APPH 4200
Physics of Fluids

Review
October 21, 2010

1. Review
2. Problems from old Midterms
3. Midterm 2008

Review

- Introduction
- Tensors, vectors, symmetric and antisymmetric tensors, vector calculus, Gauss' and Stokes' Theorems, ...
- Streamlines, pathlines, convective derivative
- Definitions: strain-rate tensor, vorticity, circulation, rigid rotation, stream function

Review

- Conservation of mass, Navier-Stokes, Newtonian fluids, Stokes' model for stress tensor, deformation work, viscous dissipation
- Bernoulli's principle for inviscid, irrotational flow
- Rotating frame of reference, centrifugal force, Coriolis force

Review

- Vorticity equation, Kelvin's theorem, vorticity dynamics
- Potential flow, 2D Euler flow, complex velocity potential, Blasius theorem
- Dimensional analysis, Reynolds number
- Steady, laminar flow with strong viscocity

Midterm 2006

- 4 problems. Closed book
- Conservation of mass
- Bernoulli's principle
- Viscous flow
- Vorticity conservation

Midterm 2007

- 4 problems. Closed book
- Flow near a stagnation point
- Viscous flow (Couette)
- Circulation, mass flow, and energy
- Bernoulli's principle

Problem 1

Read the questions carefully. Notice that the fluid is viscous in some problems, inviscid in others. Inviscid means the viscosity is zero, $\mu = \nu = 0$. Good luck.

1. (20 points)

   (a) Determine the vertical velocity, w, such that the three-dimensional flow field whose other two components are

   $$u = x^2yz, \quad v = -y^2x$$

   satisfies the incompressible mass conservation equation. You may neglect any constants of integration.

   (b) An idealized velocity field is given by the formula

   $$\mathbf{u} = txy\hat{\mathbf{x}} - t^2y\hat{\mathbf{y}} + xz\hat{\mathbf{z}},$$

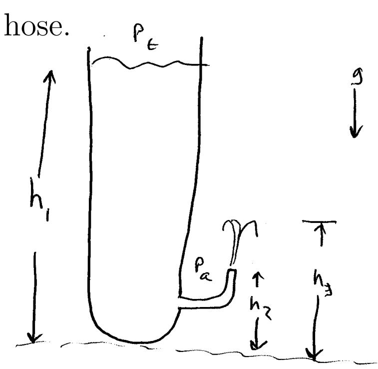
   where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ are the unit vectors, and t is time. Compute the x-component of the acceleration vector $\frac{d\mathbf{u}}{dt}$.

Problem 2

2. (25 points) Fluid fills a tank to a level h_1 above the bottom. The hose emits fluid from a level h_2 above the bottom. The pressure in the atmosphere outside the tank is p_a, while the pressure above the water surface is p_t, different from p_a. The force of gravity points downward, with constant gravitational acceleration g. Consider the fluid to be incompressible, constant density, and inviscid, and assume the flow to be steady. Find the following quantities in terms of the given constants:

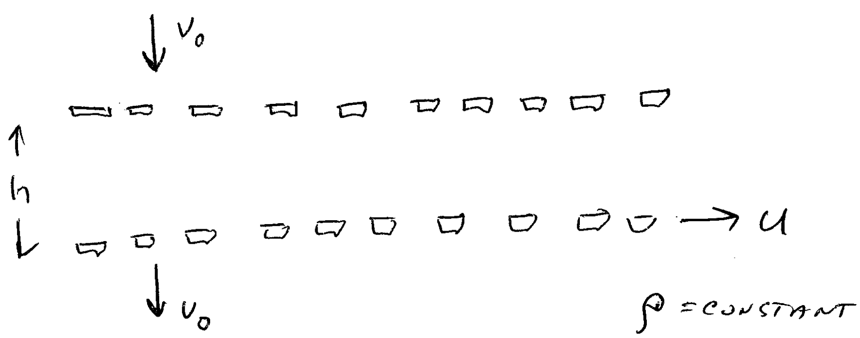
   (a) The height to which the water stream rises, h_3 in the figure.

