# Entanglement in Topological Phases

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# Overview of topics

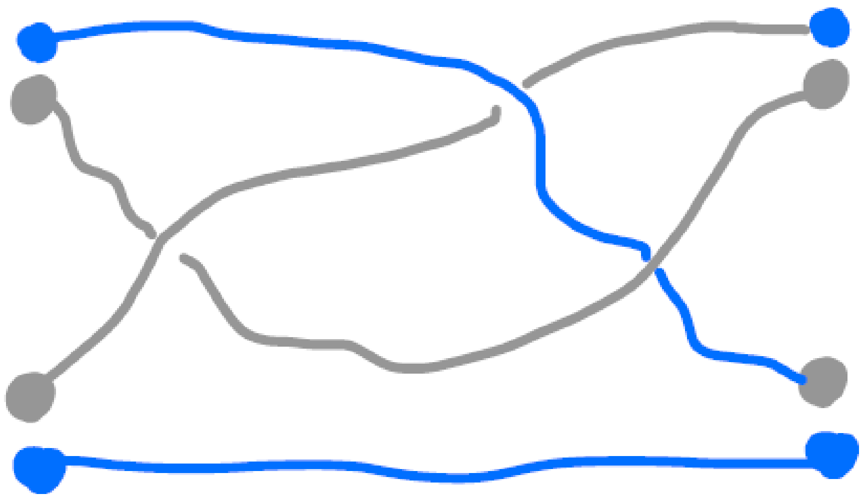
- Topological Phases
- Aspects of Entanglement
- Examples and my work

# Topological Phases

- Examples include: fractional quantum Hall, chiral spin states, etc
- Cannot be described by Landau symmetry-breaking
- Characterized by other properties: non-Abelian geometric phase, fractional statistics/charge, topological entanglement entropy
- Where does the name come from?

# Why do we care about them?

- It means Landau symmetry-breaking is incomplete
- Topological quantum computing



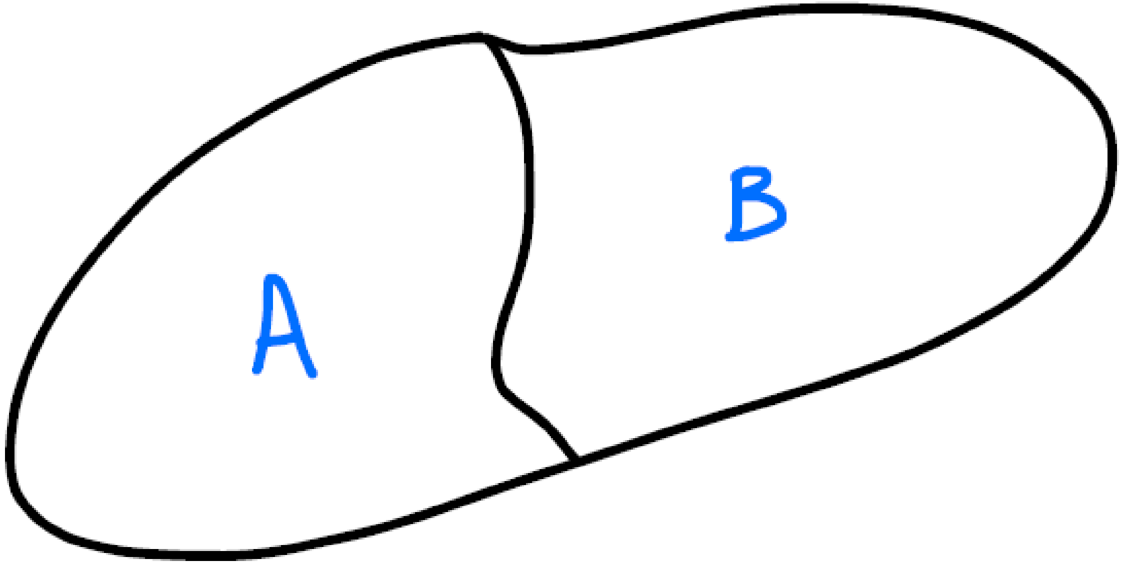
# What is entanglement?

