ of

$$\delta=\frac{\omega_g}{2\nu_0}\left\{1\pm\left[1-\left[\frac{2}{\omega_g\tau_s}\right]^2\right]^{1/2}\right\}. \tag{24}$$

With a given storage time, the lowest resonant frequency is obtained at $\omega_g\tau_s=2$, $\delta=\frac{1}{2}\omega_g/\nu_0$. Different offsets then tune the optical system to a higher resonant frequency. With the same optics, this will give a sensitivity gain S for these higher frequencies which is the same as that at $\omega_g\tau_s=2$ [to see this, substitute (24) into (19)]. In other words, a resonant recycling detector which is tuned to a frequency a factor of 2 higher than that for which it was optimized will have a shot-noise-limited sensitivity a factor of 2 worse than if it had been optimized for that frequency.

The bandwidth of the detector increases as $\delta\rightarrow0$: if the lowest possible resonant frequency with a particular optical system is ν_{g0}, so that $\omega_{g0}\tau_s=2$, then the bandwidth when the detector is tuned is

$$\Delta\nu_g=\frac{2\nu_0A^2}{\pi}\frac{\nu_g}{\nu_{g0}}. \tag{25}$$

In the limit of $\delta\approx0$, resonance can be obtained as long as $\omega_g\tau_s\gg1$; the sidebands will then have their phase inverted on reflection virtually independent of their frequency. Choice of $\omega_g\tau_s=(\omega_g/2\nu_0A^2)^{1/2}$ and $T_0^2=4/\omega_g\tau_s$ will then give a detector with bandwidth $\Delta\nu_g\approx\nu_g$ and sensitivity gain S just equal to that obtainable from standard recycling (13). Thus, resonant recycling can be made broadband.

TUNED RECYCLING

Just as resonant recycling can be made broadband, it is possible to make standard recycling narrow band. One scheme, known variously as tuned or detuned recycling, was suggested recently by Brillet:[7] it uses the same optical arrangement as standard recycling (Fig. 1 with mirror M_0) but the cavities are adjusted so that it is one of the gravitational-wave-induced sidebands, rather than the laser light, which is on resonance with the isolated cavities. This gives a phase shift on the light of one-half the maximum value (cf. 18 with $\delta=\omega_g/\nu_0$) while the losses for the laser light are reduced, allowing a larger build up of intensity in the center cavity and an improved shot-noise-limited sensitivity. It is also possible to view this arrangement as a coupling of the center and interferometer cavities, leading to a two-mode system. More quantitatively, the maximum power gain P in the center cavity is

$$P=\frac{1}{1-R_{\text{eff}}^2}=\left[A^2\left[\eta+\frac{4F/\pi}{1+(\omega_g\tau_s)^2}\right]\right]^{-1}, \tag{26}$$

where ηA^2 is the loss associated with one round trip of the center cavity; increasing the intensity in the center cavity enhances the importance of any losses there. If the center cavity losses are negligible, then the maximum sensitivity gain $S=P^{1/2}\,\delta\phi/(h\omega/\omega_g)$ is obtained [cf. (18)] when $T_1^2=A^2$, i.e., the highest storage time possible without losing signal. In this case, combining (26) and (18) gives [remember $(F/\pi)\omega_g/\nu_0=\omega_g\tau_s$]

$$S=\frac{\pi\nu_g}{4\nu_0 A^2} \tag{27}$$

which is exactly the same as for resonant recycling. The bandwidth is the same, also. Similarly, it is possible to choose cavity storage times so that the sensitivity gain is somewhere between the maximum value (27) and the broadband value (12), with a bandwidth such that the gain-bandwidth product is constant.

A problem with tuned recycling is that it will be hard to ensure that the losses in the center cavity are negligible; it will only be possible to obtain the maximum sensitivity gain at low frequency, where the cavity losses are more important:

$$\nu_g \ll \frac{\nu_0}{\pi}\left[\frac{A^2}{\eta}\right]^{1/2} = 100\left[\frac{l}{1\text{ km}}\right]^{-1}\left[\frac{A^2}{5\times10^{-5}}\right]^{1/2}\left[\frac{\eta}{10}\right]^{-1/2}\text{ Hz}\,. \tag{28}$$

Broader band operation will be possible at higher frequencies and will probably be the most useful way of running tuned recycling.