   (b) The velocity of the water stream as it exits the hose.



Problem 3

3. (35 points) Fluid with constant density, ρ, and constant, nonzero kinematic viscosity, ν, flows between two rigid boundaries $y = 0$, $y = h$. The lower boundary moves in the x-direction with speed U, the upper boundary is at rest. The boundaries are porous, and the vertical (y) velocity is $-v_0$ at each one, v_0 being a given positive constant, so that there is an imposed flow across the system. You may take the system to be uniform in x and y, assume steady flow, and neglect gravity. Find the flow (both the x and y components). Hint: do the y-component (v) first.



Problem 4

4. (20 points) Consider a large lake, initially at rest. In the center, the water is stirred by a solid object inserted into it (e.g., a spoon or egg beater). The object is then removed.

   (a) If the fluid is inviscid, explain why no vorticity can be generated by the stirring.

   (b) If the fluid is viscous, vorticity can be created. Assume that this occurs, but only in a small region around the object. Explain why after the removal of the object, the net vorticity created must still be zero, that is, the areal integral of the vertical vorticity over the entire lake must vanish,

   $$\int_{lake} \omega \; dA$$

   where

   $$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

   is the vertical vorticity.

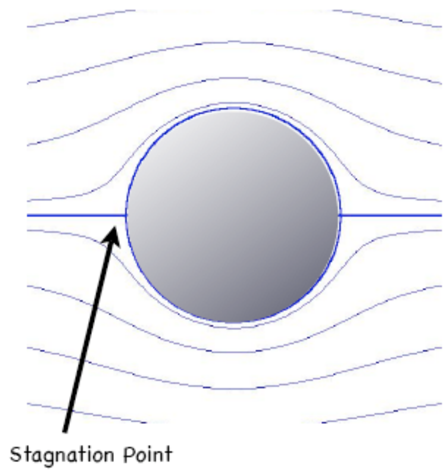
In both parts, you may assume that the water has constant density.

Problem 5

1. Describe the steady, inviscid flow near the stagnation point in front of a sphere moving at constant speed through an incompressible fluid of uniform density.

   [Suggestions: In the small region near the stagnation point, you should treat the surface of the sphere as a flat plane. Understand the symmetry of the problem. You may wish to use cartesian, (x, y, z), coordinates.]

Problem 1



Problem 6

3. Consider two concentric rotating cylinders with radii, r_1 and r_2.

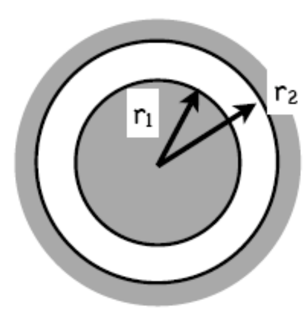
   (a) How fast should the two cylinders rotate to make the fluid rotate as a rigid body?

   (b) How fast should the two cylinders rotate to make the fluid rotate with uniform circulation?

   (c) For each case above, what is the mass flow rate, Q, per unit height of the cylinders?

   (d) Also for each case, what is the mechanical energy per unit height of the cylinders?

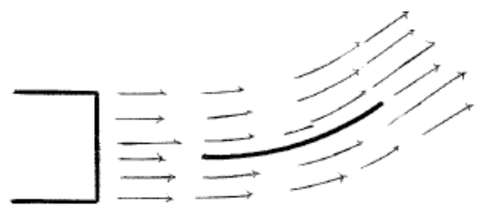
Problem 3



Problem 7

4. A curved plate causes a stream of fluid to curve. (See attached figure.) Estimate the pressure difference between the inside (convex) side of the plate and the outside (concave) side of the plate if the following conditions apply: the flow far from the plate has a uniform speed U_0, the fluid is inviscid, the radius of the curved plate is R, and the plate is thin with thickness, δ.