- Concept first discussed by EPR, term coined by Schrödinger
- Basically a result of quantum superposition
- Bell's inequality – no local hidden variable theories

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right]$$

# Schmidt Decomposition

- Singular value decomposition for Hilbert spaces
- Write the space explicitly as a product



# Entanglement Entropy

- Consider the density matrix of one block
- Calculate the von Neumann entropy
- Scaling law emerges with multiple components

$$S_A = \alpha L - \gamma + \mathcal{O}\left(\frac{1}{L}\right)$$

# Entanglement Spectrum

- Write the density matrix in a suggestive form
- Consider the spectrum of this operator
- Reveals additional information & properties

# Recent Developments

- Structure of ES matches spectra for edge
  - Haldane & Li conjecture
- For real-space partition, better matching

# Example

- Laughlin 1/3 state

$$\text{1 particle:} \quad |m\rangle = \frac{z^m}{\sqrt{2\pi 2^m m!}} \exp\left\{-\frac{1}{4}|z|^2\right\}$$

$$\text{N particles:} \quad |m, N\rangle \propto \prod_{i<j} (z_i - z_j)^m \exp\left\{-\frac{1}{4}\sum_{i=1}^{N}|z_i|^2\right\}$$

# Example cont'd

- On a sphere!

$$\left(a_1^\dagger b_2^\dagger - a_2^\dagger b_1^\dagger\right)^3 = \underbrace{\left[\left(a_1^\dagger\right)^3\left(b_2^\dagger\right)^3 - \left(a_2^\dagger\right)^3\left(b_1^\dagger\right)^3\right]} + 3\underbrace{\left[\left(a_1^\dagger\right)\left(a_2^\dagger\right)^2\left(b_1^\dagger\right)^2\left(b_2^\dagger\right) - \left(a_2^\dagger\right)\left(a_1^\dagger\right)^2\left(b_2^\dagger\right)^2\left(b_1^\dagger\right)\right]}$$

$$|-3, 3\rangle$$

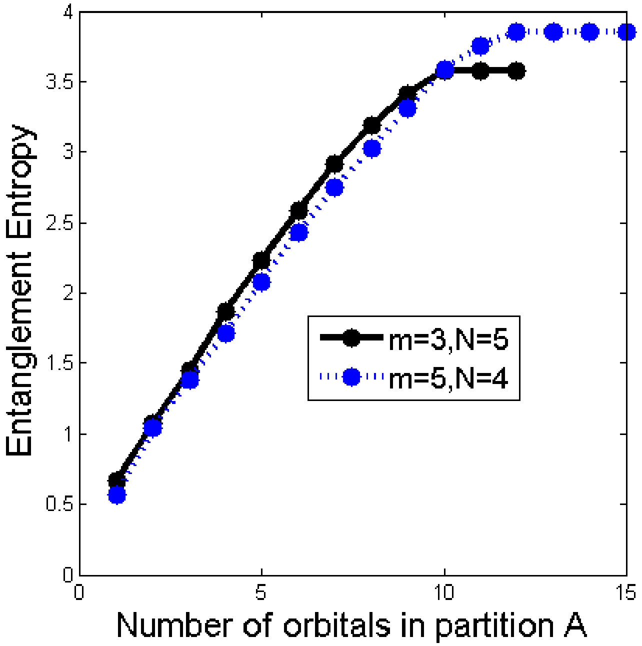
$$|1, 0, 0, 1\rangle$$

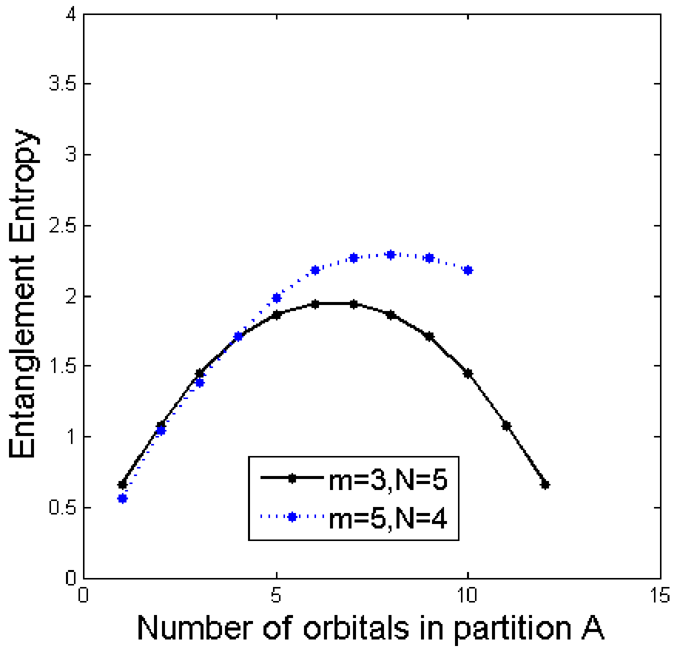
$$|-1, 1\rangle$$

$$|0, 1, 1, 0\rangle$$

# My work

- Can handle up to N=5, close to EE results in ref. 4



# References and Acknowledgments

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