A NEW TECHNIQUE: DUAL RECYCLING

The amplitude-phase diagram of the light emerging from a multipass delay line Michelson interferometer is shown in Fig. 3. It can be seen that when the storage time of the delay line is comparable to the gravitational-wave period, the phase changes (or sidebands) induced on the light no longer have the correct relative phase to add most efficiently. Resonant recycling is a method of making the resultant $\delta\phi$ add coherently, but there is another approach—that of attempting to add the phase changes induced on each pass so that they always have the correct relative phase to increase the signal. A proposed method of doing this is shown in Fig. 4. In its simplest form, this consists of a simple Michelson interferometer with both the standard recycling mirror M_0 and a new recycling mirror M_3 in the output port of the interferometer. When the interferometer is operating on a dark fringe, the laser frequency is directed toward M_0 while the sidebands travel to M_3, where they are recycled; the transmitted sidebands constitute the signal. It might be thought that this arrangement is equivalent to putting mirrors M_0 and M_3 into the arms of the interferometer to form optical cavities which would then enhance the signal by the effective number of bounces in the cavity (at least for $\tau_s \ll \tau_g$). However, the system of Fig. 4 contains an additional degree of freedom: namely, the position of M_3 (relative to the image in the beamsplitter of M_0). This allows the phase of the recycled sideband reflected off M_3 and reentering the interferometer to be adjusted so that it has exactly the correct phase to add coherently with the sideband being produced by the gravitational wave. In this way, the signal is increased by the effective number of bounces (set by the losses) even for a total optical storage time longer than the gravitational-wave period. Since there are two recycling mirrors, one recycling intensity and one signal, this arrangement may be termed "dual recycling".

If the arms of the interferometer have an arbitrary optical length l_{opt} then only one sideband can, in general, resonate: the mode frequency spacing $c/2l_{\text{opt}}$ only equals the sideband spacing $2\nu_g$ when $\tau_s=\frac{1}{2}\tau_g$. For a simple, single pass Michelson, the effective phase change per pass from each arm is, therefore [cf. (2)],

$$\delta\phi_1=\frac{h\omega}{4\omega_g}\omega_g\tau_s\,. \tag{29}$$

The amplitude E_s, of the single sideband emerging

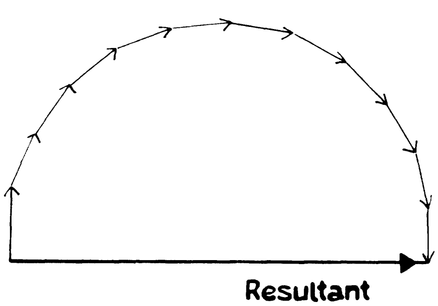


FIG. 3. An amplitude-phase diagram for the light emerging from a delay line Michelson interferometer with $\tau_s=\frac{1}{2}\tau_g$. Each vector corresponds to the phase change induced by a continuous gravitational wave on one traverse of the interferometer. The change in angle of each arrow is produced by a change in phase of the gravitational wave. For $\tau_s>\tau_g$, the curve is a spiral (or a circle if there are no losses).

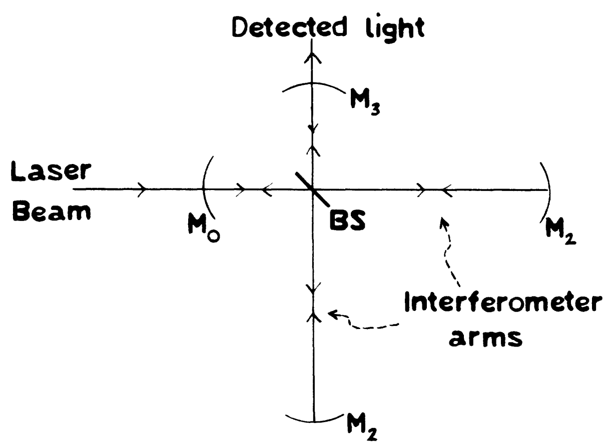


FIG. 4. The proposed new arrangement for recycling ("dual recycling"): the new element is the mirror M_3 which ensures that both signal and intensity are recycled. The optical elements in the arms of the interferometer can be either single or multipass delay lines or cavities. Viewed from M_3, the interferometer must operate on a dark fringe if differential motion of the two arms is to be enhanced.