Problem 4



Midterm 2008

- 3 problems. Closed book
- 2D Potential Flow
- Bernoulli's principle
- Poiseuille Flow

Problem 1

1. (30 Points) For two-dimensional potential flow, the divergence and curl of the flow vanishes, *i.e.* $\nabla \cdot \mathbf{U} = \nabla \times \mathbf{U} = 0$. The steady flow can be represented by a velocity potential, $\mathbf{U} \equiv \nabla \phi$, or a stream function, $\mathbf{U} \equiv -\hat{\mathbf{z}} \times \nabla \psi$. Since ϕ and ψ satisfy Laplace's equation, the flow can be represented on the complex plane, $z = x + iy = re^{\imath\theta}$, by an analytic function called the "complex velocity potential", $w(z) \equiv \phi + i\psi$.

   When $w(z) = Az^n$, $w(z)$ represents the flow around or inside a corner located at $(x, y) = (0, 0)$. Show that (i) when $n < 1$, the flow passes over the "outside" of a corner having an angle greater than 180°, and (ii) when $n > 1$, the flow passes on the "inside" of a corner having an angle less than 180°.

   For each case, $n > 1$ and $n < 1$, determine whether the fluid velocity *increases* or *decreases* when approaching the corner.

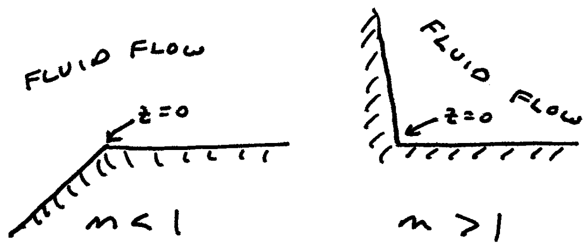


Figure 1: For Question 1. Two-dimensional potential flow near corners.

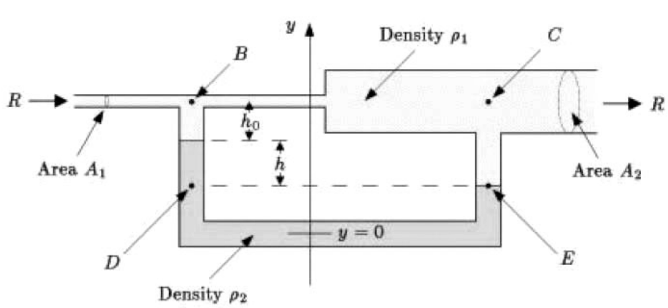
Problem 2

2. (35 points) An incompressible liquid of density, ρ_1, is flowing through a tube at a volumetric rate of Q. The tube cross-sectional area as two parts. On the entrance side, the area is A_1, and on the exit side the area is A_2 (with $A_2 > A_1$.)

   A "U-tube" is connected to the tube as shown in the figure below. The U-tube is partially filled with a liquid of density ρ_2, which is denser than the fluid flowing through the tube, $\rho_2 > \rho_1$. The two fluids do not mix.

   The differerence between the heights of the upstream and downstream sides of the U-tube is h. *Your problem*: Derive an expression for h in terms of Q, A_1, A_2, ρ_1, ρ_2, and g, the acceleration of gravity.

   [Hint: The fluid in the U-tube is in a state of hyrdostatic equilibrium, where pressure gradients balance gravitational forces. The pressure must be constant where the U-tube connects to the tube with flowing fluid.]



Problem 3

3. (35 points) A viscous fluid flows at a steady rate through a tube with an elliptical cross-section. See figure below.

   What is the relation between the volumetric flow rate, Q, through the tube and the axial rate of pressure drop, dP/dz, in terms of the dimensions of the tube and the fluid viscosity. (You may leave integrals unevaluated, provided they are defined fully.)

   Show that the pressure gradient is balanced by a viscous force.

   [Note: You do not have to evaluate any expressions for any integral. Just define them fully.]