APPH 4200 Physics of Plasmas: In-Class Worksheet

Answer the following without looking at your notes or textbooks. (These are questions 10 and 11 from Chapter 2.)

Question

With $\phi(x, y, z)$ a scalar function, prove the following

$$\nabla \times \nabla \phi = 0$$

and with $\mathbf{A}(x, y, z)$ a vector field, prove the following

$$\nabla \cdot \nabla \times \mathbf{A} = 0$$

You may use Einstein notation (or any other method)

$$\begin{aligned} \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) &= \epsilon_{ijk} a_i b_j c_k \\ (\mathbf{a} \times \mathbf{b})_k &= \epsilon_{ijk} a_i b_j \end{aligned}$$

with $\epsilon_{ijk} = 1$ if ijk are cyclic, $\epsilon_{ijk} = 0$ if any ijk are repeated, and $\epsilon_{ijk} = -1$ if ijk are anti-cyclic.

Answer

The proof of the identies is ...