through mirror M_3 is enhanced both by the resonance of the intensity and of the sideband, being increased by a factor

$$E_s/E_{s0}=\left[\frac{T_0R_D}{1-R_0R_D}\right]\left[\frac{T_3R_D}{1-R_3R_D}\right], \tag{30}$$

where R_D is the amplitude reflectivity of the interferometer arm, including any losses in the beamsplitter. If M_0, M_2, and M_3 have equal losses A^2 and $R_D^2=1-\eta A^2$ ($\eta>1$ allows for losses at the beamsplitter), then (30) may be rewritten as (with $\eta A^2 \ll 1$)

$$E_s/E_{s0}=\left[\frac{T_0}{\frac{1}{2}T_0^2+\frac{1}{2}A^2(\eta+1)}\right]\left[\frac{T_3}{\frac{1}{2}T_3^2+\frac{1}{2}A^2(\eta+1)}\right]. \tag{31}$$

The sideband amplitude is a maximum if the recycling mirrors are chosen so that

$$T_0^2=T_3^2=(\eta+1)A^2 \tag{32}$$

giving

$$E_s/E_{s0}=\frac{1}{(\eta+1)A^2}. \tag{33}$$

The shot-noise-limited sensitivity is determined by the sideband amplitude. The sensitivity gain $S_{\rm DR}$, normalized to the storage time limited case with no recycling (when $\delta\phi=h\omega/\omega_g$) is then

$$S_{\rm DR}=\delta\phi_1\left[\frac{\omega_g}{h\omega}\right]\frac{E_s}{E_{S0}}=\frac{1}{4(\eta+1)A^2}\omega_g\tau_s, \tag{34}$$

i.e.,

$$S_{\rm DR}=\frac{\pi\nu_g}{2(\eta+1)\nu_0A^2}. \tag{35}$$

Thus, if $\eta=1$ (i.e., the mirror losses are dominant), the sensitivity gain is just that obtainable from resonant recycling with a cavity system. This result is not surprising: dual recycling may be regarded as another way of arranging a coupled cavity system so that both the laser light and one sideband are resonant.

At frequencies slightly different from the resonant frequency, any sidebands will not resonate perfectly in the cavity formed by M_3 and the mirrors M_2, being suppressed by a factor $[1+(n_{\rm eff}\Delta\phi)^2]^{1/2}$ relative to the center frequency, where $\Delta\phi=(4\pi l/c)/\Delta\nu_g$ is the phase difference produced on a single traverse of the interferometer between the center frequency and the detuned frequency, the difference between enhanced by the effective number of bounces

$$n_{\rm eff}=\frac{R_3}{1-R_3R_D}. \tag{36}$$

Thus, the bandwidth of the resonant system is determined by $n_{\rm eff}\Delta\phi<1$ or, in the case of the maximum gain case $T_3^2=(\eta+1)A^2$,

$$\Delta\nu_g=\frac{(\eta+1)\nu_0A^2}{\pi}. \tag{37}$$

Comparison with (23) reveals that this is the same as with a cavity resonant recycling system, as long as $\eta=1$.

The bandwidth may be increased by increasing the transmission of the mirror M_3, at the cost of reducing the peak sensitivity gain. The choice of $n_{\rm eff}=\nu_0/\omega_g$ gives a system with bandwidth $\Delta\nu_g\approx\nu_g$ and a sensitivity gain just equal to that obtainable from standard recycling. Thus, greatly differing sensitivity-bandwidth combinations may be obtained by varying the transmission of a single mirror (M_3).

Another significant operational difference between this dual-recycling arrangement and resonant recycling is the possibility of tuning over a large range of gravitational-wave frequencies. In resonant recycling, signal build up requires a 180° relative phase shift between signal and carrier on reflection from a delay line or cavity. For a delay line, this requires $\tau_s=\frac{1}{2}\tau_g$ so a change in tuning requires a change in the storage time, or number of reflections. A cavity system can satisfy the resonance condition over a wide range of frequencies, but there is a minimum frequency ($\omega_g\tau_s=2$) below which the system cannot be tuned without changing the cavity mirrors. In contrast, the new dual recycling arrangement only requires that the Michelson interferometer be operating on the null of a fringe: it will work for any storage time in the arms of the interferometer and can thus be tuned to any gravitational-wave frequency. This is a qualitatively new feature: it is no longer necessary to have a storage time in the delay line (or cavity) which is comparable to the gravitational-wave period in order to obtain good sensitivity. The consequent reduction in the required number of reflections should enable delay line systems to operate well at much lower frequencies. Also, any dual recycling system should have considerably improved operational flexibility.

