APPH4200 Physics of Fluids: Homework 1

Question 1

Consider the two-dimensional, incompressible flow

$$\mathbf{u} = (u, v, w) = (\cos \alpha t, \sin \alpha t, 0)$$

Give a complete description of the streamlines and the trajectory of a particle released at the origin at $t = 0$.

Extra credit for the brave: Dye is injected into the flow at the origin beginning at $t = 0$ and continuously thereafter. Describe the streaklines traced out by the dyed fluid, for all time $t \geq 0$. How does the streakline which initiates at the origin at $t = 0$ differ from the trajectory which originates at the same place and time?

Question 2

Consider the steady, two-dimensional, incompressible flow

$$\mathbf{u} = (u, v, w) = (-x, y, 0)$$

on the half-plane $(0 < x < \infty, -\infty < y < \infty)$.

1. Sketch the streamlines for the flow.
2. Compute the strain and rotation tensors (if it's not easy, you're doing it wrong).
3. A spatially distributed source releases a passive chemical tracer, with mixing ratio c, into the flow. The tracer obeys

$$\frac{dc}{dt} = S,$$

with S the source and $d/dt = \partial/\partial t + \mathbf{u} \cdot \nabla$. In this case assume that $S = xe^{-x}$. Find a solution for the c field as a function of x, y, assuming that it is in steady state. (*Hint*: you will find the equation much easier to solve if you first make an educated assumption about the symmetry of the solution. You only need to find *a* valid solution, not necessarily the most general.)

4. The steady state exists despite the fact that the tracer is being continuously added everywhere. How is the budget of tracer mass closed, that is, where is the dye going?

Question 3

Prove the identity

$$\mathbf{u} \cdot \nabla \mathbf{u} = (\nabla \times \mathbf{u}) \times \mathbf{u} + \nabla(\frac{1}{2}|\mathbf{u}|^2)$$

(This is a vector equation with three components; it is sufficient to work out one component out of three).

Question 4

Kundu & Cohen, Chapter 3, problem 6.

Question 5

Kundu & Cohen, Chapter 4, problem 2.

NOTE: I will re-use these questions. Please do not share your answers or the class solutions with students from previous or future classes!