So far, the discussion of dual recycling has concentrated on the case of single-pass delay lines in the arms of the Michelson interferometer. It will work equally well, however, with either multipass delay lines or optical cavities. Such a choice has the advantage of reducing the significance of any losses at the beamsplitter. For a delay line with $\tau_s \ll \frac{1}{2}\tau_g$ but whose losses are greater than those at the beamsplitter, Eq. (34) holds with $\eta=1$. If $\tau_s=\frac{1}{2}\tau g$, then both sidebands are resonant but the resultant phase change is reduced by $\pi/2$ due to the curvature of the amplitude phase diagram [Fig. 3 (2)], giving a maximum gain in shot-noise-limited sensitivity of

$$S=\frac{\nu_g}{\nu_0A^2} \tag{38}$$

which is the same as that obtainable from resonant recycling [cf. (15)].

With cavities in the arms of the interferometer, the situation is similar. In the absence of recycling, two sidebands emerge from the cavities and leave the interferometer, each with an amplitude given by (18). In dual recycling, mirror M_0 is then adjusted to optimally recycle the intensity and M_3 is adjusted to optimally recycle a sideband of a particular frequency. The positioning of M_3 must take into account the frequency-dependent phase shift (17) of light on reflection off a cavity. It is this which ensures that only a narrow range of frequencies (and only one sideband) are in resonance. The choice of cavity storage time and phase offset is not at all critical: application of (18) to (34) shows that as long as the recycling mirrors are chosen such that

$$T_0^2=\frac{4FA^2/\pi}{1+F'\sin^2\delta/2} \tag{39}$$

and

$$T_3^2=\frac{\dfrac{4FA^2}{\pi}}{1+F'\sin^2\left[\dfrac{\delta-\omega_g/\nu_0}{2}\right]} \tag{40}$$

then the peak gain in shot-noise-limited sensitivity is

$$S=\frac{\pi\nu_g}{4\nu_0 A^2} \tag{41}$$

which is the same as for a delay line with $\tau_s \ll \tau_g$ and for a cavity resonant recycling system.

There are many ways in which one can imagine operating a practical dual recycling system. As an example, consider a choice of cavity storage time such that $\omega_g\tau_s=1$ at the frequency of interest. With the cavities operating on resonance ($\delta=0$) and $T_0^2=4FA^2/\pi$ but without mirror M_3, this arrangement is optimized for

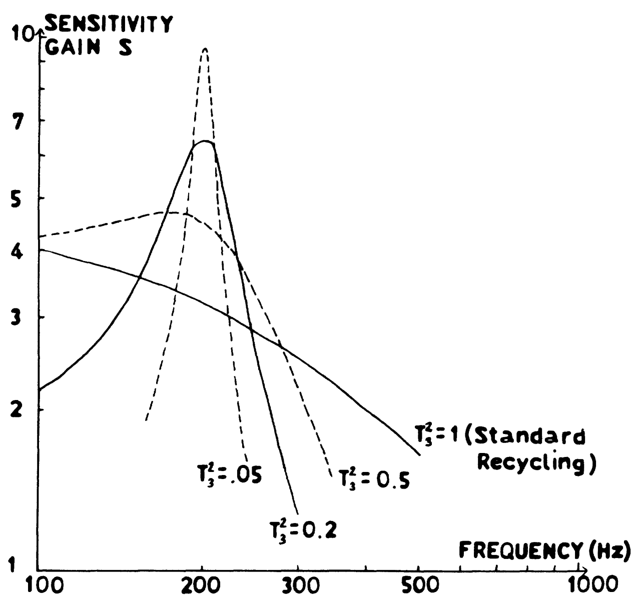


FIG. 5. The gain S in shot-noise-limited sensitivity with dual recycling, compared to a nonrecycled system, as a function of gravitational-wave frequency ν_g. The optical element in the interferometer arms is assumed to be a cavity which is optimized for standard recycling ($\omega_g\tau_s=1$) at 200 Hz, with a length $l=1$ km and mirror loss coefficient $A^2=10^{-4}$. The different curves show the effect of increasing the reflectivity of the dual recycling mirror M_3 from zero, keeping it tuned at 200 Hz: the sensitivity is enhanced within a bandwidth which decreases as the reflectivity increases.

standard recycling and broadband operation. If mirror M_3 is inserted, the peak sensitivity is increased and the bandwidth narrowed. This is illustrated in Fig. 5, which shows the effect of decreasing the transmission of M_3 while keeping the rest of the optics fixed. It can be seen that, as well as optimizing the sensitivity at one frequency or for a wide range of frequencies, it is possible to considerably enhance the sensitivity within a significant bandwidth. For example, in the case of $l=1$ km, $A^2=10^{-4}$ as shown in Fig. 5, it is possible to arrange (with $T_3^2=0.2$) for the sensitivity to be better than with standard recycling between 150 Hz and 250 Hz, with an improvement by a factor of 2 at 200 Hz. Such an arrangement would be eminently suited to searches for pulses or chirps from coalescing binaries.

The sensitivity in these situations may be calculated by a parallel argument to that applied to the simple Michelson, essentially combining (30) and (18). This gives

$$S=\frac{S_{\max}^{1/2}T_3}{T_3^2+(2F/\pi)A^2}=\frac{S_{\max}^{1/2}T_3}{T_3^2+\frac{1}{4}S_{\max}}, \tag{42}$$

where $S_{\max}$ is given by (40). The bandwidth is just

$$\Delta\nu_g/\nu_g=\frac{T_3^2+\frac{1}{4}S_{\max}}{(1-T_3^2)^{1/2}}. \tag{43}$$

The pulse sensitivity $S\Delta\nu_g^{1/2}$ is virtually independent of bandwidth, only time resolution is lost as the bandwidth is narrowed.

The different combinations of sensitivity gain and bandwidth are obtained, in dual recycling, by changing the transmission of the mirror M_3. This may be done physically, by changing the mirror. Alternatively, it may be possible to use a variable reflectivity mirror: one way of realizing this is to make M_3 an optical cavity. Such a cavity might have mirrors of low loss but quite high transmission. If the mirrors are of equal transmission, then, when the cavity is on resonance, virtually all of the light is transmitted through it, $T_3^2\approx 1$. When the cavity is far off resonance, $R_3^2\approx 1$. So, tuning the length of this cavity alters the transmission of the "mirror" M_3 and thus the bandwidth of the detector.

It is interesting to calculate the best combination of sensitivity and bandwidth when searching for a stochastic background of gravitational radiation that may be a relic of the big bang or of oscillations of cosmic strings.[1] In such an experiment, the signal from two detectors would be cross correlated to pick out the common background; the resultant signal-to-noise ratio scales as[1,2]

$$S/N\propto S(\Delta\nu_g\tau_{\text{int}})^{1/4}, \tag{44}$$

where τ_{int} is the integration time. Thus, when designing a detector, $S\Delta\nu_g^{1/4}$ should be maximized. The optimum combination may be found using (42) and (43):

$$T_3^2(\text{opt})=\frac{1}{2S_{\max}}=\frac{2\nu_0A^2}{\pi\nu_g}, \tag{45}$$

$$\Delta\nu_g=\frac{3\nu_0A^2}{\pi}, \tag{46}$$

which gives a cross-correlation sensitivity 1.04 times better than if the detector operated at the peak narrow-band sensitivity within its (narrow) bandwidth. The resulting sensitivity to a stochastic background in a bandwidth $\Delta\nu_g \approx \nu_g$ is

$$h = \left[\frac{4\hbar\lambda c \nu_g}{\pi\epsilon I_0}\right]^{1/2} \left[\frac{4}{3c\tau_{\rm int}}\right]^{1/4} \left[\frac{A^2}{l}\right]^{3/4} . \quad (47)$$

The equivalent detectable energy density in gravitational waves[1] Ω_g, in terms of the critical density for closure of the Universe, is, therefore,

$$\Omega_g = 10^{-10} \left[\frac{\nu_g}{200\ \rm Hz}\right] \left[\frac{\epsilon I_0}{100\ \rm W}\right]^{-1} \left[\frac{A^2}{5\times 10^{-5}}\right]^{3/2} \times \left[\frac{l}{1\ \rm km}\right]^{-3/2} \left[\frac{\tau_{\rm int}}{10^7\ \rm s}\right]^{1/2} . \quad (48)$$

Another property of interest is the possible sensitivity when the dual recycling system is tuned to another frequency by moving M_3. For a fixed cavity system, different sideband frequencies have different reflectivities off the cavities [cf. (10) with $\delta = \omega_g/\nu_0$]; if the transmission of M_3 is variable, it can be reoptimized for each frequency and the maximum sensitivity gain is just given by (41), the optimum value. If T_3 is fixed at the value for maximum sensitivity gain at, say, $\omega_{g0}\tau_s = 1$, then the sideband build up will not quite be optimum: the sensitivity gain $S(\nu_g)$ compared to the maximum value for that frequency $S_{\rm max}$ is

$$\frac{S(\nu_g)}{S_{\rm max}} = \left[\frac{8[1+(\nu_g/\nu_{g0})^2]}{[3+(\nu_g/\nu_{g0})^2]^2}\right]^{1/2} . \quad (49)$$

Thus, at frequencies other than the optimized frequency ν_{g0}, the sensitivity gain is slightly less than optimum: this is illustrated in Fig. 6. It can be seen that the possible sensitivity is hardly reduced at all for frequencies lower than ν_{g0}; this is a result of the small change in cavity reflectivity for $\nu_g < \nu_{g0}$ and the way the sidebands resonate in the cavity. For $\nu_g > 2\nu_{g0}$ the possible sensitivity falls off more quickly, as the cavity reflectivity decreases, but the fall off is modest, much less than with resonant recycling. This results from the liberation in dual recycling from the resonance condition of resonant recycling, with the consequent change in reflectivities for both carrier and sideband as the system is tuned: in dual recycling it is only the sideband which is not recycled optimally when the tuning is altered. Thus, as can be seen from Fig. 6, a dual recycling system with fixed optics is able to tune over a factor of 20 in frequency while maintaining a sensitivity within 10% of the optimal value.

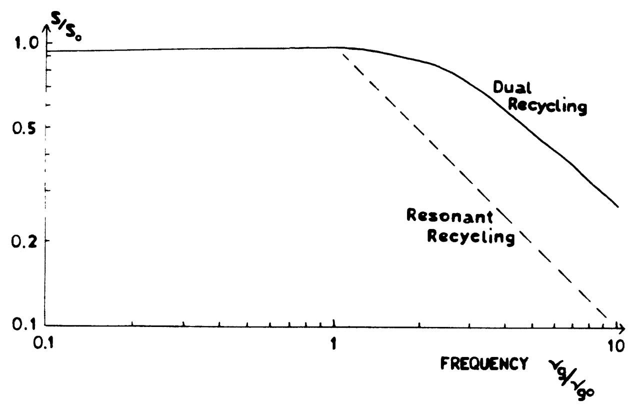


FIG. 6. The gain S in shot-noise-limited sensitivity as a function of its maximum possible value S_0 at a gravitational-wave frequency ν_g, for a fixed optical system optimized for $\nu_g = \nu_{g0}$.

THE EFFECT OF GEOMETRICAL IMPERFECTIONS

Another important feature of any recycling system is its sensitivity to geometrical imperfections in the optics: these might be misalignments of the mirrors, deviations from flatness, or birefringence. In standard recycling, be it broadband or tuned, the effect of such imperfections is to reduce the accuracy of overlap, at the beamsplitter, of the beams from the two arms of the interferometer. This increases the rate at which light leaks out of the interferometer, tending to reduce the sensitivity of the detector both by limiting the build up of intensity in the recycling system and by degrading the contrast of the interference fringe at the output. For example, consider the case of a misalignment by angle θ of the two arms of the interferometer; this will reduce the amplitude E_0 of the fundamental mode seen by one of the beams[5] to

$$E_0 \propto e^{-(\theta/\theta_c)^2} , \quad (50)$$

where θ_c is the characteristic angle of the beam:

$$\theta_c = \frac{\lambda\sqrt{2}}{\pi w} \approx \left[\frac{\lambda}{l}\right]^{1/2} , \quad (51)$$

with w the beam radius and λ, again, the wavelength. A fraction $1-\exp[-(\theta/\theta_c)^2]$ of the amplitude will therefore emerge from the output port of the beamsplitter rather than be recycled. If the sensitivity is not to be degraded, this fraction must be considerably less than the losses in the cavities (or delay lines) or equivalently

$$\theta/\theta_c \ll 1/S_{\rm opt} ,$$

where $S_{\rm opt}$ is the best possible gain is sensitivity with broadband recycling. The pointing accuracy of the masses has, therefore, to satisfy

$$\theta \ll \left[\frac{c\lambda A^2}{2\nu_g l^2}\right]^{1/2} \sim 3\times 10^{-6} \left[\frac{\nu_g}{1\ \rm kHz}\right]^{-1/2} \left[\frac{l}{1\ \rm km}\right]^{-1} \left[\frac{A^2}{10^{-4}}\right]^{1/2} \times \left[\frac{\lambda}{5\times 10^{-7}\ \rm m}\right]^{1/2} \rm rad . \quad (52)$$

While it is probably possible to achieve such pointing accuracy, the requirements may be relaxed by using dual recycling. In a sense, this is because the rejected light is being fed back in; more precisely, any higher modes of the beam which are required to express an optical imperfection will be resonantly suppressed by the recycling cavity formed by both M_0 and M_3 of Fig. 4. This is the same phenomenon that occurs when a simple two-mirror cavity is, say, misaligned. The effect of the misalignment is to simply reduce the amplitude of the fundamental mode, as described by (50). The requirement that the sensitivity is not significantly degraded then becomes

$$\theta \ll \theta_c \sim 2\times 10^{-5} \left[\frac{l}{1\ \text{km}} \right]^{-1/2} \left[\frac{\lambda}{5\times 10^{-7}\ \text{m}} \right]^{1/2} \text{rad} . \tag{53}$$

This result is only strictly true if the cavity formed by the recycling mirror M_3 and the interferometer arms has a high finesse and higher modes are nonresonant. For lower finesses, the amplitude of the higher modes will be suppressed by a factor approximately equal to this finesse, and the required angular stability (53) will be relaxed by the same factor.

The mode-cleaning action of the dual-recycling system ensures that the beam emerging from the output mirror M_3 will be pure fundamental mode, giving good fringe contrast and hence good shot-noise-limited sensitivity.

Resonant recycling is also tolerant of geometrical imperfections, for two main reasons. First, the build up of intensity in the recycling system is determined by reflection at the recycling mirror (as in a simple cavity) and is not dependent on interference at a beamsplitter. Second, the two countercirculating beams travel through the same optics, so the quality of the final interference is not sensitive to geometrical imperfections.

Thus, both dual recycling and resonant recycling place significantly less stringent requirements on the geometrical quality of the optics than does standard recycling.

CONCLUSION

It has been seen that the shot-noise-limited sensitivity of a laser-interferometric gravitational-wave detector may be greatly enhanced by using recycling, but that this improvement may be obtained in several ways. In each case there is no significant difference in the possible sensitivity if delay lines or cavities are used in the arms of the interferometer. The arrangements known as standard or broadband recycling, resonant recycling, and the new technique of dual recycling all give, at least in principle, the same performance both when operating in broadband (12) and narrow-band (22) modes. Dual recycling does, however, have some desirable features. First, the optical elements in the interferometer arms, be they cavities or delay lines, are no longer required to have a storage time comparable to the gravitational-wave period in order to get the best sensitivity. Not only does this give the system great flexibility, the reduction in the required number of reflections in a delay line may make the low-frequency operation of such an arrangement considerably easier. Second, while resonant recycling can be tuned over a range of frequencies, it has been seen that dual recycling has a less restricted tuning range and gives a better performance when tuned away from its optimum frequency. Third, the sensitivity-bandwidth combination of dual recycling may be changed by altering the transmission of a single mirror. Fourth, while dual recycling retains the basic optical arrangement of standard recycling, it has a greatly reduced sensitivity to geometrical imperfections in the optics. Thus, while much work needs to be done in order to know how to implement recycling, it seems likely that the use of dual recycling will considerably enhance the operational performance of a gravitational-wave observatory.

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